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Hydrografiska avdelningen, Göteborg

Ekman's Theory for Upwelling

in Shallow Waters

by Artur Svansson

May 1979

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Ekman's Theory for Upwelling in Shallow Waters. by Artur Svansson, National Board of Fisheries, Hydrographic Department

1. Summary.

Upwelling of deeper waters at the west coast of Africa and America is well known and conspicious. Upwelling of smaller magnitude is, however, common at most coastlines. Johansson (1977) found that according to temperature variations at coastal stations upwelling is a common feature along the Swedish coast of the Baltic. In very many cases the upwelling had been caused by a nearly alongshore wind (with the coastline to the left of the wind). There were, however cases with other wind directions. Bladh et al. (1978) presented salinity measurements made every second day during one year in the Hanö Bight as function of wind directions. See also Svansson (1975).

Smith (1968) gave a general review of upwelling. As is usually the case the depth at the coast is supposed to be of the order of or larger than the Ekman depth d (see Chapter 2). The purpose of this paper is to investigate the case of shallow depths in the Ekman sense. What is presented here is actually only Ekman's (1905) results, given, however, with more details. Chapter 2 contains basic Ekman theory with derivations similar to those used by Welander (1957). The theory assumes the water to be homogeneous and that the important coefficient of vertical eddy viscosity is constant from surface to bottom. The theory does not contain time variations.

In Chapter 3 we restrict the theory to deal with a long strait coast with constant depth (i.e. a bottom profile of no inclination). The conditions are that a) the total transport perpendicular to the coast , U , is zero and

b) that there is no alongshore sea level gradient.

The last condition may be disputable in an enclosed sea like the Baltic. Ekman (l.c.) also investigated the case of an enclosed sea of constant depth with conditions U = V = 0. But numerical calculations with barotropic models usually show, that a longitudinal wind brings with it a net transport along the coast and that the necessary return transport takes place in the usually deeper parts of the basin middle. If a numerical model is available alongshore level gradients could be extracted and substitute condition b). As these models usually are time-dependant such a procedure would introduce time variability in a rough way.

An eddy viscosity constant from surface to bottom may be a serious restriction. From North Sea M2 tidal data Kraav (1970) derived a horizontal variation of ν - values from 0.0001 to 0.09 m²/s (d - values between 4 m and 120 m). Davies (1979) found difficulties in applying Kraav's coefficients in a 3-dimensional meteorologically induced circulation of the North Sea. Kullenberg (1971) showed that for the surface layer ν may be expressed

 $\nu = \frac{8.1 \cdot 10^{-5} W^2}{0.05 \left| \frac{du}{d_{7}} \right|}$

where W is the wind velocity and 0.05 is supposed to be a representative Flux Richardson number. The conclusion may be that the surface layer eddy viscosity varies with the wind speed and du/dz, whereas the bottom layer coefficient may be much more constant in time. The Kullenberg formula does not contain stratification explicitly but in stratified water du/dz is probably larger than in homogeneous water. Smaller eddy viscosity coefficients are expected in the surface layer

2.

in the summer season.

The most important results of the Ekman computations made are as follows. Fig. 2 A shows that for small H/d (=H') the most effective wind-direction is a perpendicular one, but that for increasing H'the angle changes more and more, and for H' \geq 1.5 an alongshore wind is the most effective one to produce upwelling. Fig. 2 B shows the surface layer transport as function of wind direction. (There are difficulties to draw the curves for some angles. This is explained by the existance of more than two flows, see Table 1 and Fig:s 3 g, h, and i). Fig:s 3 show details of (non-dimensional) velocities u'and v'es function of depth for three different H' and for three wind directions. Whereas conditions for $\varphi = 0$ and for $\varphi = 90^\circ$ are simply two-layered, for $\varphi = 160^\circ$ there are at least 3 currents, explaining why it is difficult to draw integral curves in Fig. 2 B for some angles.

When trying to apply the theoretical results to reality, we first translate non-dimensional units to dimensional ones. Accepting all the simplified conditions of the theory, there is unfortunately uncertainty about the magnitude of Ekman depths in the Baltic Sea Area. It is often assumed that in homogeneous ocean water d is about 100-150 m. In stratified water, however, d is smaller , e.g. in the Kattegat where Jacobsen's (1913) investigations gave d-values between 2 and 13 m, some other work (Svansson and Szaron 1979) a little higher, about 18 m. When working with a one-dimensional barotropic model for the Gulf of Bothnia in the northern Baltic for a case of 11 days in October 1958 , Svansson et al. (1974) found the best results for an eddy viscosity coefficient of 0.o15 m²/s, corresponding to an Ekman depth d of 50 m. During the season of a thermocline it is probable that the surface Ekman depth is smaller possibly equal to the depth of the thermocline, about 20 m. As this is also a typical bottom depth along the coast, we may speculate in a summer Ekman depth half of the winter value. This could lead to the result that an off-shore wind in winter is a more effective upwelling producer than an alongshore one, which on the other hand would be more effective in summer.

A comparison between the Ekman theory and the results presented in the two papers mentioned above, viz. Johansson (1977) and Bladh et al. (1978) does not give clear conclusions. Most cases treated in the first mentioned paper have their origin in June - Sept., i.e. in summer, a fact which may explain that most cases were combined with along-shore winds. There are however cases with wind angles larger than 90° even, indicating the influence of along-shore level gradients. Whereas Johansson (l.c.) used temperature as indicator, Bladh et al. (l.c.) show relations between wind direction and salinity in the Hanö Bight in the southern Baltic. Unfortunately this area seems to have a too complicated morphology without a welldefined "long strait coast".

Walin (1972) who made daily observations of temperature variations on a Hanö Bight section during two summer months 1968, found that off-shore winds were the most effective upwelling producers. Walin (l.c.) explains this discrepancy from a large depth Ekman theory by suggesting traveling internal Kelvin waves along the coast.

More investigations of the relation between wind and upwelling indicators are necessary to clarify if the Ekman theory is applicable at natural conditions.

<u>Note</u> that v'is positive in Figs:s 3 d and 3 i (scale reversed by mistake).

2. Basic Ekman equations.

In considering the wind action on the sea Ekman (1905) used a model described by the following two equations of motion for steady state:

$$\begin{pmatrix} -fv = -g \frac{\partial h}{\partial x} + v \frac{\partial^2 u}{\partial z^2} \\ fu = -g \frac{\partial h}{\partial y} + v \frac{\partial^2 v}{\partial z^2} \end{cases}$$
(1)

The boundary conditions are

$$\mathcal{V}\left(\frac{\partial u}{\partial z}\right)_{z=0} = \mathcal{T}_{x}; \quad \mathcal{V}\left(\frac{\partial u}{\partial z}\right)_{z=0} = \mathcal{T}_{y} \quad (2)$$

$$\mathcal{U}_{Z=-H} = 0 \quad : \quad \mathcal{V}_{Z=-H} = 0$$

Here x, y are horizontal locally Cartesian coordinates and u, v are the corresponding velocities. z is the vertical coordinate counted positive upwards. The free surface is at z = h(x,y) and the bottom at z = -H(x,y), see Fig. 1., f is the Coriolis parameter, g the acceleration of gravity and \mathcal{V} the coefficient of vertical eddy viscosity. These are considered as constants.

 Z_{χ} , $\overline{C_{\gamma}}$ finally, are the components of the wind stress acting on the sea surface (density of sea water = 1 ton/m³). It is assumed that the water is homogeneous, that the pressure is hydrostatic, and that lateral friction and non-linear acceleration terms can be neglected.

The subsequent derivation of equations is similar to that one used by Welander (1957):

W = u + i V $\frac{\partial h}{\partial n} = \frac{\partial h}{\partial x} + i \frac{\partial h}{\partial y}$ $\overline{U} = \overline{U}_{\chi} + i \overline{U}_{\chi}$

6.

(3)

Equation (1) can then be written:

 $\begin{array}{l}
\mathcal{V} \frac{\partial^2 w}{\partial z^2} - i f w = g \frac{\partial h}{\partial n} \\
\text{with the boundary conditions} \\
\left(\mathcal{V} \left(\frac{\partial w}{\partial z} \right)_{z=0} = z \\
\mathcal{W}_{z=-H} = 0 \end{array}$ (4)

Consider both the wind stress \mathbb{Z} and the surface slope $\frac{\partial f_1}{\partial n}$ as prescribed functions of x and y. The general solution to eq. (3) is then, $\mathcal{A} = \frac{\pi}{f} / \frac{2\nu}{f}$

being the so called Ekman depth:

 $w = C_{e} e^{\frac{\pi}{4}(1+i)z} + C_{2} e^{-\frac{\pi}{4}(1+i)z} + \frac{ig}{5} \frac{\partial h}{\partial h}$

The boundary conditions (4) determine C_1 and C_2 uniquely:

 $W = \frac{\pi(1-i)}{fd} \frac{\sinh\left(\frac{\pi}{d}(1+i)(2+H)\right)}{\cosh\left(\frac{\pi}{d}(1+i)\cdot H\right)} \cdot \overline{Z} -\frac{ig}{f}\left(\frac{\cosh\left(\frac{\pi}{d}(1+i)z\right)}{\cosh\left(\frac{\pi}{d}(1+i)H\right)}-1\right)\frac{\partial h}{\partial n}$ (5)

U and V of the total transport W from surface to bottom are

 $\left(\mathcal{U}=\frac{1}{F}\left(\left(T_{X}-DT_{Y}\right)+\frac{gH}{F}\left|E\frac{\partial u}{\partial x}-F\frac{\partial u}{\partial y}\right|\right)$ $V = \frac{1}{E} \left(D \overline{t}_{\chi} + C \overline{t}_{q} \right) + \frac{9}{F} \left(F \frac{2k}{f_{\chi}} + E \frac{2k}{f_{q}} \right)$

Where C, D, E and F are functions of H/d. The full expressions are found in Welander (1.c.). 3. Long Strait Coast.

3.1. Equations.

$$U = 0$$
, $\frac{\partial h}{\partial y} = 0$

The first eq. (6) will then be changed into

$$O = \frac{1}{f} \left(CT_{x} - DT_{y} \right) + \frac{gH}{f} = \frac{2h}{2x}$$

or

$$\frac{\partial h}{\partial x} = \frac{1}{gH} \frac{D\bar{c}_y - C\bar{c}_x}{E}$$
(7)

$$+ \frac{i(cT_{x}-DT_{y})}{fHE} \left[\frac{\cosh \pi \overline{d} \cos \pi \overline{d} + i \sinh \pi \overline{d} \sin \pi \overline{d}}{\cosh \pi \overline{d} \cos \pi \overline{d} + i \sinh \pi \overline{d} \sin \pi \overline{d}} - 1 \right]$$

$$If T = T_{0} (\cos t + i \sin t) and u, v = \frac{2\pi T_{0}}{4\pi d} u, v'$$

 $(\gamma = 0^{\circ} \text{ W-wind}, \gamma = 90^{\circ} \text{ S-wind etc. ----})$ we have

 $\begin{aligned} \mathcal{U}' &= \frac{(\ddot{A}+\ddot{0}')(P_{cos}f+Q_{Rin}f') - (\ddot{0}'-\ddot{A})(P_{Rin}f-Q_{cos}f')}{2(P^{2}+Q^{2})} + \frac{(C_{cos}f-D_{Rin}f')(TQ-\ddot{A}P)}{2\pi H' E(P^{2}+Q^{2})} \\ (9) \\ V' &= \frac{(\ddot{0}'-\ddot{A}')(P_{cos}f+Q_{Rin}f') + (\ddot{A}'+\ddot{0})(P_{Rin}f'-Q_{cos}f')}{2(P^{2}+Q^{2})} + \frac{(C_{ain}f-D_{Rin}f')(TP+A\dot{A})}{2\pi H' E} - \frac{(C_{cos}f-D_{Rin}f')}{2\pi H' E} \end{aligned}$

Inserting (7) into eq. (6) for the total volume V we have

3.2. Results of Computations.

Fig:s 3 show examples of distribution of u' with depth z' (= -z/H) for some H' (= H/d). The values of u' is equivalent with u if

$$\frac{2\pi}{df} = 1$$

If we use for \mathcal{C}_{o} , $2 \cdot 10^{-4}$ MTS (equivalent with a wind velocity of ≈ 10 m/s, d will be (f = 1.26 $\cdot 10^{-4}$ s⁻¹)

$$d = \frac{2\pi Z_{s}}{f} \approx 10m$$

If for the same \mathcal{T}_o , d were e.g. 20 m, u would be half the value of u'.

<u>Transports</u>: To represent transports 0.5 H'2 u' was computed:

As $\Delta Z = H' \cdot d \cdot \Delta z'$ and $\Delta z'$ was chosen = 0.05 we get $\sum_{u} \Delta z = \frac{2\pi Z_{o}}{df} \cdot H' \cdot d \cdot 0.05 \cdot \sum_{u'} u' = \frac{2\pi A_{os} \cdot Z_{o}}{f} H' \cdot \sum_{u'} U'$ Supposing $T_0 = 2 \cdot 10^{-4}$, we get $\Sigma u \Delta z = 0.5 H' \Sigma u'$

The transports were grouped according to sign. (The first and last u' value in a sign group were halved). For each H' the angle \mathscr{P} was searched that corresponds to maximum positive transport from surface to depth of sign shift. In Fig. 2a this angle as well as the transport itself are shown as function of H'.

Fig. 2b shows the variation of the surface transport as function of angle \mathscr{P} . Some parts of the curves have been omitted: when direction changes from plus to minus, the number of positive and negative transports increases from just one negative and one positive to about two of each sign. In this case a more close study of the computed value is necessary, see Table 1. The transports in Table 1 were evaluated from profiles resulting in a better accuracy than the $\sum u'$ procedure.

3.2.1. Large H'.

Let us a moment look at a solution without wind stress 2.

 $\overline{z} = 0 \quad ; \quad \frac{\partial h}{\partial y} = 0 \quad ; \quad \frac{\partial h}{\partial x} = \frac{f}{g} V_g$ $\begin{cases} \mathcal{U} = H \cdot E \cdot v_{g} \\ V = H F \cdot v_{g} = H \cdot v_{g} - H v_{g} (I - F) \end{cases}$

For large H', H(1 - F) $\rightarrow \frac{d}{2\pi}$ HE $\rightarrow \frac{d}{2\pi}$

$$\mathcal{U} = -\frac{V_q d}{2\pi}$$

$$V = H v_g - \frac{v_g d}{2\pi}$$

U is here confined to the bottom layer. It can be shown that in case of upwelling the bottom transport is still = $-\frac{V_q}{2\pi} \frac{d}{d}$. v_g is determined from eq. (10) :

$$f H v_g = T_o \left(H^* \cos \varphi + G \sin \varphi \right)$$

Then

$$\mathcal{U} = -\frac{V_{g}d}{2\pi} = -\frac{T_{o}d}{2\pi} \left(H^{*} \cos \theta + 6 \min \theta \right)$$

In the case of upwelling the same amount is transported by the wind in the surface layer in the opposite direction. 3.2.2. Small H'.

For small H'the solution for $\mathscr{S} = 0^{\circ}$ is similar to that one for a channel perpendicular to the coast, U being = 0.

$$0 = -g \frac{\partial h}{\partial x} + U \frac{\partial^2 u}{\partial z^2}$$

with $\left(\mathcal{V} \frac{\partial \mathcal{U}}{\partial z} \right) = Z_0$ and $\mathcal{U}_{2=-14} = 0$

Then:

or

 $u' = \frac{3}{4} \pi H' (1-z') (\frac{1}{3}-z')$

 $u = \frac{g}{2u} \cdot \frac{\partial h}{\partial x} \left(z + H \right) \left(z + \frac{f}{3} H \right)$

Fig:s 3 a, b, and g show that there is change of direction at $Z' \approx -\frac{H'}{3}$ and that

Uo ~ H' II where u = 2TT To . u'

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m ² /s	for H' = 2.5.			
	Surface - Z ₁ '	z ₁ '- z ₂ '	Z ₂ '- Z ₃ '	Z ₃ '- bottom
0 5 0 5	0.53 0.63 0.73 0.80 0.89 1.03 1.13 1.20 1.30 1.35 1.40 1.55 1.60 1.65 1.65 1.65 1.65 1.65 1.65 1.65 1.65 1.65 1.65 1.65 1.65 1.65 1.55 1.43 1.30 1.28 1.13 1.28 1.13 1.28 1.13 1.28 1.13 1.28 1.13 1.28 1.13 1.28 1.28 1.13 1.28 1.28 1.295 -0.05 -0.10 -0.18 -0.25 -0.33 -0.40	-0.53 -0.45 -0.43 -0.33 -0.33 -0.25 -0.25 -0.25 -0.15 -0.15 -0.10 -0.10 -0.05 -0.10 -0.05 -0.10 -0.05 -0.05 +0.05 +0.02 +0.90 +0.83 +0.75 +0.68 +0.58		-0.10 -0.30 -0.40 -0.55 -0.70 -0.80 -1.25 -1.25 -1.25 -1.25 -1.50 -1.50 -1.50 -1.55 -1.60 -1.65 -1.65 -1.65 -1.65 -1.45 -1.45 -1.45 -1.45 -1.45 -1.45 -1.45 -1.45 -1.45 -1.55 -1.55 -1.55 -1.60 -1.65 -1.55 -1.55 -1.60 -1.55 -1.55 -1.55 -1.60 -1.55 -1.55 -1.55 -1.60 -1.65 -1.55 -1.55 -1.60 -1.55 -

Table 1. Transversal Transports 0.5 H' Zu'

Depths Z_i are counted consecutively from surface to bottom. The interval $Z_2' - Z_3'$ is the layer of the deep current, see e.g. Fig. 3f.







Fig. 1.



Fig. 2 a.



HyA/78/5



Fig. 3a-c



Fig. 3d-f



Fig. 3g-i

