

Correctness of mathematical reasoning is important for mathematicians and computer scientists alike: proofs may contain mistakes and programs may contain bugs. One way to increase the confidence in proofs and programs is to formalize them using proof assistants on a computer. It is then the task of the computer to mechanically check that the reasoning in each of the steps is correct. The core of many modern proof assistants is dependent type theory, a foundation for constructive mathematics.

Recently a new connection between type theory and a seemingly unrelated field, homotopy theory, was discovered, shedding new light at one of the most intricate concepts of type theory: equality. This connection suggested Voevodsky's Univalence Axiom which implies that two isomorphic structures can be identified. This axiom is justified using a model of type theory based on simplicial sets. This model is however not effective and thus doesn't explain how to incorporate this axiom into type theory such that important computational properties are preserved. This thesis investigates Voevodsky's axiom from a computational perspective by building new models based on cubical sets which are effective. Inspired by one of the models, the thesis also presents an extension of type theory possessing good computational properties and in which the Univalence Axiom becomes provable, making it suitable as the foundation of future proof assistants.

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Cubical Interpretations of Type Theory



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