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CANAL MODELS OF SEA LEVEL AND SALINITY VARIATIONS IN THE BALTIC AND ADJACENT WATERS

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The vignette on the title-page represents Bronze Age fisherman: from a rock-carving at Ödsmål, parish of Kville, Bohuslän.
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Contents

Abstract

Many years ago the present author started work on sea level (SL) problems (SVANSSON 1959). The Skagerrak, the Kattegat, the Belt Sea and the Baltic were treated as canals. Sea levels and water transports were computed by numerical integration (an explicit method), the sectioning being mostly copied from NEUMANN (1941). Later the method was improved (SVANSSON 1966 and 1968) but only some parts of the area were included.

In this paper the results of the numerical computations of sea levels and transports hitherto published by the present author are summarized (Ch. 25). Then work with another numerical method, an implicit one allowing long timesteps applied on a simple system of canals, is presented (Ch. 26). Furthermore this model is combined with a model for salinity variations, among other things used in an attempt to explain an interesting connection between the variations of the sea level of the Baltic and of the salinity of the Kattegat. The mathematical background and the numerical scheme will be found in Chapters 21—23, while Chapter 1 is a descriptive part presenting background information.

1. Descriptive Part

Chapter 11 below summarizes the ideas of various authors about the problem of the water exchange of the Baltic. It is intended to be a piece of background information particularly relevant to the computations with the salinity model, see Chapters 265—267.

In Ch. 12 the idea is presented that there is a close connection between the salinity variations of the strongly stratified water in the Belt Sea, the Kattegat and partly the Skagerrak on one hand and, on the other the SL variations of the Baltic. Also some of the consequences of this idea are briefly touched upon especially in connection with the old problem of the internal waves in the Gullmar fiord.

Chapter 13 summarizes the present knowledge of the strong permanent currents in the Skagerrak. It supplies the information necessary for the understanding of the special conditions in the SE corner which are described in Ch. 14. In this area the permanent current is strongly disturbed probably by the SL variations of the Baltic. Particularly the 5-day period, described by MAGAARD and KRAUSS (1966) for Baltic SLs, is shown to be existent also in the Kattegat and the Eastern Skagerrak.

The results of the current measurements during the international co-operation in August 1964 are only briefly touched upon as they seem to be too complicated to fit into the canal concept of this paper.

Figs. 1: 1—1: 5 are maps containing all information on positions and places referred to in the text and also the sectioning described in Ch. 254.

11. The Water Exchange of the Baltic

In many respects the Baltic can be considered an estuary with a large mouth, the latter consisting of the Belt Sea, the Kattegat and parts of the Skagerrak. Fresh water of an amount of approximately 500 km^3 pr year

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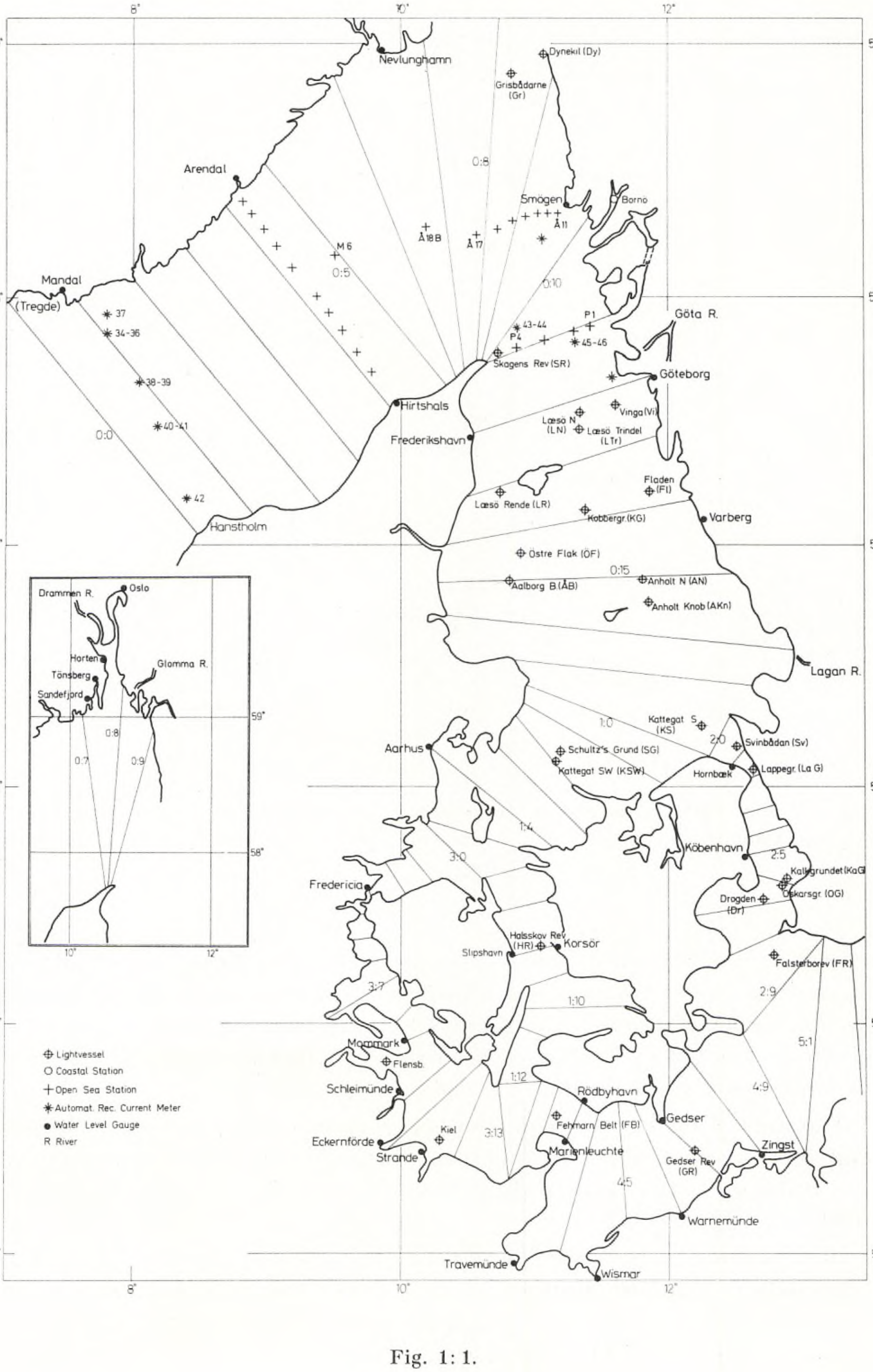
Fig. 1: 1. Map of the Western part of the seas concerned. The purpose of the sectioning is described in Ch. 254. It is here used to approximately indicate the limits of the various sea areas, a division suggested by WATTENBERG (1949): the Skagerrak (0: 0—0: 11), the Kattegat (0: 11—0: 18 and 1: 0—1: 2, 3), the Baltic (4: 8—, 2: 8—,) and the Belt Sea. The Belt Sea consists of the Samsö Belt (1: 2—1: 6), the Little Belt (3: 1—3: 9), the Great Belt (1: 6—1: 12), the Bay of Kiel (3: 9—3: 13, 4: 0—4: 3), the Bay of Mecklenburg (4: 3—4: 8) and the Sound (Öresund, 2: 0—2: 8).



Nevlunghamn
Dynekil (Dy)
Grisbådarne (Gr)
Arendal
Smögen
Borno
Mandal
(Tregde)
Göta R.
Skagens Rev (SR)
Göteborg
Hirtshals
Vinga (Vi)
Læsö N (LN)
Læsö Trindel (LTr)
Frederikshavn
Fladen (Fl)
Læsö Rende (LR)
Kobbergr. (KG)
Varberg
Hanstholm
Östre Flak (ÖF)
Aalborg B. (ÅB)
Anholt N (AN)
Anholt Knob (AKn)
Lagan R.
Drammen R.
Oslo
Horten
Glomma R.
Tönsberg
Sandefjord
Kattegat S (KS)
Schultz's Grund (SG)
Svinbådan (Sv)
Aarhus
Kattegat SW (KSW)
Hornbæk
Lappegr. (La G)
København
Fredericia
Drogden (Dr)
Oskarsgr. (OG)
Halsskov Rev (HR)
Korsör
Slipshavn
Falsterborev (FR)
Mommark
Flensb.
Schleimünde
Rödbyhavn
Fehmarn Belt (FB)
Gedser
Eckernförde
Kiel
Strande
Marienleuchte
Gedser Rev (GR)
Zingst
Warnemünde
Travemünde
Wismar
Lightvessel
Coastal Station
Open Sea Station
Automat. Rec. Current Meter
Water Level Gauge
R River

Fig. 1: 1.

Fig. 1: 2. Map of the Baltic proper. Sections 11: 0—11: 10 lie in the Gulf of Riga. Symbols etc. are explained in Fig. 1: 1. For the explanation of symbols of the anchor stations 1964, see Fig. 143: 3.

(15 000 m³/s) flows from this area to the ocean. Due to the topography and the mixing conditions there is a transport of saline ocean water in the opposite direction governing a pattern of salinity ranging from 0 ‰ in the innermost part of the Baltic to ocean salinity (approximately 35 ‰) in the outermost part of the mouth.

The water exchange problem is rather complicated and it is not astonishing that there is more than one approach to it. First a few words about the classical approach of MARTIN KNUDSEN presented in two papers in 1899 and 1900.

It is assumed that in the strait between the ocean and an enclosed sea filled with brackish water there are two layers, a top one consisting of outflowing brackish water and a bottom one of much higher salinity and flowing inwards (Fig. 11: 1). It is furthermore assumed that at a certain sec-

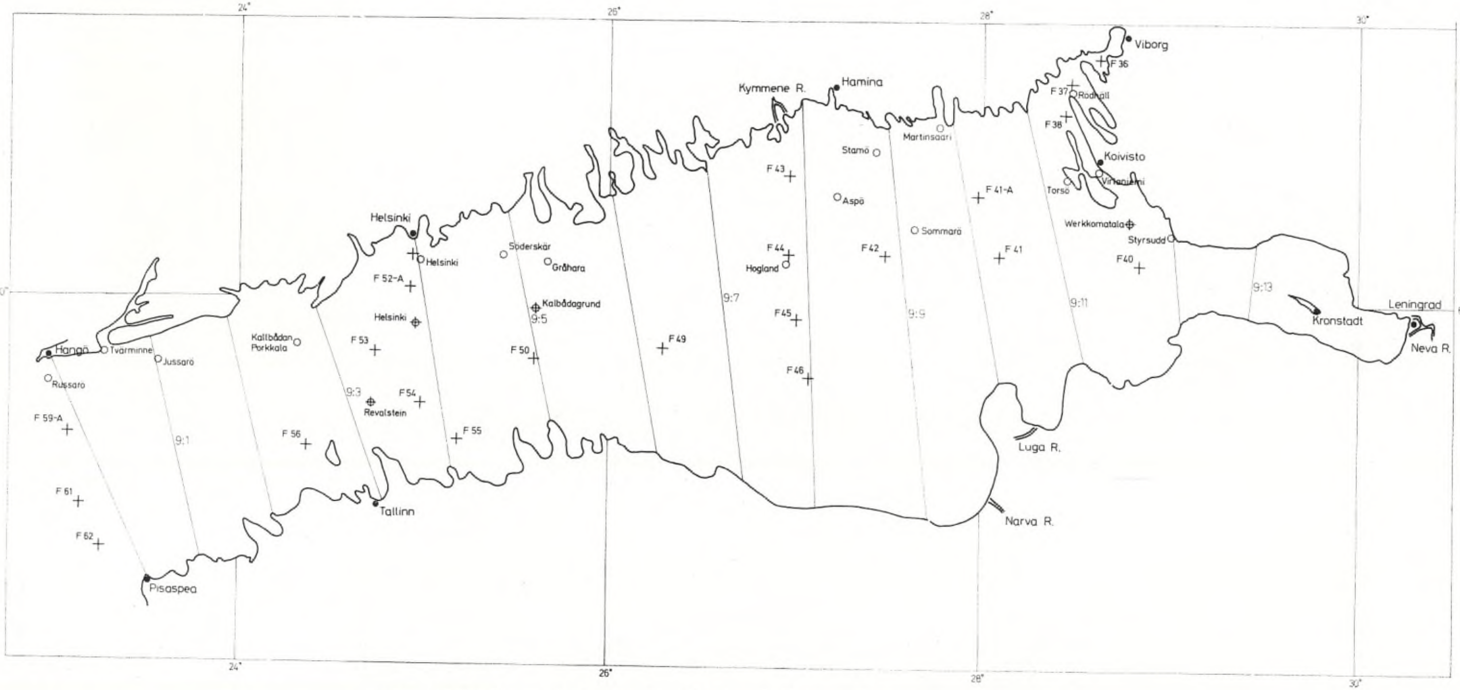


Fig. 1: 3. A Map of the Gulf of Finland. For explanation see Fig. 1: 1.

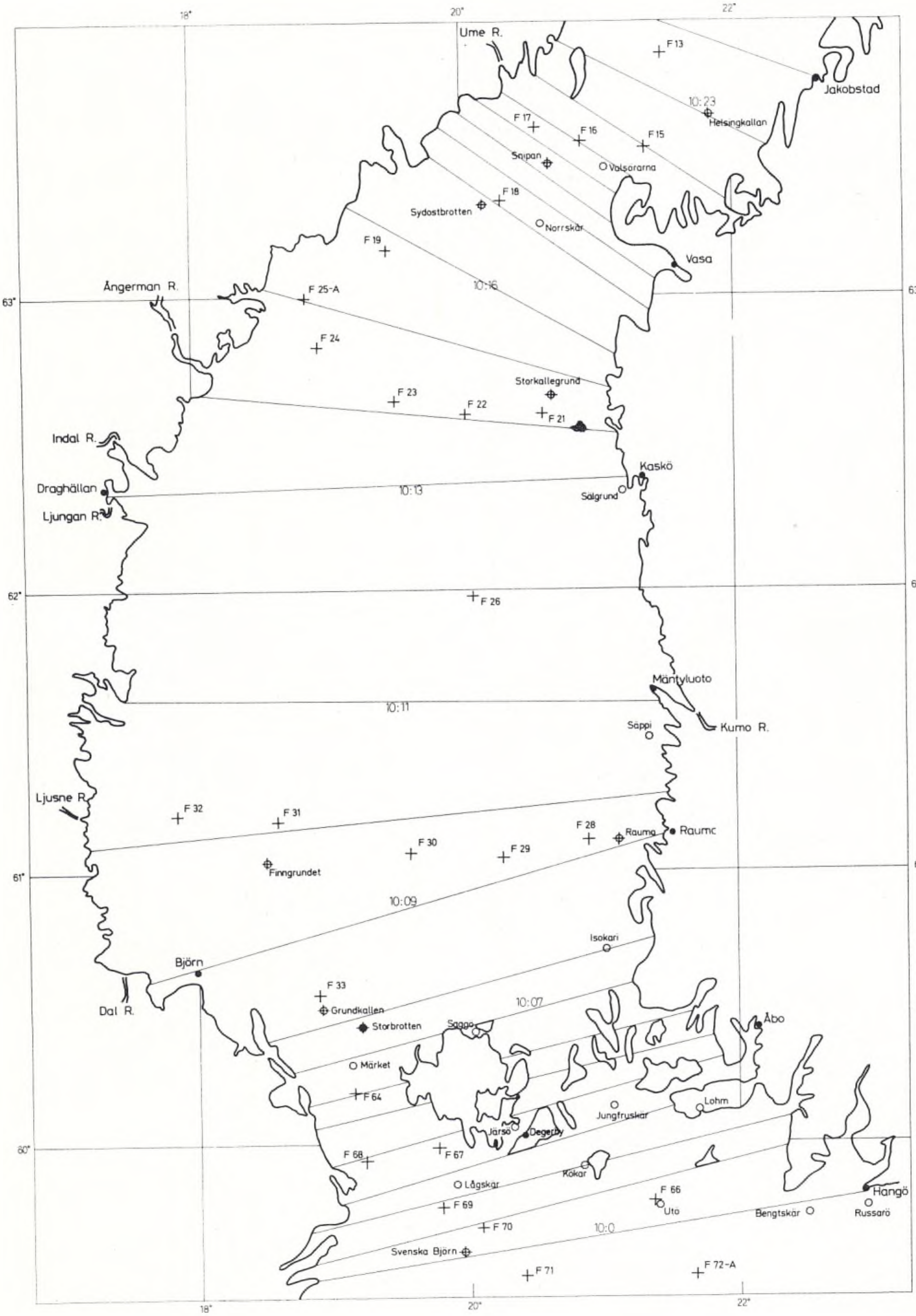


Fig. 1: 4. A Map of the Bothnian Sea. For explanation see Fig. 1: 1.

Fig. 1: 5. A Map of the Bothnian Bay. For explanation see Fig. 1: 1.

tion we can distinguish between the two regimes and also determine their respective salinities. Finally assuming the salt transport to be zero we obtain the KNUDSEN relations

$$U_1 = \frac{S_2}{S_2 - S_1} \cdot Z$$

$$U_2 = \frac{S_1}{S_2 - S_1} \cdot Z$$

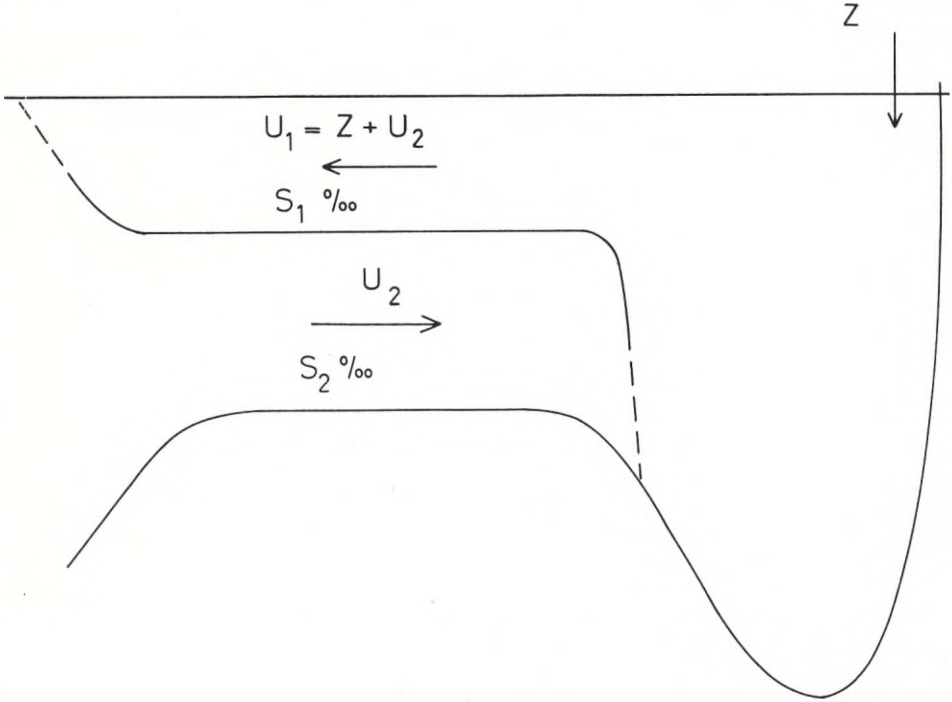


Fig. 11: 1. Schematic figure of an enclosed sea with the fresh water supply Z and the salinity S_1 ‰ connected through a strait with an ocean of the salinity S_2. U_2 is the compensation transport.

The symbols are explained in Fig. 11: 1. KNUDSEN applied the formulae at many sections, the most interesting one being the Darsser Schwelle section (see Fig. 1: 1, section 4: 8) at the smallest depth (a sill depth of 18 m) between the Baltic and the ocean. For the period 1877—1897 KNUDSEN found in the scientific literature 19 measurements of the salinity at the sill depth. Of these he kept 13 values disregarding all salinities below 15.5 ‰ because "these salinities cannot renew the deep waters of the Baltic"! So for S_2 he obtained 17.4 ‰ and without going much into detail S_1 was put=8.7 ‰. Thereby the compensating inflowing current would be of the same magnitude as the fresh water supply Z.

STOMMEL and FARMER (1953) and, in a slightly different manner, KULLENBERG (1955) derived a relation between the transports U_1 and U_2 as functions of the fresh water supply Z for an estuary assumed to contain well-mixed water. The solution of the problem is such that U_2 as function of Z first increases from zero (for $Z=0$) up to a maximum, thereafter decreases to zero again for $Z=Z_{max.}$. The salinity of the estuary, S, however, does not assume any extreme value but decreases steadily from the ocean salinity S_2 to zero for $Z=Z_{max.}$. It is not quite unrealistic to assume the Baltic to be

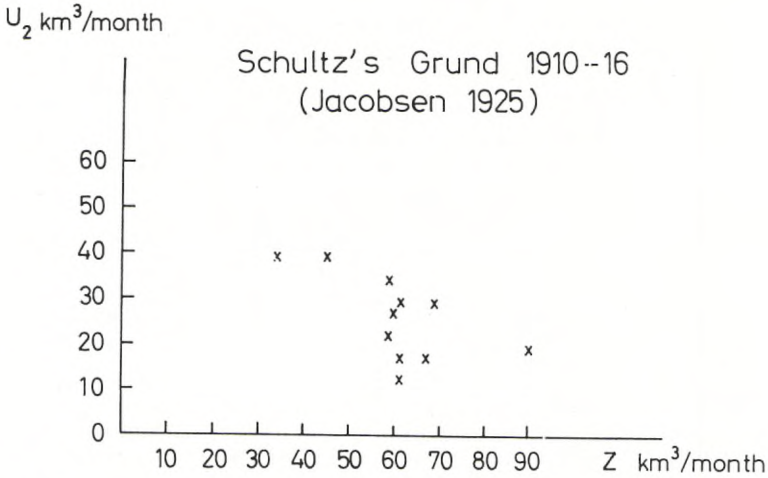


Fig. 11: 2. The ingoing transport in the deep of the Great Belt (Fig. 1: 1) as function of the fresh water supply Z to the Baltic.

wellmixed; the deep basins (FONSELIUS 1969) are relatively small in volume. In Fig. 11: 2 the U_2s computed by JACOBSEN (1925) from current measurements from Schultz's Grund light-vessel (see Fig. 1: 1) 1910—1916 are plotted. If the computation of U_2 is to be trusted and if it is allowed to use monthly means in this way it would indicate that the maximum point occurs for $Z \leq 30$ km^3/month.

It is quite clear that in reality there are difficulties to find the right salinities to enter into the Knudsen relations. Furthermore there seem to be few cases when there are currents in opposite directions on top of each other.

Table 11: 1 shows mean values of Danish current measurements determined at both surface and non-surface horizons. While the data of the light-vessels (L/V) Laesö Rende and Lappegrund clearly reveal outgoing (in the surface layer) and ingoing (in the deep) currents, the outgoing currents at the L/V Anholt Knob and the L/V Schultz's Grund are rather weak. Anholt Knob is often assumed to be situated in some kind of "counter-current" in the Kattegat (DIETRICH 1951, SVANSSON 1968).

In his large work SOSKIN (1963) more or less disregards the 2-layer system. Instead he assumes the transport through the Belt Sea to be either completely outwards or completely inwards. Already JACOBSEN (1925) and WYRTKI (1954 a) presented formulae to compute the transport when the surface currents were known at some Danish light-vessels. SOSKIN furthermore improved the formulae mostly by separating ingoing and outgoing transports and obtained one formula for each direction. Then Soskin computes the transport for every year 1898—1944. The difference between out-

Table 11: 1. Mean Currents at Danish light-vessels in cm/s (positive values outgoing currents, negative incoming).

Depth m	0 1901—1930*	2.5	5	10	15	20	25	References 2.5 m—25 m
Läsö Rende (N-component)								
5/9 1912—14/11 1913 ..	22.0	26.0	24.7	9.7	—0.2	—2.2	—	ROSSITER (1968)
Anholt Knob								
17/6—17/9 1910	—5.0	1.7	—2.3	—4.3	—5.1	—4.3	—3.9	JACOBSEN (1913)
Schultz's Grund								
1910—1916	10.0	2.4	0.4	—9.4	—18.2	—19.0	—15.0	JACOBSEN (1925)
Lappegrund								
1/9—22/11 1909 .... 22/6—17/8 1912	35.0	27.0	17.5	—10.3	—13.2	—11.3	—9.0	JACOBSEN (1925)
Halsskov Rev (N-component)								
July 1969—Jan. 1970 .. April 1970—May 1970	13.0	15.0	13.0	12.0	9.0	—	—	HERMANN (1971)

* DIETRICH (1951)

going and incoming transport is called water exchange (Fig. 11: 3). The fluctuations are really large; one asks if it is possible that some years there is no net outflow at all. It seems quite clear that various types of atmospheric circulation, zonal with a large amount of precipitation and meridional with smaller amounts of precipitation are most responsible for the variations. As

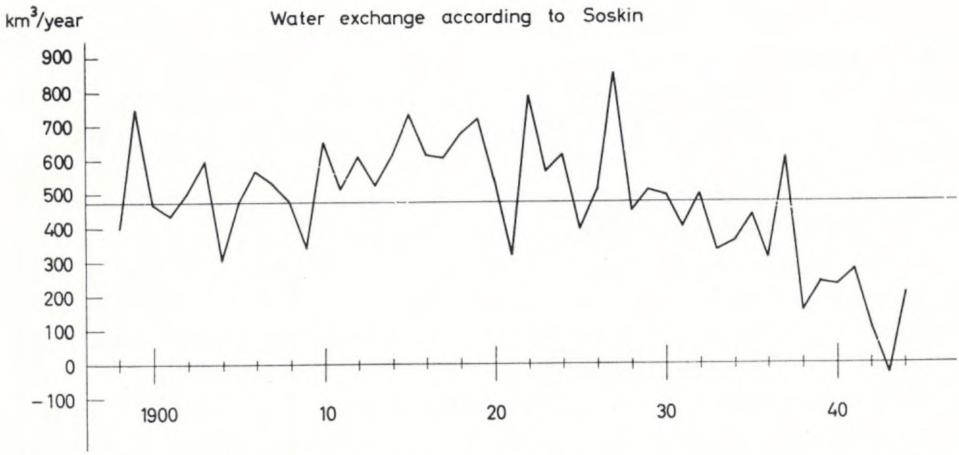


Fig. 11: 3. Annual water exchange values computed by SOSKIN (1963) from data of currents measured at Danish light-vessels.

mentioned above we have values of the total fresh water supply only for shorter periods, but we can study the outflow from some large river like FONSELIUS (1969) did (see Fig. 265: 1). However, the river transport hardly goes down to zero. Of course there is also a much lower precipitation over the Baltic itself during a period of low fresh water supply, but still there are difficulties to arrive at very low values if the evaporation figures generally used are to be trusted. The method used by BROGMUS (1952) and others are more or less confirmed, however, by PALMÉN and SÖDERMAN (1966) by quite a new manner of derivation (Flux of water vapor in the atmosphere).

SOSKIN (1963) indicates, but does not use, another method of determining the fresh water supply by using the sea level difference between the Baltic and the Kattegat.

Lately MIKULSKI (1970) has determined the total fresh water supply for the period 1951—1960. He arrived at 440 km^3/year or 92 % of what BROGMUS (1952) got for a period, which for many rivers was 1910—1940 but for some important ones only 1921—1930. It should be recalled that SOSKIN (1963) got 473 km^3/year or practically the same as BROGMUS (1952) as a mean value of the water exchange 1898—1944.

In Ch. 265 computations are presented with a model to describe long term salinity fluctuations. It is shown that variations of the fresh water supply only, are sufficient to generate salinity variations in the system which are not at all unrealistic. There is also some correlation between surface salinities and deep salinities of the Baltic (See Fig. 265: 1): when the salinities in the Kattegat get higher than normal, it is easier for water of higher salinity to enter the deep basins on occasions of intrusion, which occasion often is equivalent with a high sea level (See next chapter).

12. The Relation Between the Variations of Salinity and Sea Level

Fig. 12: 1 shows the variations during one year (1964) of the daily means of the SL at Landsort (hourly readings) and the surface salinity measured once a day at the L/V Kattegat SW. The two curves often follow each other rather well and the reason is not difficult to understand. The SL of Landsort can be assumed to represent the SL of the whole Baltic fairly well (see Ch. 24). If the SL there rises from —40 cm to +40 cm, which sometimes happens, it means that half of the water in the Kattegat must have been drawn into the Baltic. It is, however, probable that more surface water than bottom water is withdrawn and therefore, during e.g. some exclusive inflow situation like in December 1951 (WYRTKI 1954 b), one gets the impression that there has been a movement from the north of the Kattegat to the south

of the Belt Sea of all the Kattegat surface water, while the SL of the Baltic rose from —35 cm to +55 cm during a fortnight.

The idea that there is a connection between the variations of salinity in the Kattegat and the SL of the Baltic is implicitly presented in many works (HELA 1944, WYRTKI 1954 b) and practically explicitly written down in SOSKIN (1963). Nevertheless some of the many consequences have not been investigated before.

Many attempts have been made to find a relation between herring fishery at the West Coast of Sweden and some hydrographic parameters (ANDERSSON 1960, SVANSSON 1965). One thing seems to be rather clear. If there is too much Baltic water in the Eastern Skagerrak and the fiords of Bohuslän, the herring will leave these areas (ANDERSSON op. cit.). O. PETTERSSON and G. EKMAN (1897) could draw this conclusion from salinity measurements in a famous example when, after a long period of herring winters, in December 1896 the herring disappeared. Looking now at the SLs of Landsort for the period concerned it is quite evident that the SL was low during December 1896 and January 1897. While, however, a high SL of the Baltic is a necessary condition for good herring fishery it is not at all sufficient.

OTTO PETTERSSON (1914) presented daily observations of salinity in the Gullmar fiord during 1909—1911. He described the great vertical variations of the isolines as internal waves, driven by tidal forces, the period of importance being around a fortnight. Later HANS PETTERSSON (1916 and 1920) showed rather a high correlation between this phenomenon and the wind. JERLOV (JOHNSSON 1943) found cases when the correlation with the atmospheric pressure was high on an occasion when ice covered the whole Eastern Skagerrak.

The present author wants to incorporate this phenomenon into the general horizontal movements in and out by the Baltic water. Fig. 12: 2 shows some parameters measured 1909. Of the Bornö station data, only 3-day means could be found of the depth of the 31 ‰ isohaline. It is evident that there is a 14-day period of approximately the same phase in nearly all the curves, namely the SLs in Varberg (Kattegat) and Landsort (Baltic proper), the surface salinities in the Kattegat and the Öresund and also the depth of the 31 ‰ isohaline at Bornö station (Skagerrak). As it is probable that the variations of the SL in Varberg kept in step with the variations of the atmospheric pressure (high pressure — low sea level and vice versa) this time a small depth of the isohaline at Bornö was simultaneous with a low atmospheric pressure. JERLOV (JOHNSSON 1943) showed the opposite: that a small depth of the isohalines is simultaneous with a high atmospheric pressure. Of importance is apparently if the characteristic period of the variations is of the order of magnitude of a fortnight or a week. In the former case we have the direct correlation, in the latter the indirect one of JERLOV's (JOHNSSON 1943). The latter case which occurs much more often will be taken up in Ch. 14.

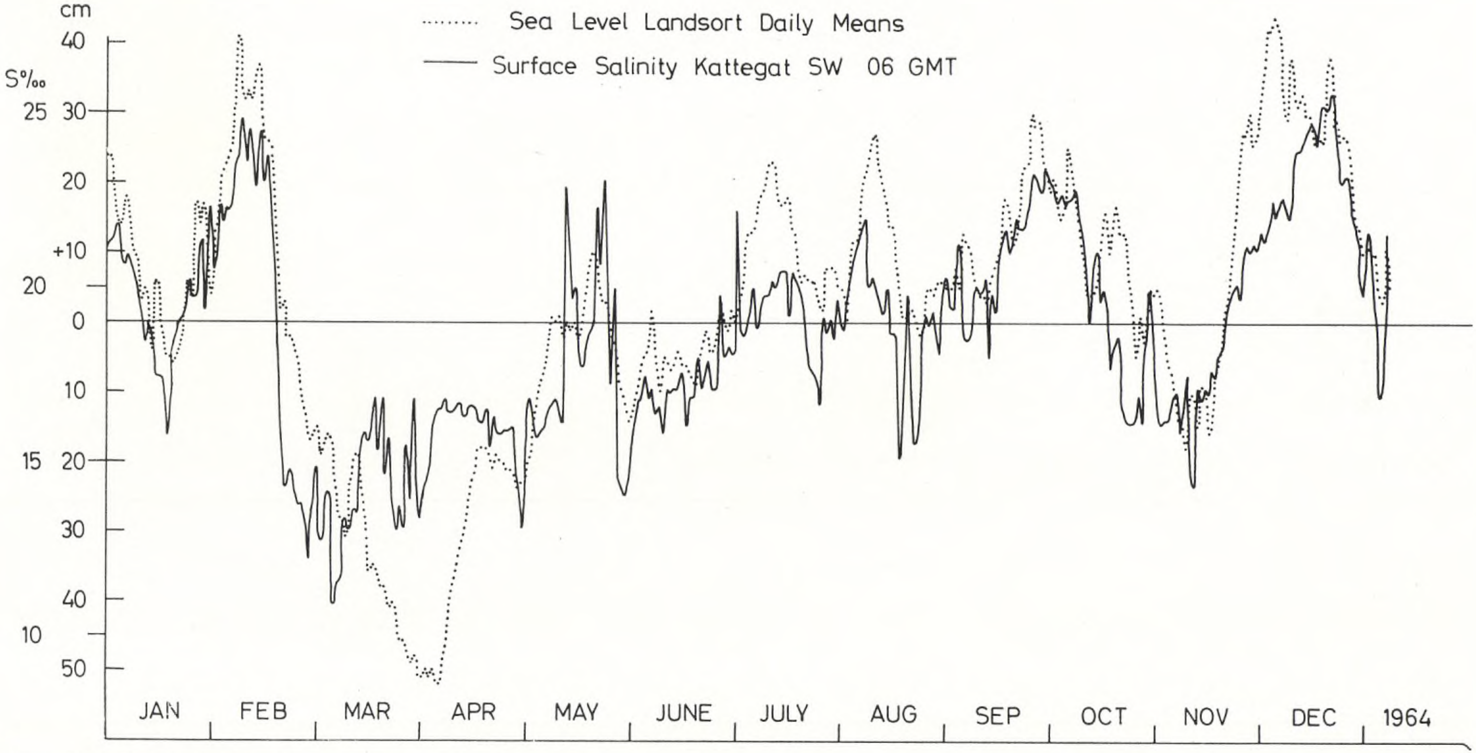


Fig. 12: 1. A comparison between the variations during one year (1964) of the daily means of hourly readings of the sea levels at Landsort (Fig. 1: 2) and of the surface salinities measured once a day at the L/V Kattegat SW (Fig. 1: 1).

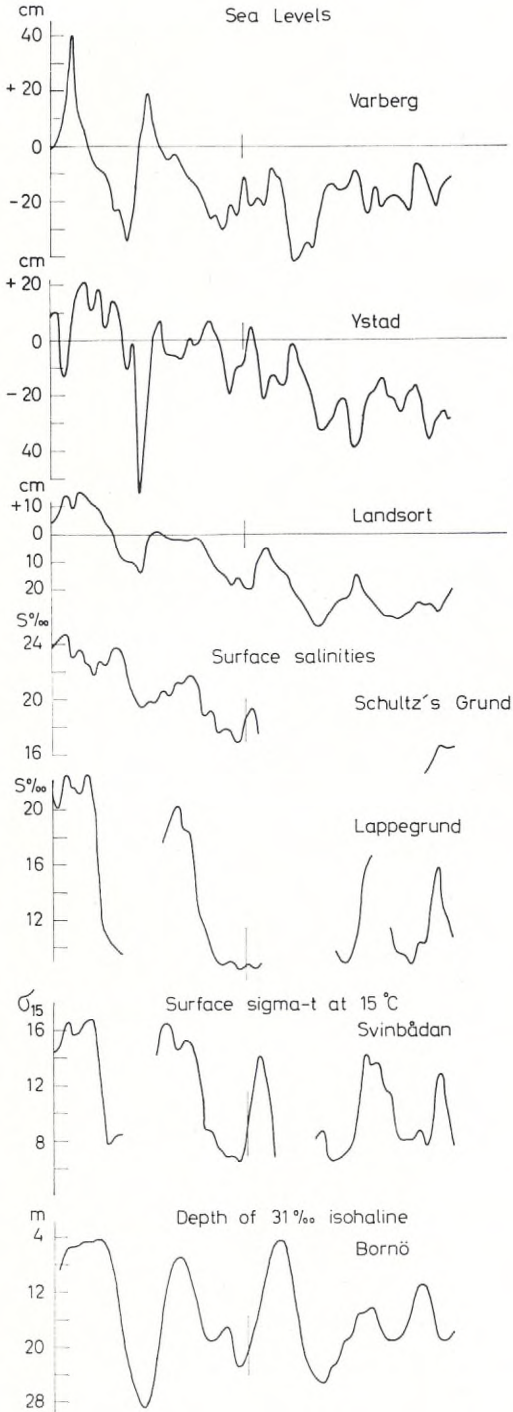


Fig. 12: 2. A comparison between some sea levels and surface salinities on an occasion when the phases were approximately the same.

13. The Permanent Currents of the Skagerrak

Fig. 13: 1 shows a simplified map of the surface currents of the Skagerrak. It has been compiled from BÖHNECKE (1922), TAIT (1930 and 1937) and others. On the Danish side there is the incoming Jutland current and from the Kattegat comes the Baltic current. Along Sweden and Norway the two currents unite and leave the area together in the NW corner flowing along the Norwegian coast even in the North Sea. From the large mixing zone in the SE corner of the Skagerrak a small amount of water probably flows southwards into the Kattegat as a "countercurrent" (cf. Ch. 11). At non-surface horizons we know much less, but some measurements were made by means of automatically recording devices and also from anchored research vessels. The data shows that the currents usually run in the same direction from surface to bottom (HELLAND-HANSEN 1907, SVANSSON 1961, Anon. 1969). Therefore it is maybe less advisable to use the method of a layer of no motion to compute geostrophic currents from data of temperature and salinity like KOBE (1934) and TOMCZAK (1968) did. SVANSSON and LYBECK (1962) tried to compute the geostrophic transport by referring to measurements of surface currents in calm weather. They got a transport of approximately 1/2 million m^3/s for both in- and outgoing currents (the difference, 15 000 m^3/s from the Baltic, is too small to be found in this rough calculation). Fig. 13: 2 shows the daily mean values of July 9 during the international cooperation 1966. This type of circulation is probably rather common. It is evident from this figure as well as from the salinity maps in the Atlas from the cooperation (Anon. 1970) that a great deal of the water circulating in the Skagerrak comes from the Norwegian Sea along the isobath of 150—200 m, but in the surface layer there is probably also a transport from the Southern North Sea (JACOBSEN 1913).

Table 13: 1. Monthly means of the N-Component of the Current at a Depth of 50 m SW off Smögen (Measuring interval 20 minutes).

April	1971	17 cm/s	September	1967	14 cm/s
June	1967	19 ,,	October	,,	29 ,,
July	,,	8 ,,	,, ...	1971	37 ,,
August	,,	9 ,,			

Below will be shown that the SE corner of the Skagerrak is disturbed by the Baltic (Ch. 142), but if we use monthly means this disturbance may disappear. Table 13: 1 shows the monthly mean values of the N-component of data from a currentmeter SW of Smögen (SVANSSON 1969 a). These monthly means are all positive and approximately 15 cm/s.

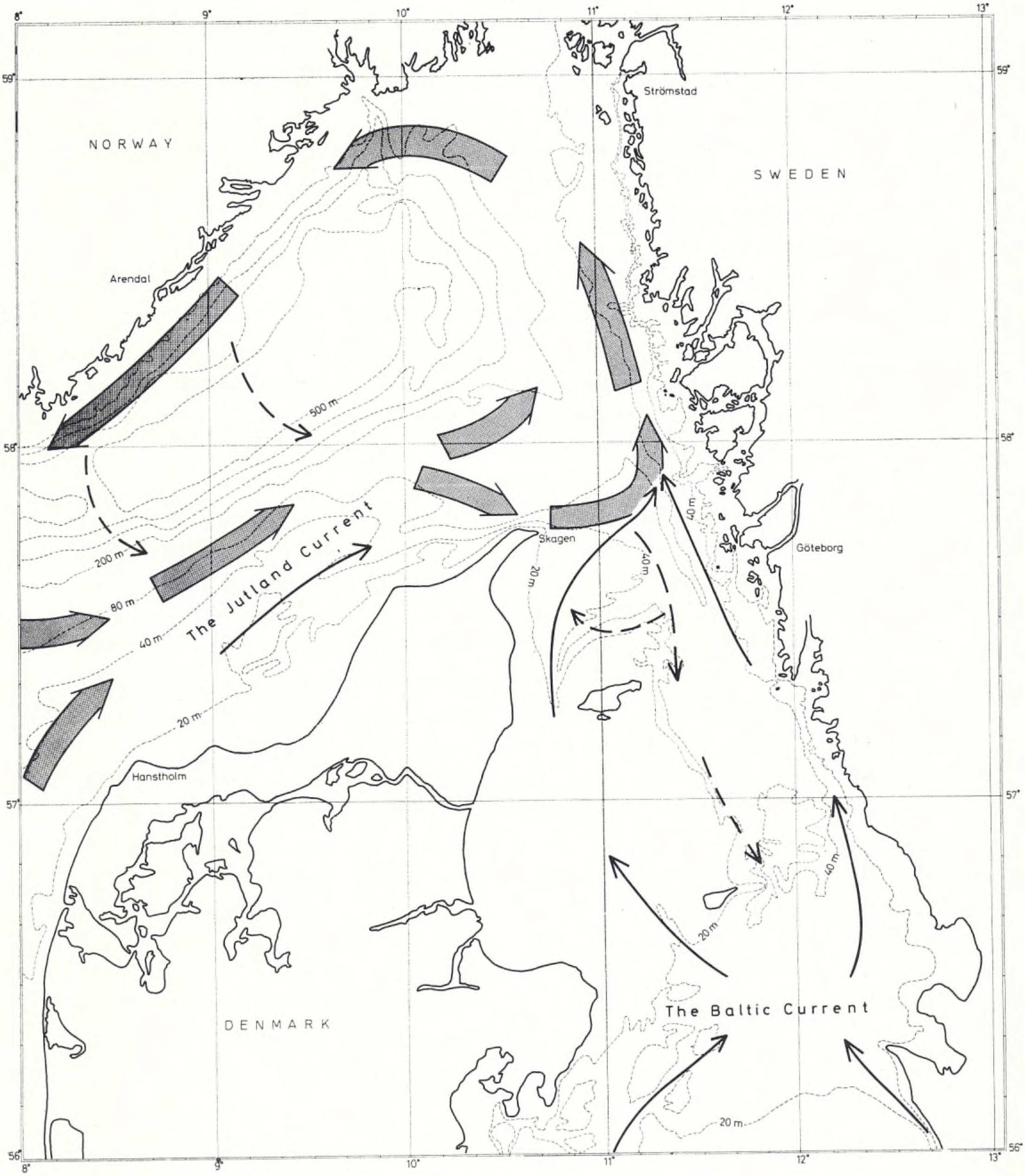


Fig. 13: 1. A simplified map of the surface currents of the Kattegat and the Skagerrak. In the Kattegat is indicated the "countercurrent", mentioned in Ch. 11, which originates from the large mixing between the Jutland current and the Baltic current. The main bulk of this fusion is, however, leaving the Skagerrak along the coasts of Sweden and Norway.

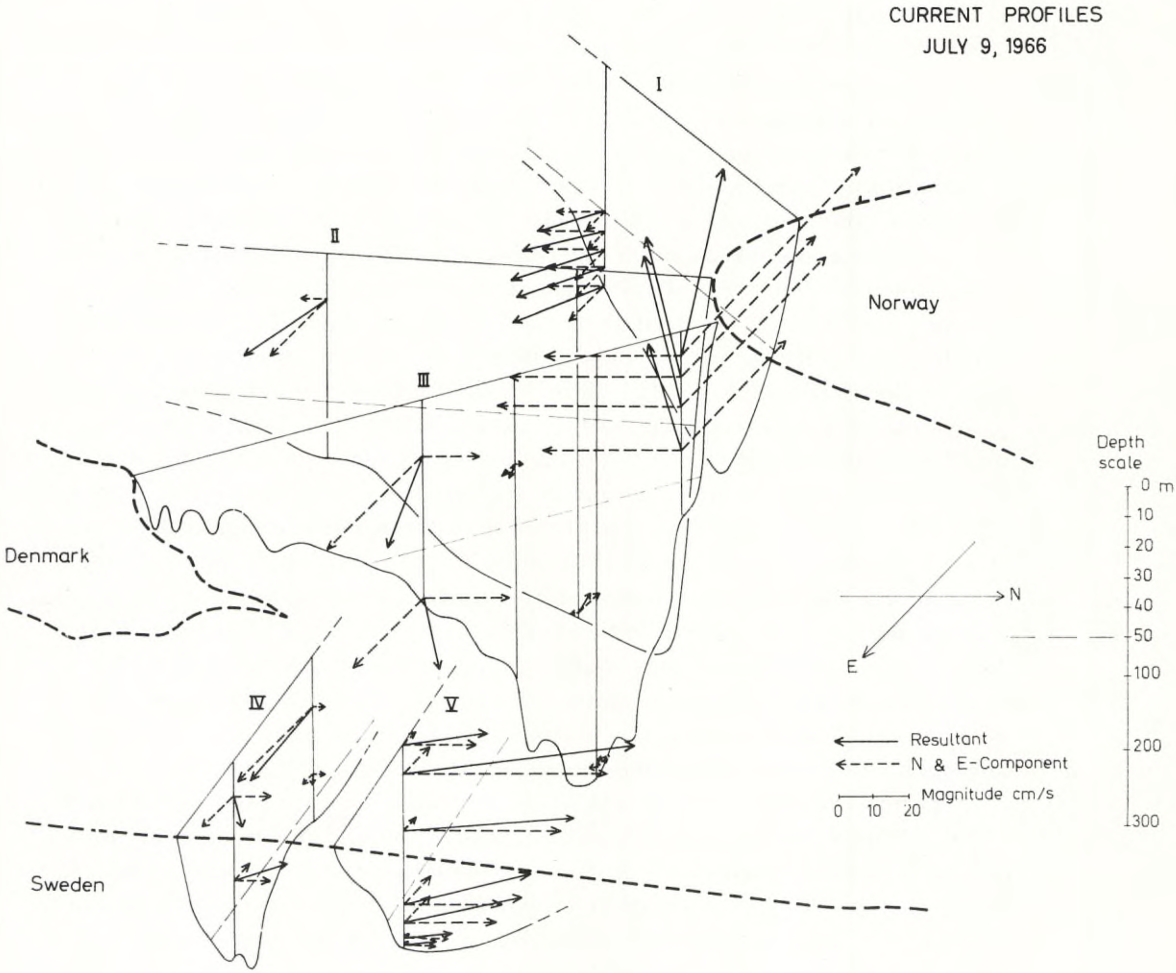


Fig. 13: 2. Daily means of currents measured on July 9, 1966, during the International Skagerrak Expedition. Starting at section I we can see the current flowing into the Skagerrak along the southernmost parts of sections II and III. At the eastern vertical of section IV between Denmark and Sweden the current is seen to turn northwards, at section V the direction is nearly northwards and at section III at the vertical nearest to Norway the current leaves the area.

14. A Study of some Periods of Numerous Observations in order to Disclose Large Scale Events

During two weeks in the beginning of 1964 an international cooperative study of the Baltic was carried out from 6 anchored and 2 moving research-vessels. Furthermore were set out two anchored masts with recording current meters (Anon. 1968).

In order to better understand the variations of currents and sea levels the present author also collected SL observations (hourly means) from the Skagerrak, the Kattegat and the Belt Sea. These together with the data of atmospheric pressure are used below in Ch. 2 to test the numerical models. In this chapter the daily means are studied. As, however, there is nearly no information of what happened in the Skagerrak during this period in 1964, first are studied two other periods viz June—July 1966 when there was an international program in the Skagerrak and 1967 when there was a long Swedish series of current measurements in the Eastern Skagerrak.

It is a well known fact that the semidiurnal and diurnal tidal amplitudes which in the Kattegat attain magnitudes of 20—30 cm are strongly attenuated in the Belt Sea. Fig. 14: 1 showing the amplitudes and phases of M2, was compiled by means of DEFANT (1934 and 1961). It is, however, probable that longer periods are not filtered that effectively (Chapters 2126 and 263). Moreover the characteristic period of the whole system Skagerrak—Baltic, regarded as a semi-open canal, is probably of the order of magnitude of two weeks according to a calculation described in Ch. 268. Such a high value of the characteristic period may seem improbable when considering that the period of the closed Baltic is of the order of 2 days (NEUMANN 1941). That shallow straits may cause highly increased periods, however, was shown by NEUMANN (1944). As an example we can take the lakes Michigan-Huron with a period of 48 hours, the periods being only 9 hours for the closed Michigan and 7 hours for the closed Huron (ROCKWELL 1966).

A spectral analysis of Baltic sea levels was made by MAGAARD and KRAUSS (1966). While peaks at the periods of approximately 260 hours (11 days) are hard to find, maybe due to the fact that there are very few points in this region of the spectrum, there is everywhere, except in the Gulf of Finland, a very clear peak at 120 hours (5 days). From the data shown in the present paper it is clear that 5-day periods are common and that we also find them in the Belt Sea, the Kattegat and the Skagerrak. There seem to be two nodal lines, one in the Belt Sea and one in the northern Baltic proper. Apparently, however, it is not a characteristic period of the system (Ch. 268).

Comparing the SLs in the Skagerrak with the atmospheric pressure there is a negative correlation. While the SLs at Mandal seem to be ordinary in the sense that a change of one millibar of the atmospheric pressure gives a change of approximately 1 cm of the sea level, the records at Smögen and particularly Hirtshals show that the change is at least 2 cm/mb (cf. Fig. 262: 1). To explain this fact one can imagine that low

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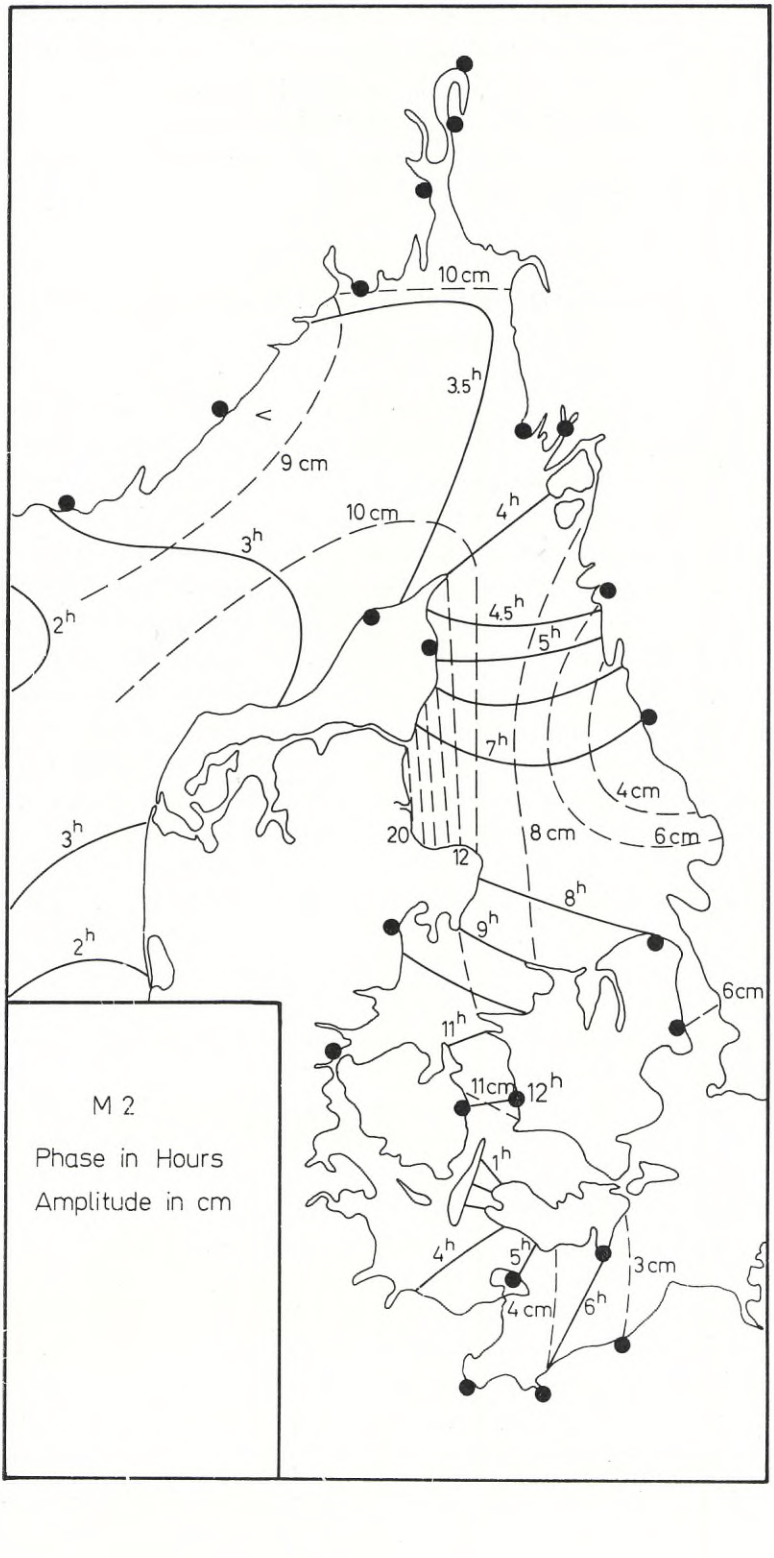
Fig. 14: 1. Phases and amplitudes of the tidal component M2 (12.42 hours). No isolines have been drawn in the W. Baltic due to lack of observations (sea level-gauges are indicated by filled circles).


M 2
Phase in Hours
Amplitude in cm
10 cm
3.5h
9 cm
10 cm
3h
4h
2h
4.5h
5h
7h
4 cm
6 cm
8 cm
20
12
3h
8h
9h
2h
6cm
11h
11cm
12h
1h
4h
5h
3 cm
4 cm
6h

pressures are simultaneous with westerly winds raising the SLs in the North Sea and the Skagerrak. At the same time the SLs are usually high in the Northern Baltic (Kemi) but low in the Southern Baltic (Ystad). LYBECK (1968) tried to apply on the Skagerrak a theory similar to the one presented by ROBINSON (1964) and MYSAK (1967) to explain similar trends in Australian tide-gauge records (HAMON 1962).

141. The Current Records of the Joint Skagerrak Expedition in June—July 1966

Fig. 141: 1 shows the daily means of the N- and E-components of the German current meter records during the cooperation 1966 at two stations, one at the entrance into the Skagerrak of the Jutland current (stn 41) and the other on the border between the Skagerrak and the Kattegat (stn 44). The figure also shows measurements of currents from two Danish light-vessels, viz. Skagens Rev (E-component) and Halsskov Rev (N-component). The positions can be found in Fig. 1: 1. The similarity between the E-components of stn 44, at 40 m, and of the L/V Skagens Rev, at the surface, is quite evident. The strong negative correlation between these E-components on one side and the N-component of Halsskov Rev on the other are discussed in Ch. 142. There seems, however, to be hardly any similarity between the record of stn 41 and the remaining records. The period is short but the comparison gives some support to the idea, that the strong variations on the border between the Skagerrak and the Kattegat are caused mainly by the Baltic oscillations and not by something that is already in the Jutland current.

142. The Long Record of the Current at 50 m Depth off Smögen in 1967

This record made by a RICHARDSON current meter during April—November 1967 has been described in SVANSSON (1969 b) and daily means were published by SVANSSON (1969 a). Fig. 142: 1 shows the variations of the N-component during June and July (Record "Smögen"). During this time there are oscillations particularly of the 5-day type. In the figure are also included the daily means of the currents at the L/V Skagens Rev (E-component) and the L/V Halsskov Rev (N-component), further the records of the daily means of the SLs at Smögen, Ystad and Landsort and finally the depth of the 22 ‰ isohaline at the Bornö station.

Like in the 1966 case (Ch. 141) there were opposite phases between the records of the L/V Halsskov Rev and the E-component of the current on

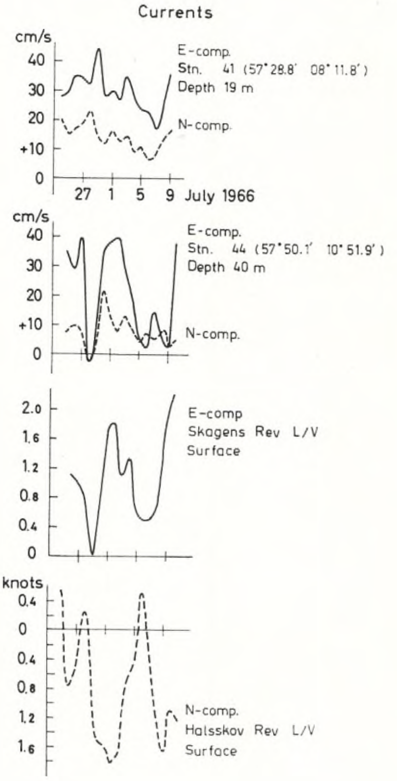


Fig. 141: 1. Daily means of currents measured during the International Skagerrak Expedition 1966. At the top a record from the Jutland current in the outer Skagerrak, in the middle two records from the border between the Skagerrak and the Kattegat and at the bottom a record from the Belt Sea (positions in Fig. 1: 1).

the border between the Kattegat and the Skagerrak (station 44 and the L/V Skagens Rev respectively). We now see that the phase is the same for the current at 50 m depth off Smögen and for Skagens Rev (and also the SLs of Smögen and Kemi). An explanation may be as follows: when the SL is high in the Western Baltic and low in the Kattegat—Skagerrak water flows back from the southern Baltic into the Kattegat—Skagerrak (see also below in Ch. 142). The Jutland current is then forced to take another direction (the N-component at the L/V Skagens Rev is sometimes enlarged on these occasions but not always). Also the current at 50 m off Smögen is weakened simultaneously.

That the isohaline of 22 ‰ at Bornö rises when the SL at Smögen is low may be explained by the removal of Baltic water at that phase of events, but regarding all remaining occurences in the SE corner of the Skagerrak it seems wise to submit a more complete explanation of the internal movements in the Gullmar fiord to a special investigation to be carried out in the future.

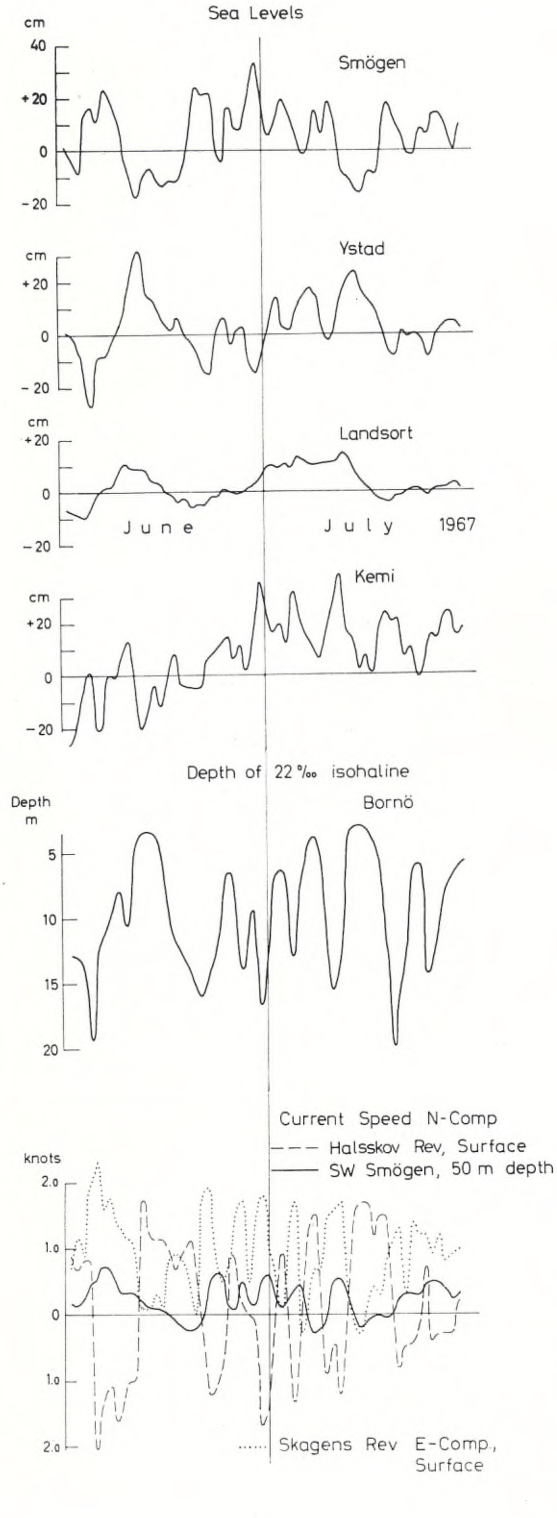


Fig. 142: 1. Daily means of sea levels (Smögen see Fig. 1: 1, Ystad and Landsort Fig. 1: 2, Kemi Fig. 1: 5), of currents (SW Smögen and light-vessels Halsskov Rev and Skagens Rev see Fig. 1: 1) and of the depth of an isohaline at Bornö hydrographic station (see Fig. 1: 1).

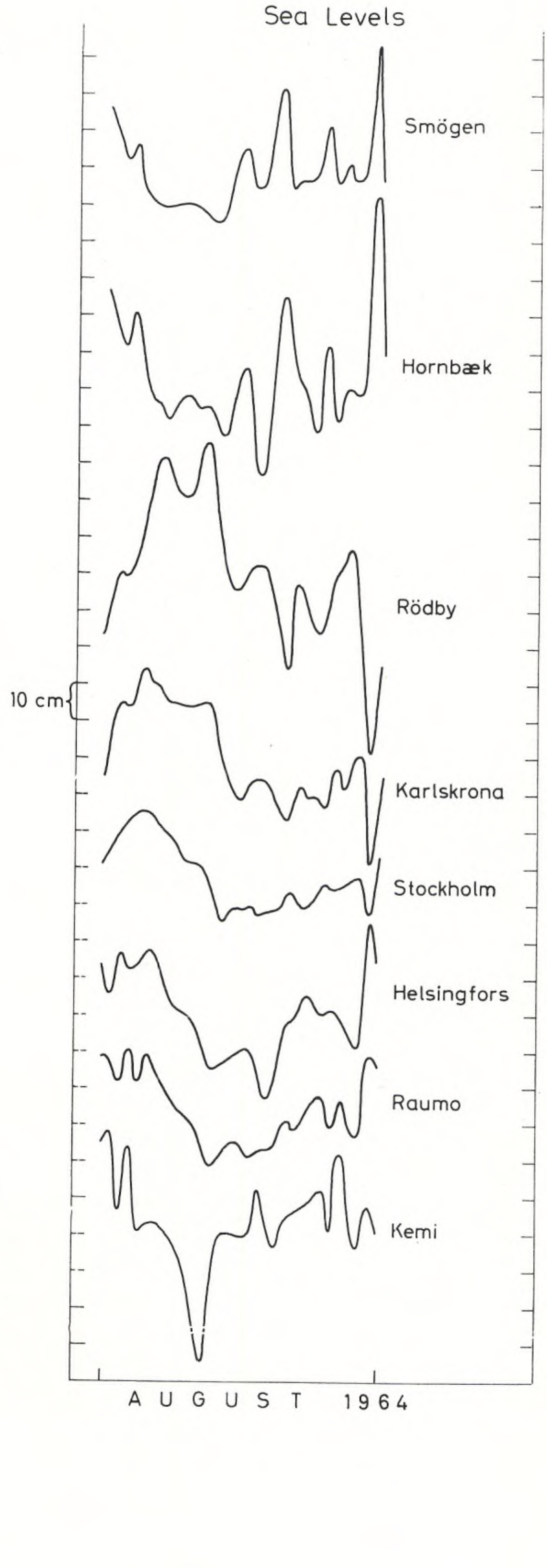


Fig. 143: 1. Daily means of sea levels from the Skagerrak (Smögen), the Kattegat (Hornbaek), the Belt Sea (Rödby), the Baltic proper (Karlskrona and Stockholm), the Gulf of Finland (Helsingfors), the Bothnian Sea (Raumo) and the Bothnian Bay (Kemi) in August 1964.

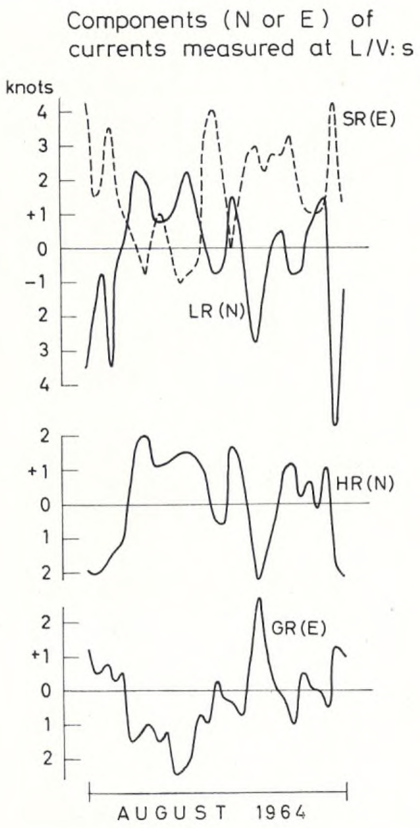


Fig. 143: 2. Daily means of currents measured at some Danish light-vessels during August 1964 (see Fig. 1: 1).

143. The International Cooperation in the Baltic in the summer of 1964

More SL and current data were collected, for a period immediately before, during and immediately after the international cooperation during August 1—13, 1964, than for any of the other cases described above. We lack information of the currents from the open Skagerrak during this period, but from the L/V Skagens Rev we have current data, the E-component of which have been shown to oscillate very much in the same manner as the N-component off Smögen (Chapters 14 and 142). — In the Baltic 6 research vessels measured currents at many horizons (see Fig. 1: 2) and in the strait between Bornholm and Sweden was anchored a mast at which were attached current meters and temperature sensors.

Fig. 143: 1 shows the daily means from some level gauges. The phases of the Smögen SLs are repeated rather unchanged through the Kattegat at least to Hornbaek, but in Rödby the phases are practically opposite to those of Hornbaek; between these ports there is an area of transition. The phases of Rödby can still be found in Karlskrona. Then comes a new area of transition

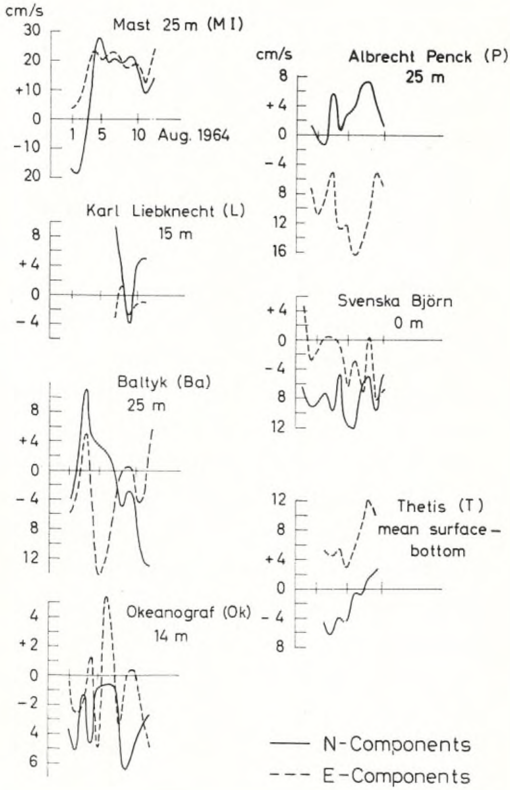


Fig. 143: 3. Daily means of some of the current measurements carried out during the International Baltic cooperation in August 1964. See Fig. 1: 2 for positions.

(Stockholm) so that the levels in the port of the Gulf's of Bothnia and Finland again have a likeness to those of the Kattegat.

Fig:s 143: 2 and 143: 3 presents daily means of some of the currents. Again there is a good negative correlation between the E-component at Skagens Rev and the N-component at Läsö Rende. Moreover this correlation is valid not only for Läsö Rende but for all the remaining Danish light-vessels (for Gedser Rev naturally the E-component). Particularly during the period August 6—12, the high levels of the SW Baltic are combined with transports of water out from the Baltic except at the mast NW of Bornholm and also at the R/V Thetis, but the currents of the central Baltic measured by many ships during the International cooperation seem, however, to be too complicated to fit into a canal model.

2. Numerical Computations

Nearly as old as the science of oceanography are the attempts to explain the variations of water movements and water levels (SLs) by applying the hydrodynamic equations. Very early, in the beginning of this century, the seiches (stationary waves) and the tides were studied in many seas by treating these as canals, and in order to take the real configuration of the canal into consideration, numerical methods were used (see e.g. DEFANT 1961). WITTING (1911) computed roughly and NEUMANN (1941) more carefully the characteristic period of the Baltic (considered closed) and LISITZIN (1943) treated the diurnal tidal component K1 of the Gulf of Bothnia likewise. HANSEN (1956) was one of the first to use an electronic computer to solve numerical hydrodynamical equations to calculate tides and Meteorological Sea Level Effects (cf. WELANDER 1961). Two-dimensional models applied on parts of the area concerned here have been made by UUSITALO (1960 and 1971), HENNING (1962), LASKA (1966), ANNUTSCH (1967), KOLTERMANN (1968), MALINSKI (1968) and others.

The present author has long worked with one-dimensional canal models applied to the seas around Sweden. In Ch. 25 the earlier parts of this work with a model using an explicit numerical method are briefly summarized. This model is possible to apply to a large number of canals but allows only timesteps of maximum 15 minutes. Additionally some future plans with this model are presented. Thereafter (Ch. 26) results with a model using an implicit method of numerical integration allowing any timestep are presented, but so far the present author has not found an easy way of applying it to the same system of approximately 10 canals as is possible with the explicit method. Furthermore this model is combined with a model for salinity variations treated with an implicit numerical method.

The mathematical background and the numerical schemes are presented as Chapters 21—23. The system of equations for the sea level problems is taken up in some detail in Ch. 21 with a derivation according to PLATZMAN (1963) and a discussion of the various terms, including those neglected, in the computations in this paper. The equation of the salinity model is derived in Ch. 22. The numerical scheme for the explicit sea level model are described in SVANSSON (1959). Here, therefore only the implicit models are described in this respect (Ch. 23). The reference sea level used is described in Ch. 24.

21. The System of Equations for the Sea Level Problems

The following system of equations has been used (symbols are explained in Ch. 5):

$$\frac{\partial U}{\partial t} = -gA\left(\frac{\partial h}{\partial x} + \frac{1}{g}\frac{\partial p_a}{\partial x} - \frac{\partial \bar{h}}{\partial x}\right) + b\tau - b\,\tau_B;$$

$$\frac{\partial h}{\partial t} = -\frac{1}{b}\cdot\frac{\partial U}{\partial x};$$

$$\left((fU = -gA\frac{\partial h}{\partial y}\right);$$

211. Derivation

A derivation of the system of equations in Ch. 21 was presented in SVANSSON (1959) except for the tidal acceleration term $\frac{\partial \bar{h}}{\partial x}$, which is, however, easily included, as described in e.g. PROUDMAN (1953). The bottom stress term is a more or less unknown term, usually made a function of the transport U, see Ch. 2126. A somewhat more complete way of taking the bottom friction into consideration was presented by PLATZMAN (1963). As his scheme is included in the planned explicit model described in Ch. 254, the derivation of it will be briefly shown. At the end of this it is easy to derive the system in Ch. 21 by some simplifications.

PLATZMAN (1963) started by the so called EKMAN equation

$$\frac{\partial w}{\partial t} = q - i\,f\,w + \frac{\partial}{\partial z}\left(\nu\frac{\partial w}{\partial z}\right);$$

$$\text{where } w = u + i\,v \text{ and } q = -g\left(\frac{\partial h}{\partial x} + i\,\frac{\partial h}{\partial y}\right).$$

Below in Ch. 212 are discussed some terms not taken into consideration. Those terms neglected already at the beginning of the derivation are the spatial acceleration term, see Ch. 2121, and the term of horizontal eddy diffusion, see Ch. 2126.

We consider the eddy diffusion coefficient ν to be constant. The bottom stress will be expressed in the following way, where $z = -H$ means bottom:

$$\tau_B = \left(\nu\frac{\partial w}{\partial z}\right)_{z=-H} = s\,w_{-H}$$

where s can assume various values: $s \to \infty$ means $w_{-H} \to 0$ a boundary condition often used, see e.g. WELANDER (1957), while $s = 0$ means zero stress.

Table 211: 1. Factors Q computed under the assumption $\nu=0.125$ MTS, $s=0.002$ m/s and $f=10^{-5}s^{-1}$.

Depth m	Ekman number	Q_1	Q_2	Q_3	$-Q_4$
11.4	0.30	1.00	1.03	0.00	18.4
15.9	0.37	1.00	1.04	0.00	11.6
27.4	0.63	1.00	1.06	0.01	6.3
55.4	1.3	0.99	1.10	0.03	2.8
78.2	1.8	1.00	1.10	0.07	1.9

Writing $(f\,i+\frac{\partial}{\partial t})=\sigma^2$ a solution is derived (after integration from surface to bottom, whereby the stress at the surface, τ, and at the bottom, τ_B, are introduced; $\overline{w}$ means the mean value of the velocity w from surface to bottom):

$$\frac{H}{\nu}\left[\sigma^2+L(\sigma)\right](H\overline{w})=Hq+\left[1+M(\sigma)\right]\tau;$$

L and M are now developed after $\sigma_o=\frac{H^2\,i\,f}{\nu}$

If only the first terms are kept we have

$$\frac{\partial(\overline{w}\,H)}{\partial t}=F'(H,\nu,s)\cdot(Hq)+i\,f\,E'(H,\nu,s)\cdot(H\,\overline{w})+J'(H,\nu,s)\cdot\tau;$$

or in components

$$\frac{\partial(\bar{u}\,H)}{\partial t}=-g\,H\frac{\partial h}{\partial x}F'_r+g\,H\frac{\partial h}{\partial y}F'_i+f\,\bar{v}\,H\,E'_r+f\,\bar{u}\,H\,E'_i+J'_r\,\tau_x-J'_i\,\tau_y;$$

$$\frac{\partial(\bar{v}\,H)}{\partial t}=-g\,H\frac{\partial h}{\partial y}F'_r-g\,H\frac{\partial h}{\partial x}F'_i-f\,\bar{u}\,H\,E'_r+f\,\bar{v}\,H\,E'_i+J'_r\,\tau_y+J'_i\,\tau_x;$$

We now restrict ourselves to one dimension by assuming the transverse velocity $\bar{v}$ and also $\frac{\partial\bar{v}}{\partial t}$ to be equal to zero. Including in the pressure term not only the sea level gradient but also the atmospheric pressure gradient and the tidal acceleration term we can write

$$\frac{\partial\bar{u}}{\partial t}=-g\left(\frac{\partial h}{\partial x}+\frac{1}{g}\frac{\partial p_a}{\partial x}-\frac{\partial\bar{h}}{\partial x}\right)Q_1+Q_2\frac{\tau_x}{H}+Q_3\frac{\tau_y}{H}+Q_4\,f\,u;$$

All the terms Q_i are functions of the position (depth), ν and s. If the Ekman number $\varepsilon=H\sqrt{\frac{f}{2\nu}}\gg 1$

then Q_1 and $Q_2\rightarrow 1$ and Q_3 and $Q_4\rightarrow 0$.

When using the equation of Ch. 21, in relation to the Platzman derivation we assume the depth to be large when considering Q_1, Q_2 and Q_3, but we do not take the consequence for Q_4. Instead we introduce some assumption meaning $Q_4 \neq 0$. (In Table 211:1 are shown the values of the Qs computed for $\nu = 0.125$ MTS and $s = 0.002$ m/s [JELESNIANSKI 1967]). Note that there is also an integration across the canal (see SVANSSON 1959), so that e.g. U means $\int_{-H}^{o} \int_{o}^{b} u \, dx \, dy$, or from the derivation above $U = A \cdot \bar{u}$.

212. The Various Terms

In this chapter the details of the treatment of the various terms in the system of equations in Ch. 21 are presented and also the importance of their presence of even non-presence. More general and complete discussions can be found in WELANDER (1961).

2121. Non-linearities

The term $u \frac{\partial u}{\partial x}$ has been left out in the computations presented in this paper. A few tests have shown that usually this term is small. But for some parts of the Belt Sea, according to some rough comparisons with the computations made, $u \frac{\partial u}{\partial x}$ may be of importance. For a future model, e.g. the explicit one as explained in Ch. 254 or, hopefully, an implicit one comprising all the canals, it will be wise to test the importance of this term.

In HANSEN's (1956) and KREISS's (1957) computations of tides in German rivers it was necessary to include such non-linear effects as $u \frac{\partial u}{\partial x}$ as well as $H = \bar{H} + h$, where $\bar{H}$ is the mean depth used in this paper. The same thing can be said about this latter non-linearity as about $u \frac{\partial u}{\partial x}$ above: tests in the Gulf of Bothnia gave no difference when including such a variable depth, but in the Belt Sea it may well be of importance.

To the third equation of the system of Ch. 21, (the one of the transversal balance) could be added a centrifugal term u^2/r, where r is the radius of curvature. We could combine it with the geostrophic term to a new term $u(\frac{u}{r} + f)$. If r is positive, as is the case when e.g. water flows from the Skagerrak to the Kattegat, the two terms are added. If we assume r to be $\approx 5 \cdot 10^4$m then $\frac{u}{r}/f = 0.02$ if $u = 10$ cm/s and 0.2 if $u = 100$ cm/s. Then one should be a little careful in using position where the canals bend strongly when comparing levels at one end of a section. The effect has not been taken into consideration in this paper.

2122. The Pressure Gradient Term

Arguments to neglect the stratification were given in SVANSSON (1959) and will in general not be repeated here. In the Kattegat and partly the Belt Sea, however, where the surface layer consisting of water of Baltic origin oscillates back and forth (See Ch. 13), the condition of barotropy might not be well fulfilled as has been assumed in all computations in this paper. The assumption of barotropy is, however, probably a good first approximation, see e.g. the results presented in Ch. 267.

2123. The Tidal Acceleration Term

The expressions for $\bar{h}$ in $\frac{\partial \bar{h}}{\partial x}$ were taken from BARTELS (1957), for M2:

$$\bar{h}=0.2426 \cdot \cos^2\phi \cdot \cos\left(\frac{360 \cdot t}{12.42 \cdot 3600}+2\chi\right)$$

and for Mf:

$$\bar{h}=0.02089 \cdot (3\cos^2\phi - 2) \cdot \cos\frac{360 \cdot t}{13.66 \cdot 24 \cdot 3600}$$

2124. The Atmospheric Pressure Gradient Term

In the open ocean the adjustment to atmospheric pressure is usually very fast, the long wave velocity being much higher than the velocity of low and high pressures areas. There it is possible to adjust all sea levels to normal atmospheric pressure, assuming that statically a change of the atmospheric pressure of one millibar means a change of the sea level of 1 cm. In the vicinity of coasts and in bays with large characteristic periods this is no longer true. The dynamic effect will be of importance and can not be neglected (See Ch. 14). In the long term computations described in Chapters 264 and 266 it is evident that the boundary values are not sufficient to drive the variations in the Baltic. The next step in computations of the type presented in those chapters should be to add the atmospheric pressure term. Then, possibly, the difference between computation and reality, which should be attributed to the wind effect, will be so small that the equation can be solved using a linear relation between the wind stress and the wind velocity (see also Ch. 2125).

2125. The Wind Stress Term

The wind stress is a function of the wind velocity. The quality and the geographical distribution of observed surface winds are such that a computation from surface atmospheric pressures is usually preferred. Unfortu-

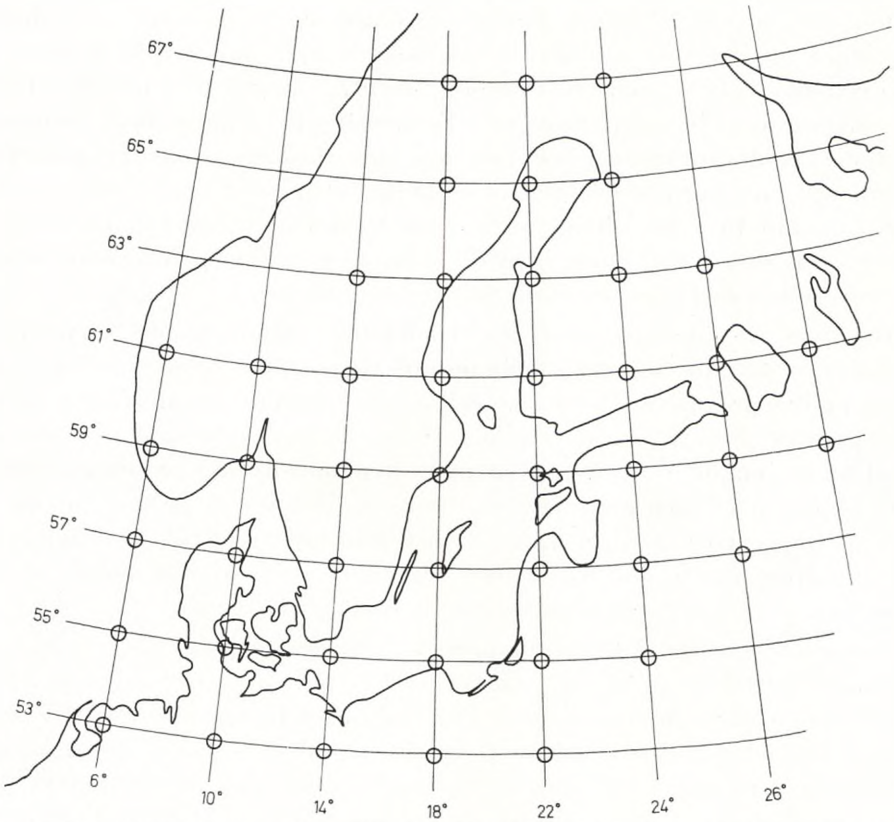
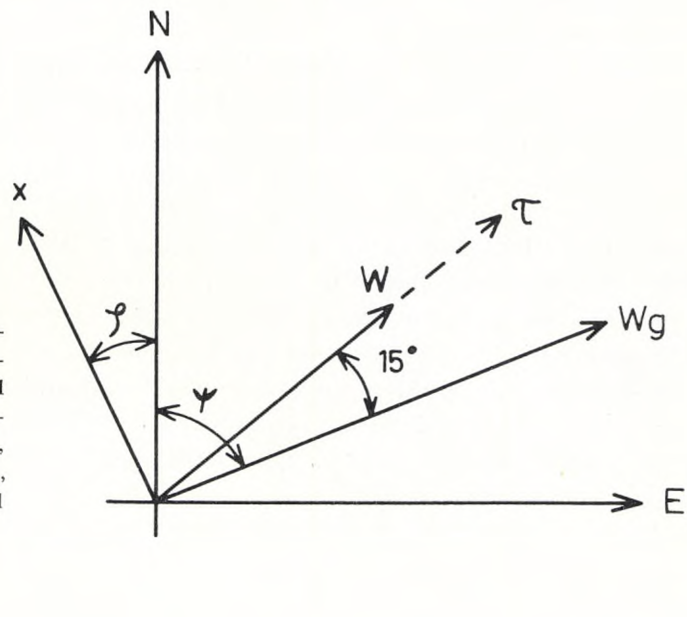


Fig. 2125: 1. Gridpoints at which were read the atmospheric pressure on surface level weather maps.

Fig. 2125: 2. Angle symbols relating the directions of the longitudinal component x_j (perpendicular to the sections j), the geostrophic wind W_G, the surface wind W, and the wind stress τ.



nately this is only a slightly better alternative as there is no good theory on which to base such a computation. Both in accordance with theory (see e.g. HAURWITZ 1941) and observations (see e.g. PALMÉN und LAURILA 1938) the surface wind in comparison with the geostrophic wind is both weakened and deflected toward the low pressure side. In this work the reduction factor was chosen to be 0.7 and the angle between geostrophic wind and the surface wind 15°. The choice of numeric values for these two parameters, however, is somewhat arbitrary as these influence the wind stress coefficient K_2, which is varied experimentally in the computations.

One may consider more complicated relations where factors such as the stability of the air and the roughness of the surface enter. This was not done in the present work. A somewhat more serious simplification is the disregard of the cyclostrophic effect. Generally it can be said that the sea level model ought to be improved hydrodynamically before the secondary effects just mentioned are included. There is, however, hope that the model will be improved to such a degree that it will allow realistic determination of the stress coefficient K_2. At that time the cyclostrophic effect should evidently be included and possibly also the stability of the air.

One usually assumes the wind stress τ to be proportional to the square of the wind speed $W: \tau = K_2 W^2$. EKMAN (1905) presented the value $K_2 = 3.2 \cdot \cdot 10^{-6}$ derived from data of a storm 1872 published by COLDING (1881). Later a very large number of K_2:s as well as other wind stress formulae have been presented (FRANCIS 1951). Interesting is the idea of WITTING (1918) that the relation is linear and can be easily combined with the direct influence of the atmospheric pressure to the so called anemo-baric effect. This would simplify the use of mean values. Actually SVANSSON (1966) presented numerical computations also using the formula $\tau = 2 \cdot 10^{-5}$ W with quite acceptable results (see also Ch. 2124).

In the more recent computations an additional program called "Gradient" has been used. From fed-in values of atmospheric surface pressures,[1] read manually from weather maps in 45 grid points of NW Europe (Fig. 2125: 1), the program computes N- and E-components of atmospheric pressure gradients in mb/m at every central point of the actual sections. The components are called DPNJ and DPEJ respectively and $\sqrt{(\mathrm{DPNJ})^2 + (\mathrm{DPEJ})^2}$ is called ROT. The absolute value of the geostrophic wind W_G will be $W_G = 0.1 \cdot \mathrm{ROT}/(q_a \cdot f)$, where q_a is the density of air (a value of 0.00125 tons/m^3 was used throughout). The absolute value of the wind stress τ is then $\tau = K_2 \cdot (0.7)^2 \cdot \cdot W_G^2$. The angle between the geostrophic wind and the N-direction is designated ψ (see Fig. 2125: 2): $\cos \psi = \mathrm{DPEJ/ROT}$ and $\sin \psi = -\mathrm{DPNJ/ROT}$ The N- and E-component of the wind stress may be written

[1] Kindly made available by the Swedish Meteorological and Hydrological Institute.

$$\tau_N = \tau \cos(\psi - 15)$$
$$\tau_E = \tau \sin(\psi - 15)$$

And finally the longitudinal component:

$$\tau_x = \tau_N \cos \varphi - \tau_E \sin \varphi$$

where φ is the angle between the N-direction and the x-direction of the section (see Fig. 2125: 2).

The atmospheric pressure gradient $\frac{\partial p_a}{\partial x}$ (called DPX) will be:

$$\mathrm{DPX} = \mathrm{DPNJ} \cos \varphi - \mathrm{DPEJ} \sin \varphi.$$

2126. The Bottom Stress Term

The derivation presented in Ch. 211, first made by PLATZMAN (1963), is an attempt to take the bottom stress $(\nu\frac{\partial w}{\partial z})_{-H}$ into consideration in a more correct way than is usually done, but as pointed out in Ch. 211, the Platzman method has not yet been applied. In accordance with experience it is doubtful whether this method will be of importance on the first approximation level. So far, therefore, only the simpler formulae

$$\tau_B = \frac{\varrho}{H^2} U = \frac{R}{H} U = \beta U$$

have been used (one at a time).

While others working in this field generally used the first one of these formulae, the one with the coefficient ϱ (e.g. FISCHER 1959, with $\varrho = 0.025$ m^2/s), the present author also experimented with β and R. Most of the computations show that rather similar results can be obtained with either of the formulae. In SVANSSON (1968) the factor gave only slightly better results than the other two and therefore β was mostly used in the work presented in this paper. The results of the computation of the tidal component M2 (Ch. 261) showed, however, much more realistic phases with ϱ than the trial with β, so that in the future if there is any doubt about which formula to use, the one with ϱ should be preferred.

The factor ϱ can be derived as $\frac{\nu \pi^2}{4 H^2}$ (integration of

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2}; \; u_{-H} = 0, \quad \left(\frac{\partial u}{\partial z}\right)_{z=0} = 0)$$

LASKA (1966), using this formula, allows ν to vary:

$$\nu = 0.54 \cdot W \cdot H \text{ for } H < 70 \text{ m}$$
$$\nu = 47 \cdot \frac{W^2}{12.3} \text{ for } H > 70 \text{ m}$$

W being the wind velocity.

HANSEN (1956) has a square law:

$$\tau_B = R' \cdot u \cdot |u|$$

R' mostly being $3 \cdot 10^{-3}$. SVANSSON (1968) also tried this formula, but obtained the best result for $R' = 15 \cdot 10^{-3}$.

As will be pointed out later in this paper, due to the simplification of the Belt Sea, the implicit model was not run to test various friction coefficients in detail like in SVANSSON (1968). Nevertheless we see that the results of Kemi shown in Ch. 262 have been derived for a smaller coefficient ($\varrho = 0.0175$ m^2/s) than would have been the case with the explicit model ($\varrho = 0.04$ m^2/s). One difference between the explicit and the implicit models is the smoothing term which is present in the former model. Instead of

$$u_j^{n+1} = u_j^n + \Delta t\,(\text{— — —})$$

is written

$$u_j^{n+1} = \alpha\, u_j^n + \frac{1-\alpha}{2}\,(u_{j+1}^n + u_{j-1}^n) + \Delta t\,(\text{— — —})$$

But the smoothing term can also be interpreted as a term of horizontal diffusion. We can write

$$u_j^{n+1} = u_j^n - \frac{(\Delta x)^2\,(1-\alpha)}{2}\left(\frac{u_{j+1}^n - 2\,u_j^n + u_{j-1}^n}{(\Delta x)^2}\right) + \Delta t\,(\text{— — —})$$

(cf. Ch. 231)

with a diffusion coefficient $K = \dfrac{(\Delta x)^2\,(1-\alpha)}{2}$

or with $\alpha = 0.75$, constantly used,

$$K = 0.125 \cdot (\Delta x)^2$$

A diffusion term $K \dfrac{\partial^2 u}{\partial x^2}$ has a dissipation of the order of $K u \dfrac{\partial^2 u}{\partial x^2}$ or

$$K\left[\frac{\partial}{\partial x}\left(u \frac{\partial u}{\partial x}\right) - \left(\frac{\partial u}{\partial x}\right)^2\right]$$

As the first term contributes only at the boundaries it will disappear because smoothing was not applied at the boundaries. What remains is a term which is always negative. One would think that the smoothing then required a smaller friction coefficient instead of a larger. As, however, there was smoothing also in the equation of continuity the problem may be more complicated (cf. FISCHER 1965).

Comparing the friction coefficient required in (1) the tidal component M2-computation (Ch. 261) with oscillations of the periodicity half a day, in (2) the computation of the tidal component Mf with oscillations of half a month (Ch. 263) and (3) the variations from month to month (Ch. 266), we see that it decreases from resp. $2.0 \cdot 10^{-5} s^{-1}$ to $0.5 \cdot 10^{-5} s^{-1}$ and finally zero.

It would, however, be very tempting to include the term $K\frac{\partial^2 u}{\partial x^2}$ which has been shown by LAMB (1932) to take care of such problems: short waves are dissipated much quicker than long ones. Actually there have been attempts to substitute the conventional friction term by this term (SVANSSON 1970). The Ks used in the salinity model (see Ch. 22) were taken as a base, all of them then multiplied by a suitable constant. It was possible to obtain results similar to those presented in Ch. 267 with the multiplication of 10^4 (in SVANSSON (1970) there is such a mistake that the K:s were multiplied by the section areas A), but when the tidal component M2 was run, there was hardly any phase differences and the experiment was stopped. A combination of the old friction term with a diffusion term is probably a better alternative, but that will mean laborious testwork.

2127. The Balance of Terms in the Perpendicular Direction

We have in this paper in the tidal computations derived two additional levels at the end points of the sections by adding or subtracting

$$\Delta h = -\frac{b\,f\,U}{2g\,A}$$

derived from the third equation of the system 31.

But if we applied the formula also to windcases we should include the perpendicular wind stress:

$$\Delta h = \frac{b}{2g\,A}(b\,\tau_y - f\,U)$$

Above (Ch. 2121) was discussed the possible inclusion of the centrifugal acceleration term.

22. The Salinity Model

This model was made and published by BOICOURT (1969). It is based on a salt continuity equation in which the seaward advection ZS (Z=transport of fresh water, S=salinity) is balanced by the turbulent diffusion $KA\frac{\partial S}{\partial x}$ towards the head of the canal. The final timedependent equation will be

$$\frac{\partial\,(AS)}{\partial t} = -\frac{\partial\,(ZS)}{\partial x} + \frac{\partial}{\partial x}\left(AK\frac{\partial S}{\partial x}\right);$$

In Chapter 23 is shown the implicit scheme of solving the equation. There is no limit of the timestep, but it is quite clear that the non-linear term with ZS may be sensitive to the timestep.

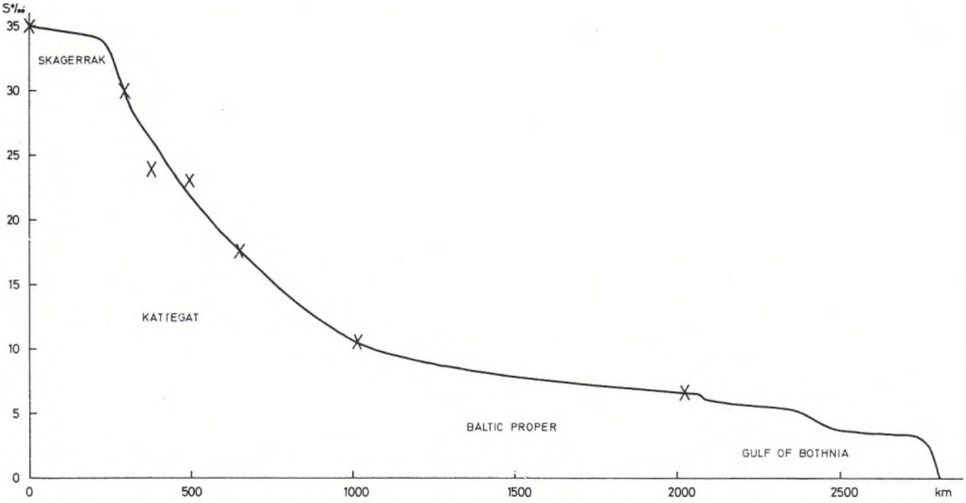


Fig. 22: 1. Long term means from surface to bottom of the salinities in the canal system Skagerrak—Belt Sea—Baltic.

In the derivation of the coefficient K it is assumed that they can be derived from the steady state equation $ZS = AK \frac{\partial S}{\partial x}$ or numerically

$$(AK)_j = \frac{(ZS)_j\ (\Delta x_{j-1} + \Delta x_j)}{S_{j+1} - S_{j-1}}$$

Boicourt (1969) also allowed dependence upon the fresh water supply Z (varying between 50 m^3/s and 15000 m^3/s) and found that K varied with Z and most at the head of the estuary. As the variation of the fresh water supply to the Baltic probably is much less than in Boicourt's case (see e.g. Ch. 265) it was assumed unnecessary to have a Z-dependence of the Ks particularly on the 1st approximation level. The salinities Sj (mean values from surface to bottom) were derived from mean values in Anon. (1933) for the Danish light-vessels and Granquist (1938) for the Northern Baltic (cf. open sea stations indexed F in the Figures 1: 2—1: 5), while the remaining salinities in the Baltic proper were interpolated (see Fig. 22: 1 for all the waters except the Gulf of Finland).

The K-vales derived are shown in Table 22: 1. Also the products (AK) j are shown; these are usually more smooth than the K-values.

A comparison between the formula above for K and the numerical scheme in Ch. 231 shows that the K:s go into the numerical equations a little differently. Therefore a computation was made with the model to allow an adjustment to new salinities. After 100 timesteps of 1-year length a new steady state was reached approximately 0.5—1.0 ‰ higher. It would probably have

Table 22: 1.

	Canal 1		Canal 2		Canal 3	
	K m^2/s	$AK \times 10^6$ m^4/s	K m^2/s	$AK \times 10^6$ m^4/s	K m^2/s	$AK \times 10^6$ m^4/s
0	0	0	0	0	—	—
1	82.4	223	471	278	4299	150900
2	79.1	415	406	797	2552	41900
3	573	2880	376	1531	3091	10820
4	641	5114	637	2503	4281	8459
5	737	6187	1677	3605	5005	10210
6	2243	10586	1645	5001	5379	11350
7	4600	12419	(2280)	8276	5485	12780
8	4714	6490	—	—	3804	11450
9	2480	2443	—	—	4483	11700
10	482	1336	—	—	5236	12619
11	213	1174	—	—	6749	11946
12	147	1490	—	—	9424	12440
13	148	2197	—	—	16410	13750
14	231	3380	—	—	13880	12660
15	529	7979	—	—	10590	10240
16	1571	22628	—	—	34210	12897
17	1732	20090	—	—	38395	12056
18	2279	9174	—	—	31580	11590
19	762	3471	—	—	62230	15620
20	990	3847	—	—	51690	14370
21	(1000)	4780	—	—	22180	12110
22	—	—	—	—	56380	11670
23	—	—	—	—	48256	12643
24	—	—	—	—	16422	10625
25	—	—	—	—	28490	10200
26	—	—	—	—	37020	12180
27	—	—	—	—	9005	12670
28	—	—	—	—	12810	12550
29	—	—	—	—	17720	13130
30	—	—	—	—	33600	13710
31	—	—	—	—	22230	16140
32	—	—	—	—	10306	25560
33	—	—	—	—	5089	26439
34	—	—	—	—	4536	34860
35	—	—	—	—	2122	34460
36	—	—	—	—	1344	27020
37	—	—	—	—	1760	35540
38	—	—	—	—	(2000)	35900

been possible to arrive near the original values of S by multiplying all the K:s by a constant factor but such a procedure would have been more time-consuming. The new S-values were used in all computations with the salinity model.

Concerning the interpretation of the coefficient or horizontal eddy diffusion, K, we cite from BOICOURT (1969): "Its physical interpretation is very elusive. The corresponding coefficient in the three-dimensional equation can be spoken of as representing non-advective fluxes over the averaging period which are due to deviation terms that intuitively are not difficult to relate to a turbulent flow. The one-dimensional K, however, obviously incorporates the effects of advective transport processes in addition to turbulent diffusion. The reason for the introduction of K is that it allows one to relate such an 'effective diffusion' term to external parameters of the estuary more readily than do the averaged crossproducts of the deviation term.

When a transport U, computed by means of a sea level equation, was added to the fresh water supply Z, the interpretation of K may be even more elusive, particularly if K is not adjusted to the averaging period. Such an adjustment was not made in the computations presented in Chapters 265—267, but should be considered when leaving the first approximation level.

23. The Numerical Schemes for the Implicit Models

231. The Numerical Scheme for the Salinity Model

$$A_j \frac{S_j^{n+1}-S_j^n}{\Delta t}=\frac{Z_{j-1}^n S_{j-1}^n+Z_{j-1}^{n+1}S_{j-1}^{n+1}-Z_{j+1}^n S_{j+1}^n-Z_{j+1}^{n+1}S_{j+1}^{n+1}}{2\,(\Delta x_{j-1}+\Delta x_j)}+$$

$$+\frac{(AK)_{j+1/2}\dfrac{S_{j+1}^n+S_{j+1}^{n+1}-S_j^n-S_j^{n+1}}{2\Delta x_j}-(AK)_{j-1/2}\dfrac{S_j^n+S_j^{n+1}-S_{j-1}^n-S_{j-1}^{n+1}}{2\Delta x_{j-1}}}{\dfrac{\Delta x_{j-1}+\Delta x_j}{2}};$$

Reference is made to Figures 26: 1 and 232: 1.

The numerical calculations are made according to RICHTMYER and MORTON (1967, p. 198 ff). Assuming S (1,0) = S (2,0) = 0 and S (3,0) = 35 ‰, the factors E_j and F_j are computed successively from the beginning of the three canals respectively to the branching point. Now with the assumption that the salinity is the same in this point in all three canals and further $\Sigma\, AK \frac{\partial S}{\partial x} = 0$ at the branching point this salinity can be computed. Thereafter it is possible to compute all the other salinities backwards (in Russian texts referred to as the PROGANKA method). The trunction errors were not investigated.

232. The Numerical Scheme for the Sea Level Model

$$\frac{U_j^{n+1}-U_j^n}{\Delta t}=-g\,A_j\frac{h_{j+1/2}^{n+1}+h_{j+1/2}^n-h_{j-1/2}^{n+1}-h_{j-1/2}^n}{\Delta x_{j-1}+\Delta x_j}-A_j\left(\frac{\partial p_a}{\partial x}\right)^{n+1/2}$$

$$+g\,A_j\frac{\bar{h}_{j+1}^{n+1/2}-\bar{h}_{j-1}^{n+1/2}}{\Delta x_{j-1}+\Delta x_j}+b_j\,\tau_{xj}^{n+1/2}-\beta_j\frac{U_j^{n+1}+U_j^n}{2};$$

$$h_{j-1/2}^{n+1}=h_{j-1/2}^n-\frac{\Delta t}{\Delta x_{j-1}(b_{j-1}+b_j)}\left(U_j^{n+1}-U_{j-1}^{n+1}+U_j^n-U_{j-1}^n\right);$$

See also Fig. 232: 1.

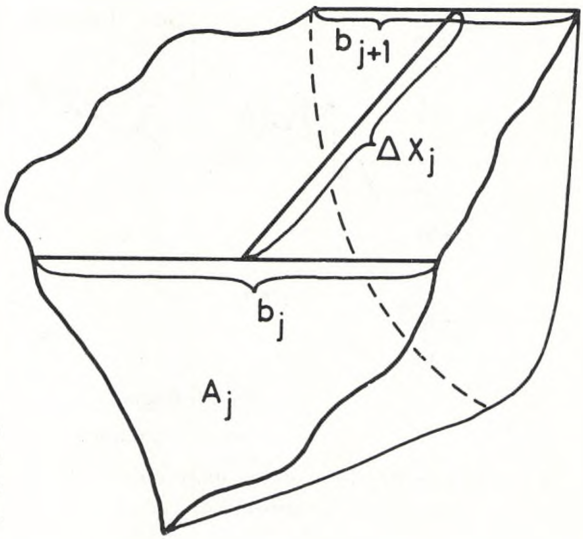


Fig. 232: 1. A figure showing the positions of the cross section area A, the width b and the distance Δx_j between the sections j and j+1.

The expression of $h^{n+1}_{j-1/2}$ as well as one of $h^{n+1}_{j+1/2}$ are introduced in the first equation. Again the PROGANKA method is applied to this equation in U: $U(1,0) = U(2,0) = 0$; at section (3,0) we feed in $\frac{\partial h}{\partial t}$ or $\frac{h^{n+1}_{1/2} - h^{n}_{1/2}}{\Delta t}$

$$\text{As } b\frac{\partial h}{\partial t} = \frac{\partial U}{\partial x} \quad \text{we can write}$$

$$U\,(3,0)^{\,n+1} = U\,(3,1)^{\,n+1} + 2\,b_{1/2}\,\Delta x_0 \frac{h^{\,n+1}_{1/2} - h^{\,n}_{1/2}}{\Delta t} - U\,(3,0)^{n} + U\,(3,1)^{\,n}\,;$$

Now we can start the computation of E_j and F_j. When we arrive at the branching point it is assumed that the level h is the same in all three canals and further that $\Sigma\, U = O$. The three Us (and h) at the branching point can be computed, and thereafter all the remaining Us (and hs from the continuity equation).

24. The Reference Levels Used

Following the advice of Dr. E. LISITZIN (pers. comm.) a mean value of the annual levels during 1931—1960 was computed for the Swedish (and the Finnish) sea level recording stations. Land uplift coefficients were taken from ROSSITER (1967):

Ratan:	413.31 cm	+0.754	(Year—1945)
Draghällan:	384.06	+0.774	„
Björn:	404.35	+0.590	„
Stockholm:	336.95	+0.397	„
Landsort:	462.03	+0.302	„
Kungsholmsfort			„
(Karlskrona):	419.70	+0.032	„

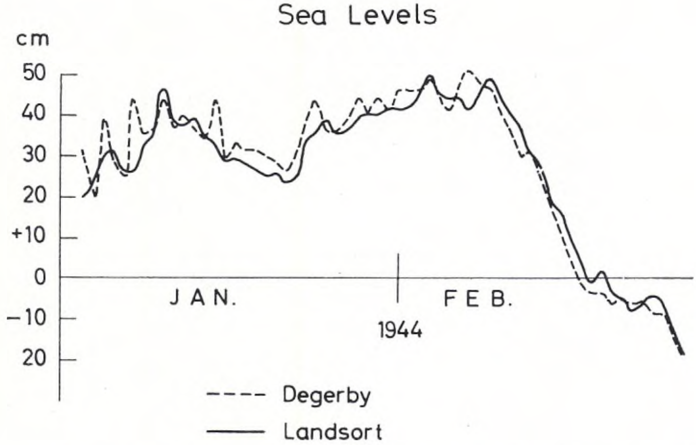


Fig. 24: 1. A comparison of the daily means of the sea levels recorded at Degerby (Fig. 1: 4) and Landsort (Fig. 1: 2) during January and February 1944.

Ystad:	399.52	−0.086	(Year—1945)
Varberg:	466.73	+0.058	,,
Smögen:	365.62	+0.237	,,
Kemi:	175.6	−0.674	,,
Jacobstad:	168.2	−0.783	,,
Kaskö:	173.6	−0.754	,,
Raumo:	177.2	−0.603	,,
Degerby:	190	−0.409	,,

Note that the Swedish SLs are measured in relation to an index situated above the sea. For the Danish stations the mean sea level annotated on the forms of observations was used.

For the remaining sea level stations used (Leningrad and Swinoujscie) a reference level was determined under the assumption that the mean value during August 1—12, 1964, was the same as the one of a neighbouring Swedish or Finnish station. — For the period August, 1—12, 1964, the so called anemobaric effect was applied, —7.5 cm at Kemi, —8.5 cm at Leningrad and —1.2 cm at Ystad—Swinoujscie, because the mean sea level is higher in the North than in the South due to a higher frequency of SW-winds (Witting 1918).

In this paper the sea level at Landsort has been used to represent the whole Baltic on a few occasions. The reason for using Landsort and not Degerby, as suggested by Hela (1944), is partly because the series extend further back in time at Landsort, partly because it was more practical to use Swedish level data, the difference between Landsort and Degerby being small; Fig. 24: 1 shows level variations (daily means) during January and February 1944. While the level at Ystad and Kemi (not included in the figure) usually fluctuate a great deal, often with opposite phases, the levels at Degerby and Landsort are nearly free from these variations of shorter periods and furthermore follow each other closely.

25. Results from the Explicit Model

In a work published more than ten years ago the present author treated the seas around Sweden as one long canal (Svansson 1959). Numerical computations by means of an explicit model were made of the SL and transport variations during one week of December 1932. In a later work (Svansson 1966) the study was focused on two parts of the seas, one of them was the Gulf of Bothnia (meteorological sea level effects during December 1958 and tides) and the second one the Belt Sea (tides only). In Svansson (1968) the meteorological sea level effect case in 1958 in the Gulf of Bothnia was further studied.

251. The Meteorological Sea Level Effects in Decmber 1932

This was the first attempt by the present author to numerically compute levels and transports induced by the wind and the atmospheric pressure gradient (Svansson 1959). For every 6^{th} hour 7 (i.e. not for every section) wind stresses and 7 atmospheric pressure gradient values were fed in. The windstress τ was computed to be $3.2 \times 10^{-6}\, W^2$, where W is the wind speed, and a friction coefficient $R = 0.2 \times 10^{-3}$m/s was used (Ch. 2126). The results of the sea levels of Ystad were astonishingly positive; in the Gulf of Bothnia the damping was clearly too small and the results in the Gulf of Finland were directly negative (the latter gulf was influenced by the main canal through the boundary condition but not vice versa).

252. The Meteorological Sea Level Effects in October 1958

To improve the numerical model a smaller part, the Gulf of Bothnia, was chosen as a well defined canal and also a more recent wind surge case was used i.e., that of October 1958 with atmospheric pressure data available every 3rd hour. Furthermore these data were transformed to wind stresses and pressure gradients at every section separately.

In the case of 1932 (Ch. 251) the friction coefficient was derived from a β-value (cf. Ch. 2126) of $0.5 \times 10^{-5} s^{-1}$ published by Neumann (1941). Now it was attempted, however, to determine a friction coefficient by calculating by the same numerical model the tidal component K1 (period 23.97 hours). If the section Ratan—Jacobstad is used as an adjustment section, a value= $5 \times 10^{-5} s^{-1}$ gave the best results (Svansson 1966). Then with this β-value different wind stress coefficients were tested and for $\tau = 1.75 \times 10^{-6} W^2$ a rather good result was achieved (see Fig. 252: 1 reproduced from Svansson 1966).

Compared with the β-value of NEUMANN (1941) the one found by SVANSSON (1966) is rather high. Still higher friction coefficients were found appropriate in an investigation published in SVANSSON (1968): "For the first five days of the period in October 1958 the mean square deviation (MSD) between computed and measured SLs has been determined for various combinations of the bottom friction constant (β, R or ϱ) and the wind stress constant (K_2 in $\tau = K_2 W^2$). A minimum value of MSD is supposed to indicate the wind stress searched for". Fig. 252: 2 reproduced from SVANSSON (1968) shows the result of the investigation with the friction coefficient ϱ. For the section Draghällan—Kaskö (h6, mean depth 80 m) K_2 is approximately 3.5×10^{-6}, for the section Jacobstad—Ratan (h15, mean depth 40 m, in the vicinity there are depths of 10 m) 2.1×10^{-6} and for Kemi (h23, mean depth 10 m) $K_2 = 2.8 \times 10^{-6}$. The result is ambiguous. It would be tempting to accept the large value of 3.5×10^{-6} as the most correct K_2-value because the results h6 seem very little influenced by the choice of friction coefficient. On the other hand if we choose a smaller value of K_2 we much better satisfy the shallow parts of the Baltic which usually have the higher sea levels. In Ch. 262 therefore a $K_2 = 2.0 \times 10^{-6}$ has been used.

It should be borne in mind that there is always in this computation with the explicit model a smoothing coefficient 0.75 (Ch. 2126), which probably changes the friction coefficients.

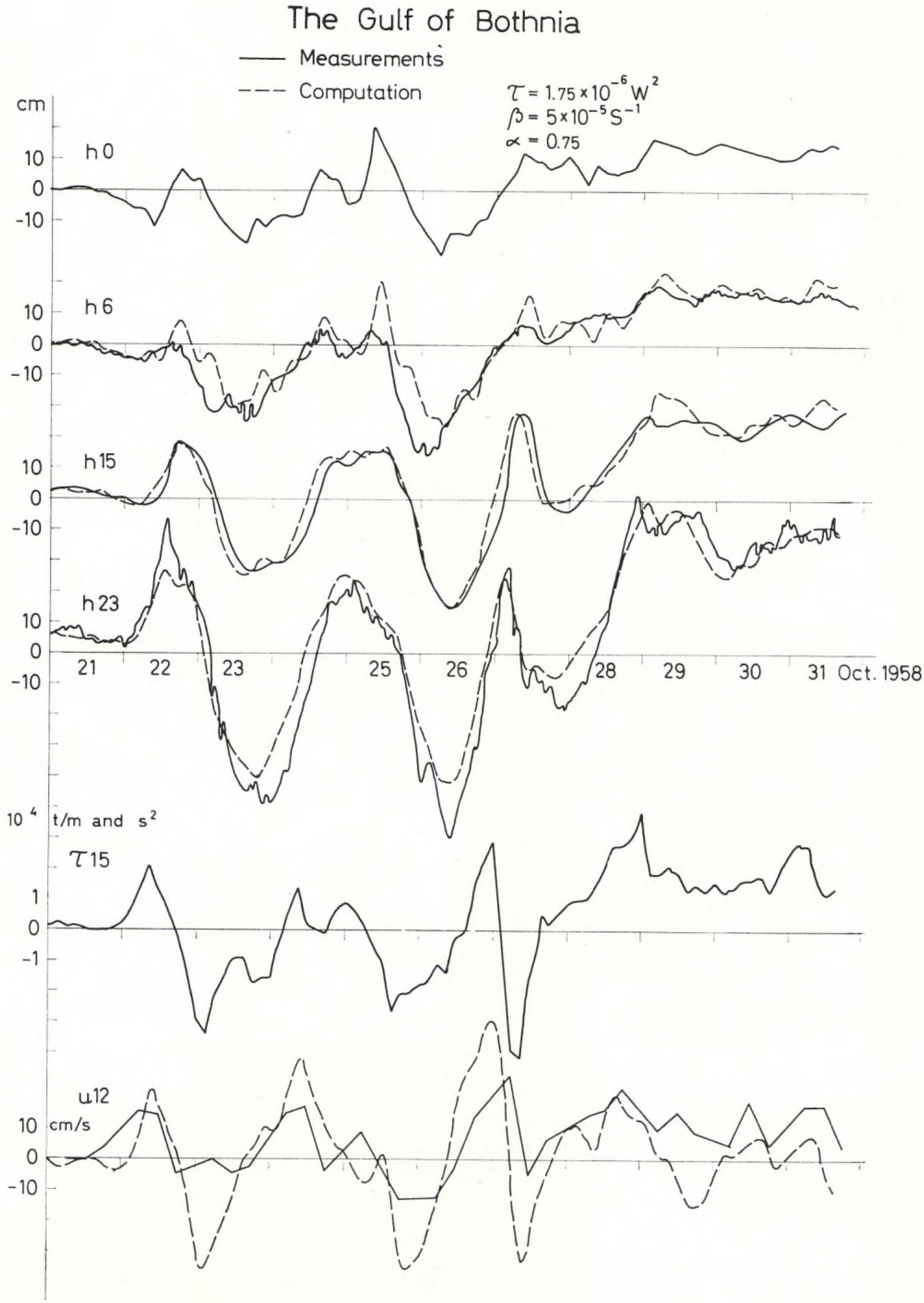
On this occasion the program to convert atmospheric pressures in a grid over Northwestern Europe to atmospheric pressure gradients in the central points of the sections was introduced (Ch. 2125).

253. Computations of Tides in the Kattegat and the Belt Sea

Calculations of the tidal component M2 (period 12.42 hours) in a system of canals consisting of the Kattegat and the Belt Sea were made by means of the explicit model. Practically the same sectioning as shown in Fig. 1: 1 was used. Boundary values of known SLs were applied at the sections 0: 12 and 5: 2. The boundary conditions at the junctions of the canals were the same as those described in Ch. 254. "Different values of the friction coefficient were tested, that value was chosen which made the amplitude at the section Slipshavn—Korsör (1: 8) coincide with the mean value 11.4 cm. This

---

Fig. 252: 1. Comparison between measured and computed sea levels for a meteorological sea level effect case in October 1958. As a boundary condition was taken the mean value of the sea levels of Björn and Raumo (h0, section 10: 09 in Fig. 1: 4) and comparisons are made at h6 (10: 13 in Fig. 1: 4), h15 (10: 24 in Fig. 1: 5) and h23 (10: 32 in Fig. 1: 5). The wind stress variations $\tau 15$ at section 10: 24 are shown as well as a comparison between computed velocities u12 at section 10: 19 and measured velocities at the Finnish L/V Snipan.



The Gulf of Bothnia
Measurements
Computation
$\tau = 1.75 \times 10^{-6} W^2$
$\beta = 5 \times 10^{-5} S^{-1}$
$\alpha = 0.75$
cm
h 0
h 6
h 15
h 23
10
0
-10
21
22
23
25
26
28
29
30
31 Oct. 1958
10^4 t/m and s^2
τ 15
1
0
-1
u12
10 cm/s
0
-10

Fig. 252: 1.

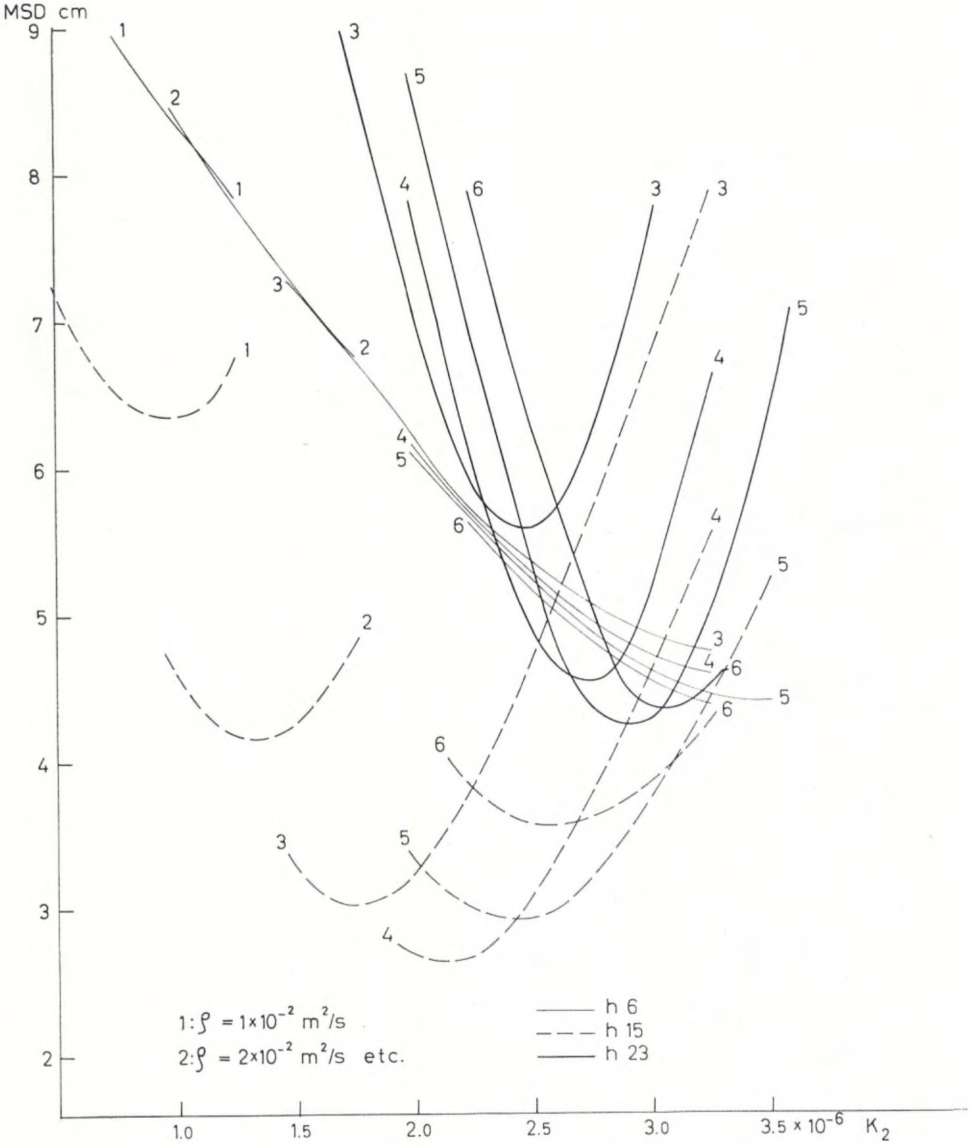


Fig. 252: 2. The mean square deviation (MSD) between computed and measured sea levels during the first five days in October 1958 at h6 (10: 13 in Fig. 1: 4), h15 (10: 24 in Fig. 1: 5) and h23 (10: 32 in Fig. 1: 5).

occurred for $\beta=3.0\cdot10^{-5}s^{-1}$ and also for $\varrho=0.01$ m^2/s" (SVANSSON 1966). The fact that a smaller value of friction could be used here than in the Gulf of Bothnia (Ch. 252) may indicate that the Gulf of Bothnia with its more homogeneous water has a higher friction.

254. Plans of the Use of an Explicit Model for All the Seas Around Sweden

An explicit model with all the "facilities" of the earlier explicit models is under construction. As can be seen in Fig. 1: 1—5 the area is subdivided into 12 canals. At nearly all junctions between the canals the boundary conditions are (1) the same SL and (2) a continuity in water transport. The friction coefficients β, R or ϱ can be applied but alternatively also the friction of type Platzman (see Ch. 211). MSD-tests will be possible to carry out at approximately ten sections in the general case as well as in a tidal computation. The atmospheric pressure gradients are computed from the special "gradient"-program mentioned above (Ch. 2125).

26. Results from the Implicit Models

While the explicit model described in Ch. 25 does not permit time-steps longer than approximately 15 minutes, an implicit model offers quite different possibilities (Richtmyer and Morton 1967), but as it would probably be rather laborious to solve the system of equations created when all the canals were included as planned in Ch. 254, only a system of three canals (the Gulf of Bothnia, the Gulf of Finland and as the third canal the remaining parts of the area) has been treated. In this case a method described in Richtmyer and Morton (1967) can be used to solve the system of equations. As the reduction of canals described above probably is rather a great simplification, the results in this chapter should be considered to be a 1st approximation.

Fig. 26: 1 shows the three canals with sections. The subdivision was originally made with the idea that the sectioning should be dense where the horizontal salinity gradients are large and vice versa. During the computation it has been clear that the subdivision could have been made in other ways to better fit the requirements, but once settled it was better to stick to the original scheme.

First the tidal component M2 is studied (Ch. 261). That friction coefficient which makes the results fit best with observations is then used in a computation of the variations during the international cooperation in August 1964 (Ch. 262). The fortnightly tidal component Mf is studied likewise (Ch. 263). Here a smaller friction coefficient is required. This coefficient is also used in a computation of the variation from day to day during July—November 1964 (Chapters 264 and 267); in this case the variations are only generated at the mouth of the system and not at all by winds and atmospheric pressure gradients.

It should be borne in mind that the method of first computing the friction coefficients for the "pure" tides, which are small in the Baltic, and then

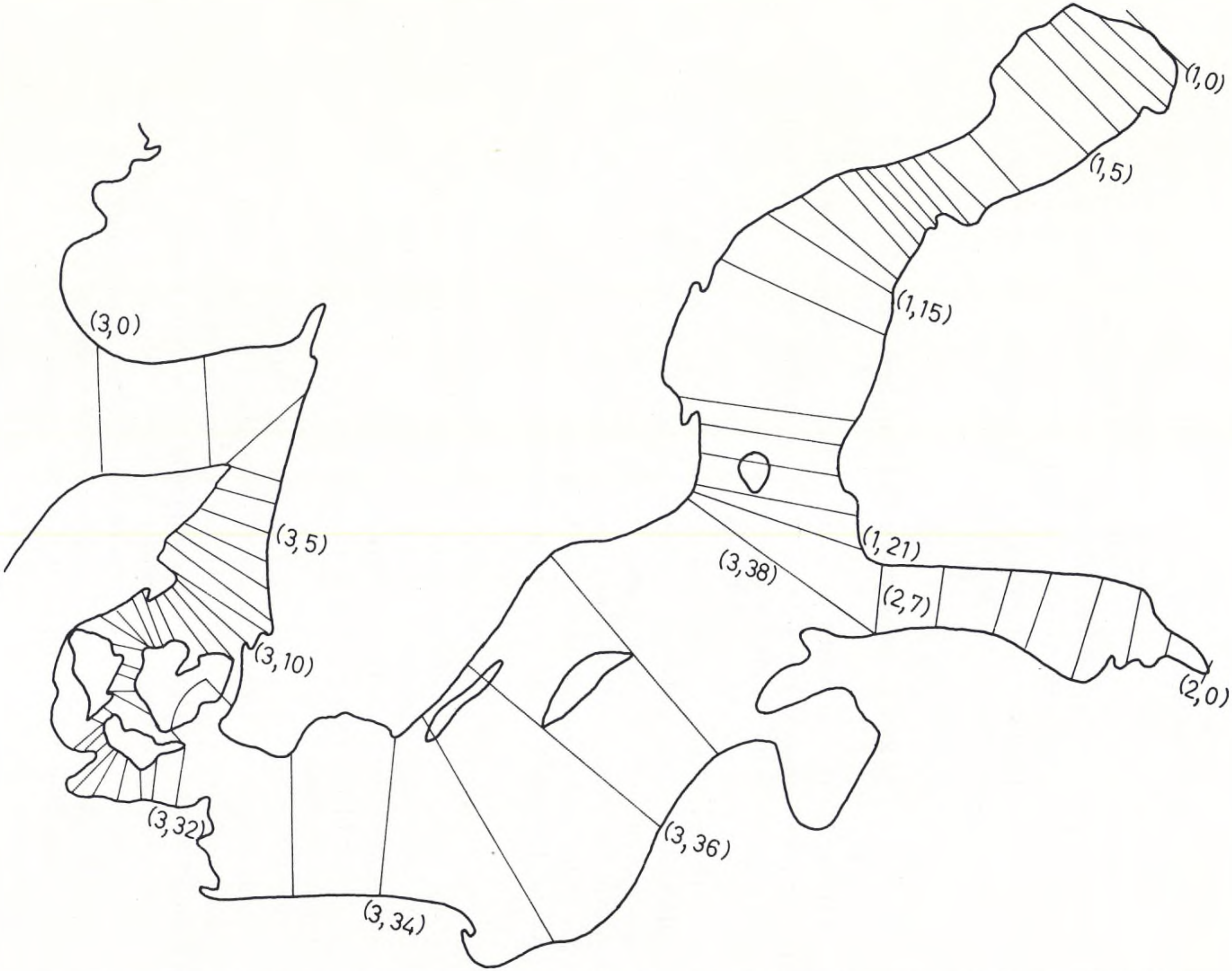


Fig. 26: 1. The sectioning used for the implicit models. The cross sections A_j can be derived from Table 22: 1.

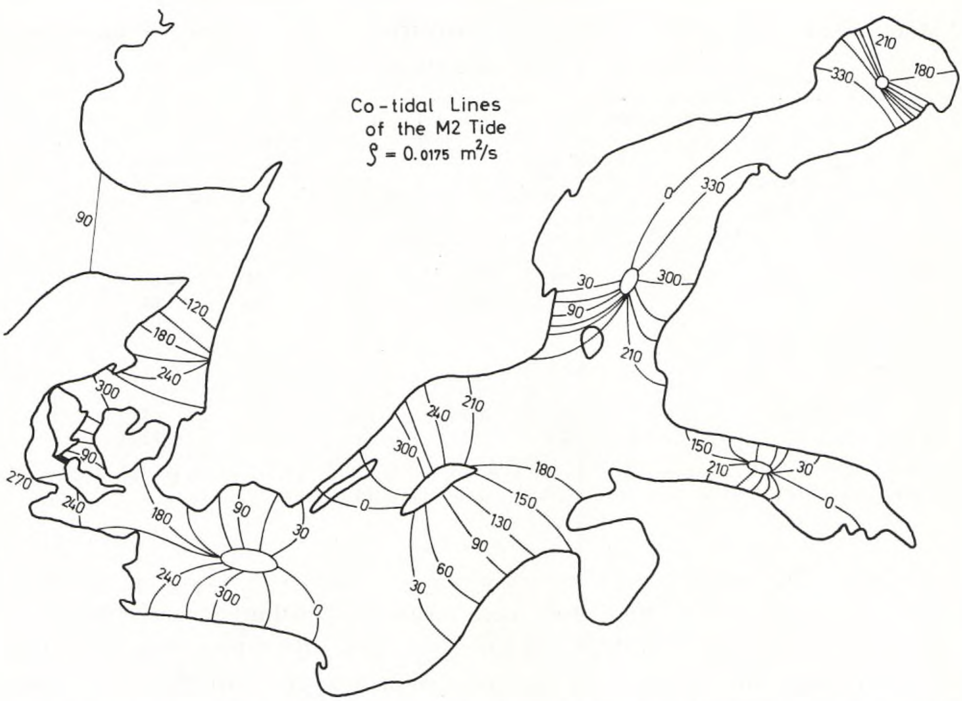


Fig. 261: 1. The distribution of phases (degrees) resulting from a computation of the tidal component M2 (period 12.42 hours).

using these coefficients for the computations of meteorological sea level effects which are at least one order of magnitude larger may be dangerous. One disturbing factor for small amplitude tides may be the tidal movements of the solid earth. This is one reason for the alternative use of the MSD method of computing the friction coefficient as described in Ch. 252.

Not only an implicit sea level model but also an implicit model to describe salinity variations have been presented (Ch. 22). This latter model has been used both in an attempt to describe long term variations as a function of the fresh water supply (Ch. 265) as in combination with the level model to compute short term salinity variations (Chapters 266 and 267).

261. Amplitudes and Phases of the Tidal Component M2

The investigation of a tidal component was started in order to obtain a friction coefficient to be used in the general case of August 1964 (Ch. 262). The tidal acceleration term was included in the equation of motion and as a boundary condition the harmonic values of Smögen (0.0956 m, 90°) were used. The validity of using the value of Smögen will be discussed below

Table 261: 1. Comparison between "measured"[1] and computed harmonic constants of the tidal component M2.

	Amplitude cm				Phase degrees (Greenwich)			
	"Measured"	Computed			"Measured"	Computed		
		$\beta=2.0\times 10^{-5}\,s^{-1}$	$R=5.0\times 10^{-4}$m/s	$\varrho=1.75\times 10^{-2}m^2/s$		$\beta=2.0\times 10^{-5}$	$R=5.0\times 10^{-4}$	$\varrho=1.75\times 10^{-2}$
Kemi	*0.25*	0.25	0.38	0.36	*181*	224	195	185
Björn	*0.51*	0.12	0.24	0.37	*42*	313	33	13
Kronstad ..	*1.71*	1.53	2.06	1.78	*276*	44	24	12
Helsinki ...	*0.31*	0.35	0.71	0.68	*121*	184	159	138
Landsort ..	*0.86*	0.75	1.04	0.87	*144*	269	223	196
Karlskrona .	*0.62*	2.2	2.4	2.0	*88*	62	60	55
Korsör	*10.7*	13.4	7.0	2.7	*348*	68	46	36
Varberg	*3.8*	4.7	2.7	2.1	*205*	294	292	271

[1] The "measured" values have been determined by harmonic analysis of sea level records as published by CRONE (1906), WITTING (1911) and LISITZIN (1943).

(Ch. 262). The timestep was one lunar hour (3726 seconds) and approximately 300 steps were run before the harmonic constants were computed. (A computation with 600 steps did not show any differences from the first run.) By using the geostrophic approximation also the two "beach" values were computed for every section (cf. Ch. 2127).

All three friction coefficients β, R and ϱ were tested roughly. For $\beta=3.5\times10^{-5}$ (cf. Ch. 253) the amplitudes at Korsör—Slipshavn (section 3, 19) were quite acceptable but most amplitudes in the Baltic were too small. A decrease to $\beta=2.0\times10^{-5}s^{-1}$ increased the amplitudes at Korsör—Slipshavn somewhat but gave resonable values for most parts of the Baltic.

As R and ϱ gave much smaller values at Korsör—Slipshavn it was first planned to use $\beta=2.0\times10^{-5}s^{-1}$ below in Ch. 262, but when the results were more carefully scrutinized it turned out that $\varrho=0.0175$ m^2/s gave much more reasonable results of phase distribution, see Fig. 261: 1.

The amphidromies are approximately the same as in MAGAARD and KRAUSS (1966) except for one in the vicinity of Gotland which is new. A more narrow spacing of the sections in the Baltic proper will be necessary to achieve a more reliable calculation. Table 261: 1 shows results for some different friction coefficients. The discrepancy for Varberg may not be completely due to the simplification of the Belt Sea, but a hint that the boundary values are not quite correct.

262. Variations of Sea-Levels During August 1—12, 1964, the Wind Stress and the Atmospheric Pressure Gradient Taken into Consideration

The program for converting atmospheric pressures in a grid over north-western Europe to atmospheric pressure gradients in the central points of

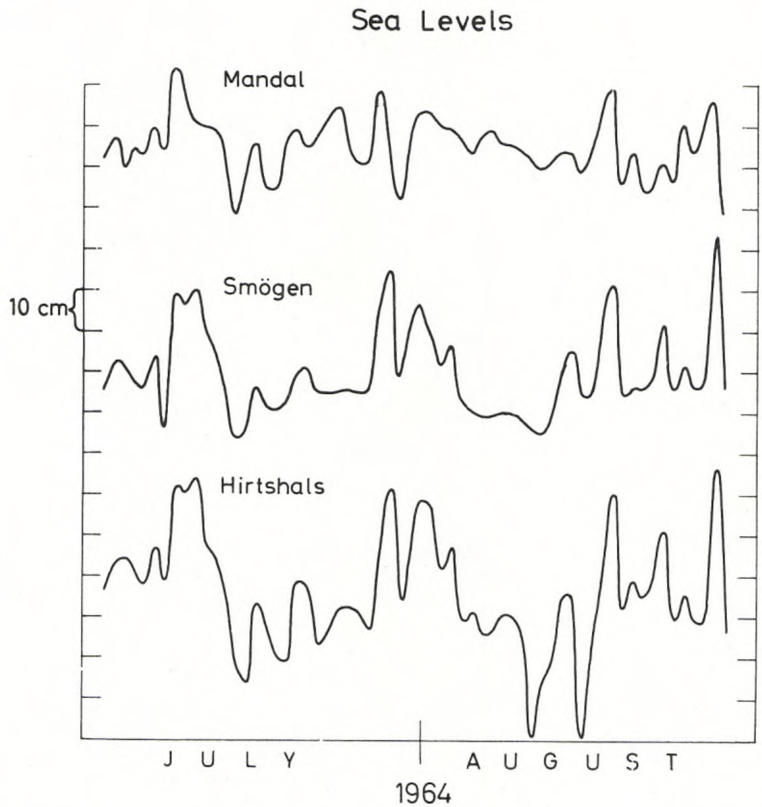


Fig. 262: 1. A comparison between the sea levels at three different places in the Skagerrak (Fig. 1: 1).

the sections (Ch. 2125) was run also for the period August 1—12, 1964 when there was an international cooperation in the Baltic. The gradient were used in the implicit program in one computation only with the friction coefficient $\varrho = 0.0175$ m²/s (cf. Ch. 261). The timestep was 3 hours (the same as the distance in time between atmospheric pressure observations, except on one occasion pr. 24 hours when there were 6 hours between the observations). As a boundary condition at section (3,0) was taken the SL at Smögen; there is no tidal acceleration term in the equations of motion. The wind stress was assumed to be $2.0 \cdot 10^{-6} W^2$ (see Ch. 252). The SLs at the beginning of the computation were approximately the correct ones.

The use of the SL measured at Smögen as a boundary condition is supposed to be a good provisional arrangement. The best choice would be a mean value of the SL at Mandal (Tregde) and Hanstholm, but unfortunately there are no SL records from the latter port. As Fig. 262: 1 shows the SLs at Hirtshals vary much more than those at Mandal and also more than those at Smögen.

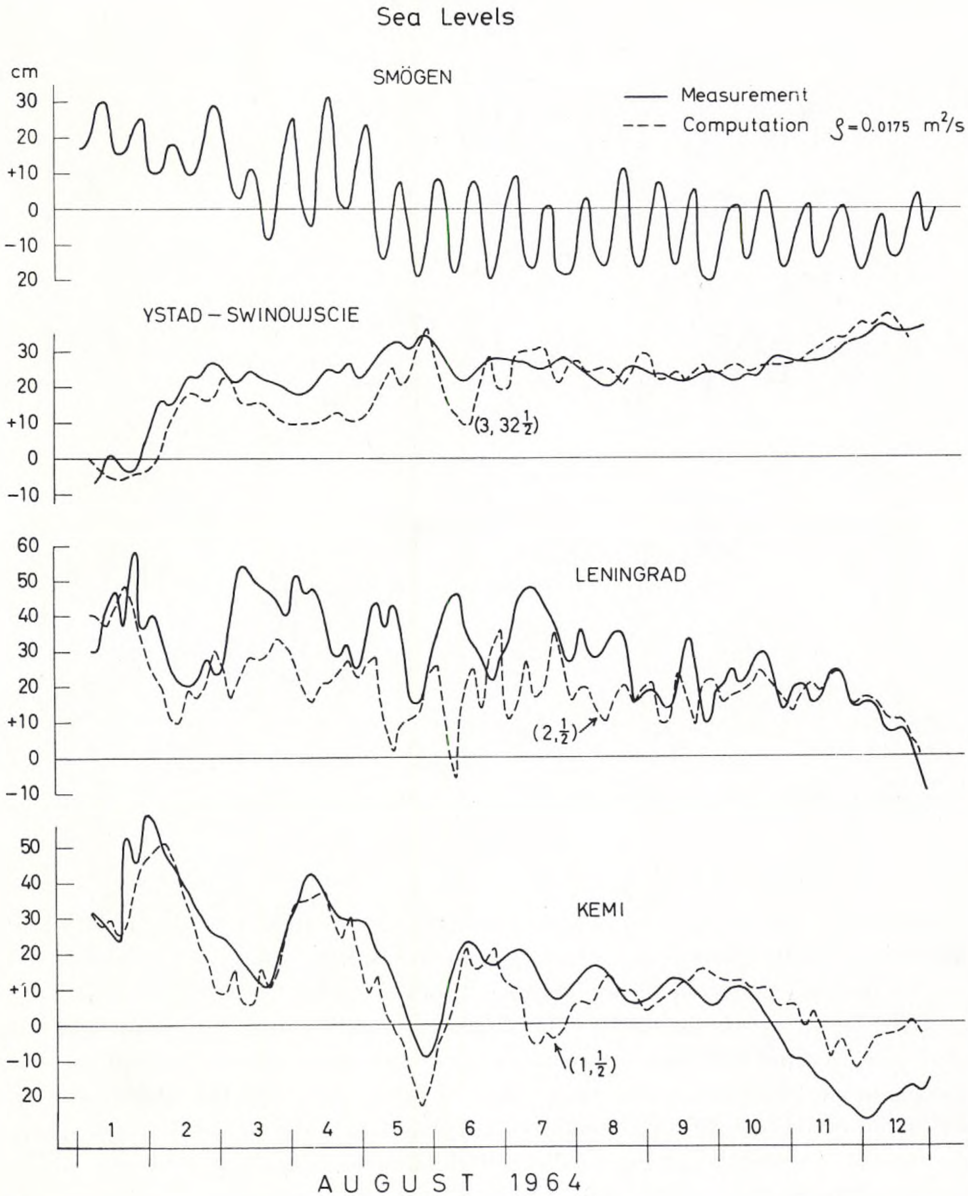


Fig. 262: 2. A comparison between measured and computed sea levels. As a boundary condition was taken the value of the sea level at Smögen (See Fig. 1: 1) and comparisons are made between the computations at three sections (see Fig. 26: 1) and the measured sea levels at Ystad—Swinoujscie (mean value of the two; for position see Fig. 1: 2), Leningrad (see Fig. 1: 3) and Kemi (see Fig. 1: 5). The wind stress and the atmospheric pressure gradient were taken into consideration.

Table 263: 1. Amplitudes in cm of Mf, Msf and M2.

	Mf	Msf	M2	
Smögen	3.52	1.66	9.56	Tidal Inst. Liverpool, pers. comm.
Kronstadt (Leningrad)	2.15	2.64	1.71	WITTING (1911)
Landsort	2.62	0.39	0.86	„
Libau (Liepājā)	0.33	2.36	0.09	„

The result is shown in Fig. 262: 2 for Kemi, Leningrad and Ystad—Swinoujscie. While the result for Kemi is quite acceptable and not bad for Ystad—Swinoujscie it is fairly negative for Leningrad. In the last mentioned port it looks as if the tidal amplitudes were too high while the characteristic oscillations of the Gulf of Finland with a period of approximately 24 hours (LISITZIN 1959) were suppressed. We may interpret this result in such a way that the friction coefficient should really have been smaller. The necessary damping of the boundary wave must then be operated through e.g. a more realistic description of the Belt Sea.

263. Amplitudes and Phases of the Fortnightly Tidal Component Mf

A computation with no friction revealed a characteristic period of 18 days (see Ch. 268). As it also became clear that the friction coefficient must be chosen smaller if studies are to be made of events where the characteristic period is larger (see Ch. 2126), there were reasons to study the tidal period Mf (13.66 days). There are not many ports for which this period has been investigated, see Table 263: 1 where also MSf (14.77 days) and, for comparison, M2 are

Table 263: 2. Harmonic constants of Mf, computed with $\beta=0.5\times10^{-5}s^{-1}$.

	Amplitude cm	*Phase degrees*
Kemi (1,0)	1.63	329
Draghällan (1,15R)	1.91	335
Leningrad (2,0)	2.05	336
Landsort (3,37L)	2.09	341
Ystad (3,32L)	2.60	3
(3,23R)	7.26	257
Varberg (3,6L)	2.05	215
Smögen	3.52	221

Section numbers refer to Fig. 26: 1.
R=Right "beach" point of the section
L=Left „ „ „ „ „

included. We see that while at Smögen the amplitudes of the fortnightly components are only one third of that of M2, in the Baltic they are usually double the amount of M2. This fact is probably due to the nearness in period between the characteristic oscillation and the tidal component.

With a timestep of 1/12 of the period 13.66 days, 120 steps were run before the harmonic constants were determined. Only two β:s were tested. For Kronstadt the best result, an amplitude of 2 cm, occurred for $\beta = 0.5 \times 10^{-5} s^{-1}$, while $\beta = 0.3 \times 10^{-5} s^{-1}$ gave a 3 cm amplitude. The former β was chosen for the subsequent computation (Ch. 264). Table 263:2 shows some harmonic constants from a computation with $\beta = 0.5 \times 10^{-5}$. Similar results were achieved with $\varrho = 0.00035$ m²/s.

264. Variations from Day to Day During July—August 1964

It seemed interesting to investigate how far one could go by neglecting the anemo-baric forces (see Ch. 2125) and see the result of boundary variations only as the cause of sea level changes. As the characteristic period of the system is approximately 1/2 month it is further appropriate to have a rather long time series and 24 hours was chosen as timestep. The friction coefficient of Ch. 263 ($\beta = 0.5 \times 10^{-5}$) was used.

The period July—September 1964 was run. Contrary to what was the case in Ch. 262, the starting values of the sea levels were zero instead of the actual ones. Thereby it may take at least one month for the adjustment and in Fig. 264: 1, which shows the results, the first one third of the curve should not be taken too seriously. Looking at the remaining part, the following could be said. The coincidence is best at Landsort (the geostrophic effect was not taken into consideration) while the levels at Ystad and Kemi deviate much more. Specifically the oscillations of a period of approximately 5 days (see Ch. 14) in the latter part of September, clearly discernable in the record of Kemi, do not turn up. Evidently the disregard of the anemo-baric forces means that very many details are suppressed, the large scale variations of the Baltic, however, are approximately described. — See further below in Ch. 267 where this type of computation is combined with salinity variations.

265. Long Term Variations of Salinity

Computations of the salinity variations were made for a period of many years. The results were compared with salinities measured regularly from 1880 and onwards at two light-vessels, one Danish, Schultz's Grund, and one Swedish, Svenska Björn. While the salinities at the Danish light-vessels are still being determined by means of a hydrometer, Sweden shifted to the method of titration in 1923. As pointed out by JACOBSEN (1908) there is a systematic error in the hydrometer determinations. This yields values

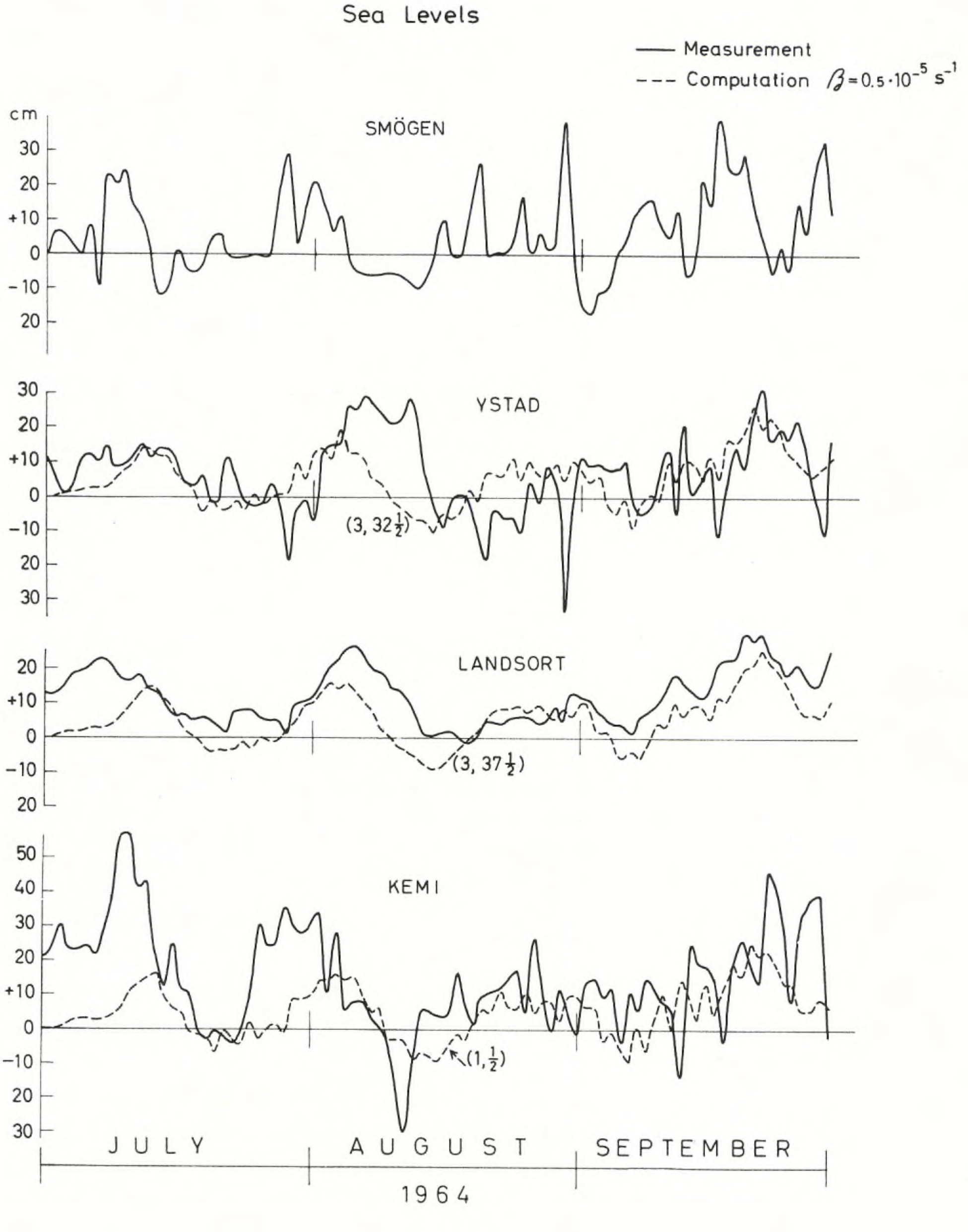


Fig. 264: 1. A comparison between measured and computed sea levels. As a boundary condition was taken the value of the sea level at Smögen (see Fig. 1: 1). No wind or atmospheric pressure were applied. Comparisons for Ystad, Landsort (see Fig. 1: 2) and Kemi (see Fig. 1: 5).

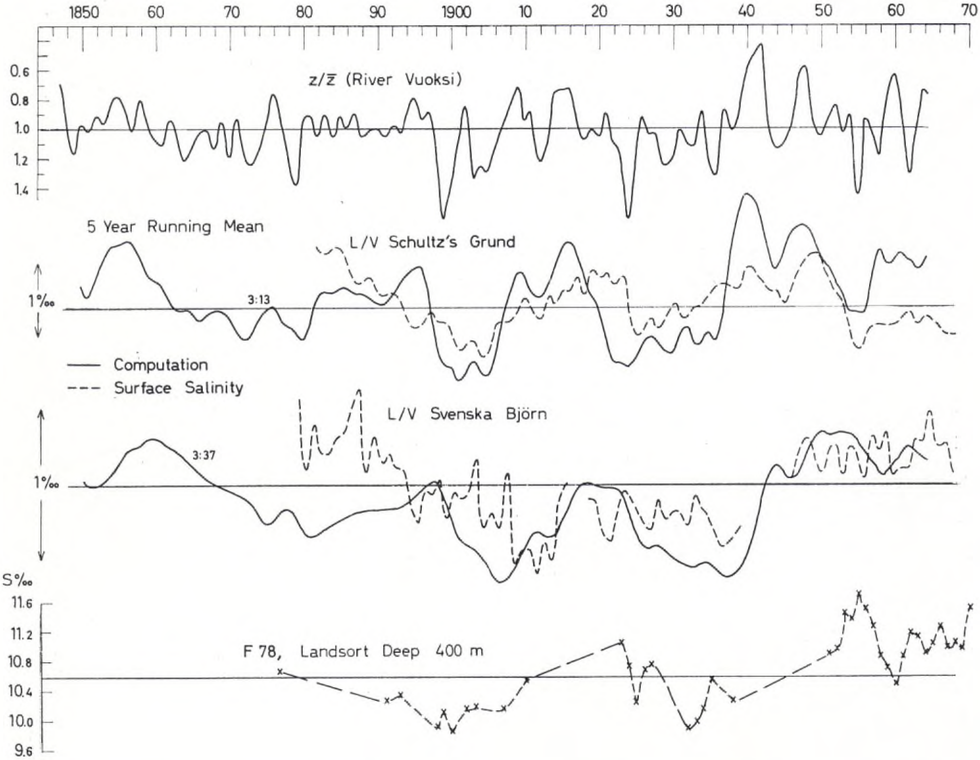


Fig. 265: 1. Comparisons between computed and measured surface salinities at the light-vessels Schultz's Grund (Fig. 1: 1) and Svenska Björn (Fig. 1: 2). At the top is shown the ratio between the annual value and a long term mean value of the runoff of the river Vuoksi. The data of the salinities in the Landsort Deep were taken from FONSELIUS (1969).

which are at least 0.2 ‰ too low, the error probably being due to surface tension. A similar error exists in the Swedish hydrometer data, the difference being approximately 0.45 ‰.

The salinities of the two light-vessels, Schultz's Grund and Svenska Björn, are inserted in Fig. 265: 1 for the purpose of comparison with computed salinities. While the Danish data are reproduced unchanged, to the Swedish hydrometer data 1880—1922 has been added 0.3 ‰. Summaries of the older Danish data can be found in JENSEN (1937) and NEUMANN (1940) and of the older Swedish data in SVANSSON (1971).

In these types of computations the mean values of the fresh water supply Z were taken from BROGMUS (1952), different for each section and month of the year.

The first computation was performed in such a manner that all these Z values were multiplied by one common factor varying from year to year like the water exchange values in SOSKIN (1963). The result was salinity variations many times larger than the measured ones. Then was tried a factor varying like the run-off values of the river Vuoksi at Imatra in Finland (SIRÉN 1958). Fig. 265: 1 shows the result: at the top the factor, then a comparison between computed and measured (surface) salinities at the position of Schultz's Grund (both as 5-year running means) and finally a comparison between computed and measured values at the position of L/V Svenska Björn.

The roughness of the model does not permit any far-reaching conclusions but some reflections may be allowed: the measured salinities during the 1880's are slightly higher than those of the 1940's and 1950's, while the computed salinities reveal a higher maximum in the 1940's than ever before. As there seems to be a correlation between the surface salinity and the salinity of the deepest parts of the Baltic deep basins (Fig. 265: 1, bottom), one could imagine the deep salinities of the 1880's to be at least as high as they have been in modern times. The only measurement we have from that time is from 1877 (EKMAN and PETTERSSON 1893) but it does not show such a high value; it fits better with the computed curve. Further studies are obviously necessary before the final answer to this interesting question can be presented.

266. Variations from Month to Month of Salinity and Sea Level During 1926—1930

The computation shown in Ch. 265 was based on rather vague values of fresh water supply. For a shorter period viz. 1926—1930 there is much more information (WYRTKI 1954 a) and therefore this period was processed. Further the SL equation was now used to produce transport values to be added to the Z-values of WYRTKI. In the SL equation there is no driving force but the boundary SLs vary as those of Smögen. Further there is no friction at all. Nevertheless the measured values of SLs are sometimes higher than the computed values (see Fig. 227: 1). The timesteps is one month in both equations.

The results of the salinity computation at section (3, 13) is shown in Fig. 266: 1 where it is compared with the measured values (weighted mean from surface to bottom) from the L/V Schultz's Grund. As the starting value of the computation was the annual mean and not the actual salinity, the curve presenting computed values has been moved 2 ‰ upwards to make the curves coincide in the beginning of the series.

Even if not particularly good the result is nevertheless interesting. Except for the uncertainties of the Ks as described in Ch. 22 and the roughness of the model particularly for the Kattegat area the discrepancies may be due to

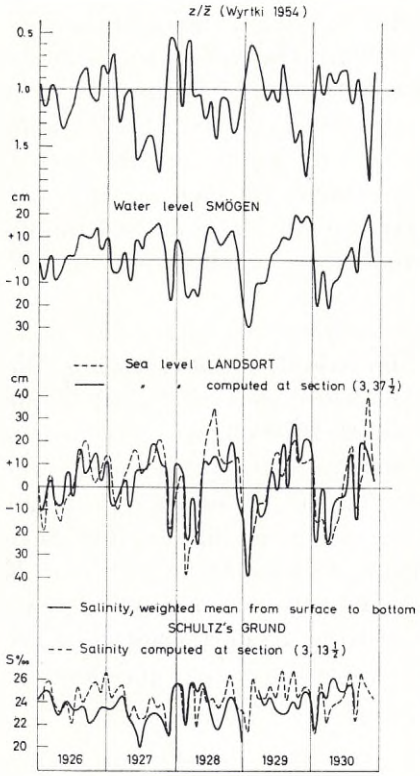


Fig. 266: 1. A comparison between computed (position of sections see Fig. 26: 1) and monthly means of measured sea levels (Landsort, see Fig. 1: 2) and salinities (light-vessel Schultz's Grund, see Fig. 1: 1). The fresh water supply variations are shown at the top; $\overline{Z}$ is a monthly mean taken from BROGMUS (1952). As a boundary condition for the sea level computation was used the sea level at Smögen; no wind or atmospheric pressure was applied. The transports resulting from the sea level computation are added to the Zs in the salinity equation.

1) the sea level model is not perfect in computing the level and transport,
2) the $Z/\overline{Z}$-ratios from WYRTKI (1954 a) may not refer to each section separately like in the computation,
3) there is a noticeable effect caused by the non-linearities.

267. Salinity Variations During September—November 1964

Salinities were computed also for the case described in Ch. 264, but as weighted means of the salinity measured at the L/V Kattegat SW were already made for another similar computation e.g. September—November, 1964, this case instead is shown (Fig. 267: 1). It is seen that the salinities can be computed fairly well, better than in the last paragraph hinting that the non-linear effect may be important.

268. Present Calculations of the Characteristic Periods

By means of the method by DEFANT as described in e.g. MAGAARD and KRAUSS (1966), the characteristic periods are being studied by the present

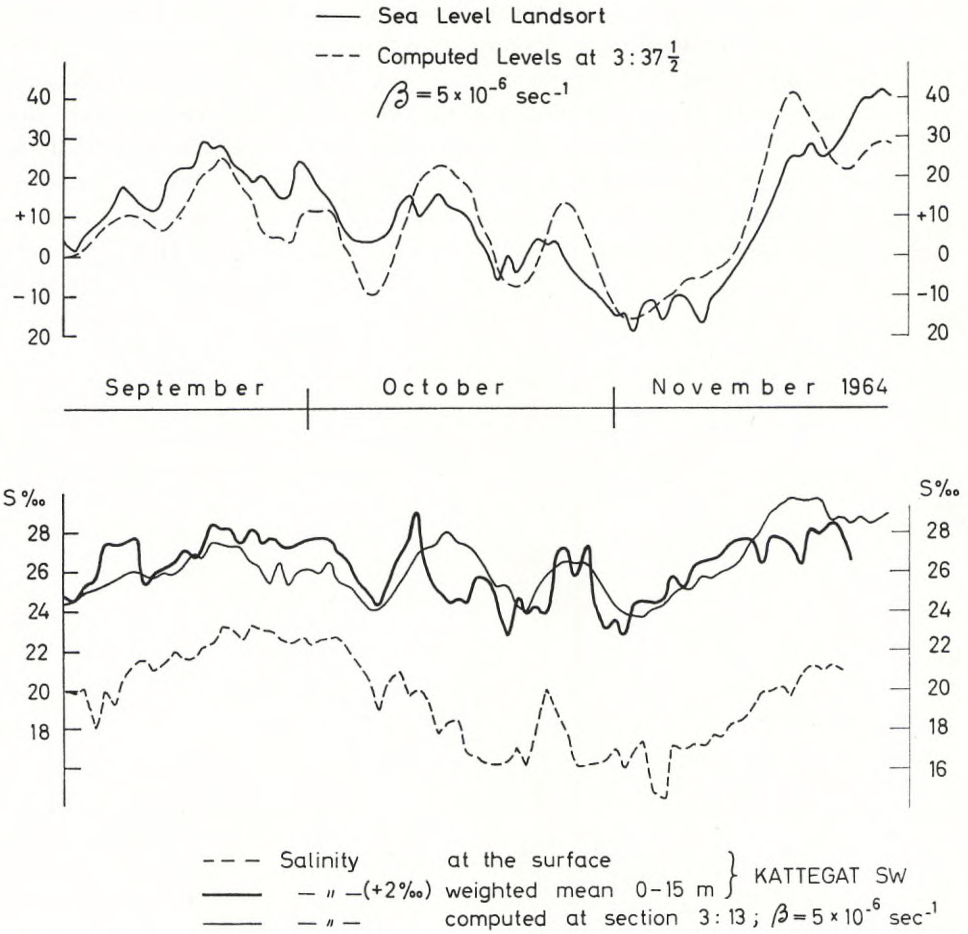


Fig. 267: 1. A comparison between computed (position of sections see Fig. 26: 1) and daily means of measured sea levels (Landsort, see Fig. 1: 2) and salinities (light-vessel Kattegat SW, see Fig. 1: 1). The fresh water supply was constantly that of October as given by BROGMUS (1952). As a boundary condition for the sea level computation was used the sea level at Smögen; no wind or atmospheric pressure were applied. The transports resulting from the sea level computation are added to the Zs in the salinity equation.

author. With a system of the three canals, open at one end, exclusively used in this article (Fig. 26: 1), the following periods were found: 18 days, 1.72 days (nodal line section 3,38), 1.40 days (nodal lines sections 1,17 and 3,33) and further 1.03, 0.93, 0.875, 0.69, and 0.64 days while other modes have not been studied.

By a way of iteration it was possible to include all the canals of the Belt Sea. Thereby the longest period decreased from 18 to 11 days, a considerable change indicating the need to include the Belt Sea correctly in computations expected to be better than the first approximation. So far the first approximation could be improved by changing the cross sections A in the Belt Sea in such a manner that the characteristic period of the 3-canal system changed from 18 days to 11 days. This would in all probability involve larger friction coefficients. — At the junction in the Western Baltic nearly one half of the transport entered the Sound and the remaining part of the Bay of Mecklenburg. At the next junction 15 % only took the passage through the Little Belt whereas the rest of the transport flowed through the Great Belt.

3. Conclusions and Discussion

In many respects the seas surrounding Sweden apparently behave like a long canal or more correct, a system of canals. This fact is very advantageous as it e.g. usually means less effort to solve one-dimensional equations than two-dimensional ones. The sea level problem has been further simplified by assuming the water to be homogeneous. The system of canals behaves like one semi-open canal with a characteristic period of approximately a fortnight.

The sea level in this canal moves up and down as a function of both the sea level variations at its mouth as well as of the driving forces on the canal itself. We can compute these variations assuming the water to be homogeneous, but at the same time we can draw conclusions of how the brackish Baltic water moves in relation to the ocean water. The Belt Sea and the Kattegat are very shallow and the Belt Sea furthermore small in width. These facts cause the long characteristic period as well as a low salinity of the Baltic water. The Kattegat and the Belt Sea are like a mouth of an estuary with a brackish layer on top of a more saline bottom layer. In the Northern Kattegat and along the Swedish and Norwegian coasts of the Skagerrak we normally find these two water masses beside and above each other. When the sea level of the Baltic rises due to the circumstances mentioned above it has such consequences that part of the brackish water in the Skagerrak, the Kattegat and the Belt Sea is withdrawn into the Baltic. The surface salinities in the Belt Sea, the Kattegat and along the Swedish and Norwegian coast of the Skagerrak rise simultaneously. When the sea level of the Baltic sinks the opposite of the above takes place and furthermore one remarkable ting: the Jutland current, which normally has a branch flowing West-East along the border between the Kattegat and the Skagerrak, on this occasion seems to be forced in another direction (Ch. 142). We know too little about the direction; this problem should be further studied by means of anchored current meters N and NW of Skagen.

While the present author in earlier papers had the ambition to determine the wind stress coefficient by means of numerical computations of Baltic sea levels, the sea level computations in this paper have been held on the 1st approximation level. The purpose has been more to show the possibilities of computing tides, meteorological sea level effects and salinity variations rather than to present more exact solutions. One reason is the fact that the complicated Belt Sea could not be well described with the implicit model so extensively used in this study.

The salinity model used here will probably be a contribution to further studies of the water exchange problem of the Baltic which practically is very important for the studies of Baltic pollution problems. It may here be possible to add also "vertical canals" to take up the problems of the deep stagnant basins of the Baltic. Until such refined methods are available we may content ourselves with the ideas put forward in this article: inflows of relatively high saline water to the deep basins of the Baltic, normally associated with a high level of the Baltic, are favoured by a small fresh water supply. In that case saline water is more accessible from the Kattegat. The lack of data of monthly means of water supply for other periods than 1926—1930 is a great drawback but might be remedied by further cooperation between Baltic hydrologists. To solve the water exchange problem by current measurements also at non-surface borizons in the Belt Sea as often suggested is unfortunately not that easy; a great effort is necessary. In this connection an investigation should be made to analyse why the current measurements at the Danish light-vessels during the late 30's and the early 40's could lead to the conclusion that the water exchange was improbably low (cf. Ch. 11).

It is suggested in this paper that special studies be initiated on the problems of the internal waves in the Gullmar fiord (Ch. 142), which apparently are related to the sea level variations of the Baltic, and also of the relation between these same sea level variations and the turnover of the bottom water in the deep basins of the Baltic (Ch. 12). The same should be said about the connection between the sea levels at the mouth of the canal and the atmospheric pressure (Ch. 14).

Concerning the numerical computations many suggestions of improvements to try are made in the text. Much effort should be devoted to solve the problem of incorporating an unlimited number of canals in the work with the implicit model. The explicit model will probably be too "expensive" to use for studies of the most important Baltic problems as a timestep of 15 minutes requires a lot of calculation time.

Parallel with these canal studies work with two- (or multi-) layer models as well as models with 2 horizontal dimensions must be encouraged. The Gullmar fiord problem is a complicated one with two water masses lying not only above but also beside each other. And the data from the joint Baltic cooperation 1964 cannot be fully used until a suitable two-dimensional model of the Baltic is constructed.

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5. Symbols

A	= cross section area	m^2
α	= smoothing coefficient (Ch. 2126)	
b	= width of section	m
β	= friction coefficient	s^{-1}
E'	= see Ch. 211	
ε	= Ekman number, see Ch. 211	—
E	= see Ch. 231	
f	$= 2\,\Omega \sin \phi$	s^{-1}
ϕ	= latitude	degrees
φ	= see Fig. 2125: 2	degrees
F'	= see Ch. 211	
F	= see Ch. 231	
g	= acceleration of gravity (9.807)	m/s^2
H	= depth to the bottom (= A/b)	m
h	= variation of sea level	m
$\bar{h}$	= height of equilibrium tide	m
j	= index of section	
J	= see Ch. 211	
K_2	= wind stress coefficient	—
K	= coefficient of horizontal eddy diffusion	m^2/s
L	= see Ch. 211	
M	= see Ch. 211	
n	= index of timestep	
ν	= coefficient of vertical eddy diffusion	m^2/s
Ω	= angular speed of the earth ($7.292 \cdot 10^{-5}$)	s^{-1}
p	= pressure	tons/m and s^2
p'_a	= atmospheric pressure	tons/m and s^2
p_a	$= p'_a/q$	m^2/s^2
ψ	= see Fig. 2125: 2	degrees
q	= density	$tons/m^3$
q_a	= density of air	$tons/m^3$
Q	= see Ch. 211	
r	= radius of curvature, see Ch. 2121	m
R	= friction coefficient	m/s
ϱ	= friction coefficient	m^2/s

R'	= friction coefficient	—
S	= salinity	$^0/_{00}$
SL	= sea level	
s	= see Ch. 211	
σ	= see Ch. 211	
τ'	= turbulent stress component	tons/m and s^2
τ	$= \tau'/q$ (τ usually means the windstress, while τ_B stands for the bottom stress)	m^2/s^2
t	= time	s
χ	= longitude	degrees
U	= transport of water in the longitudinal direction	m^3/s
u	= velocity of water in the longitudinal direction	m/s
W	= surface wind velocity	m/s
W_G	= geostrophic wind velocity	m/s
x	= coordinate in the longitudinal direction	m
Δx_j	= distance between the sections j and j+1	m
y	= coordinate in the transversal direction	m
Z	= fresh water supply (usually river water + precipitation — evaporation)	m^3/s
z	= coordinate in the vertical direction	m

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