



UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

WORKING PAPERS IN ECONOMICS

No 682

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Commuting Patterns**

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December 2016

ISSN 1403-2473 (print)

ISSN 1403-2465 (online)

Optimal Environmental Road Pricing and Integrated Daily Commuting Patterns*

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Abstract

Road pricing can improve air quality by reducing and spreading traffic flows. Nevertheless, air quality does not depend only on traffic flows, but also on pollution dispersion. In this paper we investigate the effects of the temporal variation in pollution dispersion on optimal road pricing, and show that time-varying road pricing is needed to make drivers internalize the social costs of both time-varying congestion and time-varying pollution. To this end, we develop an ecological economics model that takes into account the effects of road pricing on integrated daily commuting patterns. We characterize the optimal road pricing when pollution dispersion varies over the day and analyze its effects on traffic flows, arrival times, and the number of commuters by car.

Key Words: Air pollution, Road transportation, Road pricing, Pollution dispersion.

JEL classification: Q53, Q58, R41, R48.

*Research funding from the Sustainable Transport Initiative at the University of Gothenburg and Chalmers University of Technology and from the Adlerbertska Foundation is gratefully acknowledged.

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1 Introduction

In 2010, the health costs of air pollution due to road transportation corresponded to about USD 1 trillion in OECD countries and about USD 1 trillion in China and India alone (OECD 2014). These costs account for the effects of exposure to air pollution on the development of chronic diseases, respiratory illness, and premature mortality. Epidemiological studies have shown an approximately linear increase in health risk with increasing exposure to urban air pollutants like particulate matter, with no demonstrable threshold below which no effects are quantifiable. High spikes of pollution – rather than prolonged lower-level exposure – impose, however, the largest health hazards for those with impaired respiratory systems (Heal et al. 2012). Estimates also indicate that more than 80% of people living in urban areas that monitor air pollution are exposed to air quality levels that exceed World Health Organization (WHO) limits, that transportation contributes more than half of the many pollutants emitted into the air, and that despite improvements in some regions, urban air pollution continues to rise (WHO 2016).

Empirical evidence shows that road pricing can play an important role in reducing traffic flows and spreading traffic peaks, and thus in reducing and smoothing the emissions of several pollutants over time. The charging of fees to enter congested downtown areas in Europe and the United States has been proven to curb congestion and vehicle emissions and to spread traffic volumes by inducing intertemporal substitution toward unpriced times and spatial substitution toward unpriced roads (see e.g., Gibson and Carnovale 2015, Foreman 2013, and Daniel and Bekka 2000). Time-varying road pricing offers a more cost-effective means of reducing congestion since unlike other policy instruments that raise the cost of all driving regardless of where and when the driving occurs, they encourage people to both use less congested routes and drive a little earlier or later to avoid rush hours. The timing of emissions reduction is important because air quality does not depend only on the emission rates of pollutants, but also on pollution dispersion (see, e.g., Hayas et al. 1981, Viana et al. 2005, and Kim et al. 2012). The scientific literature shows that temporal variations in the meteorological factors that govern air mixing and thus dispersion of locally emitted pollutants (such as wind speed, vertical temperature stratification, and mixing height) can exert strong pressures on the dynamics of air quality. Due to the large temporal variation in these meteorological factors, there is strong average diurnal variation in pollution dispersion in addition to the variation in hourly traffic flows and consequently vehicular emissions (see Toth et al. 2011 and Kim et al. 2012).

This paper investigates the effects in the temporal variation of pollution dispersion on

optimal road pricing. To this end, we develop an ecological economics model of road pricing that takes into account the dynamics of transport-related air pollution. To this end, commuters make decisions about arrival times and travel mode and the regulator chooses a time-varying road charge to maximize social welfare. In particular, we assume that the total number of commuters can choose to commute by either car or public transport. Those who decide to commute by car choose a time of arrival at work and a time of arrival at home to minimize their private trip cost, which consists of three components: the travel time cost, the schedule delay cost, and the time-varying road charge. Moreover, commuters select the transport mode by comparing the cost of a round trip by car with the cost of a round trip by public transportation. Hence, the round trip by each transportation mode is not perfectly inelastic to its price since there is substitution between transportation modes. In such a setting, we characterize the optimal time-varying road charge and compare it with a charge that disregards the temporal variation in pollution dispersion.

The contribution of our paper to the literature is twofold. First, it contributes to a better understanding of economy-ecology interactions in road transportation, as well as practical policy insights since time-varying road pricing designed only to spread out congestion peaks might lead to increased traffic flows when pollution dispersion is the lowest (see e.g., Bonilla 2016).¹ Second, it contributes to the literature on transport economics since although a large literature acknowledges significant differences between morning and evening commuting patterns, the dynamic morning and evening traffic patterns have been investigated separately, and it is often assumed that they are simple mirror symmetries (e.g., Hurdle 1981, De Palma and Lindsey 2002, and Gonzales and Daganzo 2013). However, if pollution dispersion varies over the day, the environmental damage and social costs of road transportation are not symmetric even if the schedule-delay costs for morning and evening commutes are the same. When deciding whether or not to drive a car, the commuters compare the cost of driving (which includes the cost for both morning and evening commuting and is endogenous to the magnitude of the time-varying charge) with the cost of public transportation. Analyzing the effects of road pricing on a setting that captures neither asymmetries in the social cost of road transportation over the day nor the price elasticity of the endogeneously determined demand might lead to over-estimation of the magnitude of the optimal time-varying charge, affecting the political feasibility of this instrument.

¹The fact that temporally varying externalities are better addressed by instruments that follow the variation in damage (and hence the variation in the externality) is well established in environmental economics literature. See Coria (2011) and Coria et al. (2016) for practical examples of where the stringency of environmental regulations is significantly increased to account for the variability in the assimilation capacity of the environment, which poses difficult problems for pollution control policies.

To the best of our knowledge, very few previous studies have analyzed the effects of road pricing on integrated daily commuting patterns (e.g., Zhang et al. 2005 and 2008 who analyze travelers’ behavior in terms of choosing departure times for their morning and evening trips and the optimal time-varying road charges and parking fees based on users’ commuting behavior and bottleneck dynamics). Nevertheless, these studies only focus on congestion and disregard the role of road charges in reducing air pollution and how the dynamics of pollution are affected by the dynamics of travel behavior and variations in pollution dispersion. As for environmental literature, the study that comes closest to ours is one by Coria et al. (2015), who analyze how tolls could/should be designed to minimize the environmental damage from road transportation. Their results indicate that the charges should be higher at times when there are less favorable meteorological conditions for pollution dispersion and when there is an increased contribution from non-vehicle sources to pollution. In contrast to our analysis, their study relies on a series of simplifying assumptions that limit the scope of the environmental benefits derived from the charge and affect its magnitude and political feasibility. In particular, Coria et al. (2015) disregard the effect that a high charge (during either the morning or evening commute) might have on modal choice and do not characterize the first best but focus instead on estimating a time-varying road charge that ensures compliance with exogenously given air quality standards.

The paper is organized as follows. In Section 2, we formulate the model used to characterize the optimal time-varying road charge. In Section 3, we analyze the effects of time-varying pollution dispersion on traffic flows, arrival times, and the number of commuters by car. In Section 4, numerical examples are given to illustrate various equilibrium scenarios. Finally, conclusions are provided in Section 5.

2 The Model

Our analysis builds on Chu (1995) by developing an ecological economics model of integrated daily commuting patterns where the regulator aims to maximize social welfare by choosing a time-varying road charge that takes into account the dynamics of pollution. Let us assume that the total number of homogeneous commuters is N . There is a single origin–destination network connected by a traffic corridor. The origin represents a residential area and the destination a city business center. At the beginning of every day, the commuters travel to the city center for work in the daytime and return home after work. They can choose to commute by either car or by bus. The number of individuals commuting by car corresponds

to N_A (hence, the number of individuals commuting by bus corresponds to $N_B = N - N_A$). All N_A commuters travel m miles to work on the same road. Though they have a common work start time t^* , each of them can choose an arrival time at work t' to minimize the private trip cost $c(t')$, which consists of three components. First, the travel time cost $\alpha \frac{m}{s(t')}$, where α is the unit cost of travel time and $s(t')$ is the travel speed of the entire trip in miles per hour. Second, the schedule delay cost, which corresponds to $\beta [t^* - t']$ if one arrives earlier than t^* and $v [t' - t^*]$ if one arrives later than t^* . Hence, β represents the unit of cost of schedule delay early (earliness) and v is the unit of cost of schedule delay late (lateness). In line with the literature (e.g., Small 1982), we assume that commuters prefer early to late arrival. Therefore, the relative value of the schedule delay cost is such that $\beta < v$. Finally, it is the time-varying road charge $\tau(t')$.² Let t_0 and t_1 represent the times of the first and last arrivals, respectively. Thus, the private trip cost $c(t')$ can be characterized as:

$$c(t') = \begin{cases} \alpha \frac{m}{s(t')} + \beta [t^* - t'] + \tau(t') & \text{if } t_0 \leq t' \leq t^* , \\ \alpha \frac{m}{s(t')} + v [t' - t^*] + \tau(t') & \text{if } t^* \leq t' \leq t_1 . \end{cases} \quad (1)$$

Following Chu (1995), travel speed $s(t')$ is determined by the arrival flow $f(t')$ through a power speed-flow function given by:

$$\frac{1}{s(t')} = \frac{1}{S_{max}} + \left[\frac{f(t')}{R} \right]^\gamma , \quad (2)$$

where S_{max} is the free-flow speed in miles per hour, R the road capacity, and γ the elasticity of the travel delay with respect to the flow $f(t')$. Thus, the second term of equation (2) measures the travel delay associated with the flow $f(t')$.

Like Coria et al. (2015), we assume that the environmental damage from traffic emissions $D(f(t'))$ is a function of the traffic flow $f(t')$ and pollution dispersion $P(t')$ given by:

$$D(f(t')) = \phi e f(t') [1 - P(t')] ,$$

where e is the emissions per vehicle, ϕ is the damage parameter, and $P(t')$ is the rate of pollution dispersion, which can vary with time. That is, the environmental damage from traffic emissions does not depend only on emissions from traffic flow, but also on the fraction of pollution dispersed. Pollution dispersion is assumed to be exogenous and to vary over

²An implicit assumption of the model is that some drivers have flexible schedules and thus, are less constrained by a specific preferred arrival time t^* , but have the option to choose an arrival time so as to achieve better travel conditions.

time within the interval $[0, 1]$.³ Thus, the greater the pollution dispersion $P(t')$, the lower the environmental damage from traffic emissions for any traffic flow $f(t')$. Conversely, the larger the traffic flow, the greater the pollution dispersion needed to keep the environmental damage $D(f(t'))$ low.

Let us start by analyzing the choice of a one-way optimal time-varying road charge. The traffic planner chooses the traffic flow to minimize the social costs of commuting by car (which correspond to the sum of the environmental damages and the private costs of commuting) subject to the constraint that all car commuters must arrive between t_0 and t_1 , i.e., $\int_{t_0}^{t_1} f(t') dt' = N_A$. Thus, his optimization problem can be represented by means of the following Lagrangian where λ is the Lagrangian multiplier.

$$L = \int_{t_0}^{t^*} f(t') \left[\alpha \frac{m}{s(t')} + \beta [t^* - t'] + \phi e [1 - P(t')] \right] dt' + \int_{t^*}^{t_1} f(t') \left[\alpha \frac{m}{s(t')} + v [t' - t^*] + \phi e [1 - P(t')] \right] dt' + \lambda \left[N_A - \int_{t_0}^{t_1} f(t') dt' \right].$$

The first-order condition w.r.t. $f(t')$ yields:

$$\lambda = \begin{cases} \alpha \frac{m}{s(t')} + \beta [t^* - t'] + \alpha f(t') \frac{d}{df(t')} \left[\frac{m}{s(t')} \right] + \phi e [1 - P(t')] & \text{if } t_0 \leq t' \leq t^* , \\ \alpha \frac{m}{s(t')} + v [t' - t^*] + \alpha f(t') \frac{d}{df(t')} \left[\frac{m}{s(t')} \right] + \phi e [1 - P(t')] & \text{if } t^* \leq t' \leq t_1 . \end{cases} \quad (3)$$

Note that the right-hand side of equation (3) can be interpreted as the marginal social cost of arriving at time t' . Comparing the shadow social cost of driving (3) with the private trip cost in equation (1), it is straightforward to say that the optimal charge should be equal to the sum of the congestion externality and the environmental externality (which depends on the pollution dispersion at time t'). Indeed, solving for $\frac{d}{df(t')} \left[\frac{m}{s(t')} \right]$ from equation (2), the optimal charge can be represented as:

$$\tau(t') = \alpha \gamma \left[\frac{m}{s(t')} - T_f \right] + \phi e [1 - P(t')], \quad (4)$$

where T_f denotes the free-flow travel time and is equal to $\frac{m}{S_{\max}}$. Thus, the greater the

³The assumption that pollution dispersion is exogenous is a good representation of the short run. However, scientific literature shows that climate change will have a significant effect on pollution dispersion (see, e.g., Jacob and Winner 2009). Recent studies provide estimates of this climate effect through correlations of air quality with meteorological variables and perturbation analyses in chemical transport models. The results point to a detrimental effect of climate change on air quality: the future climate will be more stagnant due to weaker global circulation and a decreasing frequency of mid-latitude cyclones.

congestion externality, the larger the optimal charge. By analogy, the greater the pollution dispersion, the smaller the environmental externality and the lower the optimal charge. Moreover, even if the congestion externality is the same at two times of the day, the optimal charge may be different at these two times depending on the pollution dispersion. In particular, for the same level of congestion, a higher charge is needed at the times when the pollution dispersion is limited.

To solve for the optimal charge as a function of the parameters of the model, we assume that the pollution dispersion is a linear function of time:

$$P(t') = \rho + \theta t', \quad (5)$$

where ρ represents a background level of pollution dispersion and θ the trend over time. This is to say, pollution dispersion increases over time when $\theta > 0$, while the reverse holds when $\theta < 0$.⁴ In contrast, the optimal charge decreases over time when $\theta > 0$, while the reverse holds when $\theta < 0$.

2.1 Finding the Equilibrium for Car Commuters

Given the optimal charge (4) and our assumption regarding pollution dispersion (5), the private trip cost $c(t')$ corresponds to:

$$c(t') = \begin{cases} \alpha \frac{m}{s(t')} + \beta [t^* - t'] + \alpha \gamma \left[\frac{m}{s(t')} - T_f \right] + \phi e [1 - [\rho + \theta t']] & \text{if } t_0 \leq t' \leq t^* , \\ \alpha \frac{m}{s(t')} + v [t' - t^*] + \alpha \gamma \left[\frac{m}{s(t')} - T_f \right] + \phi e [1 - [\rho + \theta t']] & \text{if } t^* \leq t' \leq t_1 . \end{cases} \quad (6)$$

We know that in equilibrium, those who arrive at t_0 or t_1 should incur no travel delay, since otherwise they could unilaterally reduce their cost by arriving slightly before t_0 or slightly after t_1 . This implies:

$$c(t_0) = \alpha T_f + \beta [t^* - t_0] + \phi e [1 - [\rho + \theta t_0]], \quad (7)$$

$$c(t_1) = \alpha T_f + v [t_1 - t^*] + \phi e [1 - [\rho + \theta t_1]]. \quad (8)$$

To solve for $c(t')$, we use the fact that all commuters should have the same private trip cost $c(t')$ in equilibrium. Moreover, it holds that $c(t') = c(t_0) \vee t_0 \leq t' \leq t^*$, and by analogy,

⁴This simplifying assumption allows us to keep the model mathematically tractable. However, empirical evidence shows a non-linear effect of wind speed dispersing urban air pollution concentration above the background level ρ described above. We circumvent this issue in Section 3.2 by allowing pollution dispersion to vary non-monotonically over the day.

$c(t') = c(t_1) \vee t^* \leq t' \leq t_1$, which yields the following condition:

$$\frac{m}{s(t')} = \begin{cases} T_f + \frac{\beta + \phi e \theta}{\alpha[1+\gamma]} [t' - t_0] & \text{if } t_0 \leq t' \leq t^* , \\ T_f + \frac{v - \phi e \theta}{\alpha[1+\gamma]} [t_1 - t'] & \text{if } t^* \leq t' \leq t_1 . \end{cases} \quad (9)$$

Furthermore, since $c(t_0) = c(t_1)$, we know

$$t_1 = \frac{\beta + v}{v - \phi e \theta} t^* - \frac{\beta + \phi e \theta}{v - \phi e \theta} t_0. \quad (10)$$

Combining equations (2) and (9), we can solve for the traffic flow $f(t')$ as:

$$f(t') = \begin{cases} R \left[\frac{\beta + \phi e \theta}{\alpha m [1+\gamma]} [t' - t_0] \right]^{\frac{1}{\gamma}} & \text{if } t_0 \leq t' \leq t^* , \\ R \left[\frac{v - \phi e \theta}{\alpha m [1+\gamma]} [t_1 - t'] \right]^{\frac{1}{\gamma}} & \text{if } t^* \leq t' \leq t_1 . \end{cases} \quad (11)$$

Integrating $f(t')$ over the time intervals $[t_0, t^*]$ and $[t^*, t_1]$, respectively, yields:

$$\int_{t_0}^{t^*} f(t') dt' = \frac{\alpha \gamma m R}{\beta + \phi e \theta} \left[\frac{\beta + \phi e \theta}{\alpha m [1+\gamma]} [t^* - t_0] \right]^{\frac{1+\gamma}{\gamma}}, \quad (12)$$

and

$$\int_{t^*}^{t_1} f(t') dt' = \frac{\alpha \gamma m R}{v - \phi e \theta} \left[\frac{\beta + \phi e \theta}{\alpha m [1+\gamma]} [t^* - t_0] \right]^{\frac{1+\gamma}{\gamma}}, \quad (13)$$

in which we make use of the relations in (10). Recall that $\int_{t_0}^{t_1} f(t') dt' = N_A$. We know from (12) and (13) that:

$$\alpha \gamma m R \left[\frac{\beta + v}{[\beta + \phi e \theta] [v - \phi e \theta]} \right] \left[\frac{\beta + \phi e \theta}{\alpha m [1+\gamma]} [t^* - t_0] \right]^{\frac{1+\gamma}{\gamma}} = N_A. \quad (14)$$

From equation (14) we can solve for t_0 , which yields:

$$t_0 = t^* - \frac{\alpha [1+\gamma]}{\beta + \phi e \theta} \left[\frac{N_A m^{\frac{1}{\gamma}} [\beta + \phi e \theta] [v - \phi e \theta]}{\alpha R \gamma [\beta + v]} \right]^{\frac{\gamma}{1+\gamma}}. \quad (15)$$

Substituting equation (15) into equation (10) yields:

$$t_1 = t^* + \frac{\alpha [1+\gamma]}{v - \phi e \theta} \left[\frac{N_A m^{\frac{1}{\gamma}} [\beta + \phi e \theta] [v - \phi e \theta]}{\alpha R \gamma [\beta + v]} \right]^{\frac{\gamma}{1+\gamma}}. \quad (16)$$

Finally, since $c(t') = c(t_0)$, we can solve for the cost of a car commuter c_A by substituting equation (15) into equation (7), which yields:

$$c_A = \alpha T_f + \phi e [1 - [\rho + \theta t']] + \alpha [1 + \gamma] \left[\frac{N_A m^{\frac{1}{\gamma}} [\beta + \phi e \theta] [v - \phi e \theta]}{\alpha R \gamma [\beta + v]} \right]^{\frac{\gamma}{1+\gamma}}. \quad (17)$$

Note that the cost for a car commuter does not depend on the time t' , which reflects the fact that in equilibrium he/she can unilaterally reduce the travel cost by changing the arrival time. Moreover, from equations (11), (15), and (16) we can see that the background pollution dispersion ρ does not have a direct effect on the arrival times t_0 and t_1 or the instant flow $f(t')$. Nevertheless, the cost c_A is an increasing function of N_A (and the other way around) and a decreasing function of the parameter ρ . This is to say, even if the background pollution dispersion has no direct effect on the timing or density of traffic flow, it affects these factors indirectly since it reduces the overall social cost of car commuting and therefore increases the optimal number of car commuters. Recall that $\frac{\partial N_A}{\partial \rho}$ can be decomposed as $\frac{\partial N_A}{\partial \tau(t')} \frac{\partial \tau(t')}{\partial \rho}$ and $\frac{\partial N_A}{\partial \tau(t')} < 0$, $\frac{\partial \tau(t')}{\partial \rho} < 0$. Hence, background pollution dispersion ρ reduces the environmental damage from road transportation, and thereby the optimal road charge. Thus, as for the case where pollution dispersion is disregarded, accounting for background pollution dispersion increases the optimal number of car commuters. Furthermore, we have that $\frac{\partial t_0}{\partial \rho} = -\frac{\partial N_A}{\partial \rho} < 0$ and $\frac{\partial t_1}{\partial \rho} = \frac{\partial N_A}{\partial \rho} > 0$, implying that an increased background pollution dispersion will widen the time interval for commuting. Since N_A increases with ρ , so does the time flow $f(t')$. First and last arrival times and the instant flow are also affected by θ in a more complex manner to be analyzed in Section 3.

2.2 Mode Substitutability and Integrated Daily Commuting Patterns

The analysis so far reflects only the cost of a one-way trip. However, morning and evening travel differ in terms of pollution dispersion and scheduling preferences (which for the morning are defined in terms of arrival time at work, whereas preferences for the evening are defined in terms of arrival time at home). To analyze the case of round trips, let us make use of the notations (\rightarrow) and (\leftarrow) to refer to parameters and costs of the morning and evening commute, respectively. Note that if evening commuters also seek to minimize the cost of their own trip, then the user equilibrium for the evening must be a pattern of arrivals that allows no commuter to reduce his/her own cost by choosing another arrival time. Thus, the

cost of commuting by car in the morning and evening is denoted \vec{c}_A and \overleftarrow{c}_A respectively, implying that the total cost of the round trip corresponds to $\vec{c}_A + \overleftarrow{c}_A$.

In equilibrium, the cost of a round trip should be the same for both transport modes. Therefore, we can solve for the number of car commuters N_A by comparing the cost of a round trip by car with the cost of a round trip by public transportation, which yields:

$$2p = 2\alpha T_f + \phi e \left[2 - \left[\vec{\rho} + \vec{\theta} t^* \right] - \left[\overleftarrow{\rho} + \overleftarrow{\theta} t^* \right] \right] + \alpha [1 + \gamma] \left[\frac{N_A m^{\frac{1}{\gamma}}}{\alpha R \gamma} \right]^{\frac{\gamma}{1+\gamma}} \left[\frac{\left[\vec{\beta} + \phi e \vec{\theta} \right] \left[\vec{v} - \phi e \vec{\theta} \right]}{\left[\vec{\beta} + \vec{v} \right]} \right]^{\frac{\gamma}{1+\gamma}} + \left[\frac{\left[\overleftarrow{\beta} + \phi e \overleftarrow{\theta} \right] \left[\overleftarrow{v} - \phi e \overleftarrow{\theta} \right]}{\left[\overleftarrow{\beta} + \overleftarrow{v} \right]} \right]^{\frac{\gamma}{1+\gamma}} \right]. \quad (18)$$

From equation (17) it is clear that in our model, morning and evening commutes are mirror symmetries (implying the same social cost of commuting) when $\vec{\rho} = \overleftarrow{\rho}$, $\vec{\theta} = \overleftarrow{\theta} = 0$, $\vec{\beta} = \overleftarrow{v}$, and $\overleftarrow{\beta} = \vec{v}$. In such case, the pattern of trip timing in the evening is qualitatively similar to that in the morning. Since the evening peak would be a mirror image of the morning with the origin-destination matrix reversed, the number of car commuters can be solved by equalizing equation (17) to the cost of a one-way bus ticket p . However, as discussed by de Palma and Lindsey (2002), empirical differences between morning and evening peaks are apparent and have implications for the potential efficiency gains from congestion pricing, the magnitude of toll revenues, and the impact of road pricing on commuters' private costs. In particular, evening peaks typically last longer and have slightly higher travel speeds. The differences between morning and evening peaks can be explained by a series of factors, including more non-work trips and commuters making more intermediate stop in the evening (which imply more vehicles on the road and greater travel distances but also more dispersion of traffic over the road network). They can be also explained by variations in scheduling preferences by heterogeneous travellers. For instance, work hours are a dominant consideration for many commuters when choosing when to travel. The scheduling preferences of these individuals are defined mainly in terms of arrival time at work in the morning and arrival time at home in the evening (de Palma and Lindsey 2002, page 1807).

As described earlier, the aim of this paper is to investigate the effects of the temporal variation of pollution dispersion on optimal road pricing. Therefore, in the following sections, we shall conduct some comparisons between the optimal number of car commuters and trip timing in the case without pollution dispersion versus in the case with pollution dispersion that varies throughout the day.

3 Effect of Pollution Dispersion on Integrated Daily Commuting Patterns

In this section, we compare the effects of environmental road pricing on mode substitutability and intertemporal substitutability in the case of symmetric schedule preferences vis-a-vis asymmetric schedule preferences. In particular, we compare the number of people commuting by car, arrival times, and the vehicle flow per hour $f(t')$ for each case.

3.1 Symmetric schedule-delay cost and increasing pollution dispersion

We start our analysis by assuming symmetric schedule-delay cost parameters for the morning and evening trips, i.e., $\vec{\beta} = \overleftarrow{v}$ and $\overrightarrow{v} = \overleftarrow{\beta}$ (as in De Palma and Lindsey 2002), implying that the evening commute is the mirror image of the morning commute (e.g., the cost of arriving home late is the same as the cost of arriving to the office early; and the cost of arriving home early is the same as the cost of arriving to the office late). We also assume that pollution dispersion follows a constant and increasing time trend over the whole day, i.e., $\overrightarrow{\rho} = \overleftarrow{\rho} = \rho$ and $\overrightarrow{\theta} = \overleftarrow{\theta} = \theta > 0$. Under these assumptions, we have:

$$\frac{[\vec{\beta} - \phi e \overrightarrow{\theta}] [\overrightarrow{v} + \phi e \overrightarrow{\theta}]}{\vec{\beta} + \overrightarrow{v}} = \frac{[\overleftarrow{\beta} + \phi e \overleftarrow{\theta}] [\overleftarrow{v} - \phi e \overleftarrow{\theta}]}{\overleftarrow{\beta} + \overleftarrow{v}},$$

which implies that equation (18) can be rewritten as:

$$2p = 2\alpha T_f + \phi e \left[2 - 2\rho - [\overrightarrow{t^*} + \overleftarrow{t^*}] \theta \right] + \alpha [1 + \gamma] \left[\frac{N_A m^{\frac{1}{\gamma}}}{\alpha R \gamma} \right]^{\frac{\gamma}{1+\gamma}} \left[\left[\frac{[\vec{\beta} - \phi e \theta] [\overrightarrow{v} + \phi e \theta]}{[\vec{\beta} + \overrightarrow{v}]} \right]^{\frac{\gamma}{1+\gamma}} + \left[\frac{[\overleftarrow{\beta} + \phi e \theta] [\overleftarrow{v} - \phi e \theta]}{[\overleftarrow{\beta} + \overleftarrow{v}]} \right]^{\frac{\gamma}{1+\gamma}} \right]. \quad (19)$$

Equation (19) implicitly defines a function: $G(N_A, \rho, \theta) = 0$. By the implicit function theorem, we know that $\frac{\partial N_A}{\partial \theta} = -\frac{\partial G}{\partial \theta} / \frac{\partial G}{\partial N_A}$. Moreover, after some straightforward calculations (see Appendix A), one can show that $\frac{\partial N_A}{\partial \theta}|_{\theta=0} > 0$, and hence the optimal number of car commuters is larger in the equilibrium with increasing pollution dispersion (compared with the case of only background pollution dispersion). Nevertheless, as mentioned before, the effects of the parameter θ are slightly less straightforward than those of ρ , since θ has both

a direct and an indirect effect on arrival times. On the one hand, an increased value of θ increases the attractiveness of later arrival (since a later arrival will imply a lower road charge $\tau(\vec{t}')$). On the other hand, it also increases the number of commuters by car, which in turn might move the arrival time up since more car commuters need to travel in total). As shown in Appendix A, both effects set against themselves and the final outcome will depend on the extent to which each effect offsets the other. Nevertheless, our analysis suggests that first arrival time will be delayed when the environmental damage of emissions is great. Regarding the last arrival time, an increased value of θ will unambiguously delay t_1 .

A similar argument holds for traffic flows; while it is clear that increased pollution dispersion will increase the instant flow $f(t')$ at those points in time that are close to t_1 , the sign of $\frac{\partial f(t')}{\partial \theta}$ in the time interval $[\vec{t}_0, \vec{t}^*]$ is ambiguous as it depends on the sign and magnitude of $\frac{\partial t_0}{\partial \theta}$. Nevertheless, we can show that if $\frac{\partial t_0}{\partial \theta} \leq 0$, it holds that $\frac{\partial f(t')}{\partial \theta} > 0$ also in the time interval $[\vec{t}_0, \vec{t}^*]$, since the effect of pollution dispersion increasing the number of cars commuting dominates the effect of pollution dispersion delaying the trip (see Appendix A).

Note that the length of the time intervals $[\vec{t}_0, \vec{t}_1]$ and $[\overleftarrow{t}_0, \overleftarrow{t}_1]$ can be calculated as:

$$\vec{t}_1 - \vec{t}_0 = \alpha [1 + \gamma] \left[\frac{N_A m^{\frac{1}{\gamma}}}{\alpha R \gamma} \right]^{\frac{\gamma}{1+\gamma}} \left[\frac{[\vec{\beta} + \phi e \vec{\theta}] [\vec{v} - \phi e \vec{\theta}]}{\vec{\beta} + \vec{v}} \right]^{-\frac{1}{1+\gamma}}.$$

$$\overleftarrow{t}_1 - \overleftarrow{t}_0 = \alpha [1 + \gamma] \left[\frac{N_A m^{\frac{1}{\gamma}}}{\alpha R \gamma} \right]^{\frac{\gamma}{1+\gamma}} \left[\frac{[\overleftarrow{\beta} + \phi e \overleftarrow{\theta}] [\overleftarrow{v} - \phi e \overleftarrow{\theta}]}{\overleftarrow{\beta} + \overleftarrow{v}} \right]^{-\frac{1}{1+\gamma}}.$$

Since we assume symmetric schedule-delay cost parameters for the morning and afternoon commute and a constant trend of pollution dispersion (i.e., $\vec{\beta} = \overleftarrow{v}$, $\vec{v} = \overleftarrow{\beta}$ and $\vec{\theta} = \overleftarrow{\theta} = \theta > 0$), we have:

$$\frac{\overleftarrow{t}_1 - \overleftarrow{t}_0}{\vec{t}_1 - \vec{t}_0} = \frac{\left[\frac{[\overleftarrow{v} + \phi e \theta] [\overleftarrow{\beta} - \phi e \theta]}{\overleftarrow{\beta} + \overleftarrow{v}} \right]^{\frac{-1}{1+\gamma}}}{\left[\frac{[\vec{\beta} + \phi e \theta] [\vec{v} - \phi e \theta]}{\vec{\beta} + \vec{v}} \right]^{\frac{-1}{1+\gamma}}}$$

Given that we know $\overleftarrow{v} > \vec{\beta}$, it is not difficult to show that $[\overleftarrow{v} + \phi e \theta] [\overleftarrow{\beta} - \phi e \theta] < [\vec{\beta} + \phi e \theta] [\vec{v} - \phi e \theta]$ will hold. This implies that $\overleftarrow{t}_1 - \overleftarrow{t}_0 > \vec{t}_1 - \vec{t}_0$. Thus, the evening trip will be more spread out by the increasing pollution dispersion over the day.

Let us calculate the number of cars \overrightarrow{M}_1 and \overrightarrow{M}_2 in the interval $[\overrightarrow{t}_0, \overrightarrow{t}^*]$ and $[\overrightarrow{t}^*, \overrightarrow{t}_1]$ from equations (12) and (13), respectively:

$$\overrightarrow{M}_1 = \int_{\overrightarrow{t}_0}^{\overrightarrow{t}^*} f(t') dt' = \frac{N_A}{\overrightarrow{\beta} + \phi e \theta} / \left[\frac{1}{\overrightarrow{\beta} + \phi e \theta} + \frac{1}{\overrightarrow{v} - \phi e \theta} \right],$$

$$\overrightarrow{M}_2 = \int_{\overrightarrow{t}^*}^{\overrightarrow{t}_1} f(t') dt' = \frac{N_A}{\overrightarrow{v} - \phi e \theta} / \left[\frac{1}{\overrightarrow{\beta} + \phi e \theta} + \frac{1}{\overrightarrow{v} - \phi e \theta} \right],$$

Differentiating the ratio $\overrightarrow{M}_1/\overrightarrow{M}_2$ with respect to the θ yields:

$$\frac{\partial(\overrightarrow{M}_1/\overrightarrow{M}_2)}{\partial \theta} = -\phi e \left[\frac{[\overrightarrow{v} - \phi e \theta] + [\overrightarrow{\beta} + \phi e \theta]}{[\overrightarrow{\beta} + \phi e \theta]^2} \right] < 0. \quad (20)$$

The right-hand side of equation (20) is unambiguously negative. Thus, it is clear that in relative terms, trips are delayed when there is pollution dispersion to take advantage of the reduced charge. Similar arguments can be applied to the evening trip as well (through some straightforward calculation).

3.2 Symmetric schedule-delay cost and non-monotonic pollution dispersion

We keep the assumption of symmetric schedule-delay cost parameters as in Case 1, i.e., $\overrightarrow{\beta} = \overleftarrow{v}$ and $\overrightarrow{v} = \overleftarrow{\beta}$, but in contrast to that case, we assume that the pollution dispersion varies non-monotonically over the day from the background level ρ . For instance, pollution dispersion can increase in the morning due to increasing temperature

but decline in the evening due to temperature decrease (implying that $\overrightarrow{\theta} > 0$ and $\overleftarrow{\theta} < 0$). Conversely, the pollution dispersion in some cities might be decrease during the morning but increase during the evening (implying that $\overrightarrow{\theta} < 0$ and $\overleftarrow{\theta} > 0$).

Let us assume that pollution dispersion progressively increases in the morning hours (and then decrease in the evening) and that the magnitude of the variation in pollution dispersion (though not in direction) is symmetric and equal to $\theta > 0$. These assumptions allow us to specify pollution dispersion capacity as $\overrightarrow{\rho} + \theta \overrightarrow{t'}$ in the morning and $\overleftarrow{\rho} - \theta \overleftarrow{t'}$ in the evening. Equating the two at time \underline{t} (when the trend of pollution dispersion is reversed), we know that $\overleftarrow{\rho} = \overrightarrow{\rho} + 2\theta \underline{t}$. With these assumptions, and since $\overrightarrow{\theta} = \theta$, $\overleftarrow{\theta} = -\theta$, $\overleftarrow{\beta} = \overrightarrow{v}$ and

$\overleftarrow{v} = \overrightarrow{\beta}$, we can rewrite equation (18) in this case as:

$$2p = 2\alpha T_f + \phi e \left[2 - 2\overrightarrow{\rho} - 2\theta \underline{t} + \left[\overleftarrow{t^*} - \overrightarrow{t^*} \right] \theta \right] + \quad (21)$$

$$\alpha [1 + \gamma] \left[\frac{N_A m^{\frac{1}{\gamma}}}{\alpha R \gamma} \left[\frac{\left[\overrightarrow{\beta} + \phi e \theta \right] \left[\overrightarrow{v} - \phi e \theta \right]}{\left[\overrightarrow{\beta} + \overrightarrow{v} \right]} \right]^{\frac{\gamma}{1+\gamma}} + \left[\frac{\left[\overrightarrow{v} - \phi e \theta \right] \left[\overrightarrow{\beta} + \phi e \theta \right]}{\left[\overrightarrow{\beta} + \overrightarrow{v} \right]} \right]^{\frac{\gamma}{1+\gamma}} \right]$$

Note that if $\underline{t} > \frac{\overleftarrow{t^*} - \overrightarrow{t^*}}{2}$, the total cost of driving in equation (21) is lower than the total cost of driving in the absence of pollution dispersion variation. Again, let us define the implicit function $G(N_A, \rho, \theta) = 0$ from equation (21) to compute $\frac{\partial N_A}{\partial \theta} = -\frac{\partial G}{\partial \theta} / \frac{\partial G}{\partial N_A}$. Nevertheless, the effects of θ on N_A will depend on the timing of the reversal of the trend. For instance, as shown in Appendix A, a sufficient (but not necessary) condition for $\frac{\partial G}{\partial \theta} |_{\theta=0} > 0$ is that $\underline{t} < \frac{\overleftarrow{t^*} - \overrightarrow{t^*}}{2}$. This is to say, if pollution dispersion deteriorates for a significant number of hours, the derivative $\frac{\partial N_A}{\partial \theta} |_{\theta=0}$ becomes negative, which implies that the number of car commuters must be reduced to reduce the negative effects of traffic flows on (on average) a stagnant environment.

Regarding the arrival times, the effects of pollution dispersion on the last arrival time will depend on the relative magnitude of the environmental damage and on the effect of pollution dispersion on the optimal number of car commuters (see Appendix A). For instance, if $\frac{\partial N_A}{\partial \theta} |_{\theta=0} < 0$ (e.g., when $\underline{t} < \frac{\overleftarrow{t^*} - \overrightarrow{t^*}}{2}$), the first arrival to the office will be delayed. This will also hold if the environmental damage of emissions is severe. Moreover, the last arrival will be delayed if $\frac{\partial N_A}{\partial \theta} |_{\theta=0} > 0$ or the environmental damage is severe.

Let us analyze the effects of non-monotonic pollution dispersion on the length of the time intervals $\left[\overrightarrow{t_0}, \overrightarrow{t_1} \right]$ and $\left[\overleftarrow{t_0}, \overleftarrow{t_1} \right]$. Given $\overrightarrow{\theta} = \theta$ and $\overleftarrow{\theta} = -\theta$, the ratio $\frac{\overleftarrow{t_1} - \overleftarrow{t_0}}{\overrightarrow{t_1} - \overrightarrow{t_0}}$ corresponds to:

$$\frac{\overleftarrow{t_1} - \overleftarrow{t_0}}{\overrightarrow{t_1} - \overrightarrow{t_0}} = \frac{\left[\frac{\left[\overleftarrow{\beta} - \phi e \theta \right] \left[\overleftarrow{v} + \phi e \theta \right]}{\overleftarrow{\beta} + \overleftarrow{v}} \right]^{\frac{-1}{1+\gamma}}}{\left[\frac{\left[\overrightarrow{\beta} + \phi e \theta \right] \left[\overrightarrow{v} - \phi e \theta \right]}{\overrightarrow{\beta} + \overrightarrow{v}} \right]^{\frac{-1}{1+\gamma}}}$$

Since we have assumed that $\overleftarrow{v} = \overrightarrow{\beta}$ and $\overleftarrow{\beta} = \overrightarrow{v}$, this ratio is equal to one. That is, the non-monotonicity of pollution dispersion does not affect the symmetry of the patterns of trip timing in the morning and evening commute. However, this result does not hold if the time \underline{t} when the trend of pollution dispersion is reversed occurs at some point within the

time interval for the morning/evening commute. Let us assume, for instance, that it occurs in the middle of the time interval for the morning commute. In this case, the overall trend for the pollution dispersion in the morning is zero and $\overleftarrow{t}_1 - \overleftarrow{t}_0 < \overrightarrow{t}_1 - \overrightarrow{t}_0$.

Finally, let us compute the derivative of the ratio $\overrightarrow{M}_1/\overrightarrow{M}_2$ with respect to θ , which yields:

$$\frac{\partial(\overrightarrow{M}_1/\overrightarrow{M}_2)}{\partial\theta} = -\phi e \left[\frac{[\overrightarrow{v} - \phi e\theta] + [\overrightarrow{\beta} + \phi e\theta]}{[\overrightarrow{\beta} + \phi e\theta]^2} \right] < 0.$$

$$\frac{\partial(\overleftarrow{M}_1/\overleftarrow{M}_2)}{\partial\theta} = \phi e \left[\frac{[\overleftarrow{v} + \phi e\theta] + [\overleftarrow{\beta} - \phi e\theta]}{[\overleftarrow{\beta} - \phi e\theta]^2} \right] > 0.$$

This implies that consistent with the previous case, commutes are postponed to take advantage of better dispersion conditions and reduced time-varying road charges in the morning trip. For the evening trip, note we have the trend in pollution dispersion in the evening is $\overleftarrow{\theta} = -\theta$ and therefore the sign of the second equation implies that car drivers commute are relatively earlier to take advantage of better dispersion conditions and reduced time-varying road charges.

3.3 Asymmetric schedule-delay cost and increasing pollution dispersion

So far, our analysis has assumed that the schedule-delay cost parameters are symmetric. In this section, we investigate the case where the schedule-delay cost parameters are asymmetric. Let us compute the ratio $\frac{\overleftarrow{t}_1 - \overleftarrow{t}_0}{\overrightarrow{t}_1 - \overrightarrow{t}_0}$ and evaluate it when $\theta = 0$, which yields:

$$\left[\frac{\overleftarrow{t}_1 - \overleftarrow{t}_0}{\overrightarrow{t}_1 - \overrightarrow{t}_0} \right]_{\theta=0} = \left[\frac{\frac{\overleftarrow{\beta} \overleftarrow{v}}{\overleftarrow{\beta} + \overleftarrow{v}}}{\frac{\overrightarrow{\beta} \overrightarrow{v}}{\overrightarrow{\beta} + \overrightarrow{v}}} \right]^{\frac{-1}{1+\gamma}}.$$

Let us assume that $\overrightarrow{\beta} > \overleftarrow{v}$ and $\overrightarrow{v} > \overleftarrow{\beta}$, which implies that the cost of arriving home late is lower than the cost of arriving to the office early and that the cost of arriving home early is not as high as arriving to the office late. It is possible to show that for such a combination of parameters, it holds that $\overleftarrow{t}_1 - \overleftarrow{t}_0 > \overrightarrow{t}_1 - \overrightarrow{t}_0$, which is consistent with empirical evidence and

implies that the evening commute lasts longer and is more spread out.⁵ Let us now study the effects of pollution dispersion. Differentiating the ratio of the number of car commuters who arrive before and after the desired time with respect to θ yields:

$$\begin{aligned}\frac{\partial(\overrightarrow{M}_1/\overrightarrow{M}_2)}{\partial\theta} &= -\phi e \left[\frac{[\overrightarrow{v} - \phi e\theta] + [\overrightarrow{\beta} + \phi e\theta]}{[\overrightarrow{\beta} + \phi e\theta]^2} \right] < 0, \\ \frac{\partial(\overleftarrow{M}_1/\overleftarrow{M}_2)}{\partial\theta} &= -\phi e \left[\frac{[\overleftarrow{v} - \phi e\theta] + [\overleftarrow{\beta} + \phi e\theta]}{[\overleftarrow{\beta} + \phi e\theta]^2} \right] < 0,\end{aligned}$$

which implies that both during morning and evening commutes, trips are delayed when there is an increasing trend in pollution dispersion in order to take advantage of the reduced time-varying charge. Furthermore, we can show that $\left| \frac{\partial(\overrightarrow{M}_1/\overrightarrow{M}_2)}{\partial\theta} \Big|_{\theta=0} \right| > \left| \frac{\partial(\overleftarrow{M}_1/\overleftarrow{M}_2)}{\partial\theta} \Big|_{\theta=0} \right|$ if $[\overleftarrow{\beta}]^2 [\overrightarrow{v} + \overrightarrow{\beta}] > [\overrightarrow{\beta}]^2 [\overleftarrow{v} + \overleftarrow{\beta}]$, implying that the effect of pollution dispersion is larger during the morning commute. The reverse holds when this condition does not hold. Hence, the relative magnitude of the schedule-delay cost parameters will determine whether pollution dispersion increases the share of trips arriving later than the preferred time during morning or evening commutes the most.

4 Numerical Simulations

In this section, we present a numerical example to complement the analytical analysis above. Table 1 presents the parameters used in the analysis, where the parameters in the first column follow the values used by Chu (1995) and those in the second column are set by the authors.

⁵Note that under these assumptions, it holds that $\frac{\overrightarrow{\beta} \overrightarrow{v}}{\overrightarrow{\beta} + \overrightarrow{v}} = \frac{1}{\frac{1}{\overrightarrow{\beta}} + \frac{1}{\overrightarrow{v}}} > \frac{\overleftarrow{\beta} \overleftarrow{v}}{\overleftarrow{\beta} + \overleftarrow{v}} = \frac{1}{\frac{1}{\overleftarrow{\beta}} + \frac{1}{\overleftarrow{v}}}$.

Parameter	Value	Parameter	Value
N	1000	e	1 unit/vehicle
α	\$ 6.40/hour	ϕ	8
$\vec{\beta}$	\$ 3.90/hour	$\vec{\rho}$	0.3
\vec{v}	\$ 15.21/hour	θ	0.025
\vec{t}^*	8 am	\overleftarrow{t}^*	5 pm
R	3817 vehicles/hour	p	\$ 5
γ	4.08	\underline{t}	13.00
$S \max$	25 miles/hour	$\overleftarrow{\beta}$	case specific
m	15 miles	\overleftarrow{v}	case specific
T_f	37.2 minutes		

Table 1: Parameters for numerical simulation

As mentioned earlier (see Sections 3.1 and 3.2), symmetric schedule-delay cost in a round trip implies that $\vec{\beta} = \overleftarrow{v}$ and $\vec{v} = \overleftarrow{\beta}$. Therefore, let us set $\overleftarrow{\beta} = \$15.21/hour$ and $\overleftarrow{v} = \$3.90/hour$ to reflect this case. For Case 3, where we have asymmetric schedule-delay cost, we instead set $\overleftarrow{\beta} = 12$ and $\overleftarrow{v} = 2$. With these parameters and those in Table 1, one can simulate arrival times, traffic flows, optimal number of commuters, and social costs of commuting by car for our three cases. In what follows, we highlight the comparison across different cases.

	N_A	TCMC Car	TCEC Car	TC All Commutes	Revenues
<i>Case 1</i>					
Optimal Toll	635	5920	4726	14297	5277
Toll NPD	432	5588	5588	16855	7584
<i>Case 2</i>					
Optimal Toll	571	5260	5146	14697	5636
Toll NPD	432	5588	5588	16855	7584
<i>Case 3</i>					
Optimal Toll	816	7855	5792	14538	6736
Toll NPD	562	7397	7142	15487	9867

Table 2: Optimal Charge and Social Costs of Commuting by Car

Through simulations it can be found that the optimal number of car commuters for Case 1 (where we have monotonically increasing pollution dispersion capacity) is larger than in Case 2 (where pollution dispersion deteriorates from 1pm). Moreover, as shown in Table 2, in both Case 1 and Case 2, the optimal number of car commuters is larger than in the case where pollution dispersion is disregarded (e.g., 635 and 571 commuters vs. 432 commuters, which corresponds to an increase of about 32% and 25% in the number of commuters, respectively). Besides, the morning rush hour starts earlier and ends later in Case 1 than in Case 2. Figure 1 also shows the difference in the instant flow of morning trip in the two cases. Not surprisingly, the instant flow is also higher in Case 1. Thus, the results indicate that the monotonically increasing pollution dispersion capacity over the day would allow more people to drive in equilibrium. Figure 2 shows the optimal time-varying road charges for the two cases. It can be seen that during the morning trip, the tolls are higher for Case 1, whereas the tolls are higher for Case 2 during the evening trip. Specifically, as shown in the figure, the optimal toll during the morning peak is \$5.96 in Case 1 and \$5.77 in Case 2, and during the evening peak it is \$4.04 and \$5.60, respectively.

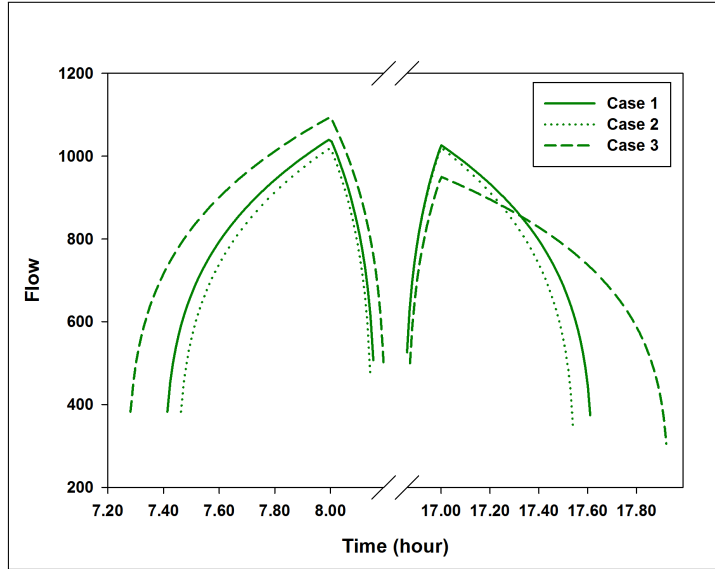


Figure 1: Traffic Flows

Regarding the comparison of Case 1 and 3, it can be found that the optimal number of car commuters in Case 3 is 816, which is larger than that in Case 1. Also, Figure 1 shows that the evening commute lasts much longer and is more spread out in Case 3 than in Case 1. Therefore, we can see that the asymmetric schedule delay-costs are very important in determining the daily commuting pattern and therefore the optimal time-varying charges over the day. The pattern of the time-varying road charge is interesting as Figure 2 shows that in Case 3, the charge must be higher during the morning commute to correct for the higher concentration of travel times and reduced pollution dispersion. The charge is greatly reduced during the evening commute (e.g., the charges during morning and evening peaks are \$6.38 and \$3.54, respectively).

Finally, note that in all cases, the optimal time variation in the charge requires the charge to be lower during the evening commute, while an analysis that disregards pollution dispersion can lead to symmetric (variable) charges. Furthermore, the fact that pollution dispersion could reduce the magnitude of the optimal charges is good news as it increases the political feasibility of this policy instrument.

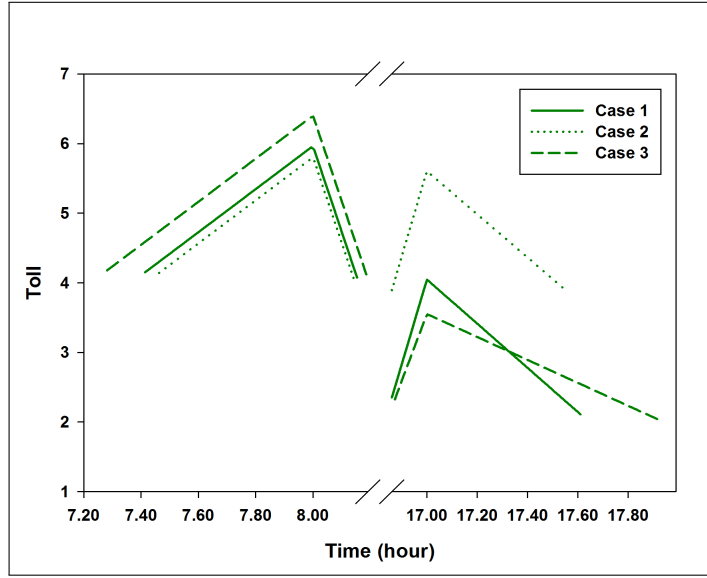


Figure 2: Optimal Road Pricing

Indeed, as shown in Table 2, taking pollution dispersion into account in the optimal design of road charges would not only allow more commuters to drive (compared with the road charges that do not take pollution dispersion into account, denoted as Toll NPD), but would also reduce the social cost of commuting by car in mornings and evenings (denoted as TCMC Car and TCEC Car in the table), as well as the overall cost of commuting (which includes the costs of those trips by bus). For Cases 1 and 2 the reduction in the social cost of commuting is about 15%, and for Case 3 it corresponds to about 7%. Since pollution dispersion reduces the optimal road charges, it does also reduce the total revenues from roads charges (which correspond to about 70% of the revenues of the case when pollution dispersion is not taken into account).

Our numerical simulation is sensitive to the magnitude of the environmental damage. If we, for example, were to increase the magnitude of the damage parameter from $\phi = 8$ to $\phi = 12$, we would find that the optimal number of car commuters is reduced in all cases (corresponding to 554, 478, and 709 for Cases 1, 2, and 3, respectively). We would also observe that greater damage leads to a more concentrated traffic flow and higher tolls compared with the reference cases (optimal tolls at the morning peak would correspond to \$7.77, \$7.58, and \$8.16 for Cases 1, 2, and 3, respectively. For evening peak, the optimal tolls are \$4.92, \$7.26, and \$4.45, respectively).

5 Conclusions

Considering the urgency of improving air quality in many cities and countries around the world, it is important to design and implement environmental policy instruments that restrict emissions when they cause the most damage. Our study generates new insights regarding how road pricing should be designed to maximize social welfare by choosing a time-varying road charge that takes into account the dynamics of pollution. In particular, our results show that by taking pollution dispersion into account, the social costs of commuting can be reduced and traffic flows can be increased. Moreover, the optimal time variation of the charge requires the charge to be lower during the evening commute, while an analysis that disregards pollution dispersion can lead to symmetric (variable) charges. Furthermore, the fact that pollution dispersion could reduce the magnitude of the optimal charges is good news as it increases the political feasibility of this policy instrument. From an analytical perspective, our results show that pollution dispersion breaks the symmetry between morning and evening commutes, even with identical schedule delay costs.

Our analysis is simplified in many respects. For instance, one critical assumption of our model is that the morning and evening travel schedules are independent of each other. That is, the morning scheduling preferences are defined in terms of arrival time at work, whereas the preferences for the evening are defined in terms of arrival time at home; the preferred morning arrival time at work and the preferred evening arrival time at home, however, are separated and predetermined. One idea for further research is to extend our analysis to the case when the morning and evening commuting decisions are more interlinked.

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Appendix A

Case 1: Symmetric schedule-delay cost and increasing pollution dispersion

N_A is determined by equation (19), which defines an implicit function $G(N_A, \rho, \theta) = 0$. By the implicit function theorem, we know that:

$$\frac{\partial N_A}{\partial \theta} = -\frac{\frac{\partial G}{\partial \theta}}{\frac{\partial G}{\partial N_A}}.$$

Differentiating $G(N_A, \rho, \theta)$ with respect to θ and N_A and evaluating when $\theta = 0$ (to account for the marginal variation in outcomes when there is no pollution dispersion variation) yields:

$$\begin{aligned} \frac{\partial G}{\partial \theta} \Big|_{\theta=0} &= -\phi e \left[\overrightarrow{t^*} + \overleftarrow{t^*} \right] < 0, \\ \frac{\partial G}{\partial N_A} \Big|_{\theta=0} &= \left[\frac{m\alpha\gamma}{N_A|_{\theta=0}R^\gamma} \right]^{\frac{1}{1+\gamma}} \left[\left[\frac{\overrightarrow{\beta} \overrightarrow{v}}{\overrightarrow{\beta} + \overrightarrow{v}} \right]^{\frac{\gamma}{1+\gamma}} + \left[\frac{\overrightarrow{\beta} \overrightarrow{v}}{\overrightarrow{\beta} + \overrightarrow{v}} \right]^{\frac{\gamma}{1+\gamma}} \right] > 0. \end{aligned}$$

Thus, $\frac{\partial N_A}{\partial \theta} \Big|_{\theta=0} > 0$, implying that the number of car commuters will be larger with an increasing pollution dispersion than with constant pollution dispersion ($\theta = 0$).

Regarding the effects of pollution dispersion variation on the first and last arrival times, we differentiate equations (15) and (16) with respect to θ and evaluate them when $\theta = 0$, which yields:

$$\frac{\partial t_0}{\partial \theta} \Big|_{\theta=0} = \frac{\alpha}{\beta} \left[\frac{m^{\frac{1}{\gamma}}}{\alpha R \gamma [\beta + v]} \right]^{\frac{\gamma}{1+\gamma}} [\beta v N_A|_{\theta=0}]^{\frac{-1}{1+\gamma}} \left[\phi e [v + \beta \gamma] N_A|_{\theta=0} - \gamma \beta v \frac{\partial N_A}{\partial \theta} \Big|_{\theta=0} \right] \quad (22)$$

$$\frac{\partial t_1}{\partial \theta} \Big|_{\theta=0} = \frac{\alpha}{v} \left[\frac{m^{\frac{1}{\gamma}}}{\alpha R \gamma [\beta + v]} \right]^{\frac{\gamma}{1+\gamma}} [\beta v N_A|_{\theta=0}]^{\frac{-1}{1+\gamma}} \left[\phi e [v + \beta \gamma] N_A|_{\theta=0} + \gamma \beta v \frac{\partial N_A}{\partial \theta} \Big|_{\theta=0} \right] \quad (23)$$

Thus, an increase in θ causes two countervailing effects on t_0 . First, it increases the attractiveness of later arrival. Second, it increases the number of people who will drive, which in turn may move the start time back (since more car commuters need to travel in total). The first effect dominates when

$$\phi e > \frac{\gamma \beta v \frac{\partial N_A}{\partial \theta} \Big|_{\theta=0}}{[v + \beta \gamma] N_A|_{\theta=0}}. \quad (24)$$

This is to say, the first arrival will be delayed (i.e., $\frac{\partial t_0}{\partial \theta} \Big|_{\theta=0} > 0$) when the environmental

effects of emissions are large. In contrast, it is clear that $\frac{\partial t_1}{\partial \theta} |_{\theta=0} > 0$, which implies that an increased pollution dispersion will delay the last arrival for sure.

By analogy, differentiating $f(t')$ in equation (11) with respect to θ yields:

$$\frac{\partial f(t')}{\partial \theta} = \begin{cases} \frac{R}{\gamma} \left[\frac{\beta + \phi e \theta}{\alpha m [1 + \gamma]} [t' - t_0] \right]^{\frac{1-\gamma}{\gamma}} \left[\frac{\phi e [t' - t_0]}{\alpha m [1 + \gamma]} - \frac{\partial t_0}{\partial \theta} \left[\frac{\beta + \phi e \theta}{\alpha m [1 + \gamma]} \right] \right] & \text{if } t_0 \leq t' \leq t^* , \\ \frac{R}{\gamma} \left[\frac{v - \phi e \theta}{\alpha m [1 + \gamma]} [t_1 - t'] \right]^{\frac{1-\gamma}{\gamma}} \left[\frac{-\phi e [t_1 - t']}{\alpha m [1 + \gamma]} + \frac{\partial t_1}{\partial \theta} \left[\frac{v - \phi e \theta}{\alpha m [1 + \gamma]} \right] \right] & \text{if } t^* \leq t' \leq t_1 . \end{cases}$$

Hence, the sign of $\frac{\partial f(t')}{\partial \theta}$ in the time interval $t_0 \leq t' \leq t^*$ depends on the sign of $\frac{\partial t_0}{\partial \theta}$. For points in time where t' is very close to t_0 , the sign of $\frac{\partial f(t')}{\partial \theta}$ will be opposite to that of $\frac{\partial t_0}{\partial \theta}$. As regards $\frac{\partial f(t')}{\partial \theta}$ in the time interval $t^* \leq t' \leq t_1$, we know that for the point in time when t' is very close to t_1 , the sign of $\frac{\partial f(t')}{\partial \theta}$ would be consistent with $\frac{\partial t_1}{\partial \theta}$.

Case 2: Symmetric schedule-delay cost and non-monotonic pollution dispersion

N_A is determined by equation (21), which defines an implicit function $G(N_A, \rho, \theta) = 0$. Differentiating $G(N_A, \rho, \theta)$ with respect to θ and N_A and evaluating when $\theta = 0$ yields:

$$\begin{aligned} \frac{\partial G}{\partial \theta} |_{\theta=0} &= -\phi e \left[\left[2\underline{t} - \left[\overleftarrow{t^*} - \overrightarrow{t^*} \right] \right] - \left[\frac{m\alpha\gamma [N_A |_{\theta=0}]^\gamma}{R^\gamma} \right]^{\frac{1}{1+\gamma}} \left[2 \left[\frac{\overrightarrow{\beta} \overrightarrow{v}}{\overrightarrow{\beta} + \overrightarrow{v}} \right]^{\frac{-1}{1+\gamma}} \frac{[\overrightarrow{v} - \overrightarrow{\beta}]}{\overrightarrow{\beta} + \overrightarrow{v}} \right] \right], \\ \frac{\partial G}{\partial N_A} |_{\theta=0} &= \left[\frac{m\alpha\gamma}{N_A |_{\theta=0} R^\gamma} \right]^{\frac{1}{1+\gamma}} \left[\left[\frac{\overrightarrow{\beta} \overrightarrow{v}}{\overrightarrow{\beta} + \overrightarrow{v}} \right]^{\frac{\gamma}{1+\gamma}} + \left[\frac{\overrightarrow{\beta} \overrightarrow{v}}{\overrightarrow{\beta} + \overrightarrow{v}} \right]^{\frac{\gamma}{1+\gamma}} \right] > 0. \end{aligned}$$

Thus, $\frac{\partial G}{\partial \theta} |_{\theta=0}$ is clearly positive when $\underline{t} < \frac{\overleftarrow{t^*} - \overrightarrow{t^*}}{2}$, i.e., when pollution dispersion deteriorates during most of the day. In such case, $\frac{\partial N_A}{\partial \theta} |_{\theta=0} < 0$, which implies that the optimal number of car commuters is reduced in order to reduce the negative effects of traffic flows in a stagnant environment. Otherwise, the sign of $\frac{\partial N_A}{\partial \theta} |_{\theta=0}$ would be ambiguous.

Regarding the effects of pollution dispersion on arrival times, the results for the derivatives ($\frac{\partial t_0}{\partial \theta} |_{\theta=0}$ and $\frac{\partial t_1}{\partial \theta} |_{\theta=0}$) in the previous section still hold. Moreover, if $\frac{\partial N_A}{\partial \theta} |_{\theta=0} < 0$ or the environmental damage of emissions is severe, it is clear from equation (22) that $\frac{\partial t_0}{\partial \theta} |_{\theta=0} > 0$, which implies that the first arrival to office will be delayed. t_1 will be clearly delayed in the cases where $\frac{\partial N_A}{\partial \theta} |_{\theta=0} > 0$ or the environmental damage of emissions is severe.

Case 3: Asymmetric schedule-delay cost and increasing pollution dispersion

N_A is determined by equation (18), which defines an implicit function $G(N_A, \rho, \theta) = 0$. By the implicit function theorem, we know that:

$$\frac{\partial N_A}{\partial \theta} = -\frac{\frac{\partial G}{\partial \theta}}{\frac{\partial G}{\partial N_A}}.$$

Differentiating $G(N_A, \rho, \theta)$ with respect to θ and N_A and evaluating when $\theta = 0$ yields:

$$\begin{aligned} \frac{\partial G}{\partial \theta} \Big|_{\theta=0} &= -\phi e \left[\left[\overrightarrow{t^*} + \overleftarrow{t^*} \right] - \Gamma \right], \\ \frac{\partial G}{\partial N_A} \Big|_{\theta=0} &= \left[\frac{m\alpha\gamma N_A \Big|_{\theta=0}}{R^\gamma} \right]^{\frac{1}{1+\gamma}} \left[\left[\frac{\overrightarrow{\beta} \overrightarrow{v}}{\overrightarrow{\beta} + \overrightarrow{v}} \right]^{\frac{\gamma}{1+\gamma}} + \left[\frac{\overleftarrow{\beta} \overleftarrow{v}}{\overleftarrow{\beta} + \overleftarrow{v}} \right]^{\frac{\gamma}{1+\gamma}} \right] > 0. \end{aligned}$$

where

$$\Gamma = \left[\frac{m\alpha\gamma [N_A \Big|_{\theta=0}]^\gamma}{R^\gamma} \right]^{\frac{1}{1+\gamma}} \left[\left[\frac{\overrightarrow{\beta} \overrightarrow{v}}{\overrightarrow{\beta} + \overrightarrow{v}} \right]^{\frac{1}{1+\gamma}} \frac{[\overrightarrow{v} - \overrightarrow{\beta}]}{\overrightarrow{\beta} + \overrightarrow{v}} \right] + \left[\frac{\overleftarrow{\beta} \overleftarrow{v}}{\overleftarrow{\beta} + \overleftarrow{v}} \right]^{\frac{\gamma}{1+\gamma}} \frac{[\overleftarrow{v} - \overleftarrow{\beta}]}{\overleftarrow{\beta} + \overleftarrow{v}} \right] > 0.$$

Thus, $\frac{\partial G}{\partial \theta} \Big|_{\theta=0}$ is negative when $\Gamma < \left[\overrightarrow{t^*} + \overleftarrow{t^*} \right]$. In such case, $\frac{\partial N_A}{\partial \theta} > 0$.