

## Interest and Engagement: Perspectives on Mathematics in the Classroom

This book is about interest and engagement in mathematics. The overall aim is to contribute to further understanding of interest manifested as student engagement in mathematics in years 6-9. In particular, the studies capture how engagement is recognised by teachers and researchers and what didactical strategies the teachers use to engage students in an introduction to algebra. Also, tasks seen by students as interesting and engaging are presented and analysed. Unlike other studies, student engagement is discussed in light of the Theory of Didactical Situations in Mathematics (TDS).

The most important results are insights into the relational constitution of engagement. These insights are visible in the interplay between the student, the teacher, the task and the mathematics. The results show that teachers have an important role in engaging students in mathematics during the didactical situation. Teachers seem to agree on how engagement is indicated in the classroom. The strategies for enhancing engagement provided and discussed by the teachers are all a part of the meso-contract. Further, working with the target knowledge in the foreground can enhance student engagement and thus contribute to the development of an adidactical situation.

These empirical findings seem to support the idea that, in order to engage students in mathematics, it is important to design didactical situations and tasks where enhancing engagement is a part of the macro-contract.



**Rimma Nyman** is a teacher and researcher in the field of Mathematics Education. With a school teacher background, she is currently employed as a lecturer in Pedagogical Work at the University of Gothenburg. Her main teaching interests concern teacher education. This project was supported by the Centre of Research in Educational Science and Teacher Research and the Department of Pedagogical, Curricular and Professional Studies at the Faculty of Education.

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INTEREST AND ENGAGEMENT



UNIVERSITY OF GOTHENBURG  
ACTA UNIVERSITATIS GOTHOBURGENSIS

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Rimma Nyman



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## Abstract

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The most important results are insights into the relational constitution of engagement. These insights are visible in the interplay between the student, the teacher, the task and the mathematics. The results show that teachers have an important role in engaging students in mathematics during the didactical situation. Teachers seem to agree on how engagement is indicated in the classroom. The strategies for enhancing engagement provided and discussed by the teachers are all a part of the meso-contract. Further, working with the target knowledge in the foreground can enhance student engagement and thus contribute to the development of an didactical situation.

These empirical findings seem to support the idea that, in order to engage students in mathematics, it is important to design didactical situations and tasks where enhancing engagement is a part of the macro-contract.



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A room with a view at Pedagogen

Gothenburg in 2017

*Rimma Nyman*

## Preface

I became interested in mathematics during a walk in Gorky Park. I was five years old and my grandfather told me about the mystery of  $x$ . Not only did the letter differ from the  $x$  in the Russian alphabet - it could also take different shapes and forms by becoming any number one wanted. As if this was not puzzling enough,  $x$  could also be squared. And cubed! Naturally,  $x$  puzzled me even more. And then, there was  $y$ ... Even though I did not like the idea of  $x$  and  $y$  being trapped in a square, (or even worse, in a cube!), I became eager to find out more about the mystical subject of mathematics, where all this magic took place.

In first grade of Soviet school, mathematics was engaging, presented as a challenging subject that every student was able to master. Our teachers had high expectations regarding student engagement. The qualities of mathematics as beautiful and at the same time useful in engineering and culture were often emphasised. Our enthusiastic teacher let us approach the board on a daily basis and share our ideas with the rest of the class, making mathematics meaningful.

In the beginning of the 90s I moved to a small Swedish town. Throughout years 4-6 being interested and engaged in mathematics was not particularly encouraged. Often negative attitudes towards mathematics were expressed: Perhaps it could be useful when you go to the store, but when else, really? Mathematics was treated with suspicion. Support groups for students who needed extra tuition were common. Mathematics was not magical and mysterious anymore;  $x$  and  $y$  seemed scary. To catch up with the native speakers, I was studying Swedish in one of those support groups. In this group I made a didactical discovery: my peers appeared interested and engaged when they understood new concepts and were able to master different challenges. I was happy to engage in explaining different concepts and showing different strategies for solving mathematical tasks. It was in this support group that I made my first attempts to make mathematics interesting and engaging.

During the secondary school years, I attended an international English-speaking class and later the International Baccalaureate Diploma Programme.

The expectations about being engaged in and learning mathematics increased: both the demanding entrance test and the final examinations in mathematics emphasised the importance of the subject.

At university, during my time as a pre-service teacher, I wondered about the connections between mathematics as a discipline and the field of mathematics education. Mathematics as a discipline was emphasised as an important part of my undergraduate programme. Mathematics education was not. Out of 90 credits in mathematics for teachers, only one course was in mathematics education/didactics. It provided 0 credits and consisted of one workshop. Burning with beginner's enthusiasm, I started to attend additional courses in didactics, listening to experienced teachers, teacher educators and researchers. I took every given opportunity to teach; one of these opportunities was provided to pre-service teachers at one of the largest high schools in the city. Engaging students in whole-class activities was a challenge. My first lesson involved 32 dropout students, who struggled to complete a course. Later, as an in-service teacher at a private school, I had a different experience of student engagement: small classes where the students "always knew everything", lost interest if the tasks were not challenging enough and refused to leave their comfort zone of silent textbook work at their own pace.

This background reveals some of my personal experiences of interest and engagement in mathematics, from a student's and a teacher's point of view. When I became a teacher educator, I started to reflect on the concept of interest and engagement from a theoretical perspective. No matter what type of school I taught in or what type of students or colleagues I met - the same question arose: How do we interest and engage students in the content matter we are about to teach? To try to answer this question, I wrote an essay on the topic of interest in mathematics, in particular looking into teachers' beliefs.

And now, nearly 30 years after my grandfather engaged me in the mystery of  $x$  and  $y$ , I explore the concepts of interest and engagement in mathematics as a researcher.

# 1 Introduction

Teacher: There is a house to build. Children get it, you have to build from the foundation before you can build a chimney and if we have not built the foundation, everything will fall apart. Interviewer: And interest is the glue that holds the walls together? Teacher: Yes, it is and it is my obligation to make sure that interest [develops]. (Emanuelsson, 2001, p. 79, my translation)

## 1.1 Why study interest and engagement in mathematics?

The gateway to this project is the oft-alleged lack of interest and engagement in mathematics in school (Mitchell, 1993; SOU 2004:97; Kim, Jiang & Song, 2015). The concepts of interest and engagement have been widely researched within a broad range of educational approaches. Dewey, for example, approached them as concerning school improvement (Dewey, 1913); Hidi, in contrast, takes a psychological approach, where cognitive and affective features of interest contribute to motivation (Hidi, 1990; Hidi, Reninger & Krapp, 2004). There are literature reviews (e.g. Silvia, 2006) that indicate a vast body of research on both interest and engagement in educational settings, covering a wide interpretation of engagement, directed towards various aspects of education. However, there are a smaller number of subject-specific studies of interest and engagement in relation to content matter and, specifically, the way content is handled in the mathematics classrooms. That literature is more thoroughly dealt with in Chapter 2. This thesis intends to add to the area of research by means of studies of interest and engagement in mathematics activities in lower secondary school seen from the perspectives of researchers, teachers and students.

Dewey (1916/1997) described interest as an active state:

To be interested is to be absorbed in, wrapped up in, carried away by, some object. To take an interest is to be on the alert, to care about, to be attentive. We say of an interested person both that he has lost himself in some affair and that he has found himself in it. Both terms express the engrossment of the self in an object. (p.126-127)



Dewey developed the idea of interest beyond personal interests and hobbies (Jonas, 2011) and his view of interest as specifically manifested through engagement was visible when he described interest as directed towards an object:

By an interest we also mean the point at which an object touches or engages a man; the point where it influences him. (Dewey, 1916/1997, p.126)

In other words, he made connections between interest and engagement: by describing a person as interested in something, it can be said that he is engaged. This way of seeing engagement as a manifestation of interest is an important insight for this thesis.

This view, that interest can be visible to an observer as expressed by engagement directed towards something, is helpful in a classroom context. As Frenzel and his colleagues (2010) conclude:

Contemporary approaches define interest as a motivational variable that refers to an individual's engagement with particular classes of objects and activities. (p. 509)

There are several reasons to study interest as engagement, the main one being their importance in relation to learning (Dewey, 1913). There is a reciprocal relationship showing that interest as an attitude affects learning and vice versa (Ma, 1997; Schraw & Lehman, 2001); there are opportunities to learn when one is interested and, likewise, when one learns, interest flourishes. Hidi and Reninger remind us: "the level of a person's interest has repeatedly been found to be a powerful influence on learning" (Hidi & Reninger, 2006, p.111). Interest is thus an important motivational factor (Schiefele, 1991, Ainley, 2012) and, seen from teachers' perspectives, engaging students is a constituent of good mathematics teaching (Wilson, Cooney & Stinson, 2005; Appleton & Lawrenz, 2011).

In Sweden, the importance of interest is emphasised on the level of national curriculum. The development of interest is one of the official aims of mathematics as a school subject. In the curriculum for the compulsory school, it is explicitly stated that "teaching should develop their [students'] interest towards mathematics" (Skolverket, 2011, p.59). Schools use the term engagement, for instance on their web pages, when they describe their work and visions related to learning. For example, one Swedish school formulates it thus:

## INTRODUCTION

The vision of our school is 'Engagement, Joy and Learning in a Safe Environment'. Activities should be permeated by the vision and include a norm critical perspective in the organisation, in daily work and teaching. (Author's translation of xx school's vision)

A central basic assumption adopted for the purposes of this thesis is that while interest is a cognitive and affective attribute of the individual, “composed of intrinsic feeling-related and value-related valences” (Schiefele, 1991, p.299), it is manifested, made visible, and made available for scrutiny through engagement in classroom activities (Dewey, 1913; Schraw, Flowerday & Lehman, 2001; Kim, Jiang & Song, 2015). Delimiting the concepts in this way strives for conceptual clarity of relevance for studies in classroom contexts, with the aim of approaching the theoretical concept of *interest* through the empirically operationalised concept of *engagement*. This assumption, that interest is manifested as engagement, is intended to help me as a researcher in capturing teachers' and students' views in a way that is relevant to classroom practices, and to contribute to further clarification of the meaning of the terms interest and engagement (Harris, 2008).

In educational research, interest is primarily assumed to be an inner state, addressed as a static attitude and therefore used as an independent variable in questionnaires (e.g. Rellensmann & Schukajlow, 2016). The meanings of the variable itself are seldom investigated or exemplified on a classroom level. Engagement, on the other hand, is approached as a classroom construct, visible to outside observers (Appleton & Lawrenz, 2011), acknowledged and reflected on by teachers (Wilson et al., 2005; Exeter, Ameratunga, Matiu, Morton, Dickson, Hsu & Jackson, 2010; Harris, 2008; 2011). It is a didactical challenge for the teachers to engage students in mathematics; indeed, as Hargreaves says, it is one of the greatest challenges in an educator's career (Hargreaves, 1986).

Teachers who are capable of identifying and acknowledging student engagement are the ones who use the most effective practices for engaging students in mathematics (Skilling et al., 2016). In previous work (Nilsson, 2009), I approached interest towards mathematics from the perspectives of experienced teachers, relating their reflections to teachers' beliefs and conceptions as described by Thompson (1992). That study showed that teachers with a problem-solving orientation within the framework of belief systems viewed interest in classroom situations as subject-specific. Also, interest towards mathematics was seen as important by teachers and, from

their point of view, beneficial for learning. Skilling et al. (2016) showed that teachers' perception of being powerless to engage students in mathematics results in limited efforts to attempt interventions.

A large-scale study (Appleton & Lawrenz, 2011) has shown that engagement can be perceived differently by teachers, students and outside observers. I aim to explore engagement from the perspectives of the researchers, the teachers and the students, with support from previous research and with tools and terminology from a theory on teaching mathematics. The intention of my contribution is to provide further insights into how interest as engagement can be developed in mathematics classrooms. An empirical approach to interest as manifested through engagement in mathematics in classroom contexts is adopted to investigate lower secondary mathematics classrooms, by using various analytical frameworks (Silvia, 2010; Helme & Clarke, 2001; Smith & Stein, 1998) and the Theory of Didactical Situations in Mathematics, TDS (Brousseau, 1986; 1997), as appropriate to different stages of the study. These will be elaborated in Chapter 3.

## 1.2 Aim and research questions

This thesis is comprised of four papers based on three empirical data sets, and this, the *kappa*. The kappa is intended to bring the four papers together as a whole, with regard to background, theory and methodology, as well as considering their results as a whole.

All of the papers are related to mathematics classrooms, positioning them in the field of mathematics education research as oriented towards practice (Wittmann, 1995). The common denominator for the four papers is this practice-orientation approach, which is taken in order to answer questions that have emerged through teaching practice and research on teaching practice as well as my own experience as a teacher and teacher educator.

The overall aim of this thesis is to gain further understanding of interest manifested as student engagement in mathematics. A first step was to identify the manifestation in classroom practice. This was investigated from both teachers' and outside observers' perspectives (Papers I and II), using previously established frameworks, with the aim of answering the following questions:

## INTRODUCTION

- What do students attend to during student-teacher interaction about specific mathematics tasks? How is interest co-constructed in such situations? (Paper I)
- How are indicators of engagement identified, negotiated and exemplified by teachers? (Paper II)

The results of these studies led to questions about teachers' roles in engaging students in mathematics during classroom practice. A third question was therefore posed and analysed in the tradition of Brousseau's TDS:

- What didactical strategies do teachers propose for engaging students when introducing algebra? (Paper III)

The results of this study showed that teachers' strategies tended to neglect the mathematical content in favour of classroom activity. Based on these results, a new study was designed to find out what students thought after being in a classroom where the teacher had specifically focused on the mathematics to provide learning opportunities and to engage them in it. Thus, new research questions were posed:

- What tasks do students identify as interesting and engaging when the teacher has deliberately brought the mathematical content into the foreground? What features are interesting and engaging in those tasks? (Paper IV)

The research questions linking the four studies together were developed in a generative process, with research questions for Papers III and IV being generated from the insights of Papers I and II.



## 2 Literature review

It is psychologically impossible to call forth any activity without some interest. (Dewey, 1903, p. 7)

This section provides a background on interest as manifested through student engagement. The concepts are put forward as a means of situating the empirical studies of this research project as a whole. Main themes in related research are presented and explicated; the concepts are discussed in light of previous studies and in relation to classroom situations.

### 2.1 Interest as manifested through engagement

Interest and engagement are closely related concepts; they both have many facets and they are hard to define in a unified manner. The word *interest* originates from Latin *inter-esse* and has the etymological meaning of being *in-between* (Dewey, 1916/1997, p.127). To engage someone means to get and keep his or her interest (Dewey, 1913; Silvia, 2006). Jonas (2011) made a conceptual analysis of how Dewey uses the term interest, and highlights the essence of the concept:

Interest acts as the psychical connector between the object and the individual; it is like a psychical bridge - it connects the consciousness with some otherwise ostensibly independent object. (Jonas, 2011, p.115)

Interest as an attitude has been interpreted within several research traditions and discourses (Silvia, 2006), in connection to many school subjects. In mathematics education, previous studies on students' attitudes towards mathematics have mainly focused on emotions and not on observable categories of attitudes (Hannula, 2002). Personal aspects of interest have been studied, for instance seeing interests as synonymous with hobbies, and how to integrate these into mathematics teaching (Ball, 1993). Interest has been approached in terms of students' activities outside the classroom context, as a student's latent, inner emotional state - an approach that is not necessarily related to interest in the subject in classroom settings (Silvia, 2006). Interest in mathematics, and especially the lack of it, has been discussed in the light of

influences of Western culture and the politics of our modern society (Valero, 2015). Even when interest has been approached as an attitude in a specific situation - so-called situational interest (Mitchell, 1993) - the questions raised have concerned the general aspects of teaching. Therefore, there is a need to specify interest in relation to mathematics teaching, focusing on subject-specific features brought up in mathematics classrooms.

My intention is to investigate how interest can be described in a content-related way, in order to actualise the concept for mathematics teachers and researchers. How can the identification of interest be operationalised on the classroom level, and hence made visible to an observer? As proposed in the introduction, interest can be approached as manifested through engagement. Dewey (1910) had this view when he emphasised the role of interest in the process of engagement. On several occasions he described interest as manifested through engagement, for example:

Children engage, unconstrainedly and continually, in reflective inspection and testing for the sake of what they are interested in doing successfully. (Dewey, 1910, p.154)

In his early work on interest and effort in education (Dewey, 1913), he specifically stressed the connection between interest and engagement, seeing engagement as evidence of interest:

Persons, children or adults, are interested in what they can do successfully, in what they approach with confidence and engage in with a sense of accomplishment. (Dewey, 1913, p.36)

His way of discussing the concept of interest opens up possibilities for empirical studies, as in the approach adopted in this thesis.

In a study of teachers' conceptions of student engagement, one of the six different conceptions found was "being interested in and enjoying participation in what happens at school" (Harris, 2008, p.65). During the interviews with the teachers, multiple participants made similar statements related to student interest, such as that interesting lessons and topics, or something that really interests the students, make students engaged. There are other empirical studies that also show that interest can be manifested through engagement in classroom activities (Schraw, Flowerday & Lehman, 2001; Kim, Jiang & Song, 2015).

Azevedo, diSessa and Sherin (2012) present a view on domain-specific student engagement as the intensity and quality of participation in a classroom activity. They model engagement in connection to students' conceptual competence in specific mathematics and physics content, for example the quality of the activities that emerged when dealing with mathematics of motion. The framework is based on a spatial metaphor, describing the mathematics classroom activity as a territory through which students move. It captures common engagement-related dynamics, the nature of the regions and overall topography of a so-called activity territory, and what kind of movement such a territory metaphor offers. This means that engagement in mathematics can be linked to the ways in which the teacher and the students deal with specific content matter during interaction.

Another important aspect of the research presented in this thesis is the idea of approaching student engagement as a dynamic process, which is in line with previous research (Wilson et al., 2005; Harris, 2008; 2011), as well as with my professional experience of the complexity of mathematics classrooms. By operationalising interest through the concept of engagement, I am making the assumption that studying interest in mathematics will be more fruitful if approached as mediated through a more empirically grounded construct. This approach gives an opportunity to build on empirical research within the field of student engagement.

### **2.1.1 Interest and engagement in relation to learning mathematics**

In this thesis, interest and engagement are in focus because they are seen as beneficial for learning (Wilson et al., 2005; Exeter et al., 2010; Harris, 2011). Interest as an attitude has a reciprocal relationship to learning, a relation that Ma (1997) has established by means of a questionnaire on students' attitudes towards mathematics and mathematics achievement tests. In that study, structural equation modelling showed that interest as an attitude can contribute to learning (Figure 1).



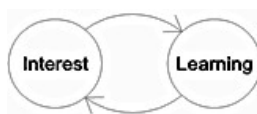


Figure 1: The reciprocal relation of interest and learning. Interest contributes to learning and vice versa.

Further, Ma and Kishor (1997) performed a meta-analysis of 113 studies on attitudes towards mathematics, where interest in mathematics was one of the factors linked to learning. They showed that interest in mathematics could statistically be linked to performance and achievement. This meta-analysis treats interest as affect and emotion, with an emphasis on interest as a form of enjoyment, for instance when a student states “I like mathematics”. How interest is expressed in classroom settings was not in focus in any of the studies comprising the meta-analysis, but Ma (1997) stresses that students with high levels of mathematics achievement do not automatically enjoy mathematics. The teacher’s role is important, since “instructional measures that help students enjoy learning mathematics can make a difference in mathematics achievement” (p.288).

Samuelsson (2011) shows a statistically significant correlation between interest and test results, which yet again underpins the important role of interest in relation to learning. Similarly, Baumert and Schnabel (1998) promote the importance of interest for academic achievement, and base their arguments on empirical findings. Their investigation concerned the relationship between academic interest and achievement in the subject of mathematics.

Interest is considered to be a driving force in learning by OECD (2004). This is based on PISA results of students’ responses to a series of research-based questions, where students with a negative score responded less positively to mathematics than students on average across different OECD countries. Likewise, the results showed that a student with a positive score responded more positively than an average student (OECD, 2004). Suggestions concerning how students can be engaged in mathematics are for teachers to have high expectations of their students and to actively include students in classroom practice.

Recent PISA results (OECD, 2015) show that Swedish students appear to be motivated to learn mathematics, but that at the same time student performance has declined. One explanation could be that attitudes are studied as a dichotomy: enjoyment and interest (intrinsic motivation) and/or usefulness for future studies and career (instrumental motivation). The students claim that they are interested, which in this case might mean that they have a positive attitude towards mathematics, but do not necessarily express it on a classroom level, i.e. do not engage, and therefore do not learn.

There have been studies leading to evidence-based results on how interest in a situation is a condition for learning. Bikner-Ahsbals (2002; 2004; 2015) shows how interest in mathematics emerges in situations where there is interaction between teacher and students on a collective level. She developed a Theory of Interest-Dense Situations, which treats interest as a psychological construct, “a personal or social feature reflecting a genuine engagement in mathematical activity” (Radford, 2008). The phenomenon referred to as an interest-dense situation captures how students get involved in an activity and become a part of a dynamic and epistemic process. Situations including these processes are of such a nature that collective interest emerges (Bikner-Ahsbals, 2004); the students reach deeper mathematical meanings together. This theory also connects the concepts of interest and engagement, by showing that interest-dense situations are situations where more and more students also start to engage in mathematics. The density of interest is seen as high on a group level when most students engage in the content matter. A situation is interest-dense if students indicate interest-based actions, for instance expressing the will to learn and to understand, to actively report a completed project, asking questions about mathematical content matter, sharing ideas, expressing a will to learn and understand. Also, in order for a situation with high density of interest to occur, the students must develop further knowledge in a common mathematical content. When a student consciously experiences involvement and meaningfulness concerning the content, one can claim that he/she is interested. In a classroom, students can express and share their mathematical ideas with each other and the teacher. The teacher’s role, according to Bikner-Ahsbals’ (2004) study, is to initiate interest as a part of the learning process, making more and more students collectively engaged.

Mitchell (1993) studied how different components of the classroom environment affected situational interest in mathematics. He puts forward a

hypothesised construct of catching and holding interest, by means of meaningfulness and involvement in different activities. Based on this construct, he developed a survey including components such as group work, computers and puzzles. He tested the model on 350 high-school students from three different high schools in USA and found that active involvement is important when catching and holding interest. Unfortunately, student engagement was not studied as a subject-specific construct in Mitchell's study. The model of situational interest leaves room for different types of subject-specific elements that can catch and hold interest rather than assuming that the student is either involved in an activity or not.

Similarly to Mitchell, I seek specific components of the classroom environment that elicit interest and engagement, but in connection to mathematics as content matter and identified by outside observers, teachers and students. For the purpose of my studies, it is appropriate to approach interest empirically as being manifested by student engagement and therefore observe it.

### **2.1.2 Interest as a research theme**

Historically, interest has been a source of fascination. Early ideas about interest in educational settings can be traced back to Herbart and Smith (1895) as well as Dewey (1913), all of whom emphasise interest as an important factor in relation to learning. Herbart and Smith had many concerns about the concept of interest and the role of interest in education in general as well as in mathematics teaching and learning in particular. They specified interest as being important in educational settings, speaking of interest in general terms, as altruistic or selfish. Their definition of interest was a psychological state, a latent attribute, compared to thoughts and desires, in connection to action and different interaction of concepts. Their main idea was that the teacher should make subject matter interesting to the students by appealing to their emotions and imagination. Another contribution they made to the field of interest is the link between interest and attention; attention and expectation are the two aspects of interest, which "belong likewise to the fundamental notions of general pedagogy." (p. 259).

Dewey (1913) defined interest as a guarantee of attention, highlighting the relation between interest and engagement by saying that engagement can

serve as evidence of student interest. Brousseau (1997) connects the degree of interest of a problem to engagement:

The didactical interest of a problem will depend in an essential way on what the student will engage in, what she will out to test [sic], what she will invest. (p.83)

In other words, Brousseau (1997) also linked interest to student engagement. In his view, interest is connected to a specific activity; it is an active state, meaning that we take interest in a problem or a mathematical task by engaging with it.

Dewey (1913) compared interest-oriented learning, where students' interest is in play, with effort-based learning, which is a mechanical activity. He distinguished direct interest from indirect interest, where direct interest is an emotional state within an individual. Indirect interest is also an emotional state, but is instead developed in a context. Here is an example provided by Dewey (1913):

Many students of a so called practical make-up, have found mathematical theory, once repellent, lit up by great attractiveness after studying some form of engineering in which this theory was a necessary tool. (p. 22)

In order to interest students in classroom settings, specific teaching strategies are applied, for instance “providing students with a variety of materials and educational opportunities that capitalized on their existing preferences and motivation” (Schraw & Lehman, 2001, p. 25). Dewey argued that when choosing subject matter, a teacher could make it interesting by selecting the content with the students' experience and pre-knowledge in mind.

Dewey was one of the first to talk about intrinsic qualities of interest as a motive for attention, that is, the inner factors that make us pay attention. He explained that it is not enough to catch someone's attention in order to claim that that person is interested; the attention must be sustained.

In the work of Dewey as well as in that of Herbart and Smith, interest is split into two main categories: internal and external. Internal interest is connected to direct interest within the person, and external is indirect, including outside influence contributing to stimulating the direct, inner interest. This dualistic view on interest is described as including intrinsic (inner) and extrinsic (outer) elements. This point of view emphasises the inner quality of interest. Later research also distinguishes between the two domains,

for instance using the terms individual/personal interest and situational interest (Hidi, 1990; Renninger, 1992). In research connected to the individual, examples of interest as an inner quality emerged. Even though studies continue to deal with interest as a dichotomy, describing it as an internal/external state, the term has developed from being somewhat trivial to becoming content specific (Hidi, 1990; Bikner-Ahsbahs, 2003). This type of research has been conducted in many different areas in the field of education (Silvia, 2006), for example, attempts to bridge the dualistic view of the concept of interest in theoretical as well as empirical studies (Bikner-Ahsbahs, 2003; Krapp, 2007).

The overemphasis on the role of individual interest in mathematics has been questioned, by for example Firsov (2004). When he established the principle of interest, based on teaching experience and research, he stated that interest leads to learning - if a student is interested in a subject, he/she will succeed. He criticised the intention to maximise individual interest, a pre-existing personal interest, which is an attribute that is already present when a student attends a lesson. According to Firsov (2004), individual interest in mathematics is rare and not a necessary condition for students to succeed. He questioned the positive effects of actions that try to maximize this type of interest. The question of whether interest is a condition for learning must be posed in a different way, he states, to focus on aspects other than students' personal interest in mathematics.

Instead of pursuing an ambitious goal of developing “fundamental” interest in mathematics, we could pay more attention to a modest goal of making a particular math lesson more interesting for an individual child. (Firsov, 2004, p. 333)

In other words, Firsov (2004) favours teachers focusing on situational interest rather than assuming individual interest. Other attempts to approach situational interest have shown how a more stable individual interest develops (Hidi, Renninger & Krapp, 2004). A four-step model of interest development in learning situations (Hidi & Renninger, 2006) consists of four interrelated categories of interest: triggered, maintained, emerging and well-developed interest. Interest is seen as qualitatively different on different levels and includes affective as well as cognitive factors.

This model of interest was applied by Samuelsson (2011) in a study of interest development, where 219 students (age 13-16) in 10 different classes in

10 different upper secondary schools participated. By using PISA 2003 questionnaires and pre/post tests, he found a strong correlation between interest as an affective factor and students' achievements. He writes about the differentiation between interest as an inner state and an outer process, and tries to bridge the two by using the above-mentioned four-step model, according to which situational interest in mathematics can develop into a more stable individual interest.

Interest is a motivational factor, since it is a central component for the student to be motivated in learning (Dewey, 1913; Renninger, 1992). Findings show that interest is related to self-oriented, intrinsic motivation (Schiefele, 1991). However, when interest is studied in motivation research, it is linked to a set of underlying motives that contribute to participation in activities. Motivational psychology includes quantitative research where interest is treated as curiosity, with *motives* in focus. This focus helps in answering the question of *why* students are interested, instead of *how* interest is visible or *what* the students are interested in. The question *why* is often answered in terms of students' goals, and goals of such a nature could be non-mathematical: the student is interested in order to get good grades, to win the teacher's approval, to get home earlier or to impress other students.

The present thesis does not neglect such goals, but they are not in focus. The focus is rather on what happens in the classroom: in which ways students are interested and engaged, how students deal with certain tasks and content matter, and how certain tasks can be structured to be interesting and engaging. In other words, whatever the motives of the students are, the didactical question remains: What can be done within the limits of a lesson in order to interest and engage the students in mathematics?

### **2.1.3 Student engagement as a research theme**

Student engagement has been an object of study where the interactive aspects of interest are analysed. Engagement is generally described as a multidimensional construct (Harris, 2008), including behavioural, cognitive and emotional components (Fredricks, Blumenfeld, & Paris, 2004). On a classroom level, cognitive engagement has been described as deliberate task-specific thinking that a student expresses by participating in a classroom activity (Helme & Clarke, 2001; 2002).

In order to engage students in mathematics, the purpose rather than process must be promoted (Schoenfeldt, 1992). He defines a mathematical problem as a task where one condition is for the student to be “interested and engaged and for which he wishes to obtain the resolution” (p.72). The purpose needs to be visible and clear to the students, an important insight when aiming to understand the nature of interest and engagement in mathematical content.

Weiss (1990) claims that mathematics teaching must focus on active involvement and that student-centred activities are advantageous when attempting to interest and engage students. Examples of such activities in the mathematics classroom are experiments, workshops and projects. The question remains as to how such activities should be structured and what content matter they should contain in order to raise the level of interest and to engage students. How can a student-centred approach be combined with a content-centred one?

Boaler (1999) presented findings about participation in different classroom activities from longitudinal case studies. With support from data in two mathematics classrooms over a period of two years, she showed that students who engage in their mathematics learning, rather than simply practising procedures, were able to achieve good results. Later, Boaler (2000) also conducted interviews with 76 students from six different schools and found how the classroom communities, the environment and the activities the students participate in are of great importance. She found that algebra was a challenging area to engage students in, since it was difficult to relate the content to students’ everyday life or the outer world.

Azevedo and his colleagues (2012) studied student engagement on a classroom level and according to their results, engagement is a function of students’ conceptual competences in specific content (Azevedo et al., 2012). Some activities are instantly engaging: “students show excitement and commitment to ideas they generate” (p. 276). The initial engagement in an activity can be developed into a sustained engagement. In their model, engagement is seen as a function of students’ conceptual competences in a specific content area. Here, similarly to the four-step model of interest development discussed previously (Heidi & Renninger, 2006), initial engagement in an activity can develop into sustained engagement, with support from a teacher. A teacher can support students by allowing students to problematise the content, by empowering them to address the problems with their own authority and by providing relevant resources.

Liljekvist (2014) stated, based on the results of several empirical studies, the importance of the kind of mathematical tasks students engage in and what within those tasks they engage in. For instance, task design that encouraged engagement in creating one's own solutions contributed to better performance on tests than tasks with given methods. This reasoning leads to further questions about what specifically within a task can be perceived as interesting and engaging.

Teachers' views on student engagement influences their teaching, such as their responses to students and their efforts in the classroom (Skilling et al., 2016). For instance, perceptions of being powerless to engage students in mathematics resulted in teachers' limiting their own efforts to attempt interventions. In spite of this, teachers themselves have specific suggestions on how to engage students in mathematics. In a study by Wilson et al. (2005), nine experienced mathematics teachers are interviewed about what good teaching is. Suggestions on how to engage students are made, with emphasis on group work, moving students around in the room, meeting the students at their mathematical level but at the same time later challenging them. Traditional ways of teaching mathematics, such as teacher lecturing were not considered as engaging as group work and opportunities for the students to exchange ideas, explain to each other how to solve problems. The emphasis was on the level of classroom management and did not involve any intra-mathematical, content-related suggestions.

#### **2.1.4 What makes algebra engaging?**

Since the body of research on student engagement in general, and in mathematics in particular, brings out strategies related to classroom organisation, one can wonder if mathematical content in itself can be engaging for the student. Rellensmann and Schukajlow (2016) found that students experience high levels of interest when solving purely intra-mathematical problems. Looking at algebra teaching and learning, there are different interpretations of what algebraic thinking is and what can make it engaging. Several experts in the field of mathematics education suggest that *generality* is the core of algebraic thinking:

At the very heart of algebra is the expression of generality. Exploiting algebraic thinking within arithmetic, through explicit expression of generality makes use of learners' powers to develop their algebraic thinking and hence to appreciate arithmetic more thoroughly. Algebraic symbols are



a language for expressing generalities. As fluency and facility with expressions of generality develops, the expressions become more succinct, and hence manipulable. (Mason, Graham & Johnston-Wilder, 2005, p. 310)

Similarly, Vance (1998) highlights generality by defining algebra as generalised arithmetic or as a language for generalising arithmetic, emphasising that algebra is more than a set of rules for dealing with symbols: it is also a way of thinking and making connections. Kriegler (2016) suggests that in order for students to take an interest in and engage themselves in algebra, to meaningfully utilise it, it is essential that teaching focuses on sense-making and not merely symbol manipulation. Kaput (1999) points out active exploration and conjecture as the most important aspect of algebraic thinking, providing opportunities to become interested and engaged.

Kriegler (2016) describes central definitions of algebraic thinking, given by several experts in the field (Table 1). Those definitions provide a nuanced picture of algebraic thinking for years 6-9.

*Table 1: Aspects of algebraic thinking summarised from (Kriegler, 2016).*

<b>Herbert &amp; Brown (1997)</b>	<b>Usiskin (1997)</b>	<b>Kieran &amp; Chalouh (1993)</b>
Algebraic thinking is using mathematical symbols to analyse situations by: - Extracting information from the situation - Representing that information mathematically in words, diagrams, tables, graphs, equations - Interpreting and applying mathematical findings, such as solving for unknowns, testing conjectures, identifying functional relationships.	Algebra is a language, consisting of five major aspects: - Variable and variable expressions - Unknowns - Formulas - Generalised patterns - Placeholders - Relationships	Algebraic thinking involves the development of mathematical reasoning within an algebraic frame of mind by building meaning for the symbols and operations of algebra in terms of arithmetic. It includes: - Relations (not only calculations) - Representing (not only solving a problem) - Equal sign is structural (not only dynamic) - Letters/unknowns, variables, parameters (not only numbers) - Operations and inverse operations, such as doing or undoing.

In other words, different experts express aspects of algebraic thinking in different ways, and all of these aspects need to be considered in terms of whether or not they are engaging for the students. I compared reasoning on algebraic thinking (Table 1) for the purpose of this thesis, to see which aspects can become visible when researchers and teachers identify student

engagement, what can be pointed out as essential in connection to intensity and quality of participation in classroom activities, and what is emphasised in strategies used to enhance engagement by the teachers in this study.

## 2.2 Concluding remarks

Although widely used, interest and engagement are not well-defined, unambiguous concepts, and researchers do not have a common ground when it comes to definitions of those concepts (Harris, 2008). Looking at the literature review, it can be concluded that:

- Despite the lack of conceptual clarity, there are results showing that interest and engagement are beneficial for learning (Ma, 1997; Harris, 2008; Exeter et al., 2010).
- If interest is seen as manifested through engagement (Dewey, 1913), it is possible to observe and discuss interest on a classroom level (Frenzel et al., 2010).
- To engage students in mathematics, a teacher can promote purpose rather than process (Schoenfeldt, 1992), focus on active involvement and student-centered activities (Weiss, 1990), connect to everyday life (Boaler, 1999), support conceptual competences (Azevedo et al., 2012), encourage active involvement (Mitchell, 1993), allow students to problematise the content, empower them to address the problems using their own authority and provide relevant resources (Heidi & Renninger, 2006). These strategies tend to focus on classroom activities rather than the content, in line with what Wilson et al. (2005) have also shown.
- In algebra, examples of engaging strategies are to focus on sense-making and not merely symbol manipulation (Kriegler, 2016) as well as active exploration and conjecture (Kaput, 1999).

Student engagement is studied as an ongoing process of interplay between the actors in educational settings. In other words, student engagement is not studied as an ontologically determined phenomenon, something that *is*; but instead as a dynamic process that *develops* during classroom interaction.

There is a deficit of studies on engagement in mathematics at the classroom level, focusing on specific content in mathematics. A way to operationalise interest is by following Dewey's (1910; 1913; 1916/1997) view on interest as manifested through engagement. This approach opens up for

empirical investigations related to classroom context from different perspectives.

## 3 Theoretical background

In this section, the analytical frameworks used in Papers I and II, and the theoretical framework used in Papers III and IV are presented. I intend to return to the theoretical framework in the discussion section and describe the results of this thesis in the light of it.

### 3.1 Three analytical frameworks

In Paper I student engagement in connection to knowledge was related to the Gaps of Knowledge model (GOK), in which the view of knowledge originates from Information Gap Theory (Silvia, 2006), with an assumption that knowledge is something that one can be aware of or unaware of having or not having. It is based on Loewenstein's (1994) theory, which "views curiosity as arising when attention becomes focused on a gap in one's knowledge." (p.87). When interest manifested as student engagement was initially approached empirically in this thesis, it was seen as a process that may be described using the Gap of Knowledge as a metaphor, between what the student is aware of knowing and aware of not knowing (Loewenstein, 1994). The student's attempt to bridge the gaps of knowledge was seen as a sign of engagement and that was how it became visible in the empirical results.

In Paper II, engagement is seen as the deliberate task-specific thinking that a student expresses when participating in classroom activities (Helme & Clarke, 2001; 2002). In the model of cognitive engagement (CE) presented by Helme and Clarke (2001; 2002), engagement is seen as an act of participation. They developed the model through analysis of interviews and classroom data in the form of video-recorded lessons, which resulted in a set of indicators in different settings: during individual work, group work with and without the teacher, and during whole-class interaction. Within each type of interaction, 5-6 qualitatively different indicators were found, all connected to active participation: asking and answering questions, verbalising thinking and completing teachers' utterances, as well as contributing ideas and enhancing ideas, justifying an argument and being resistant to

distractions or interruptions. Further, the students described engagement as an effort, as when one really puts one's mind into mathematics. CE is visible to an observer and Helme and Clarke's (2002) study helps us to see student engagement in mathematics as an active form of involvement in the process of learning mathematics. By analysing recorded lesson sequences, Helme and Clarke (2002) showed that it is possible to approach student engagement in mathematics empirically, on a classroom level, and therefore this model was chosen for this thesis.

In the third study, reported in Paper IV, the Mathematical Task Framework (MTF) is used when analysing the level of challenge of a task. This is done because the level of challenge of a task could be one reason that students find it engaging. Stein, Grover and Henningsen (1996) have identified various patterns of student engagement when students worked with tasks on the highest level of cognitive demand, and Smith and Stein (1998) have developed this framework to make it possible to analyse the level of challenge of a task. In this framework, the level of challenge is referred to as a task's cognitive demand, implying that the demand increases gradually from "Memorization" (1) to "Procedures without connections" (2), followed by "Procedures with connections" (3) and at the highest stage there are tasks labelled "Doing Mathematics" (4). At the lower levels of cognitive demand, when memorizing and carrying out procedures without connections, a student can write down the answer to a task based on the definition or on algorithms, or because they have previously seen analogous tasks and answers. Smith and Stein (2011) point to examples such as stating decimal and percentage equivalents for a fraction as tasks with a lower level of challenge. The third level requires students to use different procedures to develop an understanding of mathematical concepts and ideas. In order to reach this level of cognitive demand, students must select suitable strategies to solve and provide explanations. As mentioned, Stein et al. (1996) have identified various patterns of student engagement when students worked with tasks on the highest level of cognitive demand, that is, with tasks that were set up to encourage "Doing mathematics" (4). In summary, according to MTF, a task is of the highest level, (4), if it:

- Requires complex and non-algorithmic thinking. There is no predictable approach explicitly suggested by the task instructions.
- Invites exploration and understanding of the nature of concepts, processes and relationships.

- Demands self-monitoring or self-regulation of student's own cognitive processes.
- Requires relevant knowledge and experience, and making appropriate use of them.
- Opens up for analysis of task constraints that may limit possible solution strategies and solutions.
- Includes the unpredictable nature of the process leading to the solution(s) and requires considerable cognitive effort.

Stein et al. (1996) describe a task that serves as an illustration of high cognitive demand in year 4 (10 year olds):

A fourth-grade class needs five leaves each day to feed its 2 caterpillars. How many leaves each day would they need to feed 12 caterpillars? (Smith & Stein, 1998, p. 347)

This task was found to be cognitively demanding by students in year 4, based on the assessment of students' results using the MTF analysis guide, showing that only 6% of pupils in year 4 found a solution.

## 3.2 A theoretical framework of didactical situations in mathematics

Unlike the analytical frameworks presented in previous section, the Theory of Didactical Situations (TDS) fits under the description of a theory, being a body of concepts organised with the purpose of explaining a phenomenon (Johnson & Christensen, 2010). The theory is used in Papers III and IV in the process of systematically formulating ideas and explanations in relation to student engagement in mathematics and when analysing the role of the teacher and the task in identifying and enhancing student engagement. Initiated by Guy Brousseau in the early 70s, TDS is a theory of teaching mathematics that has proved useful in describing what happens in mathematics classrooms, due to its unique conceptual tools for analysing didactical aspects.

The epistemological assumptions of TDS are based on seeking answers to the question "Under what conditions does acculturation of a particular knowledge of the mathematical community occur?" (Brousseau, personal communication, February 13, 2016). The foundation of TDS framework is empirical, based on experiments, as described by Margolinas and Drijvers (2015). It is helpful in attempts to better understand mathematics teaching, in particular through the way the relationships and

interplay between the teacher, the student and the mathematics are modelled by the didactical triangle (Figure 2).

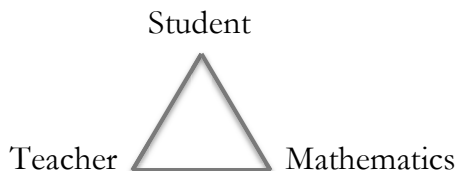


Figure 2: An illustration of a didactical triangle (Brousseau, 1997; Hansson, 2011)

The didactical triangle illustrates the notion of a *didactical situation*. In this situation, interplay between the teacher, the student and the mathematics takes place. In this situation it is “a teacher’s responsibility to create didactical situations that involve the students and allow for alternative solutions to a problem” (Hansson, 2011, p. 37). The core concepts of didactics are grounded in a basic epistemological assumption that the *didactical transposition* of mathematical knowledge is the core of teaching the subject of mathematics (Brousseau, 1997; 1999). The idea of the didactical transposition is central to TDS (Chevallard, 1992), referring to the way in which *target knowledge*, a mathematical idea, is transposed, modified by the teacher to fit into the classroom context (Hansson, 2011, p.37).

For example, Strømskag (2015) presents target knowledge in algebra in one of her studies:

The target knowledge in this case is the equivalence statement: “the sum of the first  $n$  odd numbers is equal to the  $n$ -th square number”, potentially represented by  $1+3+5+\dots+(2n-1)=n^2$  (p. 478)

In the example, knowledge is seen as intentional and possible to target. Didactical transposition of target knowledge is “based on the assumption that knowledge selected to be taught in an educational institution has a pre-existence outside the institution, and in order to be teachable it has to be adapted depending on the constraints given in the didactic system” (Jablonka & Bergsten, 2010, p.35). In the case of “the sum of the first  $n$  odd numbers...” the adaptation can take place by using a number of post-it notes as representations for each odd number, rearranging them in ways that illustrate the equivalence (Figure 3).

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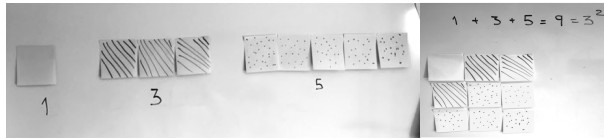


Figure 3: A representation of the sum of the first three odd numbers.

The transposition allows the mathematical idea to be tried out in classroom context. Since the representation is a modification of a mathematical idea, made to fit the classroom, understanding the representation in Figure 3 does not necessarily mean understanding the mathematical idea of the equivalence between the expressions.

The didactical transposition of the target knowledge begins with the establishment of a *didactical contract*, the set of subject-specific norms and rules negotiated between teacher and students. Those are implicit rules of a didactical situation, where the teacher sets the scene and makes his or her intentions explicit. Within a didactical situation, *devolution of an adidactical situation* takes place, if the teacher manages to provide the important conditions for the students to accept the task as their own. The teacher leaves the scene and the student(s) work with the task without the teacher's interference. This interplay between the student and mathematics is referred to as the *adidactical axis* (Hansson, 2011). In a *situation of formulation* the teacher re-enters the scene and interacts with the students (Brousseau, 1997). When the teacher re-enters the scene, the students have an opportunity to become engaged in a specific piece of mathematical knowledge, a process that can be driven both by the student and the teacher during that situation.

Mathematics lessons are seen as shaped by didactical structures (Mason, 1988; Brousseau, 1997), in the interplay between the student, the teacher, the mathematics and, as described in Rezat and Sträßer (2012) in a recent theory development of TDS, an artefact, creating a three-dimensional didactical tetrahedron, where tools or artefacts “are considered to be the fourth fundamental constituent of a didactical situation in mathematics education” (Rezat & Sträßer, 2012, p. 649). By adding an extra node to the original triangle, four triangular faces are formed and can be used as platforms for analysing the didactical interplay. In my adaptation of this model, the artefact is a task, similarly to the way Gallos Cronberg (2016) introduces the textbook as the artefact vertex (Figure 4).



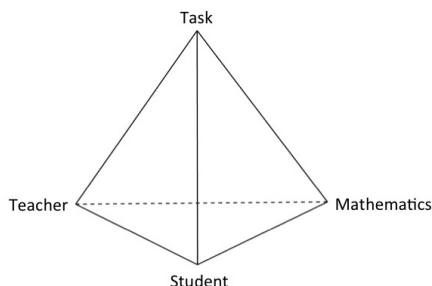


Figure 4: An adaptation of the didactical tetrahedron (Rezat & Sträßer, 2012).

In contrast to the original two-dimensional model of the didactical triangle, the target knowledge can now be analytically separated from the task, and the mathematics in relation to the task is represented by an edge of the tetrahedron. This is done in order to differentiate the target knowledge from other elements of a task, and for that purpose a specific axis connecting the artefact (the task) and the mathematics (the target knowledge) is needed.

Brousseau (1997) suggested, similarly to Dewey (1913), that interest is linked to student effort and investment. The most interesting task or problem will, according to Brousseau (1997), be the one that permits overcoming an obstacle, based on what the student is ready to invest in working with target knowledge. The target knowledge with its specific mathematical notions and properties is intentionally brought into a classroom context by transposition: objects from the discipline of mathematics are transposed by the teacher, in order to be dealt with in the school context. Student engagement in this study is considered to occur in the interplay between the teacher, the student, the task and the mathematics. TDS approach provides linguistic tools to analyse lessons, lesson episodes, classroom situations and tasks, providing opportunities to point out essential moments and details using didactical concepts. TDS is used as an analytic tool in this thesis, which may be seen in some of the research questions posed.

### 3.3 Didactical contract

In order for the transposition to be successful in the classroom, interrelations between the involved parts must be established; a mutual agreement on the nature of such interrelations is known as a *didactical contract*. A didactical contract is a set of implicit and explicit rules, consisting of attitudes expressed in classroom interaction:

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We are especially interested in what is specific of the knowledge to teach: we call didactical contract the set of specific attitudes that the student expects from her teacher, and the set of specific attitudes the teacher expects from her student. (Brousseau, 1997, p. 88)

In other words, interest as an attitude can be considered a part of the didactical contract. The notion of the didactical contract is one of the key elements in TDS. When students meet their teacher's expectation, concepts and behaviours become a part of the contract in the classroom. Implicitly, a part of the didactical contract can also be cultural traditions and codes that are embedded in classroom discourse, but most importantly it is about how the mathematical content is handled in the classroom.

Further development of TDS shows that a didactical contract can be of different types: macro, meso and micro (Hersant & Perrin-Glorian, 2005a):

The macro-contract is mainly concerned with the teaching objective, the meso-contract with the realization of an activity, e.g. the resolution of an exercise. The micro-contract corresponds to an episode focused on a unit of mathematical content, e.g. a concrete question in an exercise. (Hersant & Perrin-Glorian, 2005a, p. 119)

Hersant and Perrin-Glorian (2005b) made an attempt to illustrate the different components of the didactical contract as a whole by developing a model, which I have adapted for my purposes (Figure 5).

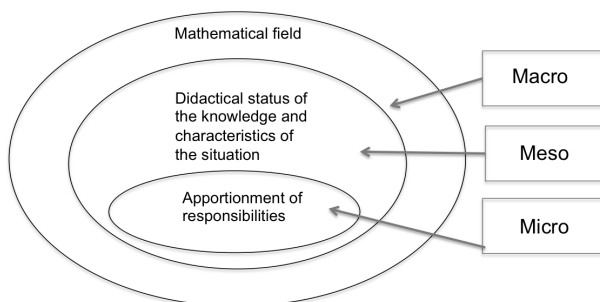


Figure 5: My adaptation of Hersant and Perrin-Glorian's (2005b, p.120) model of the didactical contract. The macro-, meso- and micro contracts are marked with boxes to the right.

In this thesis, the term *level* is used in Paper III (Nyman & Kilhamn, 2015) to describe the different contracts. Since the model in Figure 5 has not been widely used, I further elaborate on it and exemplify the different contracts with examples in the forthcoming section.

### 3.3.1 The macro-contract

*The macro-contract* focuses on the teaching objectives, the target knowledge from the mathematical field and how it is put forward in curriculum, how the choice of the target knowledge is made by the teacher (Hersant & Perrin-Glorian, 2005a). In the case of algebra, Selling (2016) showed that strategically selected patterns engage students in representational practice. Mason et al. (2005) suggest that an interesting and engaging task includes the process of generalisation. Hunter (2010) provides an example from introductory algebra, a task involving two open number sentences with missing parts,  $3 \times \_ = 6$ ,  $6 \div \_ = 2$ . The teacher chose this task to engage the student using the general relationship between multiplication and division.

### 3.1.2 The meso-contract

*The meso-contract* is the level of lesson activities, that is the implementation of the target knowledge by the teacher (Hersant & Perrin-Glorian, 2005a). This is the level on which the activities are carried out. While the macro-contract specifies what target knowledge to aim for, the meso-contract is where the teacher sets the norms and discursive traditions of how to deal with the target knowledge (Miyakawa & Winslow, 2009). Returning to Hunter's example (2010), when the task  $3 \times \_ = 6$ ,  $6 \div \_ = 2$  was dealt with in the classroom, the teacher posed the question: "What do you notice about those two? Discuss in pairs". Here, the discussion in pairs and the way the task was presented illustrates the meso-contract. The teacher could also use the opportunity brought into the classroom by the target knowledge by engaging the whole class in a prolonged discussion about the general relationship between multiplication and division.

### 3.2.3 The micro-contract

*The micro-contract* is the distribution of responsibilities in the classroom, which is negotiated, as described by Hersant and Perrin-Glorian (2005a), in terms of how the students are expected to act during the interplay with the target knowledge. An example of a tradition in Swedish mathematics classrooms is to work silently and individually in textbooks, with each student working at their own pace, relying on the answer key in the book to check the work. Different solutions and student responses to the activity can be considered to make up the micro-contract, or as Miyakawa & Winslow (2009) describe it: the level that is "crucial to explain the variety and quality of the reasoning put forward in the class" (p.212). They provide examples of the micro-contract such as when a student explains another student's ideas, or corrects

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another student. Returning once more to Hunter's (2010) case of  $3 \times \_ = 6$ ,  $6 \div \_ = 2$ , the micro-level is the way the open sentences are handled by the students working in pairs, whether this means treating them as merely containing a blank space or an unknown, labelling it x or y, or even, as one student suggested, saying that the two sentences are somehow opposites of one another.



## 4 Methodology

In this chapter the choices of methods used in the empirical studies that form the thesis will be described. This chapter provides on the one hand a description of what has been done in the three studies and on the other hand a reflection on different approaches that can be used to observe what happens in the classroom and different techniques that can be used when interviewing teachers and students. Experience from large-scale, multiphase video projects in collaborative workgroups and small-scale individual interviews are described.

### 4.1 The empirical approach to studying engagement

The four appended papers are based on three different empirical data sets. The studies take place in Swedish settings, involving content matter dealt with in years 6-9 (students age 12-16). The overall design of the thesis is of an explorative character (Silverman, 2010), driven by the research questions rather than in favour of a particular method or theory. Therefore different empirical approaches were considered for the different studies.

In order to investigate interest as manifested through engagement in mathematics classrooms, a starting point was an empirical study of student-teacher interaction, with focus on different aspects of a given task. A study of student-teacher interaction requires rich classroom data that makes it possible to observe different details within a lesson. Therefore, priority was given to using available high quality data, which was relevant and rich.

Two of the data sets chosen consist of a substantial amount of various high-quality video data, namely the LPS data described in Section 4.3 and the VIDEOMAT data described in Section 4.4. Compared to field notes, video data provides “incomparably richer records” (Jordan & Henderson, 1995). The data used here can be seen as having high quality since it was thoroughly planned and systematically gathered, using professional equipment in the form of several cameras.

At the initial stage of exploratory work, the researcher needs to focus on sharpening research questions and on planning the analysis of relevant data rather than on data collection. Making a secondary analysis of existing data, such as LPS, provides such opportunities:

There are no 'brownie points' given by most disciplines for having gathered your own data. [...] You may condemn yourself to have less time to engage in the much more important activity of data analysis. (Silverman, 2010, p. 57)

Data production is a valuable experience for a researcher, and it can be accomplished both by generating or collecting data and by producing categories from existing data. As stated earlier, the choice of methods used in this thesis is guided by process-oriented research questions.

When I embarked on the task of identifying interest in mathematics during engagement in classroom interaction, the initial step was to find material relating to mathematics lessons that was relevant to the research focus. I intended to consider different methods and started out by observing a variety of mathematics classrooms and lessons. In order to find out more about interest as manifested by student engagement in classroom settings, observations were made within an observational spectrum from unstructured observation, spontaneously made in two different classrooms, to structured observations documented by field notes, video- and/or audio recordings. In other words, I approached interest as manifested by engagement ethnographically, within the observation spectrum.

## 4.2 Pilot study - A researching teacher's dilemma

At the initial stage of the empirical approach to student engagement, the choice of method and data collection was considered. In this section, I provide a short description of an ethical dilemma faced during the pilot observations from two different year 8 classrooms at a local school and the conclusion drawn from this experience.

I contacted experienced teachers, Tina (who later participated in Paper IV) and Johan<sup>1</sup>. Tina, who has taught mathematics for over 20 years in years 6-9 and Johan, a teacher educator and an experienced colleague who has taught mathematics and natural science for over 20 years, were observed for one lesson each. To get a holistic view, I started by observing the environment, the students and the teachers. This school had a stable organization from preschool up to year 9, with socially well-functioning classes and well-developed, research-based teaching methods.

One major limitation noticed during pilot observations was in line with that described by Jordan and Henderson (1995) - the hindrance caused by the dual role of a researcher with a teaching background. As the individual work began, two students approached me for help. Even though I had been presented as a researcher, students

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<sup>1</sup> Tina and Johan are fictive names.

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immediately approached me for help while working on their own, as shown in these field notes from Tina's lesson:

Student: What are you doing? (Elev: Vad gör du här?)  
Researcher: I am a researcher. I am looking around, studying this lesson to see if I can find something... (Forskare: Jag är forskare. Jag tittar mig omkring för att se om jag hittar något...)  
Student: But do you know some mathematics? Look here! Please! (Elev: Men kan du lite matte? Kolla här! Snälla!)  
Researcher: Let's wait for your teacher. (Forskare: Vi får vänta på din lärare).  
Student: Come on! (Elev: Kom igen då!)

Incidents of this kind hindered observations of other conversations occurring at the same time. The same situation occurred in Johan's classroom. One student had an inventive strategy to involve me in individual work, as shown in field notes from Johan's lesson:

Student: Hi, can you come here! (Elev: Du, kom hit!)  
Researcher: Me? Yes. (Forskare: Jag? Ja.)  
Student: Look here, do you know the answer? [10% of 40 crowns]  
(Elev: Kolla här, kan du svaret?)  
Researcher: Yes, as a matter of fact I do. (Forskare: Ja, det kan jag faktiskt.)  
Student: What is it then? (Elev: Vad är det då?)  
Researcher: 4 crowns. Do you know how to calculate that? (Forskare: 4 kronor. Vet du hur man räknar ut det?)  
Student: Yes, I sure do, I wrote it here [wrote:  $10\% \cdot 40 = 4$ ] but do YOU? But can you do it in you head? Everybody should know that. (Elev: Ja, klart JAG kan, men kan DU det? I huvudet? Alla borde kunna det.)  
Researcher: You think so? How do you solve it? (Forskare: Tycker du? Hur gör du då?)  
Student: I won't tell you. And now that you are here, help me with the next one. (Elev: Säger jag inte. Men nu när du är här, hjälp mig med nästa.)  
Researcher: But that (is the task) you are supposed to do in your groups. (Forskare: Men den skulle ni ju arbeta med i era grupper.)  
Student: I know, I know, just checking... (Elev: Jag vet, jag vet, kollar bara...)

At times it seemed unethical to deny the students assistance during individual work. It is a question of balance between closeness to and distance from the research object and the classroom practice (Gustafsson, 2008; Hansson, 2011), adopting an analytical role rather than following a teacher's instinct.

The conclusion of the pilot observations was that the dual role of a researcher with a teaching background has its advantages in the analytical process, but needs to be considered carefully when collecting data. The observer sees only a small part of what the teacher is doing (Brousseau, personal communication, February 19, 2016) and in mathematics classrooms a teaching background can undermine the role of the



researcher. Therefore, I chose not to pursue observation as a data generation approach.

### 4.3 Study 1: The LPS video data

In this section, I will elaborate on the choice of data for Paper I, by pointing out advantages of secondary data analysis in general and the analysis of the LPS data in particular. I obtained access to video data from the Learner's Perspective Study (LPS), which consists of mathematics lessons and video-stimulated interviews with students in year 8 in 15 countries including Sweden<sup>2</sup> recorded in 2003 and 2004 (Clarke, Emanuelsson & Jablonka, 2006). LPS was initially created as a platform for the work of an international community of classroom researchers (Clarke et al., 2006), designed to analyse mathematics classrooms in year 8. In contrast to the Trends in International Mathematics and Science Study (TIMSS) video study, where randomly sampled single representative lessons were recorded, the LPS provides authentic sequences of lessons, recorded in naturalistic classroom settings (Niss, Emanuelsson & Nyström, 2013). A research team collected the Swedish classroom data in three Swedish schools.

The data within the LPS project is technically advanced and thorough, and communities of researchers all over the world are familiar with it. If attempts to record lessons had been made at the initial stage of work for this thesis, it would have shifted focus from the analysis of details to the extensive production of video material. However, the LPS data can be used to shed light on phenomena that were not previously considered.

Using video-recorded data has many advantages. When making observations it is hard to attend to the many things that may take place simultaneously. Taking notes while observing involves, to some degree, instant interpretation of what is occurring (Hägström, 2008). A researcher's attention can only be directed towards one conversation or event at a time and no overview is possible while observing in the field, whereas video-recorded data, compared with field notes, gives the researcher a chance to become well acquainted with the lesson episodes. Video-recorded data provides opportunities to observe details in retrospect. Jordan and Henderson (1995) also pointed out another problem when direct observation is employed for data generation: not seeing the whole class when focusing on student-teacher conversations or private conversations between the students. This leads to a need for

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<sup>2</sup> Kult-projektet/Learner's Perspective study – <http://www.lps.iccr.edu.au>

event reconstruction of the missing parts of different conversations. Video recording using several sources can alleviate this difficulty.

The students in a classroom are attending to something all the time, but given the diversity of classrooms, the time spent by different students on-task differs (Wood & Kalinec, 2012). In Paper I, a qualitative video analysis of on-task interaction was carried out. The specific parts of the LPS data used in this thesis are eight lessons from three Swedish schools. The lessons were scrutinised repeatedly and one 60-minute lesson, filmed following the teacher and particularly rich in on-task student-teacher interactions was selected for further analysis.

### 4.3.1 Analysis of the LPS data

The analysis of the LPS data was a gradual process. Firstly, I chose episodes from different parts of the lesson, where students attended to a task while the teacher was interacting with one student, a pair or a group. In step two, I arranged sessions with a research group where I presented the episodes and we viewed the chosen episodes several times. I coded the activities that the students were engaged in during each situation. The coding process consisted of code timelines, showing the different aspects of the mathematics in which the students were engaged (Figure 6).



Figure 6: Screen shot of the coding.

This coding is used to summarise the observations made. After the coded situations were clustered, they were validated during several meetings within the research group. Event sequence analysis was applied, that is, a timeline was used for clarification of key events and sequences (Miles & Huberman, 1994). During the analysis a software program developed for qualitative in-depth video analysis, StudioCode, was used. This

system made it possible to cluster the chosen video sequences. An expert on Studiocode, who was familiar with the LPS data, validated my coding of the sequences. The instances of on-task interaction in the empirical data were classified into six different categories, and from the six clusters of on-task engagement, three themes were developed. In the development of the themes the focus was on what students attend to, bringing out possible aspects in student-teacher interaction that can be interpreted as student engagement. In summary, it was found to be fruitful to analyse the LPS data of student-teacher interaction by approaching student engagement in terms of focus of attention. The analysis based on this data set provided an extensive overview and ideas for further decisions concerning methods, and the results of Paper I are summarised in Chapter 5.

#### 4.4 Study 2: The VIDEOMAT data – Video-stimulated focus-group interviews with teachers

Papers II and III are based on parts of a large international video study VIDEOMAT, where teachers in different countries compare and discuss video-recorded classroom data. In the VIDEOMAT project, lessons were recorded and focus-group interviews with teachers were carried out and analysed with regard to instructional practices and the use of artefacts and written work. In contrast to the LPS data, for the Swedish VIDEOMAT data the topic of student engagement was added as a specific research focus, in questions handed to teachers before the focus-group sessions and as the topic of discussion in one of the sessions.

I was involved in the fieldwork by video recording lessons at one of the schools. Thereafter I worked on collaborative analysis in a working group. Sessions were held regularly, analysing recordings or transcripts. The group worked together, producing and validating codes, lesson graphs and interpretations. There were also monthly videoconferences involving all the participating countries, providing an opportunity to present analysis and validate results. Further, I was involved in the development of coverage codes and content logs for the lessons where I was involved in filming. I constructed lesson graphs (LG) (Appendix 1), based on the original lesson graphs from the TIMSS video studies, constructed to display the structure and content of a lesson, as well as the artefacts used and what the teacher is doing, saying and writing on the board. The LGs were based on a coding system, using Coverage Codes (CC) to mark the type of interaction. CC's were developed within the research team soon after the fieldwork (Kilhamn & Røj-Lindberg, 2013). I chose to add screen-shots of the interaction to some LGs in order to illustrate certain content specifics and types

of activities. Preparations for the focus-group sessions were made by handing over videos of lessons to the teachers for the focus group in person.

Filming of focus-group interviews took place, where I moderated group interviews together with another researcher who was observing, taking notes. Questions of engagement in lesson sequences were discussed. After the focus-group sessions were transcribed and analysed, I presented on-going work or results at group meetings and conferences in Gothenburg (Sweden), Vasa (Finland) and Cambridge (England) during 2012 and 2013.

The specific parts of VIDEOMAT data used in the second study are focus-group interviews on the theme of student engagement, which constitute data for Papers II and III. These were carried out using a method of interviewing that originated in the late 1940s (Merton, 1946) and is seen as an effective qualitative method for gathering verbal data from specific target groups. The main difference in comparison to individual interviews is the collective response, the sharing of ideas and the way participants influence each other's perceptions of a concept shared by a group. The development of focus-group interviews contributed to more effective and beneficial ways for a specific group, in this case experienced teachers, to generate data by sharing professional insights (Morgan, 1996). Cohen, Lawrence and Morrison (2007) also point out that during group interaction, valuable data emerges, making it possible to access details and develop themes.

In the focus-group interviews, experienced teachers participated in two groups consisting of five and three teachers respectively. The teachers, of various ages, and with various backgrounds and teaching experiences, taught in years 6 and 7 at four different Swedish schools and were competent in the subject of mathematics, being general teachers (years 4-6) and mathematics teachers with university credits in mathematics or mathematics education. They all consented to participate in focus-group sessions and expressed a hope that they would develop their professional knowledge in mathematics education through their participation. The focus-group sessions were framed in the following way:

- **STEP I:** Three different cameras (whole-class camera, teacher camera and focus-group camera) were used to film a set of five lessons per teacher on introductory algebra.
- **STEP II:** All the lessons were mapped by constructing LGs showing time intervals, type of interaction, description of content and illustration of tasks and activities.
- **STEP III:** Each teacher was handed recordings of their own lessons with corresponding LGs, along with practical guidelines for selecting episodes from

the films that they wanted to discuss. To direct the focus-group discussion towards engagement, the teachers were asked to select episodes where the students were engaged in algebra and to consider how a teacher can engage students in the algebra content they are dealing with. The teachers were given three weeks to prepare for the sessions by choosing sequences from the video recordings of their lessons. The episodes were intended to illustrate student engagement in algebra (Nyman, 2015), or an attempt by the teacher to enhance student engagement (Nyman & Kilhamn, 2015). These episodes were the starting points for the focus-group sessions.

During the focus-group sessions, the moderator introduced the topic, aiming to set a positive tone and open up the discussion. One of the teachers continued the session by showing his or her chosen episodes to the group, thus initiating a discussion about how engagement was seen and how it was or could be enhanced by teachers. In turn, each teacher showed his or her episodes, explaining and discussing why they had been chosen and in what way they make engagement visible.

#### **4.4.1 Analysis of the VIDEOMAT data**

The analysis of the VIDEOMAT data was made after the two focus groups had recognised and discussed situations of student engagement. The focus group discussions were transcribed, resulting in 407 turns in focus group 1 and 181 turns in focus group 2. Two researchers who had been present during the focus-group interviews analysed and discussed them, revisiting both the transcripts and the videos several times in an iterative process. An overview of the teachers was made with respect to their school, background and years of experience. In the transcripts, the researchers identified indicators of engagement corresponding to Helme and Clarke's (2001, 2002) model, and didactical strategies brought up by the eight teachers, both strategies shown in the episodes and strategies that emerged as a reaction to the episodes chosen by the other teachers in the group.

The two transcripts from the two focus groups were analysed together, treated as one data set. We used a multi-step process of data analysis (Miles and Huberman, 1997). The first step was noting patterns and themes in the data, thereafter making sense of the patterns in relation to the theoretical constructs chosen, in this case the Theory of Didactical Situations. Different aspects of the didactical contract and the didactical strategies were noted, and finally, chunks of reasoning that represent different aspects of the didactical contract and different didactical strategies used to enhance engagement were identified. In practice, this meant that different utterances

were clustered and placed under different headings, summarising the findings emerging from the teachers' examples.

### 4.5 Study 3: Individual student interviews

In the interview study that led to Paper IV, conditions for finding evidence of student engagement related to the specifics of mathematical target knowledge were optimised, now focused on a student perspective. Individual students in year 9 were interviewed, looking into what tasks and what specifics of those tasks from years 7-9 students found interesting and engaging. Participants were chosen from two year 9 groups who had been taught by a teacher with over 20 years of experience, who is in addition a PhD candidate in mathematics education and supervises professional development courses. In her choice and design of tasks, she explicitly described her intent to engage students in mathematics.

The students who were interviewed commented on the teacher's competence, rating it as high or very high. In other words, she is seen as particularly skilled by the local community, in line with the teachers for the LPS and the VIDEOMAT studies (Clarke et al., 2006; Kilhamn. & Røj-Lindberg, 2013). She taught two classes on the same level through years 7-9. Once a week, those two classes were mixed, working in smaller groups, choosing the group themselves according to which grade they aimed to achieve by the end of year 9: A (the highest), C or E.

The teacher was asked to select the student participants for this study, by choosing 4 or 5 students from each group. This choice was based on the teacher's knowledge of students' ability to verbally express their reflections. She asked them if they wanted to participate in the interview and all 15 students (5 from each achievement group) agreed. The students' names were coded as A1-5, B1-5 and C1-5, for anonymous reference in Paper IV.

The teacher was interviewed for 40 minutes, during which time she described the students as engaged on a whole-class level, with a range of individual variation. She had chosen to specifically take on the least engaged classes at her school at the beginning of year 7 and teach them through years 7-9. She indicated that she engages students by putting mathematics content to the fore, trying to optimise the possibilities of finding and designing interesting and engaging tasks, thereby actively engaging the students in mathematics. She encourages students to engage in tasks and expects participation during her lessons. The two classes she has taken on have, according to the teacher, made a long journey in their development of engagement, compared to when she first started to work with them in year 7. For her, target

knowledge, the mathematical ideas, have to come into focus, and she transposes the mathematics into tasks that she designs to engage the students, thereby enabling an adidactical situation to be created:

Semi-structured, individual interviews were carried out with the 15 selected students from the two year 9 classes, lasting around 10 minutes (within the range of 4-11 minutes). The length of the interview was constrained by the students' timetables and by their ability to respond to the questions. The key questions in the interviews were, in the first part: "Do you remember something interesting and engaging you have done in mathematics with your teacher?" "What did you think about the task(s) [you are referring to]?" "What did you learn?". Follow up questions were posed about what the students found to be interesting and engaging in the task(s) and what the students felt they had learned. In the second part, a task chosen by the teacher was presented and discussed. In the third part, the students were given an opportunity to give further examples of interesting and engaging tasks, answering the questions: "What makes the task interesting and engaging" and "What can interest and engage you in a task?" Further detailed questions were posed, where the students could once again refer to the tasks they considered to be interesting and engaging. The interview concluded with suggestions from the students about how a teacher can enhance engagement. Individual interviews in this study give a first-person perspective on what students find interesting and engaging, and why. Here, in contrast to Papers I - III the students themselves identify what engagement is.

The advantage of individual interviews are the first-person perspective, that is, the students' own view on engagement. A collective view on what tasks are interesting and engaging, as well as the reasons for them being interesting and engaging are not as accessible during individual interviews as during focus-group interviews. On the other hand, the students might have influenced each other's choices in a focus-group discussion. For this reason, in the case of the students, individual interviews were carried out.

#### **4.5.1 Analysis of the interview data**

The analysis of the interview study was a three-phase process, during which both the semi-structured interviews and the tasks that were referred to were analysed. The interviews were transcribed and the teacher provided the researcher with the tasks mentioned by the students as being interesting and engaging. The tasks were treated as items (Goldin, 2000), such that different aspects of the tasks were analysed. Phase one of the analytical process consisted of analysing the tasks brought up by the students during their interviews, and looking into the target knowledge, the context,

the level of challenge and the task structure. The analysis of the tasks was made in order to answer the question of what tasks students identified as interesting and engaging, and categories were adopted from the framework of TDS, MTF and other relevant research on task features. Phase two was a categorisation of students' utterances, where the transcripts of student interviews were thematically analysed in order to shed light on the research question about what students identify as interesting and engaging in those tasks. Looking for words in students' utterances related to task features is the focus of this analysis, to see if details in the target knowledge, the context, level of challenge (cognitive demand) and task structure (openness, routine, scaffolding) or other details from the theoretical framework would be revealed. Examples of student utterances in the results illustrate what students identify as interesting and engaging. Several quotes illustrating the same type of reasoning were included in the final draft of the paper, to strengthen the researchers' interpretations and to show a variety within the same theme. Phase three consisted of an iterative process, where going back and forth between phases one and two connected the features found during phase one with findings during phase two.

## 4.6 Quality - Reliability, validity and ethics

### 4.6.1 Reliability and validity

Video and audio are useful in data collection for capturing interactive aspects and utterances. The type of data within the LPS and in VIDEOMAT corresponds well with the aim of analysing lessons with the purpose of answering questions about student engagement in the mathematics classroom. In this section I discuss reliability, different observer perspectives and different concepts of validity. First, I take up the ways in which the data and results of the three studies were validated.

Categories of task-specific attention emerged from the data for Study I. To ensure the quality of my results, after choosing sequences from the LPS data I invited colleagues and fellow researchers to participate in video-analysis sessions. I presented video sequences and we discussed the potential categorisation. Further discussions with those who were well acquainted with the data and those for whom this data was new, were held and final categories were arrived at.

In the VIDEOMAT project, Study 2, collaborative teamwork meetings were arranged on several occasions. The aid of the collaborative development of content logs, coverage code development and coding the lessons gives a solid ground for the



quality of observations. I presented video sequences, and we discussed my analyses within the team.

When it comes to the interview study, Study 3, the categories suggested by the researcher were presented in a seminar to a group of experienced teachers and researchers within the field of mathematics education and reworked in the light of their comments. A draft of the manuscript was also provided to the participating teacher, and read by her on several occasions to validate the results. She provided respondent validation on the analysis, suggesting changes were needed, and strengthening argumentation concerning the discussion of the results.

External validity addresses the issue of generalisation. Through the process of naturalistic generalisation (Eisner, 1991), what one can learn from a particular classroom or a set of classroom situations is also relevant to classrooms other than the particular ones being studied. The use of recorded material and analysing in focus groups strengthens the external validity of the results. When it comes to the events illustrating task related attention, student engagement has been recognisable by researchers and teachers, both by participants in the studies, but also an outside audience. Validating results from all four papers with teachers, in research groups and research communities has been a part of the working process in this research project. To test reliability, I have continuously arranged video sessions in different forums. There are always situation specifics in studies of qualitative character that limit the generalisability. However, the criteria for naturalistic generalisation are fulfilled and the results are in resonance with others' understanding, and are therefore not only valid for a specific instance or situation since they are recognisable to others.

In this thesis, the question of validity touches also on the quality of communication. Communicative validity is about the explanatory power of the text, the quality of the dialogue between the researcher and the participants, between the researcher, the readers and various discussants (Kvale, 1994). Clarifications and detailed explanations of every step of the methods are intended to be of informative value to the reader. Sequences from all the studies were continuously presented in classes, in working groups and at conferences where they were tested on a broader audience. Students, teachers and fellow researchers, both those who were and were not acquainted with the data, validated teacher-chosen sequences of student engagement. All the papers in this thesis have been through the peer-review processes of the anthology and the journals they are published in, and suggestions for revision and improvement were taken into consideration. I see peer-review process as a test of communicative validity and studies that are accepted for publication as passing that test, and therefore as communicatively valid.

#### 4.6.2 Ethical considerations

Throughout all the empirical studies in this thesis, ethical issues are taken into consideration, both on a general and on a specific level. On a general level, decisions for every empirical approach were based on a consideration of ethical issues concerning whether or not my actions would be appropriate. On the specific level, I made choices regarding recorded data, adopting an ethical view based on the balance between humanistic values and a pragmatic research approach, where a pragmatic approach "[...] judges actions according to their specific consequences – benefits and costs – for various audiences; the researcher, the researched, colleagues, the public" (Miles & Huberman, 1994).

There are established ethical principles in the research community that guide ethics in the field of Swedish educational research (Swedish Research Council, 2011), which at the time of this thesis provides guidance both on the juridical and individual level. These principles have been followed throughout the research process, during data collection and data analysis. One of those principles touches upon the issue of respect and avoidance of harm to participants. The LPS and VIDEOMAT data are both high quality data sets, where ethics were taken into consideration, and the data has undergone thorough scrutiny. In recorded data from LPS, the research teams follow requirements for good ethical principles both on the group level and on the individual level (Häggström, 2008). In both cases, the Swedish team leader gave consent to use the Swedish data. In all the studies, both teachers and students gave written consent, knowing that the purpose of the data collection was for research. I also informed students in Study 3 that the results would not be recognisable on an individual level (Appendix 2), neither by their teacher nor by the other readers, through appropriate anonymisation.

Certain precautions have been taken when familiarising ourselves with and analysing the recorded material of LPS and VIDEOMAT data, as well as audio data from the interviews. For one thing, analysis of the data was made from a password-protected source, thereby avoiding multiple copies of the material or copied material going to unprotected sources. Pseudonyms were used when referring to the students and the teachers. Some of the students gave permission to be a part of the research, but not for material that was to be used at conference presentations and similar events. Therefore, when I shared sequences in working groups, with supervisors or conference participants, I only selected students who had given permission to use the material more publicly. When analysis and text was shared in working groups, with supervisors or conference participants, I made it clear that the manuscripts are a work in progress, not to be spread further or cited.

These considerations correspond well with the requirements of confidentiality and utility (Swedish Research Council, 2011), i.e. that individuals are protected from identification and not exploited in non-scientific activities. Since this dissertation is of an explorative character, not all ethical aspects are obvious from the beginning.

## 5 Results

This chapter summarises the three studies and the four associated papers that make up the empirical part of the thesis. The separate results of each paper are presented in sections 5.1-5.4. Paper I has the outsider's perspective, observing parts of a lesson as it progresses. Papers II and III capture the perspectives of the teachers on engagement in their own and their colleagues' teaching. Paper IV captures the perspectives of students on their own interest and engagement. In Section 5.5, the results of the four papers are presented in relation to one another and to the overall aim of the thesis, in terms of the Theory of Didactical Situations. Table 3 provides an overview of the papers, identifying the analytical perspectives taken. A summary of the most important contributions to the field of research ends the chapter.

### 5.1 PAPER I

Nyman, R & Emanuelsson, J. (2013). What do students attend to? Students' Task-Related Attention in Swedish Settings. In B.Kaur, G.Anthony, M. Ohtani & D. Clarke. (Eds.), *Student Voice in Mathematics Classroom Around the World* (pp 115-132). Rotterdam: Sense Publishers.

Paper I describes a study in a Swedish year 8 algebra classroom. By analysing video data from the Learners Perspective Study, the learners' interest and engagement were analysed as task-related attention. The aim of the paper is to show what students attend to in student-teacher interaction during algebra lessons, when dealing with tasks on mathematical relations. Video sequences from a class in an adidactical situation were chosen, focusing on the situation of formulation that took place, when a student approached the teacher. The study showed that in a situation of formulation, they engaged with the relevance of mathematical relationships in a task, the solution of a task and the validation of numerical answers. The results indicate that with the teacher's help, a student's attention can be directed towards the mathematics in a task. In other words, the student can be engaged and the adidactical situation can be restored by explaining the relevance of mathematical

relationships, by helping the students work towards a solution and by confirming or dismissing the value of a numerical answer. This study showed that it is possible to analyse what students attend to on the classroom level. Also, it led to further questions of what role the teachers has in the process and whether a distinction between the mathematics and the task can be relevant when analysing lessons.

## 5.2 PAPER II

Nyman, R. (2015). Indicators of student engagement: What teachers notice during introductory algebra lessons. *The International Journal of Mathematics Teaching and Learning*, 15(3), 1-17.

In this paper, focus is on student engagement from teachers' perspectives, providing examples of how student engagement is visible to teachers during introductory algebra. Lessons in years 6-7 were video documented in four Swedish schools. A total of 8 teachers participated in a focus-group study discussing episodes from their own classroom that they identified as featuring student engagement in algebra. The episodes were analysed using an existing model of cognitive engagement. The results show that teachers can agree upon what student engagement is: it is recognised as different types of active participation, expressed by specific student actions in the classroom. The identified indicators are: verbalising thinking, concentration, gestures expressing attention or excitement, asking and answering questions, enhancing ideas and justifying an argument. Since the chosen episodes were examples of student-teacher interaction, a discussion on the teacher's role in enhancing engagement followed, and is analysed further in Paper III.

## 5.3 PAPER III

Nyman, R & Kilhamn, C. (2015). Enhancing engagement in algebra: Didactical strategies implemented and discussed by teachers. *Scandinavian Journal of Educational Research*, 59(6), 623-637.

This paper focuses on the role of the teacher in engaging students during the introduction to algebra in years 6 and 7. This article is based on the same focus-group data as Paper II, where teachers contribute further insights into student engagement. The aim of this paper is to describe what didactical

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strategies teachers use to enhance student engagement. The data consists of a video portfolio of episodes where the teachers believe engagement is indicated and their utterances when commenting on each other's episodes. The teachers' strategies were analysed with respect to the design of didactical situations and the negotiation of the didactical contract. The strategies the teachers used were on the level of the meso-contract, the organisation of the lesson and the realisation of the activities. Strategies specifically related to the mathematical content, in this case algebra, were not forthcoming. This was the reason for designing the study, which is reported in Paper IV.

### 5.4 PAPER IV

Nyman, R. (2016). What makes a mathematical task interesting? *Educational Research and Reviews*, 11(16), 1509-1520.

This paper presents an analysis of tasks that students in year 9 identify as interesting and engaging. In semi-structured individual interviews 15 students, selected to represent a cross-section of intended achievement, were asked to recall tasks they found interesting and engaging during the past three years, and to elaborate on why. Of all the tasks dealt with during years 7-9, the students identified four specific tasks, all with target knowledge from geometry and statistics, designed by teacher(s). The target knowledge is visible in students' utterances and stands out as the main reason a task was found interesting and engaging. In all tasks, students pointed out elements of presentation as a reason for being interested and engaged. The students recalled tasks they had been working with as far back as three years ago (year 7). The results suggest that when the target knowledge is brought to the fore, and tasks include investigations and non-routine features, have a high level of cognitive demand and provide opportunities to share solutions and reasoning behind them, they lead to interest and engagement among the students.

## 5.5 Summary of the results

The three empirical studies resulted in four papers. In the papers, student engagement has been outlined from three different perspectives: as visible to observing researchers, as seen and developed by teachers during classroom interaction and as perceived by the students in mathematical tasks. Analyses on the classroom level were carried out to find what students engage in in relation to tasks on mathematical relations (Paper I). Task-related attention was connected to the relevance of a task, the process of solving and validating a task. From these results, ideas for continued research emerged concerning the teacher's role in the process of enhancing engagement.

These ideas were pursued in a study reported in Papers II and III. The teachers reached consensus on what features indicate student engagement (Paper II), and they contributed insights into their own role in the process (Paper III). The results from Paper III revealed strategies related to the activity, for instance general strategies to enhance engagement rather than specifically mathematically oriented strategies, on the level of the macro-contract.

The results from Paper III laid the foundation for the design of Paper IV, where the students' perspectives on interest and engagement in mathematics tasks were investigated in the classroom of a teacher who deliberately puts the content at the fore. The results from Paper IV show that a task can be found engaging if it has the target knowledge at the fore, includes investigations and non-routine features, has a high level of cognitive demand and a certain degree of openness and if it provides opportunities to share solutions and reasoning.

### 5.5.1 The results in terms of TDS

In Chapter 3, a didactical tetrahedron was introduced, relating student, teacher, mathematics and task. The contributions of this thesis can be located on the faces of such a didactical tetrahedron. By expanding the didactical triangle with the task-node, new faces of the figure where the teacher can make an effort to engage students become visible. In Papers I, II and III, the results are on the Student-Teacher-Task face (Figure 7).

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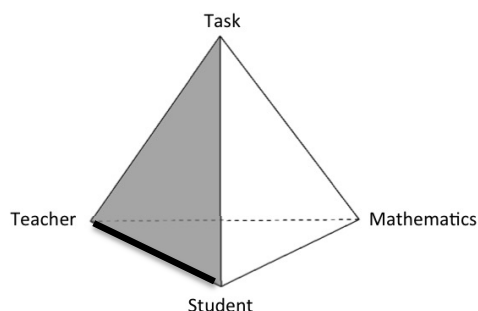


Figure 7: The results from Papers I, II and III are positioned on the didactical tetrahedron. What students attend to in the task during the situation of formulation (Paper I) and the strategies used to enhance engagement (Paper III) are placed on the Student-Teacher-Task face. The indicators of engagement (Paper II) are marked by the bold line on the Student-Teacher edge. Mathematics as such, however, is in the background.

The position of results from Paper I shows that student engagement is identifiable in relation to tasks, and that it would be relevant to make a distinction between task and mathematics when approaching student engagement in further studies.

In Paper II, the results show that teachers can identify when engagement is expressed by the students, through a number of indicators that describe behaviours observed and validated by the teachers in their own and each other's teaching. Those indicators, such as concentration and gestures, are directed towards the task or target knowledge in the tasks. However, the teachers make no connections between the indicators and the target knowledge. The indicators are connected to students' actions in the situation as such, for instance losing track of time. Therefore the results are placed on the Student-Teacher edge, marked by the bold line in Figure 7, highlighting students' actions in the classrooms.

In Paper III the strategies to engage students that the teachers described are placed on the Student-Teacher-Task face, since the teachers highlight strategies that are all part of the meso-contract, the activity level, while the teachers did not emphasise strategies connected to the Teacher-Math-Task face that could initiate engagement on the Student-Math-Task face.

In Paper IV the views of the students are placed on the Student-Task-Math face (Figure 8).



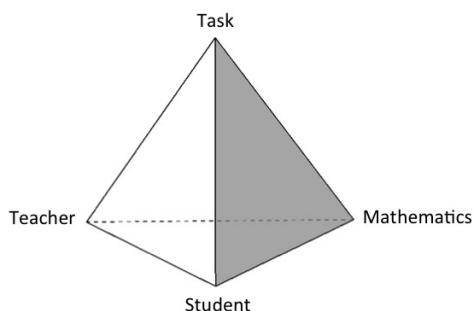


Figure 8: The shaded face represents the results from Paper IV as positioned on the didactical tetrahedron. The reasons the students give for a task being interesting and engaging are positioned on the Student-Math-Task face.

These students have been exposed to the tasks of a teacher who is explicit in her deliberate intent to engage with the target knowledge at the fore. As shown in Figure 8, the students' own descriptions of their engagement are placed on the Student-Math-Task face. The students highlight the mathematics in the tasks as a reason for what makes the task engaging.

Seen in the light of Brousseau's (1997) theory of didactical situations in mathematics, in order to engage students, it is important to prepare lessons within the Teacher-Math-Task face. When those faces are combined, they cover the devolution of an didactical situation. In other words, if the teacher plans the lessons on the Teacher-Math-Task face, by designing tasks where the mathematical target knowledge is at the fore and specific features of the task are developed to engage students, engagement on the Student-Math-Task face is made possible and an didactical situation can devolve. A schematic summary of the results is presented in Table 2.

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Table 2: Schematic summary of the papers

<b>Paper</b>	<b>I</b>	<b>II &amp; III</b>	<b>IV</b>
<b>Status (Year)</b>	Published (2013)	Published; Published (2015)	Published (2016)
<b>Data set</b>	LPS	VIDEOMAT	My own data collection
<b>Data type</b>	Video	Video	Audio
<b>Mathematics</b>	Mathematical relationships	Introductory algebra	Geometry, statistics
<b>Year</b>	8	6-7	7-9
<b>Framework</b>	GOK	CE & TDS	MTF, TDS
<b>Perspective</b>	Researchers	Teachers	Students
<b>Research questions</b>	What aspects of a mathematics task do students attend to when interacting with the teacher? And how is interest co-constructed in such situations?	II. How are indicators of engagement identified, negotiated and exemplified by teachers? III. What didactical strategies do teachers propose for engaging students when introducing algebra?	What tasks do students identify as interesting and engaging when the teacher has deliberately brought the content to the fore? What features do students identify as interesting and engaging?
<b>Main results</b>	When students approached the teacher in a situation of formulation, they engaged with the relevance of mathematical relationships in a task, the solution of a task and the validation of numerical answers.	Indicators of engagement as seen by teachers are: verbalising thinking, concentration, gestures expressing attention or excitement, asking and answering questions, enhancing ideas and justifying an argument. Strategies to enhance engagement suggested by the teachers concerned the design of the didactical situations and negotiation of the didactical contract.	A task can be found engaging if it has the target knowledge at the fore, includes investigations and non-routine features, has high level of cognitive demand and a certain degree of openness and if it provide opportunities to share solutions and reasoning.
<b>Location on the didactical tetrahedron</b>	Student-Teacher-Task	Student-Teacher/ Student-Teacher-Task	Student-Mathematics-Task
<b>The didactical contract</b>	Micro	Paper II: Micro Paper III: Meso	Macro

### 5.5.2 Main contributions

This thesis contributes to a further understanding of interest as manifested through engagement in mathematics. The main contributions are insights into the relational constitution of engagement, in the interplay between the student, the teacher, the task and the mathematics. These insights are explicated by using different faces of the didactical tetrahedron. More specifically:

- Teachers have an important role in engaging students in mathematics during **the didactical situation**. On the Teacher-Student-Task face, there are opportunities to engage students in mathematical relationships during the situation of formulation.
- While researchers' definitions of interest and engagement differ, teachers are able to agree on how engagement is indicated in the classroom, pointing out examples both in their own and in each other's teaching. They exemplify a range of strategies to enhance engagement connected to **the didactical situations** on the level of the meso-contract.
- Students brought up the target knowledge as a reason for a task being interesting and engaging. A teacher who brings the target knowledge to the fore can enhance student engagement on the level of the micro-contract and thus contribute to the devolution of an **adidactical situation**.

Central methodological contributions are firstly that data collected for other purposes can be fruitful when addressing new research topics. Secondly, the element of showing and watching concrete examples from teachers' own classrooms proved to be important. This method can show how teachers' views play out in the classroom. Compared to merely using teachers' views that are visible through questionnaires or individual interviews, the video-based focus-group discussions lend greater validity to the results, since teachers' views do not always correspond to what actually happens in the classroom (Cohen, 1990). A central theoretical contribution is the development of the didactical tetrahedron, which turned out to be useful when analysing the results using TDS. The expansion of the didactical triangle, by adding task as an extra node, proved to be helpful when separating the target knowledge from other features of the task and identifying opportunities to enhance engagement in mathematics.

## 6 Discussion

Our intentions as researchers are not to judge the teachers or the students, or correct their behaviour, but to describe and understand. (Brousseau, personal communication, February 19, 2016)

The overall purpose of this thesis has been to gain further understanding of interest manifested as student engagement in mathematics. The results contribute different perspectives on how student engagement is visible in mathematics classrooms and what teachers can do to enhance it. The findings are now discussed within the confines of the background presented above. I discuss what one can learn from the studies, compare the results with previous research, examine implications and list strengths and limitations as well as suggestions for future research.

### 6.1 What can we learn from the studies?

Looking across the empirical results presented in the papers, they all contribute to further insights on how student engagement in mathematics is visible to an observer and what teachers can do to enhance student engagement in mathematics. From my teaching background, I find the themes concerning task-related attention (Paper I), indicators of engagement in an introduction to algebra (Paper II) and didactical strategies used to enhance engagement (Paper III) recognisable and easy to relate to. The analysis of tasks pointed out by the students as interesting and engaging (Paper IV) shows that when the target knowledge is brought to the fore, it becomes visible to the students and can be an important reason for a task to be engaging.

Interest as manifested through engagement, based on Dewey's (1910, 1913, 1919/85) ideas, turned out to be helpful in conceptualising interest in an empirical approach. Papers I and II have shown, in line with Appleton and Lawrenz (2011) as well as Helme and Clarke (2001), that student engagement is identifiable on a classroom level. Seeing Paper I through the lens of TDS and GOK, engagement was visible on the Student-Teacher-Task face of the

didactical tetrahedron, in situations of formulation where the teacher re-entered the scene. Considering the level of the micro-contract, student engagement in tasks is related to relevance of the task, the process of finding a solution to a task and the validation of numerical answers. By posing a content-specific question to the teacher, the students expose a gap of knowledge (Silvia, 2006), during situations of formulation (Brousseau, 1997). Depending on how the teacher is able to discuss those questions, the didactical situation (Brousseau, 1997) can be restored. During such instances, the teacher has an opportunity to engage students in learning mathematics, for instance as mentioned in Kriegler (2016), by explaining the meaning of mathematical relationships beyond symbol manipulation.

Student engagement is expressed similarly in different situations. Papers II and III show how engagement can be visible to the teacher and enhanced when introducing algebra. The indicators identified by the teachers help in recognising students who are interested and engaged; they correspond with Helme and Clarke's results (2001, 2002), and contribute to teachers' awareness of how student engagement can be shown by students in various situations, both didactical and adidactical. However, in contrast to the model of Helme and Clarke (2001, 2002), the connections between the types of situations and the indicators differ, which implies that the nature of the indicators is dynamic in relation to the type of situation, i.e. whether or not the teacher is present. It is also noteworthy that Helme and Clarke (2001, 2002) were not dealing with the same target knowledge throughout their study, which is the case in Paper II. Instead they focused on lessons rich in student-student interaction and the aspects of engagement related to adidactical situations. Therefore, in the episodes chosen by the teachers in Paper III, indicators of engagement identified provide more information in relation to the teacher's role.

A range of strategies that teachers use and suggest in order to enhance engagement were identified, all connected to the meso-contract. In other words, they gave priority to the meso-contract rather than the macro-contract, and for the didactical situation rather than the target knowledge. They put forward issues related to the realisation of activities (Hersant & Perrin-Glorian, 2005b), while the choice of the target knowledge, what Brousseau (1997) refers to as the didactical interest of a problem, is taken for granted. In other words, the teachers' focus was, in contrast to the conditions in Paper IV, on didactical situations and not on the mathematical target knowledge.

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Paper IV shows, in contrast, that when the students explained why a task is interesting, the target knowledge was brought up as the first reason. Didactical situations of different kinds were mentioned, but the students embraced the description of the target knowledge. This was surprising, since the teachers in Paper III focus on the didactical situations rather than the target knowledge, despite the moderator's repeated attempts to re-direct towards algebra. One possible explanation is that since the teacher involved in Paper IV works with the content at the fore, the students are used to the content being highlighted and to the use of mathematical language, and therefore the content stands out and engages them.

Tasks selected as interesting and engaging by the students in Paper IV have similar features, for instance the high level of challenge. This is in line with Stein and her colleagues (2000), who state that this category of cognitive demand influences student engagement. The students found the target knowledge engaging in itself, which is in line with Rellensmann and Schukajlow's (2016) findings. Intra-mathematical aspects can be found interesting in preference to, for instance, context.

Bikner-Ahsbals (2002) emphasises engaging students on the activity level, which is also suggested by the teachers in Paper III. However, in line with Brousseau (1997), I agree that it is important to go beyond the activity level and highlight the importance of the target knowledge, as done by the teacher described in Paper IV.

Algebra-specific strategies have not been encountered in this thesis as engaging or as a means of engaging students, although all lessons discussed in the focus-group discussions in Paper III were algebra lessons. The teachers did not acknowledge any engaging intra-algebraic aspects or strategies summarised in Table 1 from Kriegler's (2016) review, even though the moderator explicitly asked for intra-algebraic details on what makes the students engaged. Further, all of the four tasks described by the students in Paper IV included target knowledge from geometry and statistics. No explicit target knowledge in algebra was chosen. Based on these results, I pose the question of whether there are strong challenges associated with making algebra engaging. There is, however, research showing that algebra can be interesting and engaging as a result of algebra-specific aspects (Mason et al., 2005). Therefore, in future research more emphasis is needed on how to make algebra interesting and engaging during algebra lessons.

Empirical studies on student engagement in this thesis benefited from being theoretically anchored through the research questions stated within the conceptual apparatus of TDS. The first attempts to approach the concept were made with the use of specific frameworks such as GOK and CE. The use of TDS and different levels of the didactical contract is a distinct example of theorising, making visible something that cannot be captured without the precision of the theoretical concepts (Jablonka & Bergsten, 2010). TDS could have been a base for all of the studies, which the analysis of the results from all of the papers made in this *kappa* confirms.

The final paper, Paper IV, has gained from the use of a well-established framework of task analysis, MTF, in combination with the solid theoretical ground of TDS. MTF helped to establish the level of challenge in every task, but TDS helped to thematise task features brought up and statements made by the students. In Papers I and II, TDS could have served as a background to what is taking place in the classroom and the ways one can look at teaching. In Papers III and IV, analytical tools from TDS are used and specific theoretical terms are investigated, for instance the fine-grained analysis on different levels of the didactical contract. The results in Paper IV show the possibilities of using TDS when dealing with interview data, bringing in the task dimension (Rezat & Sträßer, 2012), with the target knowledge made explicit. It is shown how TDS can be used when analysing interview-generated data and the findings suggest that if the target knowledge is made visible to the students, it can also be detected in their statements regarding what they find interesting and engaging.

## 6.2 How are the papers related?

The papers in this thesis are thematically and conceptually related. There are connections between the research questions, and most importantly, the results with respect to TDS and the way the results are positioned in the didactical tetrahedron.

Thematically, the research questions link the four studies together, since they were developed in a generative process. The research questions for Papers III and IV were generated from the insights provided by Papers I and II. Interest as student engagement is approached in Papers I-III as visible to observers and the results from Paper I reveal details about engagement within the meso-contract (Hersant & Perrin-Glorian, 2005b). The results in Paper III

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suggest that engagement as seen by the teachers is related to the meso-contract, the level of the activities. At this point, a legitimate question concerns the role of the target knowledge, such as aspects of generalisation or patterns (Mason et al., 2005; Selling, 2016). Can strategies described in the summary by Kriegler (2016), such as focusing on sense-making and not merely symbol manipulation, be used to engage? And what views do the students of a teacher who engages by putting the target knowledge to the fore have? This was investigated in Paper IV, which showed that the students of a teacher who puts mathematics to the fore find the target knowledge engaging. When those students described what aspects of a task make it interesting and engaging, they highlighted the mathematical content. However, target knowledge other than algebra is pointed out as particularly interesting, which leads to the question of how the topic of algebra can be made interesting and engaging. A new question for future research arose: How can teachers work with the target knowledge in algebra in order to be potentially engaging?

Further, there is a conceptual link between the papers when it comes to the use of concepts relating to student engagement. In the first paper, initial attempts were made to approach student engagement by looking at the focus of attention in connection to tasks. In Paper II an existing framework of cognitive engagement (CE) was tested as a model to analyse whether it is useful in answering the question of teachers' views on interest expressed as engagement in situ. Paper III was based on data from Paper II and was even more teacher-oriented, letting the teachers describe their own role in the process of developing interest and engagement. It provides specific examples of how to engage students on a classroom level. In Paper IV, the terms interest and engagement are both used in the interviews with the students, in order to give the students an opportunity for making rich associations and interpretations.

Papers III and IV are theory-based in that they include the specific theoretical constructs in the research questions (Radford, 2008). There is a development in the progression of approaches, from frameworks adopted in Papers I and II towards a stronger theoretical anchor in Papers III and IV, where a more established theoretical tradition of TDS in mathematics is used to analyse the outcomes. The analysis shows that the teachers emphasise student engagement as visible on the Student-Teacher-Task face of the didactical tetrahedron, and suggest strategies on the same face, in connection to the meso-contract. The students discussed interesting and engaging tasks



within the Student-Math-Task face, bringing up mathematics content as engaging. In order to engage students, preparing lessons within the Teacher-Math-Task face and the Student-Math-Task face is important, since when combined they describe the devolution of an didactical situation.

### 6.3 Implications

This research project was grounded in a didactical issue and pursued through empirical observations on the classroom level and in interviews, bringing out different perspectives on student engagement. In this thesis, the relevance of the results for the teaching profession was central. Implications for future research concern new ways of addressing the concept of interest as manifested through engagement, which adds to the body of existing research.

This thesis has didactical implications related to teachers' professional knowledge and everyday practice. Paper I describes three categories of task-related attention; by extension, this implies that there are ways of working with student engagement in relation to student questions about tasks. For instance, questions considering the use of mathematical relations in everyday life can be answered using conceptual reasoning on what mathematical relations are and how to understand those beyond instrumental use (Thompson, 1992).

In order to enhance student engagement, it is important to help the students to acknowledge what mathematical idea we are aiming for in our teaching. With respect to the results from the papers, it is possible to apply the didactical strategies brought up in Paper IV when working with tasks that enhance engagement. The main point is to provide learning opportunities, by bringing the target knowledge into the foreground in the classroom. If the content is neglected, the didactical situation might suffer. Adding to this line of reasoning, I suggest that student engagement could be enhanced in the planning stage, the Teacher-Mathematics-Task face, so that specific mathematical ideas initially become visible for the students and are meant to be engaging in themselves.

A teacher has opportunities to enhance engagement on all faces of the tetrahedron – when planning lessons on the Teacher-Math-Task face, during the introduction or discussions of mathematical ideas on the Teacher-Student-Mathematics face, when discussing tasks with the students on the Student-Teacher-Task face and when the didactical situation takes place, on the

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Student-Mathematics-Task face. It is important to be aware of this, so that the emphasis does not land solely on the meso-contract, in such a way that the didactical situation, when the students can engage without the teacher's involvement, is lost.

One challenge has been to define and operationalize interest, since there is a lack of consensus on this issue among researchers (Harris, 2008). In this thesis, this was done in line with Dewey's reasoning, through the concept of student engagement. The episodes depicting student engagement presented in this thesis are not intended to provide a universal definition of student engagement. They aim to contribute to conceptual clarity by bringing forth illustrations of interest as manifested through engagement in mathematics classrooms from different perspectives. During such a process, "the researcher is a learner, seeking the meaning and structure of her phenomenon." (Marton & Booth, 1997, p. 133).

A possible research outlook includes aspects of how to further approach student engagement in connection to mathematical target knowledge, on the Teacher-Math-Task face of the didactical tetrahedron. I agree with Harris (2011), who suggests that student engagement is a useful concept when describing student experiences and learning.

The multi-method approach including different perspectives has generated new, relevant research questions. This richness is visible in the video analysis in Paper I, which serves as a methodological example of the possibilities of the practical use of existing data sets and how data can be used in new ways. Paper I shows, in line with Silverman (2010), that it can be advantageous when several researchers with different relationships to the data collaborate during the analysis – both the researchers who are acquainted with the data and those to whom the data is new.

Even though insights can be gained from secondary analysis of a data set, fieldwork provides opportunities for the data generation process to be influenced directly by the research themes and questions. The research questions addressing the design of the empirical study contribute to a more specific approach, which can be developed and adjusted during the course of the data collection. As I see it, both secondary and primary data sets are valuable, but for novice researchers it is especially important to acknowledge collection of empirical data as a learning opportunity.

Teachers benefit from sharing their teaching with one another and discussing issues that are relevant to their profession. As shown in Papers II

and III, specific questions about student engagement were posed to the teachers: for instance, the moderator stressed the possibility of engagement being related to the inner qualities of the target knowledge. The lack of intra-mathematical strategies raises further question about what makes algebra interesting in itself. The research approach resulted in a direct negotiation of meanings and interpretations. Several steps led to the focus interviews (Merton, 1946), where the teachers chose sequences to share and discuss with colleagues, and where professional insights into teachers' practices were obtained.

The teacher plays an important role in the collection of student data. Paper IV shows that there is a need for awareness that fieldwork is dependent on collaboration with the participating teacher. In this case, the teacher provided necessary information about the students, made an effort to select them from different groups and made the practical arrangements so that each and every student could participate in the interview.

## 6.4 Strengths, limitations and outlook

This thesis contributes potentially valuable professional insights on student engagement and the teacher's role when enhancing engagement in mathematics. Different methods of a qualitative nature used in order to portray student engagement are favoured when approaching the complexity of the concept. The three different perspectives, those of researchers, teachers and students, enrich each other and contribute to rich descriptions of student engagement.

The results are strengthened by the use of rich empirical classroom data from both big and small data sets, analysed with both small-scale frameworks and a well-established theory. The studies capture different features of student engagement as researchers, teachers and learners identify them.

In this thesis, teachers in Papers II and III reached consensus on what engagement is in the classroom context. Since no study was made on the student views in direct connection with these teachers' views, it is not possible to claim that they shared the teachers' views in the situations. A study where the researchers', the teachers' and the students' perspectives within one didactical situation are included would benefit from triangulation (Bryman, 2004). Such triangulation of the same content and classroom events could be obtained in a future case study.

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This thesis took its point of departure in the beneficial role of interest to learning (Dewey, 1913; Ma, 1997; Schraw & Lehman, 2001). One of this thesis's limitations is that this specific relationship is not investigated; no conclusions can therefore be drawn about the relationship to learning based on this research project. Nevertheless, the results can be a basis for further studies on the role of student engagement in learning mathematics. With the results from this thesis, which consider different perspectives on what student engagement in mathematics is and how to enhance it, it would be possible to design a study with learning in focus. A mathematics focus can be achieved by further specifying research questions relating to elements in algebra. One way to go about that would be to investigate the development of engagement in relation to how students learn specifically chosen algebraic target knowledge over a period of time. A longitudinal study with a focus on lesson planning could make it possible to map engaging target knowledge in algebra, investigating the micro-contract (Brousseau, 1997; Hersant & Perrin-Glorian, 2005a).

Focus-group interviews in Papers II and III were fruitful not only due to the collective meaning and the consensus the teachers reached (Merton, 1949) but most of all because the teachers provided concrete examples from their teaching to support their views on what student engagement is. The focus group as a method could have been used throughout the studies of both researchers and teachers. One critical reflection is that whole-class video recordings and focus-group video recordings might have provided more details for the teachers to analyse. Since the focus of this study was on teachers' strategies as well as indicators of engagement, it was important to have access to the camera focusing on the teacher, but for future attempts a solution with multiple camera views would be considered. In Paper IV, the student interviews could be longer and in greater depth. By doing this, it would be possible to find out more about the role of the target knowledge and about engaging details in the tasks.

An overall reflection in connection to methods is that the different methods (observations, interviews in focus groups and individual interviews) were used on different categories of participant (outside observers, teachers, students). Such variety enriched this thesis, but did not provide opportunities for comparison between the perspectives. In future studies, a suggestion is to keep more factors constant. One possibility would be to compare different

views on the same sequences, such as sequences the teachers provided from their own teaching.

Looking ahead, it seems meaningful to continue studying student engagement in mathematics. The outcomes of Papers I-IV could be useful in new studies. Based on the conditions in Paper IV, where the students lacked engagement to start with, it would be fruitful to study similar cases and observe indicators of engagement found in the analysis presented in Paper II, as well as didactical strategies used by teachers presented in Paper III, and to use tasks similar to those in Papers I and IV. Further, the results could be connected to learning outcomes, to see if they can possibly be related, to study situations in which a notable shift from disengagement to engagement takes place while students are working with specially designed tasks, with features similar to those in Paper IV. Here, a case study would make it possible to further explore the teacher's role, to show different ways of designing tasks and to shed light on the described types of shift from lack of interest and engagement.

Since the didactical strategies employed by the teachers as shown in Paper III mainly focused on the meso-contract, it would be fruitful to design a study that could reveal details of the macro-contract. It is possible to investigate the mentioned aspects of the didactical contract further with the macro-contract being taken into consideration at an early stage of the research design by directing a study towards specific target knowledge: What makes algebra interesting? One possible direction would be to select other participant groups, for instance mathematicians who specialise in algebra or specific student groups and their teacher, for instance those who achieve high scores on mathematics tests or who specifically choose to study higher mathematics in upper secondary school. Another research opportunity would be to design a study with an emphasis on central aspects of algebraic thinking, such as patterns and generality (Vance, 1998; Kaput, 199; Kriegler, 2016). This can be achieved by including elements of intervention in a study where engagement is a condition for learning.

## 6.6 Conclusion

Knowledge of situational factors or the hooks that can attract students to an activity is part of the story, but for the gears to engage and generate forward movement, other processes must come into play. [...] When the content of learning activities pertains to something that is valued and/or is perceived to be enjoyable, students choose to engage and often seek to reengage if given the opportunity. (Ainley, 2012, p.300)

Each mathematics classroom is unique, and yet to some extent all classrooms are alike, sharing common features with other classrooms (Eisner, 1991). Classroom events, utterances of teachers and students as well as tasks analysed in this research project were full of surprises, but at the same time very familiar. The aim of this thesis was to gain further understanding of interest manifested through engagement in mathematics. The main questions concerned how engagement is identified in classroom settings and how teachers can enhance it. The main contributions of the empirical studies are considerations about the relational constitution of engagement in the interplay between the student, the teacher, the task and the mathematics. Positioning the findings on different triangular faces of the didactical tetrahedron has helped in further understanding student engagement expressed in the classroom - as identified by researchers, teachers and students. The results show that teachers have an important role in the process of engaging students in mathematics. The results also show that teachers can reach consensus on what engagement is, by pointing out examples from their own and each other's teaching. However, when the teachers showed and suggested a range of strategies to enhance engagement, the strategies were all connected to the meso-contract. In contrast, a teacher who puts the target knowledge to the fore can enhance student engagement within the micro-contract and thus contribute to the devolution of an adidactical situation.

The teachers did not bring up engaging algebra-specific strategies to engage students. Also, there are indications that target knowledge other than algebra stand out as more engaging to the students. Therefore in further studies it seems important to find out what target knowledge in algebra is interesting and engaging and how the macro-contract in algebra can be developed. It is possible that other types of studies are needed to find out more about intra-mathematical strategies, focusing on algebraic target knowledge.

Supported by the results of this thesis it can be said that it is an advantage if the teacher's attempts to engage students in mathematics are explicit on the Teacher-Mathematics-Task face of the didactical tetrahedron. Some suggestions about how to engage students in mathematics for pre-service and in-service teachers dealing with similar mathematics content and age groups are:

- Bringing the mathematics into the foreground.
- Providing a high degree of challenge.
- Designing tasks that allow opportunities for student influence.
- Providing opportunities for the students to present their results.

This thesis does not provide any simple solutions regarding how to engage students in mathematics. Instead, the results contribute to a deeper understanding of a complex concept. Overall, the contributions made by this thesis can serve as practically oriented support to teachers in relation to the challenges of providing engaging learning opportunities in mathematics.

The results generated new questions, potentially relevant to teachers and researchers. One such question is how to plan algebra lessons and choose the target knowledge with the intention of enhancing student engagement, thus contributing to the devolution of didactical situations.

Hopefully, the results of this thesis can help other teachers to reflect on their views on engagement and their role in enhancing it in their classrooms. Furthermore, I hope this thesis will influence other researchers to conduct further studies on student engagement in mathematics.

## 7 Swedish summary

### 7.1 Intresse och engagemang i matematik

I den här avhandlingen studeras matematikintresse manifesterat som elevengagemang. Avhandlingen har sin utgångspunkt i antagandet att intresse blir synligt när det manifesteras i form av engagemang (Dewey, 1913; Frenzel m.fl., 2010) och studierna bygger på klassrumsnära frågor: Vad kännetecknar elevengagemang i matematik? Hur kan man som lärare engagera elever i matematik? Med hjälp av resultat från empiriska studier om algebra i årskurs 6-9 belyses elevengagemang ur forskarperspektiv, lärarperspektiv och elevperspektiv. Den här sammanfattningen ger en överblick över avhandlingens centrala delar och diskuterar huvuddragen i de delstudier som ingår i sammanläggningen.

#### 7.1.1 Syfte och forskningsfrågor

Avhandlingen syftar till att vinna nya insikter om intresse manifesterat som elevengagemang i matematikklassrummet och därmed om förutsättningar för matematikundervisning. Särskilda studier ägnas åt undervisningsstrategier samt design av uppgifter i årskurs 6-9. Avhandlingen i sin helhet ämnar bidra till fördjupad förståelse av elevengagemang i matematik. De övergripande forskningsfrågorna är:

- Vad riktar elever sin uppmärksamhet mot när de interagerar med läraren om matematikuppgifter? Hur samkonstrueras intresse i sådana situationer? (Paper I)
- Hur är indikatorer på elevengagemang identifierade, förhandlade och förklarade av lärare? (Paper II)
- Vilka didaktiska strategier föreslår lärare i syfte att engagera elever i algebraintroduktion? (Paper III)
- Vilka uppgifter identifierar elever som intressanta och engagerande när läraren har matematiken i förgrunden? Vilka egenskaper identifierar eleverna som intressanta och engagerande i dessa uppgifter? (Paper IV)



## 7.2 Tidigare forskning och teoretisk anknytning

Resonemang kring intresse har förts inom det utbildningsvetenskapliga forskningsfältet under en lång tid. Redan för över hundra år sedan påtalade Herbart och Smith (1895) och Dewey (1903) att intresse var av vikt för lärande. Sedan dess har det genomförts en mängd teoretiska och empiriska studier kring intresse (Dewey, 1913; Hidi, 1990; Bikner-Ahsbaks, 2004; Krapp, 2007), främst med fokus på inre, inneboende egenskaper hos individen (Silvia, 2006). Intresse har även beskrivits som dikotomt, det vill säga som antingen personligt eller situationsbundet (Mitchell, 1993). Dewey (1910, 1913, 1916/1997) påvisade en koppling mellan intresse och engagemang såtillvida att intresse blir synligt när det manifesteras genom engagemang.

Elevengagemang har i sin tur studerats empiriskt ur lärarperspektiv (Harris, 2008; 2011) och i matematikundervisningen kopplats ihop med indikatorer synliga för observatörer (Helme & Clarke, 2001; 2002). Skilling med flera (2016) har visat att om lärare uppfattar sig själva som maktlösa och förlägger engagemangskapandet bortom sin lärarroll, begränsar det deras förmåga att engagera elever. Mot denna bakgrund vill jag undersöka om det finns aspekter av intresse som manifesteras genom elevengagemang och därmed blir synliga och möjliga att beforska på ett klassrumsrelevant sätt och ur olika perspektiv.

I de fyra artiklar som ligger till grund för denna avhandling har forskare, lärare och elever bidragit till potentiellt viktiga insikter om elevengagemang i matematikklassrummet. Analyserna har gjorts utifrån ramverk relaterade till specifika frågor som tas upp i varje artikel, och även utifrån en gemensam teoretisk anknytning. Undervisning beskrivs och problematiseras utifrån teorin om didaktiska situationer (TDS). Teorin är ursprungligen utvecklad av Brousseau (1997) och har samspelet mellan lärare, elev och matematik i fokus. En utgångspunkt för teorin är att beskriva mönster som kan uppträda i matematikundervisningen och att matematikundervisningssituationernas didaktiska fokus framhålls (Liljekvist, 2014).

Terminologin från Brousseaus teori har använts för att precisera analyser av samspelet och utsagor kopplade till engagemangskapandet i matematik. TDS erbjuder verktyg för att svara på frågor om förutsättningar för lärande (Brousseau, personlig kommunikation, 13 feb 2016). Grunden för TDS är empirisk och bygger främst på experimentella resultat, med fokus på "de villkor som tillåter elevernas kunskaper att utvecklas" (Margolinas & Drijvers,

2015, s. 896). Teorin är till hjälp för att bättre förstå matematikundervisningen och samspelet mellan lärare, elev och matematik. Samspelet förklaras genom begreppet didaktisk situation. Det är läraren som "har ansvar för att skapa didaktiska situationer som involverar eleverna och göra det möjligt för alternativa lösningar på ett problem" (Hansson, 2011, s. 37). I tidigare studier illustreras samspelet mellan lärare och elev med en så kallad didaktisk axel (Hansson, 2011, s.7). Den kunskapsteoretiska utgångspunkten inom ramen för TDS är idén om didaktisk transposition (Chevallard, 1992), vilket sker i samspel mellan lärare och matematik.

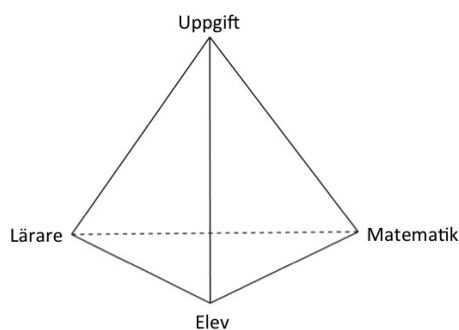
Begreppet didaktik är förankrat i ett grundläggande kunskapsteoretiskt antagande att transpositionen av matematisk kunskap är kärnan i matematikundervisningen (Brousseau, 1997, 1999), att en matematisk idé anpassas till klassrumskontexten. Denna process börjar med valet av matematiskt ämnesinnehåll och leder till en förhandling av det didaktiska kontraktet i klassrummet (Brousseau, 1997). Det didaktiska kontraktet är en uppsättning ämnesspecifika normer och regler som förhandlas fram mellan lärare och elever. Många gånger är dessa regler underförstådda och ingår i den didaktiska situation, där läraren sätter sin prägel och gör sina avsikter explicita. Det didaktiska kontraktet är indelat i tre delar (Hersant & Perrin-Glorian, 2005a; 2005b) – makro, meso och mikro. Makrokontraktet beskriver ett planeringsstadium, där det matematiska innehållet väljs ut av läraren och uppgifter designas. Mesokontraktet utgörs av aktivitetsnivån, då planen realiserar i klassrummet. Det innefattar regler och normer, elevrespons och beteenden i relation till innehåll och aktiviteter. Mikrokontraktet utgörs av elevernas resonemang och ageranden i klassrummet.

Ett illustrativt exempel är när Hunter (2010) arbetade med algebrainroduktion med hjälp av två öppna utsagor:  $3 \times \_ = 6$ ,  $6 \div \_ = 2$ . Makrokontraktet utgjordes av lärarens val av just dessa utsagor i syfte att påvisa sambandet mellan multiplikation och division. Mesokontraktet utgjordes av lärarens strategier för arbetet med det valda innehållet i klassrummet, i det här fallet diskussioner i par och sedan i helklass. Mikrokontraktet utgjordes av elevernas resonemang om vad de lägger märke till när de diskuterar uppgiften, gav förslag på samband så som att de två utsagorna är varandras motsatser.

I en didaktisk situation finns också ett adidaktiskt inslag. Adidaktisk situation utvecklas om läraren lyckas ge viktiga förutsättningar för eleverna att acceptera en uppgift som sin egen. Eleven arbetar med uppgiften utan läraren.

Begrepp som det didaktiska kontraktet samt didaktisk/adidaktisk situation möjliggör för precisa beskrivningar och analyser av vad som händer i matematikundervisningen.

Matematikundervisningens didaktiska struktur byggs upp i ett samspel mellan eleven, läraren och matematiken (Brousseau, 1997) och som det beskrivs i senare teoriutveckling av TDS (Rezat & Sträßer, 2012), en artefakt. En artefakt är "den fjärde stöttestenen som utgör en grund för en didaktisk situation" (Rezat & Sträßer, 2012, s. 649) och kan vara en lärobok (Gallos Cronberg, 2016) eller en matematikuppgift (Figur 9).



Figur 9: Den didaktiska tetraedern utifrån en modell av Rezat and Sträßer (2012).

I min anpassning av modellen är artefakten en matematikuppgift. Till skillnad från Brousseaus ursprungliga modell är det matematiska innehållet skilt från uppgiften, i syfte att kunna inkludera uppgiftens många egenskaper. Relationen mellan uppgiften och matematiken representeras av en egen kant. Genom att lägga till uppgiften bildas triangelytor som utgör nya plattformar för analys av de didaktiska relationerna mellan de tre olika aktörerna i samspelet, och tydliggör uppgiftens roll.

Elevengagemang i denna avhandling anses vara kopplat till samspelet mellan aktörerna i klassrummet (Frenzel m.fl., 2010). Brousseau (1997) uttrycker, i likhet med Dewey (1913), att engagemang också är kopplat till elevernas insatser. Den mest intressanta uppgiften kommer, enligt Brousseau (1997), att vara den som erbjuder eleven en matematisk utmaning, och intresse kännetecknas av det engagemang eleven är redo att investera i uppgiften. Grundat i det synsättet ger TDS språkliga verktyg för att analysera lektioner, lektionsepisoder, klassrumssituationer och uppgifter.

Intresse och engagemang är begrepp som används i stor utsträckning, men är varken väldefinierade eller entydiga. Den här avhandlingen beskriver hur intresse uttryckt som elevengagemang identifieras med hjälp av olika aktörer - forskare, lärare och elever. Unifrån tidigare forskning om intresse och engagemang kan man säga att:

- Intresse manifesteras genom engagemang (Dewey, 1913) och blir på så sätt observerbart (Frenzel m.fl., 2010).
- Intresse påverkar lärande och vice versa (Ma, 1997) och engagemang är fördelaktigt för lärande (Harris, 2008; Exeter m.fl, 2010).

För att engagera elever i matematik kan läraren:

- Lyfta fram syftet framför processer (Schoenfeldt, 1992).
- Satsa på elevcentrerade aktiviteter som kräver hög deltagandenivå (Weiss, 1990).
- Arbeta med uppgifter som har vardagsanknytning (Boaler, 1999).
- Utveckla elevers konceptuella kompetens (Azevedo m.fl., 2012).
- Uppmuntra elever att delta aktivt (Mitchell, 1993).
- Låta eleverna problematisera innehållet, uppmuntra dem att ta sig an problem auktoritativt och bistå med rätt resurser (Heidi & Renninger, 2006).

Många förslag på strategier har aktiviteter snarare än ämnesinnehåll i fokus (Wilson m.fl., 2005). När det gäller ämnesinnehåll kan man i algebra engagera bland annat genom att satsa på förståelse och inte enbart på symbolhantering (Kriegler, 2016) samt genom att arbeta med mönster (Selling, 2016) och generaliseringsaspekter (Mason m.fl., 2005).

Det finns många studier om intresse och engagemang, men precis som Skilling och hennes kollegor (2016) påpekar, finns det få studier som ger klassrumspektiv i relation till matematikinnehållet. Denna avsaknad leder till ett behov av studier som belyser intresse på ett klassrumsrelevant sätt.

## 7.3 Metod

Det empiriska materialet som Paper I-IV bygger på har samlats in inom ramarna för tre olika delstudier. Delstudierna är två storskaliga videostudier, Learners Perspective Study, LPS, (Paper I) och VIDEOMAT (Paper II och Paper III), samt en fristående intervjustudie genomförd med elever från två högstadielklasser (Paper IV). De empiriska analyserna har gjorts i forskargrupper, i par och individuellt.

För att få reda på vad elever riktar sin uppmärksamhet mot när de arbetar med matematiska samband har didaktiska situationer analyserats i en forskargrupp (Paper I). För att få reda på hur lärare ser på intresse och engagemang, samt hur de engagerar elever har videodata samlats in och använts i fokusgruppsintervjuer, där lärare har visat exempel från sin egen undervisning och resonerat kring begreppen (Paper II och Paper III). För att belysa intresse och engagemang ur elevperspektiv har elevintervjuer genomförts, där elever berättar om intressanta och engagerande uppgifter och ger sin syn på varför just dessa uppgifter är intressanta och engagerande (Paper IV).

I delstudierna används båda begreppen intresse och engagemang i syfte att ge studiedeltagarna ökade möjligheter till associationer. Analyser av video- och audiodata samt matematikuppgifter har gjorts utifrån olika konceptuella ramverk, i kombination med teoretiska begrepp från den tidigare nämnda ämnesdidaktiska teorin, TDS.

## 7.4 Resultat – de fyra delstudierna

Resultat från de empiriska studierna, där elevengagemang relateras till algebraundervisning i åk 6-9, redovisas i form av fyra artiklar. I tabell 3 finns en översikt över avhandlingens delstudier.

SWEDISH SUMMARY

Tabell 3: Översikt över avhandlingens delstudier.

Studie	1	2	3
Paper	I	II & III	IV
Publicationsår	2013	2015	2016
Data	LPS	VIDEOMAT	Egen
Datotyp	Video	Video	Audio
Ämnesinnehåll, årskurs	Samband, 8	Algebra, 6-7	Geometri, statistik 7-9
Ramverk	GOK	CE, TDS	MTF, TDS
Perspektiv	Forskare	Lärare	Elever
Research questions	Vad riktar elever sin uppmärksamhet mot när de ber läraren om hjälp med en matematikuppgift? Och hur samkonstrueras intresse i sådana situationer?	I. Hur är indikatorer för elevengagemang identifierade, förhandlade och exemplifierade av lärare? II. Vilka didaktiska strategier föreslår lärare i syfte att engagera elever i algebraintrödnad?	Vilka uppgifter identifierar elever som intressanta och engagerande? Vilka egenskaper i dessa uppgifter identifierar eleverna som intressanta och engagerande?
Resultat	När eleven tillkallar läraren, finns det möjligheter att engagera när uppmärksamheten riktas mot relevansstrukturen hos matematiska samband, frågor kring lösningsstrategier samt validering av numeriska svar.	Indikatorer så som verbaliserade resonemang kring vad variabler är; koncentration till den grad att man förlorar uppfattning om tid och rum; att gestikulera och visa entusiasm inför att beskriva sin förståelse av variabelbegreppet; att ställa frågor när man inte förstår; att engagera sig i andras lösningar och argumentera för sina egna. Strategier för att engagera identifierades som en del av mesokontraktet.	Elever identifierar intressanta och engagerande uppgifter som har matematiken i förgrunden, hög grad av utmaning, viss grad av öppenhet och är av icke-rutinkaraktär. Samtliga uppgifter erbjöd tillfällen att presentera lösningar och resonemang, vilket också upplevdes som intressant och engagerande.
Inplacering på den didaktiska tetradern	Lärare-Elev-Uppgift	Lärare/Elev; Lärare-Elev-Uppgift	Matematik-Elev-Uppgift

Paper I. Nyman, R & Emanuelsson, J. (2013). What do students attend to? Students' Task-Related Attention in Swedish Settings. In B. Kaur, G. Anthony, M. Ohtani & D. Clarke. (Eds.), *Student Voice in Mathematics Classroom Around the World* (pp 115-132). Rotterdam: Sense Publishers.

I den första studien problematiseras vad elever riktar sin uppmärksamhet mot när de arbetar med uppgifter om matematiska samband. Den här studien bygger på en analys av en videoinspelad, interaktionstät lektion om matematiska samband i årskurs 8, där eleven riktar sin uppmärksamhet mot en specifik del av uppgiften under ett samtal med läraren. Studiens resultat visar att elever riktar uppmärksamhet mot relevansstrukturen hos matematiska samband, ställer frågor kring lösningsstrategier samt validerar numeriska svar. I dessa situationer erbjuds läraren tillfällen att engagera elever, genom att bemöta den här typen av frågor på ett sätt som skulle kunna bibehålla uppmärksamheten och på så sätt engagera dem i ämnesinnehållet.

Paper II. Indicators of student engagement: What teachers notice during introductory algebra lessons. *The International Journal of Mathematics Teaching and Learning*, 15(3), 1-17.

I den här artikeln presenteras en studie om elevengagemang från lärarnas perspektiv. Här problematiseras frågan om hur lärare, utifrån sin egen och andras undervisning, kan avgöra om en elev är engagerad i ett visst ämnesinnehåll. Huvudfrågorna är: Vad kännetecknar elevengagemang när algebra introduceras? Hur ser en lärare att en elev är engagerad då algebra introduceras? Med hjälp av videoinspelade lektioner från egen undervisning, som lärare fick analysera och presentera för varandra, visas exempel på elevengagemang i praktiken. Fokusgruppsintervjuer med två grupper, 5 respektive 3 lärare i årskurs 6 och 7, genomfördes. Under en treveckorsperiod analyserade lärarna sin egen undervisning på egen hand, genom att välja ut lektionssekvenser ur en serie videoinspelade lektioner - lektionssekvenser där läraren ansåg att eleven var engagerad i algebra. I nästa skede presenterades sekvenserna inom fokusgrupperna och lärarna diskuterade hur elevengagemang kunde komma till uttryck. Slutligen gjordes en analys av lärarnas utsagor, utifrån ett ramverk om engagemangindikatorer utvecklat av Helme och Clarke (2002). Lärarnas indikatorer jämfördes med den befintliga modellen. Det visade sig att lärare tog upp likartade indikatorer som i

ursprungsmodellen, men att kopplingarna mellan olika interaktionsvarianter och vissa indikatorer skiljde sig åt. Lärare gav exempel på hur elever verbaliserade sina resonemang kring vad variabler är; hur de var koncentrerade till den grad att de förlorade uppfattningen om tid och rum; att de gestikulerade och visade entusiasm inför att beskriva sin förståelse av variabelbegreppet på olika sätt. Att ställa frågor när man inte förstår istället för att låtsas förstå tolkades också som en engagemangsindikator, liksom att rikta intresse mot andras lösningar och argumentera för sina egna idéer.

Paper III. Enhancing engagement in algebra: didactical strategies implemented and discussed by teachers. *Scandinavian Journal of Educational Research*, 59(6), 623-637.

I den tredje artikeln är lärarens roll i engagemangutvecklingen i fokus. I fokusgruppsintervjuer (samma som i Paper II) validerar lärare varandras resonemang och kom fram till en gemensam syn på elevengagemang. Ur den kan man utläsa ett antal gemensamma didaktiska strategier som används av lärare för att engagera elever när algebra introduceras. Didaktiska strategier som kan utläsas ur lärares svar är att använda sig av specifika typer av frågor, som exempelvis öppna frågor om vad en variabel är; ta hänsyn till val av kontext som variabelbegreppet presenteras i; introducera ett dilemma eller låta eleverna validera felaktig användning av variabler när man skriver uttryck samt att variera typer av aktiviteter (räkna i boken, använda laborativt material, ha smågruppsdiskussioner). De nämnda strategierna kan, enligt lärarna, användas för att engagera elever under algebraintroduktion. Det visade sig att lärarna använder allmänna strategier för att engagera elever i algebra, kopplade till mesokontraktet. Trots riktade frågor visade studien att strategier specifikt relaterade till algebra inte tas upp.

Paper IV. What makes a mathematical task interesting? *Educational Research and Reviews*, 11(6), 1509-1520.

I den fjärde artikeln undersöks frågeställningen om vad som gör en matematikuppgift intressant ur ett elevperspektiv. Studien genomfördes med målsättningen att maximera möjligheterna att få reda på vilket ämnesinnehåll som engagerar elever och vad i det specifika ämnesinnehållet som engagerar dem. En särskilt skicklig lärare som lyfter fram ämnesinnehållet och designar



uppgifter med avsikt att engagera elever i det innehållet ombads att välja ut 15 elever för individuella intervjuer. Eleverna fick ge exempel på engagerande uppgifter som de har arbetat med i åk 7-9 och därefter motivera sina val. Varför ansåg de att uppgiften var intressant? Vad engagerade dem? Elevuppgifterna analyserades sedan med avseende på olika aspekter av uppgiften: ämnesinnehåll, kontext, svårighetsgrad och olika aspekter av uppgiftens struktur. Sammanlagt nämndes 4 olika uppgifter, som eleverna har arbetat med i årskurs 7-9. Samtliga var examinationsuppgifter med hög grad av utmaning och viss grad av öppenhet. Det visade sig också att samtliga uppgifter handlade om geometri och statistik och att elevernas motiveringar var kopplade till ämnesinnehåll som skala, kroppar, diagram och tabeller, och till typen av didaktiska situationer som uppgifterna behandlades under, främst presentationer och redovisningar.

## 7. 5 Avhandlingens kunskapsbidrag

Den här avhandlingen bidrar till fördjupad förståelse av intresse manifesterat genom engagemang i matematik. Det huvudsakliga kunskapsbidraget är insikter om hur intresse manifesteras som engagemang i samspelet mellan eleven, läraren, matematikinnehållet och matematikuppgiften. Kunskapsbidraget tydliggörs med hjälp av den didaktiska tetraedern. Mer specifikt:

- Lärare har en viktig roll i engagemangskapandet i den didaktiska situationen. Tillfällen att engagera elever finns på axeln Lärare-Elev-Uppgift (Figur 9), till exempel när eleven ställer frågor till läraren om uppgiften.
- Medan forskarnas definitioner av intresse och engagemang inte är entydiga, kan lärare enas om hur man får syn på elevengagemang i undervisningen och peka ut dessa indikatorer i egen och andras undervisning. De exemplifierar strategier som kan användas för att engagera elever med kopplingar till mesokontraktet.
- Elever kan finna uppgifter intressanta och engagerande på grund av ämnesinnehållet. En lärare som har ämnesinnehållet i förgrunden kan bidra till engagemang på makronivå och till att den didaktiska situationen äger rum.

De huvudsakliga metodologiska bidragen är för det första att befintliga datainsamlingar är användbara även när nya frågor ställs. Vidare är konkreta exempel från lärares egna klassrum behjälpliga när lärare ska diskutera sin egen och varandras undervisning. Jämfört med individuella intervjuer och enkätundersökningar kan den här metoden ge tyngd åt lärarnas resonemang om vad som händer i klassrummet.

Det huvudsakliga teoretiska bidraget är vidareutveckling av den didaktiska tetraedern och analysen av resultaten utifrån TDS. De olika ytorna i tetraedern visade sig vara användbara när man ska identifiera och beskriva intresse som elevengagemang i matematik.

## 7.6 Diskussion och konklusion

Det övergripande syftet med avhandlingen är att empiriskt undersöka matematikintresse som manifesteras genom engagemang, med ämnesinnehåll i fokus och ur olika perspektiv. Matematikintresse och elevengagemang på klassrumsnivå behandlas som fördelaktigt för lärande, något som det finns stöd för i tidigare forskning (Ma, 1997; Harris, 2008; Exeter m.fl, 2010). Den kopplingen har varit viktig, liksom att engagemang är en dynamisk process mellan olika aktörer: lärare, elever, matematikinnehållet och uppgiften. I avhandlingens delstudier kan man läsa om hur intresse manifesterat som elevengagemang tar sig uttryck i klassrummet när man arbetar med matematiska samband i årskurs 8 (Paper I) samt under algebraintroduktion i årskurs 6 (Paper II), vad lärare har för professionssyn på engagemang i specifika undervisningssituationer (Paper II och Paper III) samt vilka typer av uppgifter som engagerar elever och vad i dessa uppgifter som är engagerande (Paper IV).

Under fokusgruppsessionerna (Paper III) var lärarna i studien överens om vad som karaktäriserar elevengagemang när de tog del av varandras undervisningspraktik. De såg det som en specifik typ av elevdeltagande med ämnesinnehållet i centrum. Anmärkningsvärt är att det i likhet med Wilson och hans kollegor (2005) samtidigt framkom att lärarna använder sig av strategier kopplade till det didaktiska kontraktet och inte ämnesinnehåll. Exempelvis visade de hur man kan engagera elever genom att lyfta fram specifika elevlösningar, något som kan göras oavsett ämnesinnehåll. Eftersom det finns forskning som lyfter fram potentiellt engagerande ämnesinnehåll i algebra (Kaput, 1999; Selling, 2016) blev lärarna under fokusgrupps-

intervjuerna särskilt tillfrågade om algebraspecifika strategier, men återkom till form snarare än innehåll i sina resonemang om elevengagemang. Det visar att engagemangskapandet lätt hamnar i Elev-Lärare-Uppgift planet av den didaktiska tetraedern (Brousseau, 1997; Rezat & Sträßer, 2012).

Elever till en lärare som arbetar med ämnesinnehållet i förgrunden redan på planeringsstadiet, det vill säga i Lärare-Matematik-Uppgift planet, blev intervjuade om engagerande uppgifter. Lärarens sätt att framhålla ämnesinnehållet speglades i elevernas resonemang om uppgifter. Elever bidrog med exempel på engagerande uppgifter genom att återge detaljer om intressanta uppgifter som de har arbetat med under högstadieåren. Deras exempel visar att lärarens sätt att arbeta med ämnesinnehållet i förgrunden och anpassa kontext, svårighetsgrad och uppgiftens struktur därefter, engagerar eleverna.

I avhandlingen presenteras olika perspektiv på elevengagemang: I Paper I är forskarnas perspektiv framträdande, i Paper II och III är lärarnas perspektiv i fokus och i Paper IV – elevernas perspektiv. Tidigare studier har visat att lärarnas, elevernas och observatörernas tolkningar kan skilja sig åt (Appleton & Lawrenz, 2011). Lärardeltagarna i studie 2 menar att lärare har tolkningsföreträde på elevengagemang i den egna klassen. De kom till konsensus om hur elevengagemang är synligt, men menar också att man som lärare har en förtrogenhet som gör att man får syn på engagemangindikatorer som oberoende observatörer kan missa. Paper IV bidrar med elevperspektiv på vad som gör en uppgift intressant och engagerande, medan Paper II och III bidrar med lärarnas professionssyn på indikatorer och didaktiska strategier som används för att intressera och engagera elever i matematik. Slutsatsen är att resonemang från olika perspektiv berikar varandra när ämnesspecifikt intresse och engagemang studeras.

En övergripande reflektion är i linje med Harris (2011) och Skillings (2016) resultat om att lärarens perspektiv och agerande spelar roll i engagemangskapandet. Utifrån den här avhandlingens resultat kan man säga att läraren har en central roll i engagemangsskapandet och att det finns specifika situationer där läraren ges tillfälle att engagera elever i matematik. Praktisknära exempel på didaktiska strategier och design av uppgifter i avhandlingens studier kan fungera som inspiration för lärare som vill engagera sina elever och som utgångspunkt för fortsatt diskussion och undersökning av intresse och engagemang i klassrumssammanhang.

En utmaning under avhandlingsarbetet har varit att hitta ett sätt att närma sig begreppet intresse på ett klassrumsrelevant sätt. Jag utgick från Deweys (1903; 1910; 1913; 1916/1997) resonemang om att intresse manifesteras genom engagemang och på så sätt blir synligt. Konsekvenser för fortsatt forskning rör nya möjligheter att ta itu med begreppet intresse som manifesterat genom engagemang, vilket kan bidra ytterligare till forskningsresultat om intresse som en samkonstruktion och inte enbart som en inneboende egenskap.

Det är viktigt att betona att resultat som presenteras i denna avhandling inte är menade att bidra till en universell definition av intresse, utan snarare visa hur det kan komma till uttryck i klassrumssammanhang och hur det beskrivs av forskare, lärare och elever. Implikationer för framtida forskning är att fortsätta närma sig intresse på ett klassrumsrelevant sätt, vilket bland annat Firsov (2004) menar är eftersträvansvärt.

De olika delstudierna innefattar olika metoder och perspektiv och har bidragit till nya frågeställningar. Denna bredd är ett exempel på hur befintliga datamängder kan användas på nya sätt. I linje med Silverman (2010) visar jag att det är fördelaktigt när flera forskare med olika relationer till data samarbetar under analysprocessen - både forskare som är insatta i datamaterialet och de som den är ny för.

Även om insikter kan vinnas av att nya frågor besvaras med hjälp av analys av befintlig data, ger fältarbete möjligheter att prägla studiens karaktär genom att påverka forskningsteman och frågor. De frågeställningar som behandlas är utformade specifikt för den empiriska studien och kan utvecklas och anpassas under datainsamlingsprocessen. Som framgår av Papper II och III, ställdes specifika frågor om intresse och engagemang till lärarna, till exempel betonades möjligheten att intresse och engagemang är relaterad till innehållsliga aspekter. Lärarna visade konkreta exempel från sin egen undervisning. Genom fokusgruppsintervjuer kan professionella insikter om lärarnas praktik utvinnas (Merton, 1946). Med andra ord kan lärare dra nytta av att dela sin undervisning med varandra och diskutera frågor som är relevanta för deras yrke.

Jag anser att det för novisa forskare är särskilt viktigt att vid någon tidpunkt under avhandlingsarbetet delta i datainsamlingsprocessen och se det som ett lärtillfälle, så som jag gjorde i studie 2 och 3. Ett konkret exempel på en insikt från min egen datainsamling från studie 3 är hur avgörande lärarens bidrag är när elevdata ska samlas in. I detta fall bistod läraren med nödvändig information om eleverna, valde ut elever till intervjuer utifrån tidigare

kännedom om deras resultat och förmåga att svara på intervjufrågor samt att hen gjorde de praktiska arrangemangen så att varje elev kunde delta i intervjun.

Avhandlingen bidrar med uppslag som rör lärares yrkeskunskaper och praxis eftersom den är förankrad i praktisknära frågor. Till exempel presenterar Paper I tre kategorier av uppgiftsrelaterad uppmärksamhet, vilket i förlängningen innebär att det finns sätt att engagera elever i liknande situationer. Till exempel kan frågor om att använda matematiska relationer i vardagen bemötas med konceptuella resonemang om vad matematiska relationer är och hur man ska förstå dem bortom en instrumentell syn (Thompson, 1992).

Om man kombinerar resultaten från alla studier är det möjligt att designa engagerande didaktiska situationer. Den viktigaste punkten ger stöd åt att ha ämnesinnehållet i förgrunden, både i klassrumssituationer och i design av uppgifter. Om man försummar innehållet finns det risk för att eleverna inte blir engagerade, vilket i sin tur kan bidra till att lärmöjligheterna går förlorade. Som matematiklärare vill man hjälpa eleven att förstå en matematisk idé, och att synliggöra det matematiska innehållet är en möjlig väg till intresse och engagemang.

Sammanfattningsvis kan det sägas att synen på intresse manifesterat genom elevengagemang i klassrummet hjälper att utvinna insikter som är klassrumsrelevanta. Det centrala i den här avhandlingen handlar om att bidra till förståelse av elevengagemang så som det identifierats av forskare, lärare och elever; hur man som lärare kan engagera elever i matematik samt vad som gör en matematikuppgift intressant och engagerande. Det har visat sig att läraren har en viktig roll i engagemangsskapandet och har möjlighet att engagera elever genom att sätta matematiken i förgrunden redan som en del av makrokontraktet. Därutöver har jag visat att lärare kan komma till konsensus om hur man identifierar intresse och engagemang i algebra, och att de använder strategier kopplade till mesokontraktet, det vill säga till aktivitetens utformning och genomförande snarare är till innehållsspecifika aspekter. När lärare arbetar med innehållet i förgrunden, kan eleverna uppfatta innehållet som intressant och engagerande.

Det finns indikationer på att annat matematikinnehåll än algebra upplevs som mer engagerande. I fortsatta studier framstår det därför viktigt att ta reda på hur just algebra kan göras intressant och engagerande, hur makrokontraktet i algebra kan utvecklas.

## SWEDISH SUMMARY

I sin helhet bidrar avhandlingen till fördjupad förståelse för intresse manifesterat som elevengagemang genom att ge begreppet en klassrumsnära innebörd. Med stöd i studiens resultat framstår det som viktigt att undervisa med matematiken i förgrunden. När uppgifter konstrueras och används i syfte att engagera elever är det också viktigt att det finns en hög grad av utmaning, utrymme för eleven att göra vissa val inom uppgiftens ramar samt att den didaktiska situationen möjliggör för elever att presentera sina resultat.

Den här avhandlingen erbjuder inga enkla lösningar på hur man engagerar elever i matematik, däremot ger den uppslag till engagerande didaktiska situationer. Avhandlingen kan vara ett stöd i lärares reflektioner över sin syn på elevengagemang och sin roll i att engagera elever i matematik. Förhoppningsvis kan avhandlingens resultat också leda till fortsatt forskning om elevengagemang i matematik.





























# Appendix 1

Example of a lesson graph (LG) from study 2, referred to in Paper III.

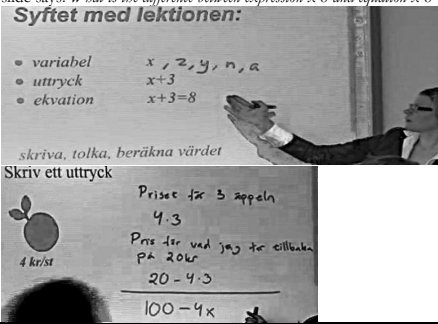
## VM – S4T3– 120130 – grade 7 (lesson 1)

[59 minutes]

1 minute	<b>NM Beginning of lesson organizing</b>																																	
15 minutes	<p><b>Whole class: IT</b> Introduction of algebra, historical background of the word, purpose of algebra – why it is useful and aim of this lesson. Variables, equations, expressions – write, interpret and calculate. The teachers talks about “the lazy mathematicians”, who according to this teacher write expressions in order to simplify what they want to write express in “mathematical language” instead of words:</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p><b>Skriv ett uttryck</b></p>  <table border="1" data-bbox="439 760 683 930"> <thead> <tr> <th>Bild</th> <th>Ord</th> <th>Uttryck för vad du betalar</th> </tr> </thead> <tbody> <tr> <td></td> <td>Kostnaden då du köper två stycken äpplen</td> <td><math>3 \cdot 2</math></td> </tr> <tr> <td></td> <td>Kostnaden då du köper fem stycken äpplen</td> <td><math>3 \cdot 5</math></td> </tr> <tr> <td></td> <td>Kostnaden då du köper tolv stycken äpplen</td> <td><math>3 \cdot 12</math></td> </tr> <tr> <td></td> <td>Kostnaden då du köper x stycken äpplen</td> <td><math>3 \cdot x = 3x</math></td> </tr> </tbody> </table> </div> <div style="text-align: center;"> <p><b>Skriv ett uttryck</b></p>  <table border="1" data-bbox="812 760 1070 948"> <thead> <tr> <th>Bild</th> <th>Ord</th> <th>Uttryck för vad du betalar</th> </tr> </thead> <tbody> <tr> <td></td> <td>Kostnaden då du köper två äpplen och fyra pennor</td> <td><math>3 \cdot 2 + 2 \cdot 4</math></td> </tr> <tr> <td></td> <td>Kostnaden då du köper tre äpplen och fem pennor</td> <td><math>3 \cdot 3 + 2 \cdot 5</math></td> </tr> <tr> <td></td> <td>Kostnaden då du köper två äpplen och sex pennor</td> <td><math>3 \cdot 2 + 2 \cdot 6</math></td> </tr> <tr> <td></td> <td>Kostnaden då du köper x äpplen och y pennor</td> <td><math>3x + 2y</math></td> </tr> <tr> <td></td> <td>Kostnaden då du köper z karameller och y pennor</td> <td><math>1z + 2y</math></td> </tr> </tbody> </table> </div> </div> <p>Tasks1 (picture): Expressing the price of different fruits,</p> <p>Task 2: The circumferences of geometrical objects, such as a square</p>	Bild	Ord	Uttryck för vad du betalar		Kostnaden då du köper två stycken äpplen	$3 \cdot 2$		Kostnaden då du köper fem stycken äpplen	$3 \cdot 5$		Kostnaden då du köper tolv stycken äpplen	$3 \cdot 12$		Kostnaden då du köper x stycken äpplen	$3 \cdot x = 3x$	Bild	Ord	Uttryck för vad du betalar		Kostnaden då du köper två äpplen och fyra pennor	$3 \cdot 2 + 2 \cdot 4$		Kostnaden då du köper tre äpplen och fem pennor	$3 \cdot 3 + 2 \cdot 5$		Kostnaden då du köper två äpplen och sex pennor	$3 \cdot 2 + 2 \cdot 6$		Kostnaden då du köper x äpplen och y pennor	$3x + 2y$		Kostnaden då du köper z karameller och y pennor	$1z + 2y$
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	Kostnaden då du köper z karameller och y pennor	$1z + 2y$																																
4 minutes	<p><b>Student work: SI</b> Students are instructed to work individually, filling in expressions in a hand out similar to the ones presented in WC. They are supposed to write an expression of oranges and bananas together. They can ask a neighbor if needed.</p>																																	
2 minutes	<p><b>Whole class: ITS</b> Discussion with whole class about a student solution</p>																																	
1 minute	<p><b>Student work: SI</b> Students continue to write expressions.</p>																																	
3minutes	<p><b>Whole class: FT</b> The teacher fills in the expressions in the gaps of the task presented earlier (in the beginning of the lesson). See Task 1, answers are filled in together with students.</p>																																	



INTEREST AND ENGAGEMENT

<p>11 minutes</p>	<p><b>Student work: SI</b></p> <table border="1" data-bbox="358 227 663 420"> <thead> <tr> <th>Namn</th> <th>Ålder</th> <th>Uttryck</th> <th>Uttryck</th> </tr> </thead> <tbody> <tr> <td>Karin</td> <td>12 år äldre än Sara</td> <td><math>x+12</math></td> <td><math>m</math></td> </tr> <tr> <td>Sara</td> <td><math>x</math> år gammal</td> <td><math>x</math></td> <td><math>m-12</math></td> </tr> <tr> <td>Emil</td> <td>5 år yngre än Sara</td> <td><math>x-5</math></td> <td><math>m-12-5</math> <math>m-17</math></td> </tr> <tr> <td>Anna</td> <td>2 år yngre än Karin</td> <td><math>x+12-2</math> <math>x+10</math></td> <td><math>m-2</math></td> </tr> </tbody> </table> <table border="1" data-bbox="676 227 1062 420"> <thead> <tr> <th>Uttryck</th> <th>Variabelns värde</th> <th>Uttryckets värde</th> </tr> </thead> <tbody> <tr> <td><math>x+3</math></td> <td><math>x=5</math></td> <td><math>5+3=8</math></td> </tr> <tr> <td><math>3a</math></td> <td><math>a=7</math></td> <td><math>3 \cdot 7 = 21</math></td> </tr> <tr> <td><math>3n-9</math></td> <td><math>n=4</math></td> <td><math>3 \cdot 4 - 9 = 3</math></td> </tr> </tbody> </table> <p>Students write expressions and calculate their value</p>	Namn	Ålder	Uttryck	Uttryck	Karin	12 år äldre än Sara	$x+12$	$m$	Sara	$x$ år gammal	$x$	$m-12$	Emil	5 år yngre än Sara	$x-5$	$m-12-5$ $m-17$	Anna	2 år yngre än Karin	$x+12-2$ $x+10$	$m-2$	Uttryck	Variabelns värde	Uttryckets värde	$x+3$	$x=5$	$5+3=8$	$3a$	$a=7$	$3 \cdot 7 = 21$	$3n-9$	$n=4$	$3 \cdot 4 - 9 = 3$
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Uttryck	Variabelns värde	Uttryckets värde																															
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$3a$	$a=7$	$3 \cdot 7 = 21$																															
$3n-9$	$n=4$	$3 \cdot 4 - 9 = 3$																															
<p>3 minutes</p>	<p><b>Whole class: FT</b> Teacher shows “how to express the value of an expression”. Edwin is <math>x</math> years, Oscar is 4 years younger (<math>x-4</math>), Alicia is 3 years younger than Oscar (<math>x-4-3</math>). One student suggests “why not just write <math>x-7</math>?”. The teacher agrees. There is a parallel small group discussion about the coming test.</p>																																
<p>1 minutes</p>	<p><b>Student work: SI</b></p>																																
<p>12 minutes</p>	<p><b>Whole class: IT</b> Teacher solves one last problem on the interactive whiteboard:</p> <p>Write an expression: Price: 4kr/ apple. Price for 3 apples is <math>4 \times 3</math>. Price for what I get back from 20kr: <math>20-4 \times 3</math>. What does <math>100-4x</math> mean? Summary, back to the aim of the lesson. What is the difference between expression and equation? Also, the aim of the lesson is written on the interactive board (picture). And the next slide says: What is the difference between expression <math>x-8</math> and equation <math>x-8=10</math>?</p>  <p><b>Syftet med lektionen:</b></p> <ul style="list-style-type: none"> <li>variabel <math>x, z, y, n, a</math></li> <li>uttryck <math>x+3</math></li> <li>ekvation <math>x+3=8</math></li> </ul> <p>skriva, tolka, beräkna värdet</p> <p>Skriv ett uttryck</p> <p>Priset för 3 äppeln <math>4 \cdot 3</math></p> <p>Pris för vad jag får tillbaka på 20kr <math>20 - 4 \cdot 3</math></p> <p><math>100 - 4x</math></p>																																
<p>4 minute</p>	<p><b>No mathematics</b></p>																																
<p>2 minute</p>	<p><b>Whole class: IT</b></p> <p>Introduction of the next day lesson. End of lesson</p>																																

RN 2012-03-24

## Appendix 2

Student consent form used in Study III. Before the form was signed, information about the study was read and explained to the students.

### Medgivande

Jag samtycker till att vara med i en inspelad intervju om matematikintresse och engagemang.

Inspelningen kommer endast att användas av forskaren och hennes handledare.

Underskrift: \_\_\_\_\_

Namnförtydligande: \_\_\_\_\_



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