Ödsmål, Kville sn, Bohuslän

Hällristning
Fiskare från
bronsåldern

Rock carving
Bronze age
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Tidal and Spectral Analyses of Kattegat Time Series of Current and Salinity

by

Henryk Bieler and Artur Svansson

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1. Introduction.

As a part of a project to determine the transport of water and matter in the northern Kattegat, automatically recording current meters, type Aanderaa, were anchored at various times and places. This paper deals with data reorded during the period 1975-04-19 -- 1975-08-19. The measuring depths were 15 m and 30 m.

Fig. 1 shows the area with depth topography. The position of the current meters was N 57°32′ E 11°19.5′ near the position of the former Läsö Nord lightship.

2. Observation Material.

Current direction and velocity were recorded every 10th minute on magnetic tape. After a first processing the data were checked and corrected. In a very few cases empty spaces had to be filled with linearly interpolated values. In a second processing N- and E-components were computed and stored on a magnetic disc.

Hourly mean values centered at whole hours were computed of both N- and E-components respectively.This procedure may introduce noise; a more sophisticated filter technique had probably been a better choice. The data sets used for the tidal and spectrum analysis consisted of these hourly values.

Two periods were investigated, a shorter one (I), 75-04-19 -- 75-06-16 and a longer one (II), 75-04-19 -- 75-08-19, the latter for the 30 m depth only however. The mean currents were as follows:

	N-comp	E-comp
15 m I	-0.06 cm/s	-8.62 cm/s
30 m I	-3.77 cm/s	-1.65 cm/s
30 m II	-2.90 cm/s	-2.35 cm/s

3. Tidal analysis.

3.1. Method.

A copy of the analysis program TIFA (Murray 1964) was kindly placed at our disposal by Dr. G. Lennon, IOS, Bidston.

The program computes the amplitude H and GMT-phase G for Doodson's (1928) constituents. For period I, 26 constituents, for period II, 28 constituents were determined.

3.2. Results.

Tables 1-3 indicate the constituents which were used in TIFA. Whereas computation I covering 1415 hours is somewhat short for the long periods Mf and Msf, these two constituents could be included in II. A comparison between the remaining constituents computed both in I and II shows small differences. Note the phase coincidence in P1 and K1 and also in the T2, S2, K2 group. Due to the shortness of the data time series (to separate T2 and S2 for instance a complete year is required) and in the absence of other additional information, relationships of the equilibrium tides for amplitudes and identical values for phase was used.

Fig:s 2 and 3 show the M2 tidal ellipses for 15 m and 30 m respectively. Surprisingly they have opposite direction of rotation, 15 m cum sole and 30 m contra solem.

The tidal currents in the Kattegat have been analyzed before. Jacobsen (1913) presented harmonic constants for M2 at L/V:s Anholt Knob (3 month's data series) and Schultz's Grund (one year) of currents measured 6 times a day at 7 depths by means of a pendulum device. Jacobsen only analyzed the longitudinal component. His determination of eddy viscosity coefficients by means of the vertical distribution of the tidal currents is a classical work. A similar time series measured at L/V Läsö Rende covering the period 1912-09-05 -- 1913-11-14 was not processed until 1968 then by Rossiter. As L/V Läsö Rende was situated in a narrow passage the currents are highly longitudinal, for M2 as follows:

depth m	H cm/s	G°
2.5	20.4	321
5	22.6	321
10	26.0	330
15	25.9	326
20	23.2	318

The vertical variations are small in contrast to conditions at L/V Anholt Knob where the surface (2.5 m) current is twice the value at 25 m depth. Defant (1934) compares measurement with results of a barotropic canal model (M2). The computed phase is approx. 325° at the border between the Skagerrak and the Kattegat and increases southwards (330° at the position of our current meter and 325° at the position Läsö Rende).

4. Spectral Analysis.

By means of power spectrum analysis the energy distribution as a function of frequency ($hours^{-1}$) was computed for the current time series used above. The tidal components computed by TIFA were subtracted before the analysis. The spectrum was determined according to Blackman and Tukey (1958), with small modifications as described in Mälkki (1975).

A program SPC computes autocovariance and spectrum of the time series at issue. The power spectrum P(f) is a Fourier transform of the autocovariance function $C(\tau)$. For a continuous case we have:

$$P(f) = \int_{-\infty}^{\infty} C(\tau) \cdot e^{-i2\pi f\tau} d\tau \qquad -\infty \leq f \leq \infty$$

$$C(\tau) = \int_{-T}^{T} P(f) \cdot e^{i2\pi f\tau} df \qquad -T \leq \tau \leq T$$

where f = frequence ($hour^{-1}$)

τ = time lag (hour)

For a discrete case is:

$$P(f) = \Delta t \sum^{N-1} c(\tau) \cdot e^{-i2\pi f \Delta\tau}$$

4.1. Computer Program.

Estimation is made of P(f) associated with a time series computed by forming autocovariances for various time lags and obtaining their Fourier transforms. In order to smooth out P(f) and C(τ) the following estimations were used:

$$C(\tau) = 1/N \sum_{\tau=1}^{N-1} (x_t - \bar{x}) \quad (x_{t+\tau} - \bar{x})$$

$$P(f) = 2 \cdot \Delta t \cdot \left[C(0) + \sum_{\tau=1}^{m-1} v(\tau)\, C(\tau) \cdot e^{-i\pi\tau/m} \right]$$

where

N = total number of observations used

$\bar{x} = 1/N \cdot \left(\sum_{j=1}^{N} x_j \right)$ = mean value of all observations

$\tau = j\,\Delta t \qquad 0 \leq \tau \leq m\,\Delta t$

$v(\tau)$ = window

$f = j/2m\,\Delta t$ = frequency interval

t = time step between two observations

m = number of lags

4.2. Data Set.

The data set consists of a time series. The following information is needed:

N, Δt and m. Furthermore a window $v(\tau)$ is chosen as a Tukey (Hanning) window:

$$v(\tau) = 0.5(1 + \cos(\pi\tau/m))$$

Blackman and Tukey (1958) show that the application of this window widens the main lobe of the equivalent analytical filter and reduces the side. Random errors in the data will introduce uncertainties into the estimates of P(f) which may be determined in terms of 95 % confidents limits. These limits have been obtained to depend upon the v (number of degrees of freedom) defined as:

$v = 2N/mb$ where b is bandwith of Tukey window.

4.2. Method.

Spectra can be very effectively computed by a "cyclic" method according to Cartright (Murray 1964):

$$w(\tau) = v(\tau) \cdot C(\tau) + 2w(\tau + 1) \cdot \cos(\pi\tau j/m) - w(\tau + 2)$$

$$w(m) = 0$$

$$w(m-1) = C(m-1) \quad v(m-1)$$

If we use the repetivite cycle (m-1) times, it finally yields w(1) and w(2), which can be shown to satisfy the relation

$$w(1) - w(2) \cdot e^{-i\pi\tau/m} = v(\tau) \cdot C(\tau) \cdot e^{-i\pi\tau/m}$$

4.3. Results.

Fig. 4 presents an example of the current data, on top in original and on bottom with TIFA computed tides subtracted, the last one used for spectrum analysis.

Fig. 5 shows the result of an analysis of the long series II East Comp at 30 m depth. There are certain peaks above the confidence limits at the following middle frequency periods: 12.1, 14.2, 17.2 and 21.7 hours.

The peak of 14.2 hours clearly originates from an intertial current. The high frequency range has a characteristic general slope. This slope, here -2 is often compared with Ozmidow's -5/3 slope in the classical theory of 3-dimensional turbulence:

$$E(k) = a\,\varepsilon^{2/3}\,k^{-5/3}$$

where

a = constant

ε = rate of energy dissipation

k = scalar wave number

Fig. 6 shows the result of the power spectrum analysis of period I, N-component of 15 m and 30 m and Fig. 7, period I, E-component of 15 m and 30 m. Note that the point density is smaller than in Fig. 5.

Also salinities measured simultaneously were processed with the power spectrum technique, Fig. 8. As no TIFA-pretreatment was carried out with this data set, a comparison with correspondingly untreated current data was assumed to be more consistent, see Fig. 9.

Acknowledgement

We express thanks to Dr. G. Lennon for placing TIFA at our disposal and also for his valuable comments on the computational results.

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Table 1

Tidal Constituents of N- and E-components of currents measured at N 57°32′ E 11°19.5′ covering period 75 04 19 - 75 06 16.
Units cm/s and degrees. Time Zone 0. 15 m (I)

Name	speed °/hour	period hours	N-comp. amplitude	N-comp. phase	E-comp. amplitude	E-comp. phase
Zo	0.0000000	0.0000	0.0619	0.00000	8.6232	0.00000
Q_1	13.3986609	26.8684	2.7224	315.16375	0.2752	51.62997
O_1	13.9430356	25.8193	4.3959	70.24478	1.5529	237.78418
P_1	14.9589314	24.0659	0.56259	275.08014	0.1812	43.36786
K_1	15.0410686	23.9345	1.6997	275.08014	0.5475	43.36786
J_1	15.5854433	23.0985	1.5737	337.54128	0.7757	224.65068
OO_1	16.1391017	22.3061	1.5888	55.23301	0.5443	319.45760
μ_2	27.9682084	12.8718	1.4074	146.07363	1.6424	28.81822
N_2	28.4397295	12.6583	1.5942	265.67922	0.8570	140.22495
M_2	28.9841042	12.4206	9.8420	358.66500	4.6768	139.30198
L_2	29.5284789	12.1916	1.6060	181.14927	0.8285	239.55282
T_2	29.9589333	12.0164	0.15198	321.35558	0.1023	111.42707
S_2	30.0000000	12.0000	2.5759	321.35558	1.7339	111.42707
K_2	30.0821373	11.9672	0.7006	321.35558	0.4716	111.42707
$2SM_2$	31.0158958	11.6069	0.2107	329.77553	0.8247	355.88236
MO_3	42.9271398	8.3863	0.3427	105.65786	0.6684	71.91571
M_3	43.4761563	8.2804	0.4250	289.34420	0.2916	239.24998
MK_3	44.0251729	8.1771	0.0532	348.85677	0.2911	158.82359
SK_3	45.0410686	7.9927	0.1179	115.44934	0.4454	293.12733
MN_4	57.4238337	6.2691	0.2173	58.47525	0.3833	86.55727
M_4	57.9682084	6.2103	0.4354	167.95451	0.2082	26.66023
SN_4	58.4397295	6.1602	0.1320	290.29264	0.2999	308.71338
MS_4	58.9841042	6.1033	0.5965	267.38229	0.6685	29.64247
S_4	60.0000000	6.0000	0.3207	244.80308	0.3015	252.08333
$2MN_6$	86.4079380	4.1662	0.3724	202.04002	0.2007	50.37555
IM_6	86.9523127	4.1402	0.3476	248.78612	0.5042	35.14531
MSN_6	87.4238337	4.1178	0.1136	187.21127	0.1264	157.23070
$2MS_6$	87.9682084	4.0923	0.2034	217.98594	0.1409	145.44192
$2SM_6$	88.9841042	4.0456	0.1105	198.82431	0.1319	280.97578

Table 2.

Tidal Constituents of N- and E-components of currents measured at N 57°32′ E 11°19.5′ covering period 75 04 19 - 75 06 16. Units cm/s and degrees. Time Zone 0. 30 m (I)

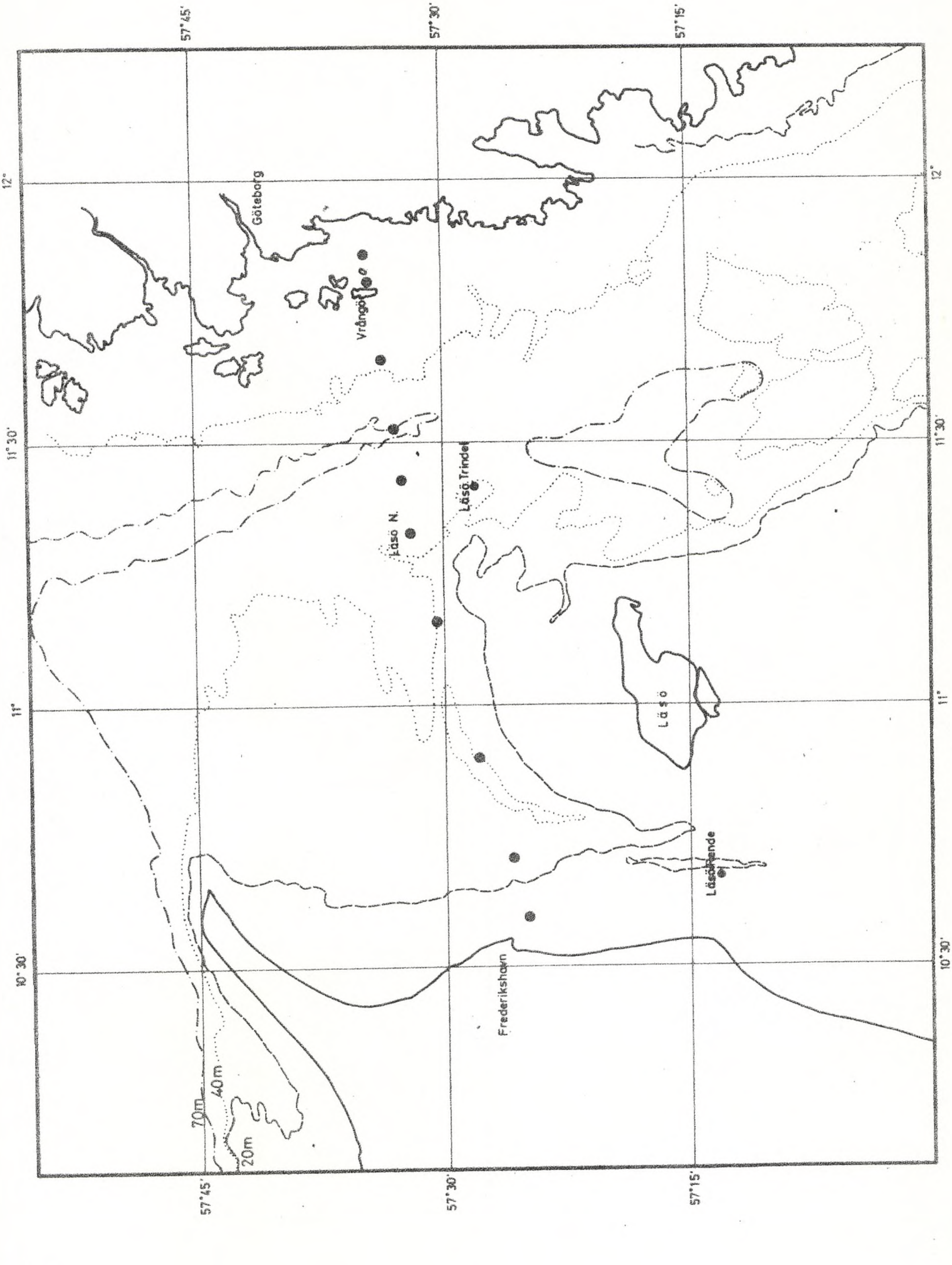
Name	speed °/hour	period hours	N-comp. amplitude	N-comp. phase	E-comp. amplitude	E-comp. phase
Zo	0.0000000	0.0000	3.7742	0.00000	1.6546	0.00000
Q_1	13.3986609	26.8684	1.6725	326.09669	0.6680	132.81609
O_1	13.9430356	25.8193	3.2795	57.90491	2.3813	231.03982
P_1	14.9589314	24.0659	0.44446	252.80514	0.3320	118.98625
K_1	15.0410686	23.9345	1.3428	252.80514	1.0031	118.98625
J_1	15.5854433	23.0985	0.4200	331.95061	0.3772	233.53144
OO_1	16.1391017	22.3061	2.2465	90.11316	1.5383	275.14835
μ_2	27.9682084	12.8718	1.2828	188.40847	1.9040	345.60612
N_2	28.4397295	12.6583	1.1961	295.71350	1.8771	149.80039
M_2	28.9841042	12.4206	4.6096	344.05909	4.9693	205.13489
L_2	29.5284789	12.1916	1.7795	352.22635	2.6564	24.93336
T_2	29.9589333	12.0164	0.01148	182.19999	0.0851	231.53248
S_2	30.0000000	12.0000	0.1945	182.19999	1.4430	231.53248
K_2	30.0821373	11.1672	0.0529	182.19999	0.3925	231.53248
$2SM_2$	31.0158958	11.6069	0.6120	291.56717	0.7982	346.70181
MO_3	42.9271398	8.3863	0.0787	315.44573	0.2564	111.21017
M_3	43.4761563	8.2804	0.5667	351.85975	0.3243	142.42725
MK_3	44.0251729	8.1771	0.4098	211.64530	0.3299	328.46406
SK_3	45.0410686	7.9927	0.3514	29.94425	0.1540	139.51828
MN_4	57.4238337	6.2691	0.1951	181.63759	0.4185	299.58442
M_4	57.9682084	6.2103	0.2438	147.43229	0.3057	294.22506
SN_4	58.4317295	6.1602	0.3716	184.37594	0.2892	263.93588
MS_4	58.9841042	6.1033	0.3617	217.93991	0.2264	161.82748
S_4	60.0000000	6.0000	0.4046	20.64837	0.3063	163.17583
$2MN_6$	86.4079380	4.1662	0.1070	116.18440	0.1347	311.61188
M_6	86.9523127	4.1402	0.2227	188.51814	0.1519	86.91631
MSN_6	87.4238337	4.1178	0.1312	104.57295	0.1617	51.47946
$2MS_6$	87.9682084	4.0923	0.2447	222.63797	0.1789	54.38920
$2SM_6$	88.9841042	4.0456	0.1498	13.91547	0.1365	161.71868

Table 3.

Tidal Constituents of N- and E-components of currents measured at N 57° 32′ E 11°19.5′ covering period 75 04 19 - 75 08 19.
Units cm/s and degrees. Time Zone 0. 30 m (II)

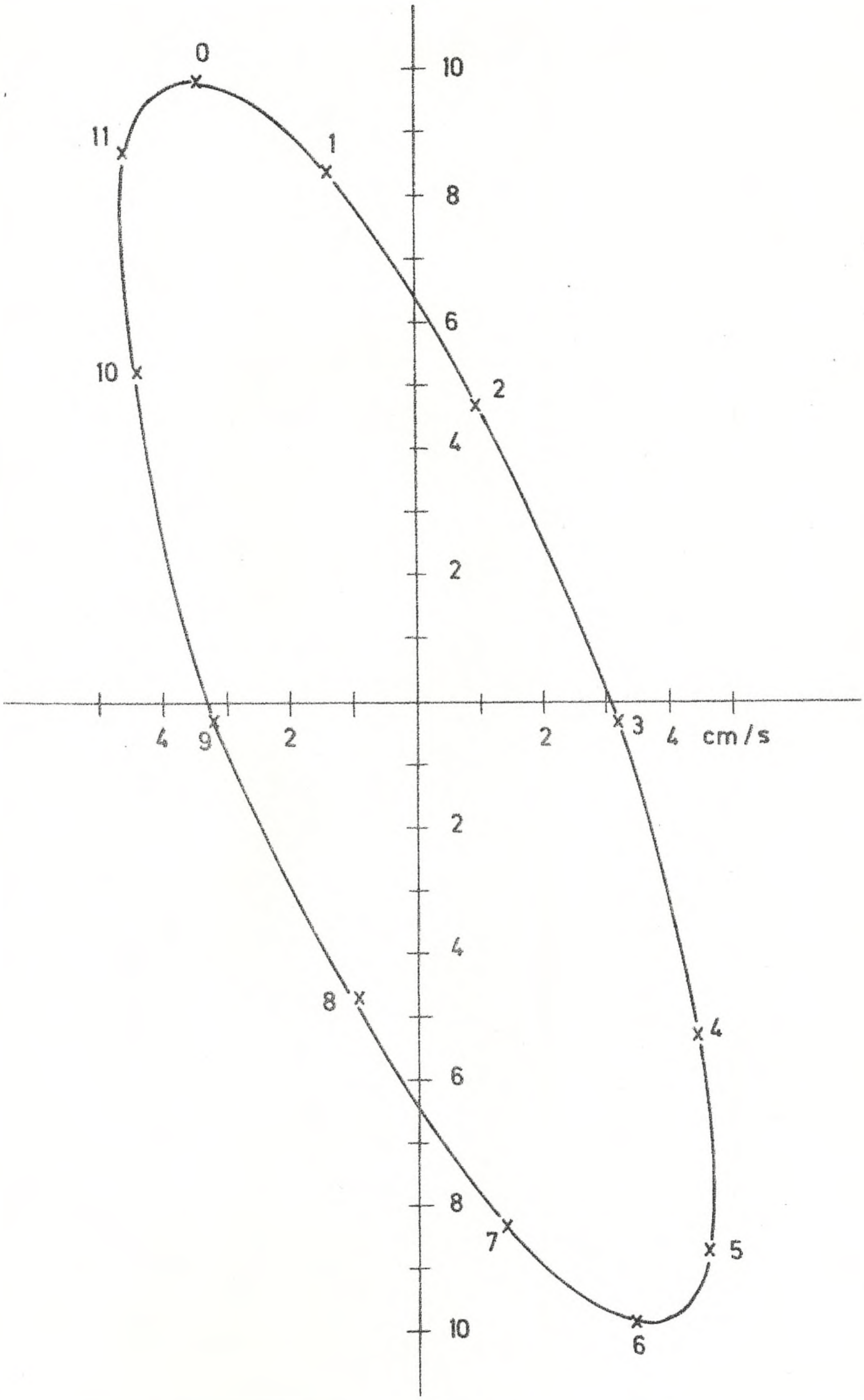
Name	speed °/hour	period hours	N-comp. amplitude	N-comp. phase	E-comp. amplitude	E-comp. phase
Zo	0.0000000	0.0000	2.8968	0.00000	2.3523	0.00000
Msf	1.0158958	14.7653days	3.4923	39.72929	0.6642	27.88773
Mf	1.0980331	13.6608days	2.4816	308.33022	3.6331	81.89451
Q_1	13.3986609	26.8684	1.2710	332.85196	0.1178	101.30363
O_1	13.9430356	25.8193	2.9254	65.89347	1.6762	227.19246
P_1	14.9589314	24.0659	0.4108	248.81725	0.2720	115.01799
K_1	15.0410686	23.9345	1.2412	248.81725	0.8219	115.01799
J_1	15.5854433	23.0985	0.2211	264.80451	0.4358	332.47019
OO_1	16.1391017	22.3061	1.2104	95.85945	1.5008	269.74208
μ_2	27.9682084	12.8718	1.2443	177.84511	1.1574	333.78779
N_2	28.4397295	12.6583	1.3168	301.38271	0.8786	145.24526
M_2	28.9841042	12.4206	4.8641	349.17469	4.5376	204.81657
L_2	29.5284789	12.1916	1.3393	2.91843	1.4680	357.74180
T_2	29.9589333	12.0164	0.0529	236.96112	0.0910	238.86457
S_2	30.0000000	12.0000	0.8959	236.96112	1.5431	238.86457
K_2	30.0821373	11.9672	0.2437	236.96112	0.4197	238.86457
$2SM_2$	31.0158958	11.6069	0.8188	301.61725	0.6436	10.28010
MO_3	42.9271398	8.3863	0.1841	19.42257	0.3783	66.79121
M_3	43.4761563	8.2804	0.4652	352.30161	0.3072	164.32377
MK_3	44.0251729	8.1771	0.1895	317.31970	0.3151	55.86164
SK_3	45.0410686	7.9927	0.1246	42.63940	0.3151	231.36549
MN_4	57.4238337	6.2691	0.1786	166.52173	0.2062	265.81305
M_4	57.9682084	6.2103	0.3010	197.22761	0.1765	392.02568
SN_4	58.4397295	6.1602	0.0620	206.29211	0.3004	271.28960
MS_4	58.9841042	6.1033	0.2948	256.90733	0.1434	122.24052
S_4	60.0000000	6.0000	0.3209	8.21907	0.2489	163.95207
$2MN_6$	86.4079380	4.1662	0.1561	196.40762	0.2568	284.82057
M_6	86.9523127	4.1402	0.2736	188.12939	0.1375	52.89149
MSN_6	87.4238337	4.1178	0.0885	191.21305	0.1506	33.36765
$2MS_6$	87.9682084	4.0923	0.1009	212.47921	0.1260	92.23225
$2SM_6$	88.9841042	4.0456	0.0601	337.13301	0.0406	283.53805

Fig. 1



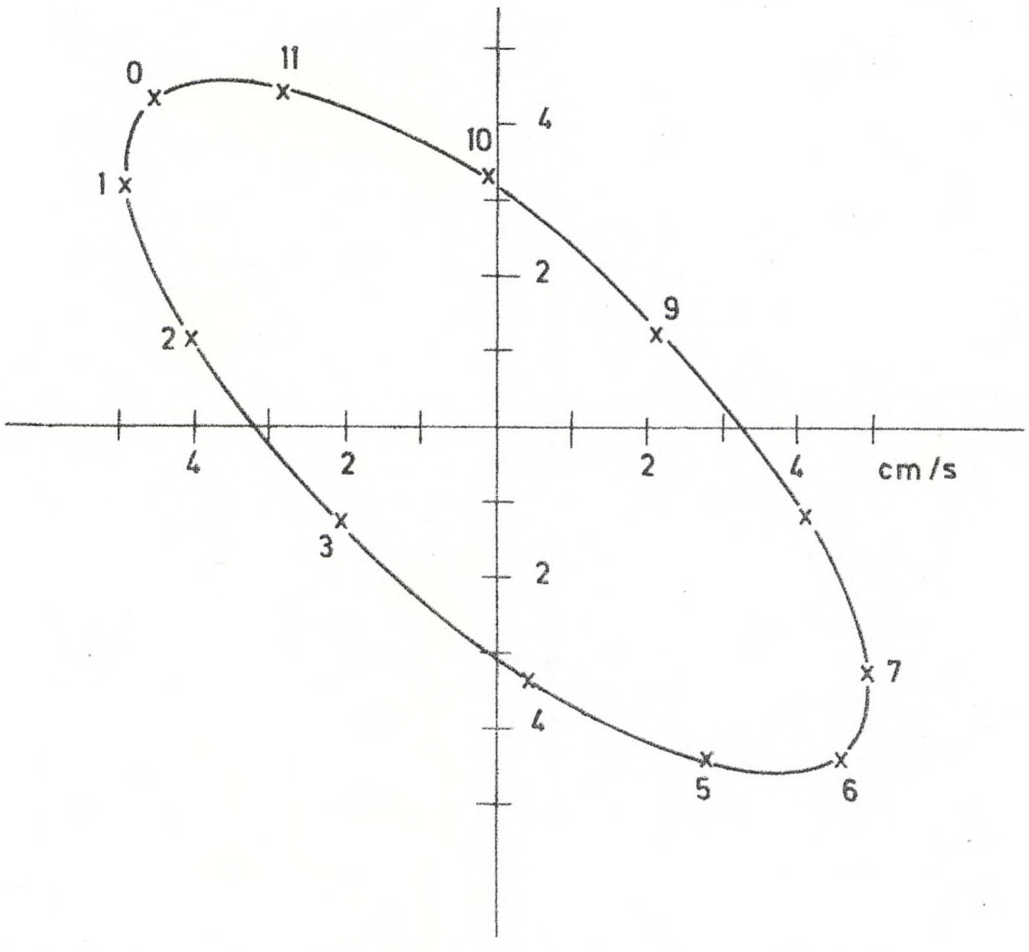
57°45'
57°30'
57°15'
12°
11°30'
11°
10°30'
Göteborg
Vrångö
Läsö N.
Läsö Trindel
Läsö
Läsö Rende
Frederikshavn
70m
40m
20m

Fig. 2



M 2 Tidal Ellipse 15 m Computational period 75 04 19 - 75 06 16

Fig. 3



M 2 Tidal Ellipse 30 m Computational period 75 04 19 – 75 06 16

Fig. 4

Hourly Means of Current Meter Data, N 57° 32' E 11° 10.'5, 15 m Depth

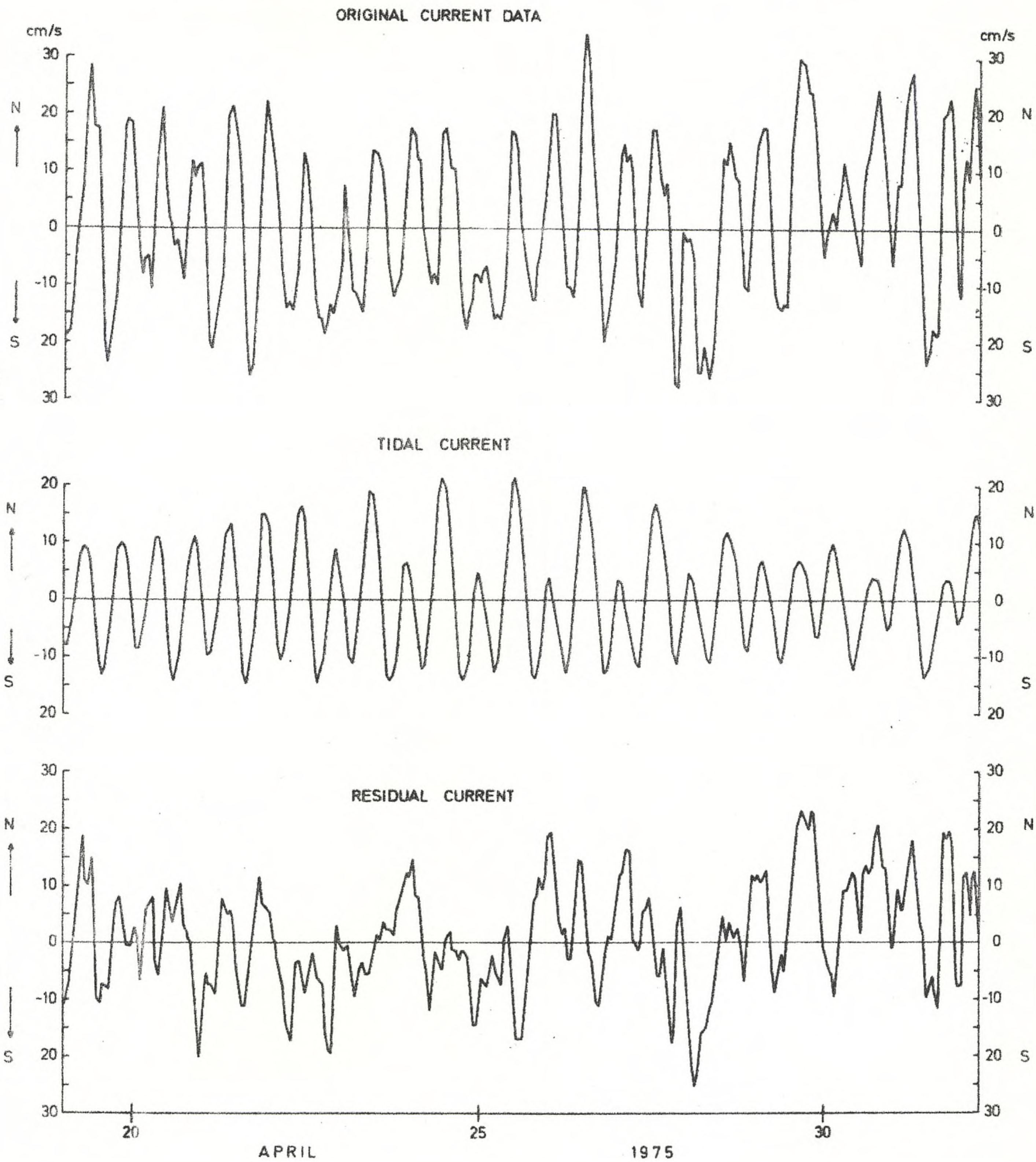
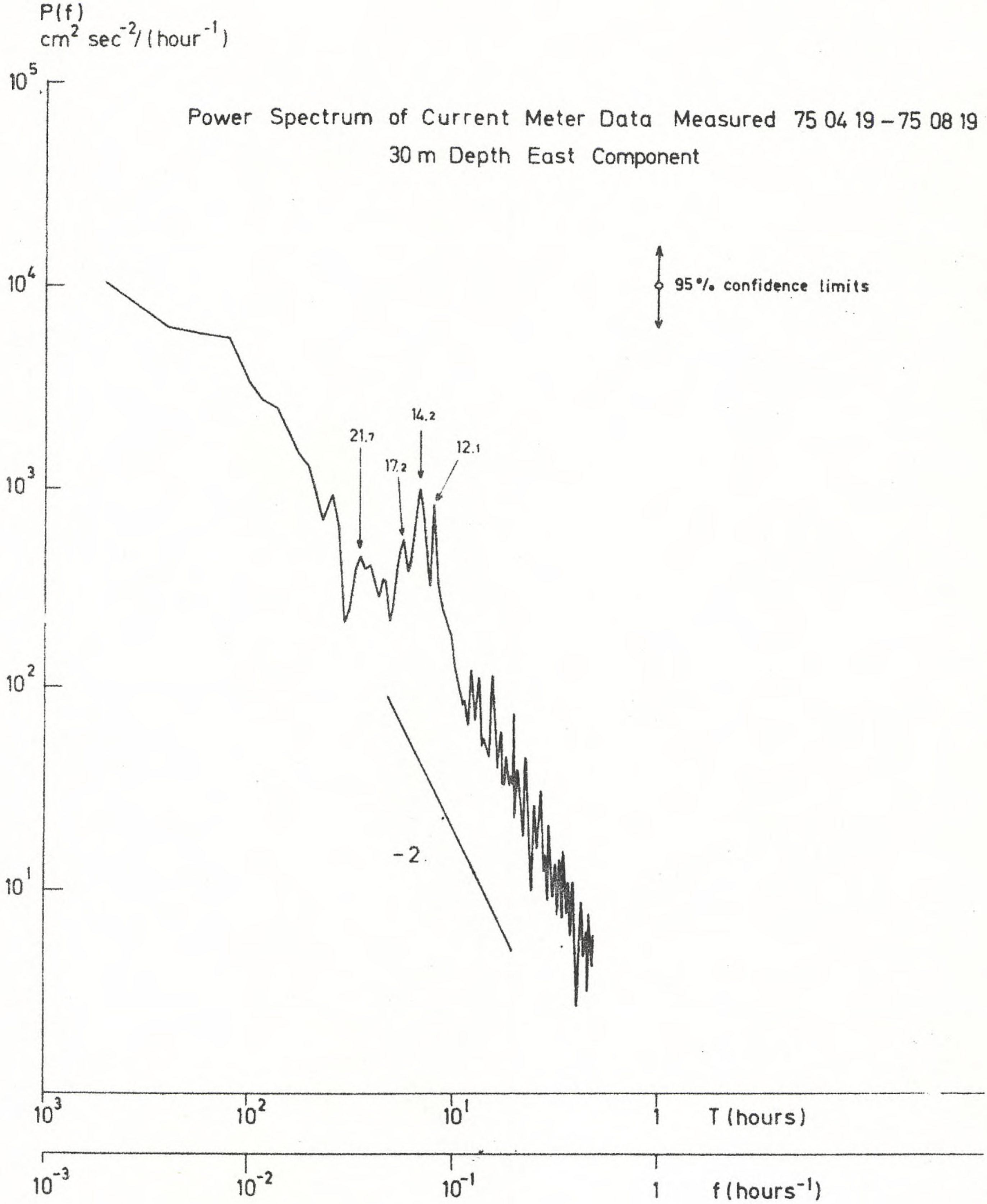


Fig. 5


P(f)
cm² sec⁻²/(hour⁻¹)
10^5
10^4
10^3
10^2
10^1
Power Spectrum of Current Meter Data Measured 75 04 19 – 75 08 19
30 m Depth East Component
95 % confidence limits
21.7
17.2
14.2
12.1
-2
10^3
10^2
10^1
1
T (hours)
10^-3
10^-2
10^-1
1
f (hours⁻¹)

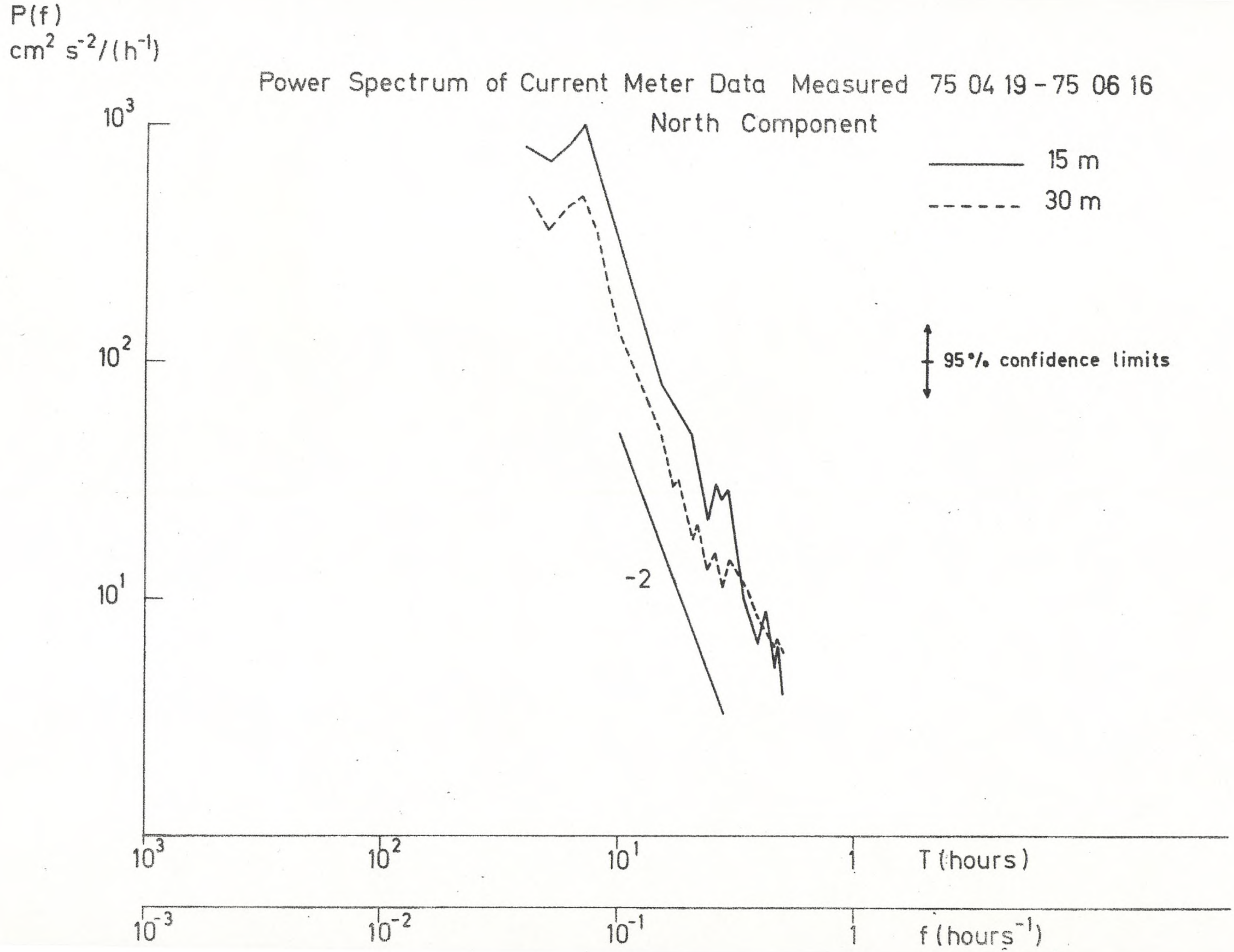
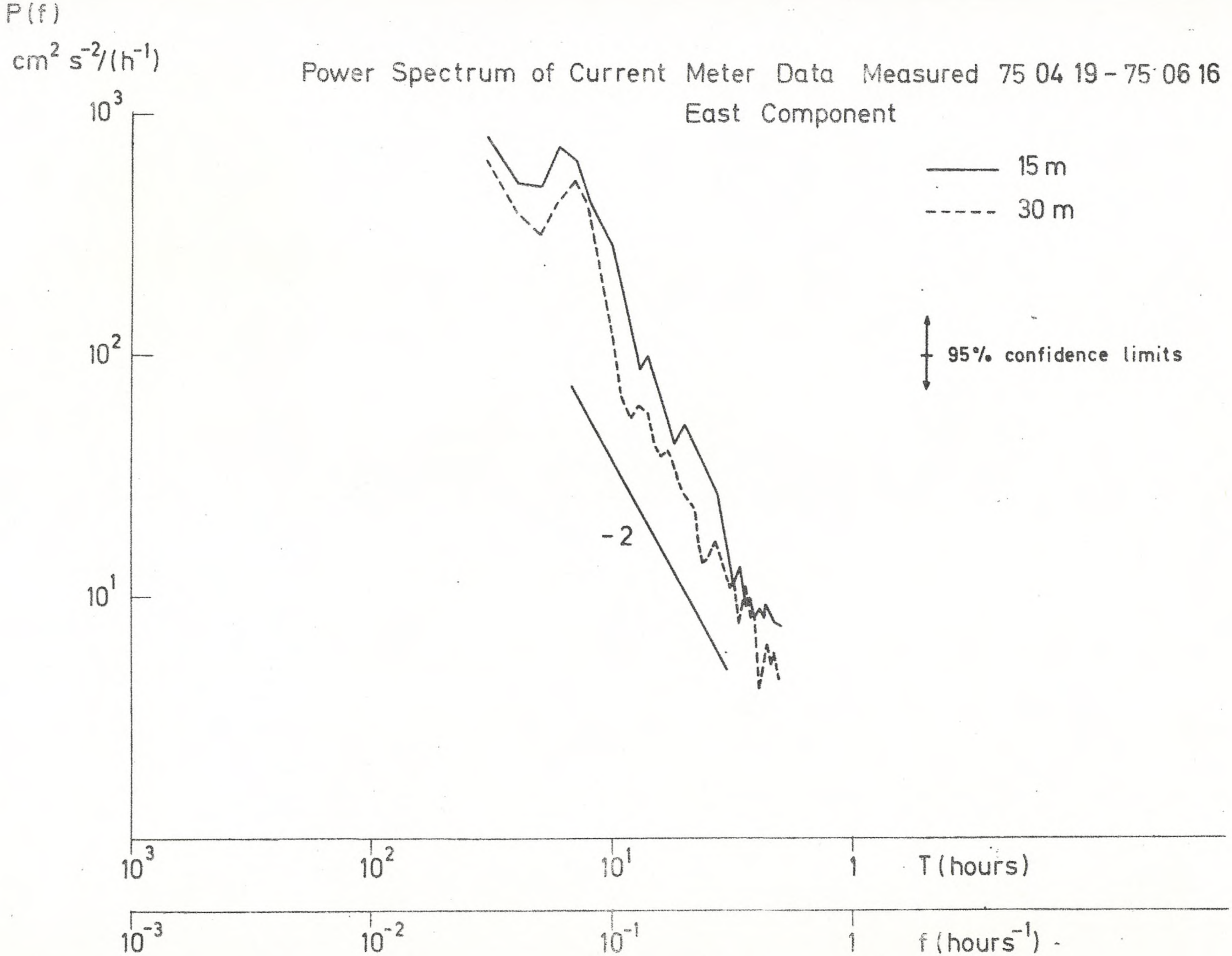

P(f)
cm^2 s^{-2}/(h^{-1})
Power Spectrum of Current Meter Data Measured 75 04 19 - 75 06 16
North Component
15 m
30 m
95 % confidence limits
-2
10^3
10^2
10^1
T (hours)
1
f (hours^{-1})
10^{-3}
10^{-2}
10^{-1}

Fig. 6

Fig. 7



P(f)
cm² s⁻²/(h⁻¹)
Power Spectrum of Current Meter Data Measured 75 04 19 – 75 06 16
East Component
15 m
30 m
95% confidence limits
-2
10³
10²
10¹
1
T (hours)
10⁻³
10⁻²
10⁻¹
f (hours⁻¹)

Fig. 8

Power Spectrum of Salinity Measured 75 04 19 - 75 06 19

15 m

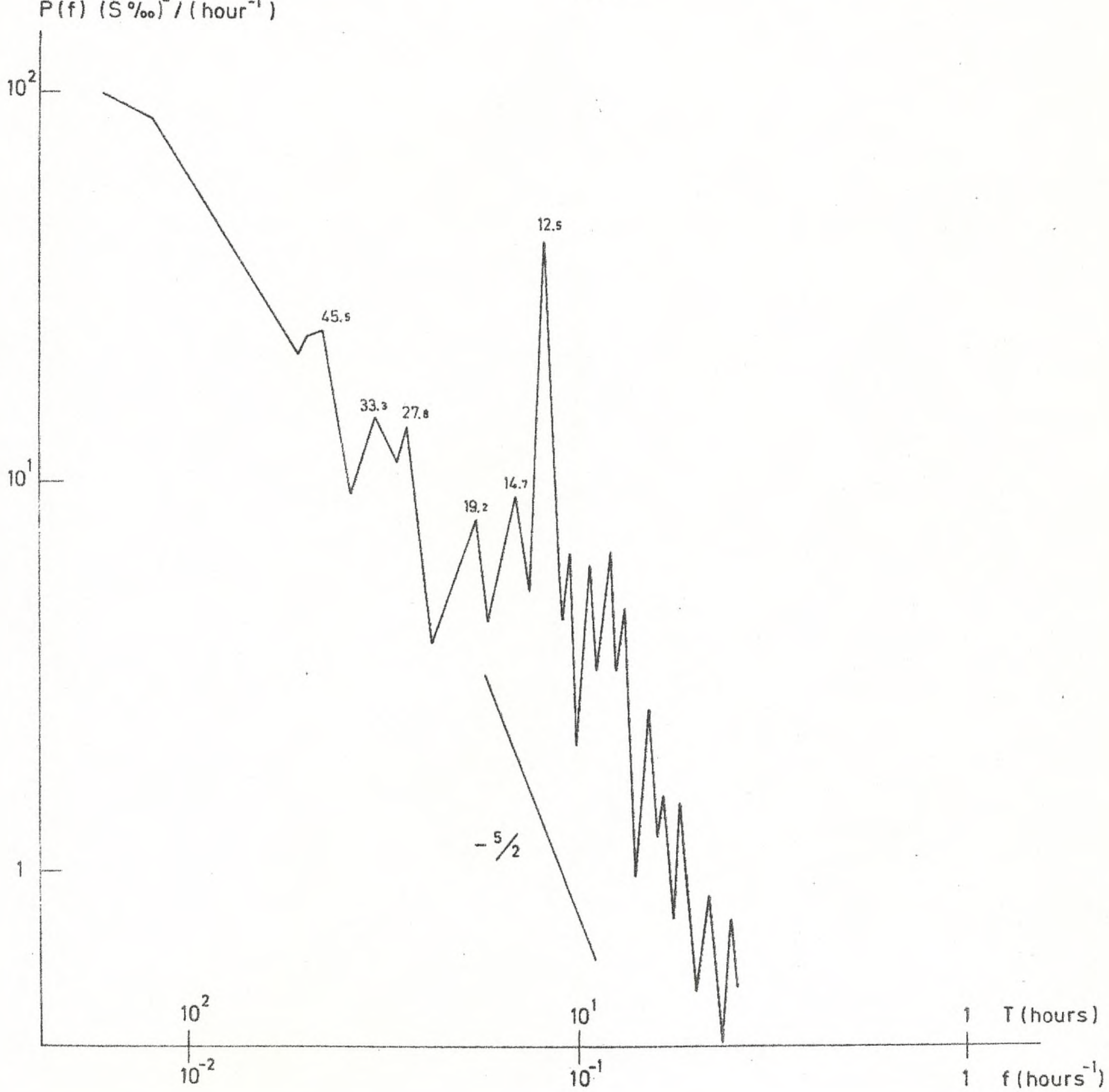


Fig. 9

Power Spectrum of Current Meter Data Measured 75 04 19 - 75 06 19
Tides included
N - Component 15 m