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A study of the Basel III CVA formula

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Abstract

In this thesis we compare the official Basel III method for computing credit value adjustment (CVA) against a model that assumes piecewise constant default intensities for a number of both market and fictive scenarios. CVA is defined as the price deducted from the risk-free value of a bilateral derivative to adjust for the counterparty credit risk. Default intensity is defined as the rate of a probability of default, conditional on no earlier default. In the piecewise constant model, the default intensity is calibrated against observed market quotes of credit default swaps using the bootstrapping method. We compute CVA for an interest rate swap in a Cox-Ingersoll-Ross framework, where we calculate the expected exposure using the internal model method and assume that no wrong-way risk exists.

Our main finding is that the models generate different values of CVA. The magnitude of the difference appears to depend on the size of the change in the spreads between credit default swap maturities. The bigger the change from one maturity to another is, the bigger the difference between the models will be.

Keywords: Basel III, Credit Value Adjustment, Counterparty Credit Risk, Credit Default Swap, Interest Rate Swap, Piecewise Constant Default Intensity, Bootstrapping, Expected Exposure, Internal Model Method.

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List of Figures

Abbreviations

BIS Bank of International Settlements

CCP Central Counterparty

CCR Counterparty Credit Risk

CDS Credit Default Swap

CIR Cox–Ingersoll–Ross

CVA Credit Value Adjustment

DVA Debit Value Adjustment

EE Expected Exposure

EMIR European Markets and Infrastructure Regulation

FRA Forward Rate Agreement

IFRS International Financial Reporting Standards

IMM Internal Model Method

IRS Interest Rate Swap

ISDA International Swaps and Derivatives Association

LGD Loss Given Default

LIBOR London Interbank Offered Rate

MTM Mark-To-Market

NPV Net Present Value

OIS Overnight Indexed Swap

OTC Over-The-Counter

PV Present Value

RSS Residual Sum of Squares

WWR Wrong-Way Risk

Contents

1 Introduction

In this section, we introduce our motivations and purpose behind this thesis, as well as stating the method used, and the structure of our study.

The 1990s saw heavy deregulations of financial markets in the western world, causing the financial industry to grow massively worldwide and enabled the banks to increasingly take on risks. A general view existed that some financial institutions were "too big to fail," meaning that the government could not let these corporations go bankrupt for fear of what it might do to the world economy. When the crisis hit in 2007-2008, governments were forced to bail out distressed banks, but when the United States government unexpectedly decided not to rescue Lehman Brothers, which were thought of as one of those institutes who were "too big to fail", the *counterparty credit risk* (CCR) associated with these entities rose sharply. CCR is the risk that a counterparty will not pay as obligated in a contract. As a consequence, all of the derivatives Lehman Brothers had sold were suddenly much riskier than initially thought. The buyers demanded that collateral should be posted. Collateral is a pledge of specific property that serves as a lender's protection against a borrower's default. This proved to be too much for these institutions who were backing the derivatives since the traded contracts were of such a nature that the seller, which typically were the big "risk-free" institutions, would have to go bankrupt if they did not have the money to meet the demands of collateral (Acharya et al., 2009). In fact, according to a *Bank of International Settlements* (BIS) press release in June 2011, two thirds of the losses that occurred during the crisis were due to the rising credit risk and devaluation of derivatives, and only one third due to actual bankruptcies (BIS, 2011).

Because of the huge effects of CCR on losses during the crisis, it is crucial that banks can accurately measure their CCR exposure. The Basel accords were updated after the financial crisis. From the Basel III accord, the important concept of *credit value adjustment* (CVA) is derived. CVA can briefly be explained as the difference between an asset's risk free value and its value including the risk of default. In other words it is a measure of CCR for bilateral derivatives. In order to maintain a stable financial system, accurately measuring the CCR is vital. In particular since the market for *over-the-counter* (OTC) derivatives has grown substantially over the last decade. OTC derivatives are derivatives traded directly between two parties, without the supervision of an exchange. The official formula for calculating the CVA is presented in BIS (2011, p. 31).

In this thesis we explain how the equation for calculating CVA in Basel III is mathematically inconsistent and examine the effects of this inconsistency. By using both market data and fictive data we calculate CVA using the Basel III equation as well as an equation modelled for piecewise constant default intensities. The default intensity is roughly defined as the rate of a default occurring in any time period, given no default up to a specific time. Piecewise default intensity means that the default intensity is constant between two maturities but changes after a maturity. We then calculate the CVA value using both models and compare the results to see if the inconsistency in the Basel equation has any significant impact on the CVA value.

The question we ask ourselves is if the inconsistency in the Basel CVA formula makes the corresponding CVA value significantly different to a model based on piecewise constant default intensities.

This thesis follows the notation of Brigo and Mercurio (2006) and much of the theoretical background is retrieved from Hull (2014). We refer to several official documents from the Bank of International Settlements such as BIS (2015, 2016) for concepts around CVA, including the official CVA formula given in BIS (2011, p. 31) and our calculations are based on the *Cox–Ingersoll–Ross* (CIR) model, introduced by Cox et al. (1985). The CIR model works better than e.g. the Vasicek model when the interest rates are close to zero, as proven by Zeytun and Gupta (2007).

We consider five scenarios, three with actual market data retrieved from Bloomberg on *credit default swap* (CDS) spread pricing and two with fictive data. A CDS is a financial swap agreement, which for the buyer of it, works as an insurance against a default for a third entity. In the first three scenarios, we use the CDS spreads of Swedbank for different maturities from times with low, high and inverted spread curves. Using fictive data we can also examine the difference in extreme scenarios, such as when the spread is constant over all maturities and when the spread changes drastically between maturities.

We use Matlab to implement the equations and to simulate the stochastic process used for calculating CVA. In Section 5 we thoroughly explain how the simulation is made. We derive default probabilities using a method called bootstrapping, which is explained in Subsection 3.2.

Possible critique of our chosen method could be about the assumptions we make, and if they are realistic. We aim to make as realistic assumptions as possible, and we also perform a sensitivity analysis of the variables that drives our simulated interest rate, in order to cover multiple different scenarios. In addition to this, we use CDS spreads from both market data, to have some realistic scenarios, and fictive spreads, to analyse the difference in extreme situations.

We choose to use CDS spreads from one single bank since it is of little importance how many different corporations we gather CDS spread data on. It does not matter whether the data is on spreads of Swedbank or Nordea, since the spreads represent the same thing in both cases. We believe that it is more relevant to have different spread curves, which is why we have five different scenarios of CDS spreads.

This thesis is structured as follows: We begin by giving a description of the financial crisis in 2007-2008 and introduce the Basel accords in Section 2, where we also discuss important concepts such as credit counterparty risk, over-the-counter derivatives, netting and central counterparty clearing. These concepts are vital to understand in order to understand the importance of CVA. In Section 3 we explain intensity based models and we describe how we calibrate our default probabilities using piecewise constant CDS spreads. Furthermore, in Section 4, we describe and discuss the different methods to calculate CVA and present the simulations we conduct in Matlab and explain the assumptions made when calculating our CVA values. The results of our comparison between the Basel model and the piecewise constant model are presented in Section 5. Lastly, in Section 6, we discuss the assumptions we make when calculating CVA, as well as the findings from our numerical studies and provide a conclusion of this thesis.

2 Theoretical Background

In this section we present concepts related to credit value adjustment (CVA) in order to build an understanding of what we aim to explain in the rest of this thesis.

2.1 Basel Regulations

In the aftermath of the financial crisis of 2007-2008, the Bank of International Settlements updated the Basel regulations with Basel III. A large part of the changes were to account for the risk of default of one's counterparties, and how to calculate and incorporate the value of these risks in the traded derivatives in a more accurate way.

Under the Basel II market risk framework, firms were required to hold capital to account for the variability in the market value of their derivatives in the trading book, but there was no requirement to hold capital against variability in the CVA. CVA is the difference between a risk-free portfolio and a portfolio value that takes into account the possibility that the counterparty might default. The counterparty credit risk framework under Basel II was based on the credit risk framework and designed to account for default and migration risk rather than the potential accounting losses that can arise from CVA (Rosen and Saunders, 2012).

To address this gap in the framework, the Basel Committee on Banking Supervision introduced the CVA variability charge as part of Basel III. The current CVA framework sets forth two approaches for calculating the CVA capital charge, namely the advanced approach and the standard approach. Both approaches aim to capture the variability of regulatory CVA that arises solely due to changes in credit spreads without accounting for the exposure variability driven by daily changes in market risk factors. Calculation of regulatory CVA is usually made using the standard approach, which can be divided into three different methods. One of these methods is the so-called *internal model method* (IMM), which requires a certain approval from supervisory authorities. The other two are so-called non-internal model methods with different degrees of complexity; the *current exposure method* and the *standardised method* (BIS, 2015). These two methods are not be used nor further explained in this thesis.

2.2 Credit Counterparty Risk

Counterparty credit risk (CCR) is the risk that the counterparty in a financial contract will default prior to the contract expiration and not make all the payments it is contractually required to make. CCR consists of two parts, credit risk and market risk. Credit risk is the risk that one party in a bilateral trade cannot uphold their part of the contract, for example by not being able to make the agreed payments, resulting in default. Market risk refers to the overall risk, such as fluctuation in prices that affects the entire market. Even though the definitions of credit risk and CCR are very similar, some differences still exist (Duffie and Singleton, 2012, p. 4).

For example, only privately negotiated contracts, traded *over-the-counter* (OTC), are naturally subject to CCR. Derivatives traded on an exchange are not subject to CCR, since the counterparty is guaranteed the promised cash flow of the derivative by the exchange itself. Two features separate counterparty risk from other forms of credit risk: the uncertainty of the exposure and the bilateral nature of the credit risk. CCR was one of the main causes of the credit crisis during 2007-2008, and as mentioned in Section 1, two thirds of the losses that occurred during the crisis were due to the rising credit risk and devaluation of derivatives, and only one third due to actual bankruptcies (BIS, 2011).

2.3 OTC-derivatives

An over-the-counter (OTC) derivative is a contract written by two private parties, a so-called *bilateral* contract. The alternative would be to buy a standardised contract by a centralised clearing house, also called *central counterparty* (CCP) (Hull, 2015, p. 390). We describe the role of a CCP in Subsection 2.10.

The OTC-market gives the counterparties the freedom to design their contracts as they desire. Regulations implemented between 2015 and 2019 require some sort of initial- and variation margin if the parties are financial institutions, or if one of the two is a systemically important institution, e.g. a very large bank. Initial margin is the collateral posted when a contract is signed and variation margin is the collateral posted based on change in the value of the derivative. If neither of the parties is a financial institution or a systemically important institution, then the parties are free to create a contract without any collateral requirements (Hull, 2015, p. 389).

The downside of a bilateral OTC contract is that the credit security provided by a CCP is lost. In 2016 the nominal value of the OTC-market exceeded 500 billion U.S. dollars (BIS, 2016). Since the 2007-2008 crisis, most financial derivatives are required to be traded through a CCP. Before this change in regulation, the OTC-market was estimated to make up 75% of the total derivatives market. Particularly popular were credit default swaps (CDS) and interest rate swaps (IRS), where the former is an insurance designed to cover defaults and the latter is a contract where two parties exchange different interest rate payments, typically a floating rate for a fixed rate (Hull, 2015, p. 389).

2.4 Forward Rate Agreements

In this subsection as well as in Subsections 2.5 and 4.4.2 we follow the notation and setup of Brigo and Mercurio (2006). All calculations in this subsection are made under "the risk neutral probability measure", also known as the "pricing measure". Such a measure always exists if we rule out the possibility of an arbitrage, see e.g. in Björk (2009). A forward rate agreement (FRA) is an OTC interest rate derivatives contract between two parties where interest rates are determined today for a transaction in the future. The contract determines the forward rates to be paid or received on an obligation starting at a future date. The contract is characterised by three important points in time (Brigo and Mercurio, 2006):

- The time at which the contract rate is determined, denoted by t
- The start date of the contract, denoted by T_1
- The time of maturity, denoted by T_2 where $t \leq T_1 \leq T_2$.

The FRA allows a party to lock in a fixed value of the interest rate, denoted by K_{FRA}, for the period $T_1 - T_2$. At T_2, the holder of the FRA receives an interest rate payment for the period. This interest rate payment is based on K_{FRA}, and is exchanged against a floating payment based on the spot rate $L(T_1, T_2)$. The expected cash flows are then discounted from T_2 to T_1. The nominal value of the contract is given by N and $\delta(T_1,\, T_2)$ denotes the year fraction for the contract period from T_1 to T_2. The FRA seller receives the amount $N \cdot \delta(T_1,\, T_2) \cdot K_{\text{FRA}}$ and simultaneously pays $N \cdot \delta(T_1,\, T_2) \cdot L(T_1, T_2)$. At time T_2, the value of the FRA, will for the seller be expressed as (Brigo and Mercurio, 2006):

$$FRA = N \cdot \delta(T_1, T_2) \cdot (K_{\text{FRA}} - L(T_1, T_2) \tag{1}$$

where $L(T_1,\, T_2)$ can be written as:

$$L(T_1, T_2) = \frac{1 - P(T_1, T_2)}{\delta(T_1, T_2) \cdot P(T_1, T_2)}.$$

Here, $P(t, T)$ for $t < T$, denotes the price of a risk free zero coupon at time t which matures at time T so that $P(T, T) = 1$. Therefore, we can rewrite Equation (1) as:

$$\begin{aligned} N \cdot \delta(T_1, T_2) \cdot & \left[K_{\text{FRA}} - \frac{1 - P(T_1, T_2)}{\delta(T_1, T_2) \cdot P(T_1, T_2)} \right] \\ &= N \cdot \left[\delta(T_1, T_2) \cdot K_{\text{FRA}} - \frac{1}{P(T_1, T_2)} + 1 \right]. \end{aligned} \tag{2}$$

The cash flows in Equation (2) must then be discounted back to time t in order to find the value of the FRA at time t as:

$$N \cdot P(t, T_2) \cdot \left[\delta(T_1, T_2) \cdot K_{\text{FRA}} - \frac{1}{P(T_1, T_2)} + 1 \right]$$

and since we know from no arbitrage interest rate theory that $P(t,\, T_2) = P(t,\, T_1) \cdot P(T_1,\, T_2)$, we can derive that the value of the FRA at time t is:

$$\begin{aligned} N \cdot P(t, T_2) \cdot & \left[\delta(T_1, T_2) \cdot K_{\text{FRA}} - \frac{1}{P(T_1, T_2)} + 1 \right] \\ &= N \cdot [P(t, T_2) \cdot \delta(T_1, T_2) \cdot K_{\text{FRA}} - P(t, T_1) + P(t, T_2)]. \end{aligned} \tag{3}$$

K_{FRA} is the unique value that makes the FRA equal to zero at time t. By solving for K_{FRA} we obtain the appropriate FRA rate (Fs) to use in the contract. At time t for the start date $T_1 > t$, and maturity $T_2 > T_1$, the FRA rate is thus given by:

$$Fs(t; T_1, T_2) = \frac{P(t, T_1) - P(t, T_2)}{\delta(t, T_2) \cdot P(t, T_2)} = \frac{1}{\delta(T_1, T_2)} \cdot \left[\frac{P(t, T_1)}{P(t, T_2)} - 1 \right]. \tag{4}$$

$Fs(\mathrm{t}; T_1, T_2)$ is here the simply-compounded forward interest rate. Rewriting Equation (3) in terms of the simply-compounded forward interest rate in Equation (4) gives:

$$FRA(t, T_1, T_2, \delta(T_1, T_2), N, K_{\text{FRA}}) = N \cdot P(t, T_2) \cdot \delta(T_1, T_2) \cdot (K_{\text{FRA}} - Fs(t; T_1, T_2)).$$

2.5 Interest Rate Swaps

In this subsection we discuss interest rate swaps (IRS). An IRS is a financial derivative where two parties agree to exchange future cash flows. Below, our notation and concepts are taken from Brigo and Mercurio (2006) and Filipovic (2009). The simplest form of an IRS is a so-called plain vanilla swap and is structured as follows: As seen in Figure 1, counterparty A pays counterparty B cash flows that equal a predetermined fixed interest rate on a principal for a predetermined time period. In exchange, counterparty A receives a floating interest rate on the same principal amount for the same period from counterparty B.

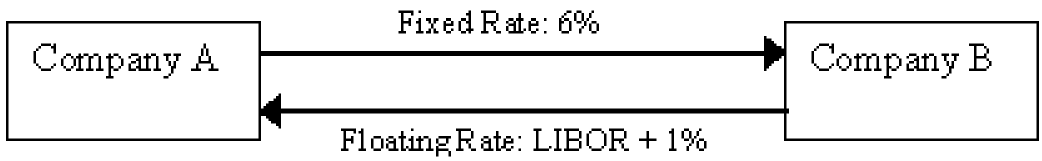


Figure 1: Example of an Interest Rate Swap (Hoffstein, 2016)

The most common IRS consists of exchanging a floating reference rate for a fixed interest rate. Historically the floating reference rate has been based on the London Interbank Offered Rate (LIBOR) but since the 2007-2008 credit crisis, other risk-free rates have been used to discount cash flows in collateralised transactions. The LIBOR is the average of interest rates estimated by each of the leading banks in London that would be charged if a bank were to borrow from another bank. In valuing swaps the cash flows have to be discounted by a risk-free rate. Hull (2014, pp. 152-153) explains that having the same rate as both the reference rate and as the discount rate simplifies the calculation.

The present value (PV) of a plain vanilla IRS can be computed through determining the PV of the floating leg and the fixed leg. Rationally, the two legs must have the same PV when the contract is entered and thus no upfront payment from either party is required ($PV_{\text{FIX}} = PV_{\text{FLOAT}}$). However, as the contract ages the discount factors and the forward rates change, so the PV of the swap will differ from its initial value. When the swap differs from its initial value, the swap is an asset for one party and a liability for the other (Kuprianov, 1993).

An IRS is equivalent to a portfolio of several FRAs. Consider the swap in Figure 1 where Company A pays a fixed interest rate and Company B pays a floating rate corresponding to the interest rate $L(T_{i-1}, T_i)$ over the contract period T_{i-1} to T_i for

T_α, $T_{\alpha+1}$, ... T_β, where $\alpha = \alpha(t)$ for each time point t, equals the integer such that the time point $T_{\alpha(t)}$ is the closest point in time to t, i.e. $T_{\alpha(t)-1} < t \leq T_{\alpha(t)}$. The maturity date of the IRS is denoted by T_β.

The party who receives the fixed leg and pays the floating, in our case Company B, is the receiver while the opposite party, Company A, is called the payer. We assume, for simplicity that both the fixed-rate and the floating-rate payments occur on the dates of the coupons $T_{\alpha+1}$, $T_{\alpha+2}$, $T_{\alpha+3}$ ... T_β and that there is no coupon when the contract is entered at T_α. The fixed leg pays the $N \cdot \delta \cdot K_{\text{IRS}}$, where, N stands for the nominal value, δ equals $T_i - T_{i-1}$, meaning it is the year proportion between T_{i-1} and T_i, and K_{IRS} is a fixed interest rate. Hence the discounted payoff at time $t < T_\alpha$ for A equals:

$$\sum_{i=\alpha+1}^{\beta} D(t, T_i) \cdot N \cdot \delta \cdot (L(T_{i-1}, T_i) - K_{\text{IRS}}).$$

The floating leg pays $N \cdot \delta \cdot L(T_{i-1}, T_i)$ which corresponds to the interest rate $L(T_{i-1},\, T_i)$. The discounting factor used to discount the payoff from T_i to today's date t, is denoted by $D(t,\, T_i)$. For maturity T_i, the interest rate $L(T_{i-1},\, T_i)$ resets at the preceding date T_{i-1}. The discounted payoff at time $t < T_\alpha$ for B is given by:

$$\sum_{i=\alpha+1}^{\beta} D(t, T_i) \cdot N \cdot \delta \cdot (K_{\text{IRS}} - L(T_{i-1}, T_i)).$$

The value of the IRS for B, $\Pi_{\text{receiver}}(t)$, is then given by (Brigo and Mercurio, 2006):

$$\begin{aligned}\Pi_{\text{receiver}}(t) = N \cdot \sum_{i=\alpha+1}^{\beta} \delta \cdot P(t, T_i) \cdot (K_{\text{IRS}} - Fs(t; T_{i-1}, T_i)) \\ = \sum_{i=\alpha+1}^{\beta} FRA(t, T_{i-1}, T_i, \delta, N, K_{\text{FRA}})\end{aligned}$$

and by using Equation (4) in the above expression we get:

$$\Pi_{\text{receiver}}(t) = N \cdot \sum_{i=\alpha+1}^{\beta} \left(\delta \cdot K_{\text{IRS}} \cdot P(t, T_i) - \frac{\delta \cdot P(t, T_i)}{\delta(t, T_i)} \left(\frac{P(t, T_{i-1})}{P(t, T_i)} - 1 \right) \right)$$

which can be simplified into:

$$\Pi_{\text{receiver}}(t) = N \cdot \sum_{i=\alpha+1}^{\beta} \left(\delta \cdot K_{\text{IRS}} \cdot P(t, T_i) - P(t, T_{i-1}) - P(t, T_i) \right). \tag{5}$$

The sum in the Equation (5) above can be separated into two sums:

$$N \cdot \sum_{i=\alpha+1}^{\beta} (\delta \cdot K_{\text{IRS}} \cdot P(t,T_i)) \ + N \cdot \sum_{i=\alpha+1}^{\beta} (P(t,T_i) - P(t,T_{i-1}))$$

where the second sum of the two, can be simplified into:

$$N \cdot \sum_{i=\alpha+1}^{\beta} (P(t,T_i) - P(t,T_{i-1})) = N \cdot P(t,T_\beta) - N \cdot P(t,T_\alpha).$$

This simplification is possible since the sum of all the terms from $i = \alpha + 1$ to $i = \beta$ cancel each other out, except $N \cdot P(t, \mathrm{T}_\beta)$ and $- N \cdot P(t, \mathrm{T}_\alpha)$. Adding the sums back together yields:

$$\Pi_{\text{receiver}}(t) = -N \cdot P(t,T_\alpha) + N \cdot P(t,T_\beta) + N \cdot \sum_{i=\alpha+1}^{\beta} \delta \cdot K_{\text{IRS}} \cdot P(t,T_i). \tag{6}$$

Equation (6) gives for the value of an IRS at time $t \leq T_\alpha$, from the receiver's point of view. Since $\Pi_{\text{receiver}}(t) = -\Pi_{\text{payer}}(t)$, the value of the swap for the payer at $t \leq T_\alpha$ is (Filipovic, 2009):

$$\Pi_{\text{payer}}(t) = N \cdot P(t,T_\alpha) - N \cdot P(t,T_\beta) - N \cdot \sum_{i=\alpha+1}^{\beta} \delta \cdot K_{\text{IRS}} \cdot P(t,T_i) \tag{7}$$

The floating leg, $N \cdot P(t, \mathrm{T}_\alpha\ )$ in Equation (7) can be viewed as a floating rate note and the fixed leg, $-N \cdot P(t,T_\beta) - N \cdot \sum_{i=\alpha+1}^{\beta} \delta \cdot K_{\text{IRS}} \cdot P(t,T_i)$ in Equation (7) can be viewed as a bond with a coupon. So an IRS can be seen as an agreement to exchange a floating rate note for a coupon bond.

A coupon bond is an agreement of a series of payments of specific amounts of cash at future times $T_{\alpha+1}$, $T_{\alpha+2}$, $T_{\alpha+3}$ ... T_β. The cash flows are in general expressed as $N \cdot \delta \cdot K_{\text{IRS}}$ when i $< \beta$ and $N \cdot \delta \cdot \beta \cdot K_{\text{IRS}} + N$ when i $= \beta$. K_{IRS} is here the fixed interest rate and N is the nominal amount. By discounting the cash flows back to present time t from the payment times T_i, the value of the coupon bearing bond at time t is given by (Brigo and Mercurio, 2006):

$$N \cdot \left(P(t,T_\beta) + \sum_{i=\alpha+1}^{\beta} \delta \cdot K_{\text{IRS}} \cdot P(t,T_i) \right).$$

Where the future discounted cash flows from the coupon payments are given by $N \cdot \sum_{i=\alpha+1}^{\beta} \delta \cdot K_{\text{IRS}} \cdot P(t,T_i)$ and the discounted repayment of the bond's notional value is given by $N \cdot P(t,T_\beta)$. The floating leg in the IRS in Equation (7), $N \cdot P(t,T_\alpha)$, can be viewed as a floating rate note, which is a contract that guarantees payments at future times $T_{\alpha+1}, T_{\alpha+2}, T_{\alpha+3}$ ... T_β of the interest rates that resets at the reset date just prior to the payment times, i.e. $T_\alpha, T_{\alpha+1}, T_{\alpha+2}$ ... $T_{\beta-1}$. Finally, at T_β, the note pays a cash flow that consists of the repayment of the notional value. The floating rate note is valued by replacing the sign of the $\Pi_{\text{reciever}}(t)$ in Equation (6), with a zero priced fixed leg and adding it to the PV of the cash flows paid at time T_β, giving (Brigo and Mercurio, 2006):

$$N \cdot P(t, T_\alpha) - N \cdot P(t, T_\beta) - 0 + N \cdot P(t, T_\beta) = N \cdot P(t, T_\alpha). \tag{8}$$

Equation (8) is convenient since a portfolio can replicate the structure of the entire floating rate note, illustrating that floating rate note always equals its notional amount when $t = T_i$ and it always equals N units of cash at its reset dates. So a floating rate always trades at par (Björk, 2009).

The forward swap rate K_{IRS} is the rate in the fixed leg of the IRS starting at time t and ending at T_β and is set so that the IRS contract value at time t is fair, i.e. so that $\Pi_{\text{receiver}}(t) - \Pi_{\text{payer}}(t) = 0$ in Equation (7) (Brigo and Mercurio, 2006), hence:

$$K_{\text{IRS}} = \frac{P(t, T_\alpha) - P(t, T_\beta)}{\sum_{i=\alpha+1}^{\beta} \delta \cdot P(t, T_i)}$$

which, assuming that the contract is written at time $t = T_\alpha$, can be reduced to:

$$K_{\text{IRS}} = \frac{1 - P(t, T_\beta)}{\sum_{i=\alpha+1}^{\beta} \delta \cdot P(t, T_i)}.$$

2.6 Credit Default Swaps

In this subsection we discuss the credit default swap (CDS), how it is constructed and valued, and how to calculate the CDS spread. O'Kane and Turnbull (2003) give an explanation of the CDS, stating that the purpose of the derivative is to give agents the possibility to hedge or to speculate in a company's credit worthiness without having to take an opposite position.

A CDS on a reference entity is a contract between two counterparties, where the seller of the CDS takes responsibility to pay the loss that the CDS buyer will suffer if the reference entity defaults. The protection buyer insures itself against a default of a third party, also known as a reference entity, by paying a fee. This fee is known as the CDS premium and is measured in basis points, where one basis point equals 0.01%. The premium is paid regularly until the contract ends or until the reference entity defaults. The CDS is often standardised in order to bring a higher liquidity and it typically has a maturity T of 3, 5 or 10 years. The reference entity is usually a bank, a corporation or a sovereign issuer. If the reference entity defaults, then the payment of the premium stops and the CDS seller fulfils its obligation by compensating the CDS buyer with the amount that the reference entity owes the CDS buyer (O'Kane and Turnbull, 2003).

Before the crisis in 2007-2008 these CDS-derivatives were trading on the OTC-market. The regulations have since then changed and regulators are now pushing for all credit default swaps to be traded via a CCP. This reduces the counterparty risk due to the CCPs ability to net the positions, which we explain in Subsection 2.9.

2.6.1 CDS Construction and Valuation

Hull (2014) describes the construction of a simple single-name CDS as follows: Company A enters into a credit default swap with insurance company B. The company which default company A insures itself against is called the *reference entity*, and the default of the reference entity is known as the *credit event*. Company A is the buyer and has the right to sell bonds issued by the reference entity in the case of a credit event to insurance company B, which is the seller of the insurance, for the face value of the bonds. The total value of the bonds that can be sold in a credit event is called the CDS's *notional principal*. A transaction of this kind, where the bonds are physically transferred between Companies A and B is called a physical settlement. An alternative to the physical settlement is the cash settlement, where B pays the net credit loss suffered by A in event of a default of the reference entity. Note that in the event of a physical settlement A has to actually hold bonds that will be delivered to B, which is not always the case. Company A could have bought insurance without actually holding any bonds, and if several parties have done the same then there would be a "short-squeeze" when everyone tries to buy the defaulted bonds in order to claim their insurance pay-out. This is not a problem if for cash settlements. The recovery rate of the bonds must however be determined, i.e. what amount company B should pay company A at default of the reference entity. This is usually solved by letting a "panel" of institutions bid on the defaulted bond, and this procedure gives the recovery rate (Herbertsson, 2016). Company A agrees to make payments to the insurance seller, typically each quarter, until the end of the CDS or until a credit event occurs (Hull, 2014, p. 548-549).

An example to illustrate the cash flows is this: Suppose that company A buys a 5 year CDS from company B in order to protect itself from a credit event by the reference entity. Suppose that they buy the CDS on March 20th 2017 and that the notional principal is $100 million. Company A agrees to pay 100 basis points per year for this protection, called the CDS spread. Company A makes payments every quarter of 25 basis points (0.25%) of the notional principal, beginning at March 20th 2017 and ending at March 20th 2022, which is the maturity date of the contract.

The amount paid each quarter is

$$0.0025 \cdot \$100,000,000 = \$250,000$$

If there is a credit event, the seller of the insurance is obligated to buy the bonds for the total face value minus the possible recovery rate. Let us assume that a credit event occurs with a recovery rate of 30%. The asset will have a value of $70 million. The CDS seller compensates the buyer with $30 million, i.e. the difference between the assets face value and current value. The CDS buyer pays the remaining accrued interest between the time of the reference entity default and the intended expiration of the contract, if the reference entity had not defaulted (O'Kane and Turnbull, 2003).

2.6.2 Calculating the Spread

As stated earlier in Subsection 2.6, the CDS spread, here denoted by S_T, is very useful when calculating the probability of a credit event for the reference entity. Let the notional amount on the bond be N. The protection buyer, company A pays $S_T \cdot N \cdot \delta_n$ to the protection seller, company B, at time points $0 < t_1 < t_2 ... < t_{n_T} = T$ or until $\tau < T$. Here τ is the time of default of the reference entity and $\delta_n = t_n - t_{n-1}$. Time T is the maturity of the contract. If default for the reference entity happens for some $\tau \in [t_n, t_{n+1}]$, A will also pay B the accrued default premium up to τ. On the other hand, if $\tau < T$, B pays A the amount $N \cdot (1 - \phi)$ at τ where ϕ denotes the recovery rate of the reference entity in % of the notional bond value. Thus, the credit loss for the reference entity in % of the notional bond value is given by $(1-\phi)$. Since S_T is determined so that the expected discounted cash flows between A and B are equal when the CDS contract is settled, we get that:

$$S_T = \frac{\mathbb{E}\left[1_{\{\tau \leq T\}} D(\tau)(1-\phi)\right]}{\sum_{n=1}^{n_T} \mathbb{E}\left[D(t_n)\delta_n 1_{\{\tau > t_n\}} + D(\tau_n)(\tau - t_{n-1}) 1_{\{t_{n-1} < \tau \leq t_n\}}\right]} \tag{9}$$

where $1_{\{\tau \leq T\}}$ is an indicator variable taking the value 1 if the credit event occurs before the maturity time T, and 0 otherwise. The discount factor $D(t)$ is dependent on the risk free rate r_t and is further explained in Subsection 2.11.2 (Herbertsson, 2016).

We can make Equation (9) a little easier to understand by making a couple of assumptions. We assume a constant recovery rate $(1 - \phi)$, that τ is independent of the interest rate, $t_n - t_{n-1} = \frac{1}{4}$, and that r_t is a deterministic function of time t, $r(t)$. Thus, S_T can be simplified into:

$$S_T = \frac{(1-\phi)\int_0^T D(t) f_\tau(t)dt}{\sum_{n=1}^{4T}\left(D(t_n)\frac{1}{4}(1-F(s)) + \int_{t_{n-1}}^{t_n} D(s)(s-t_{n-1})f_\tau(s)ds\right)}$$

where $F(t) = \mathbb{P}(\tau \leq t)$ is the default distribution and $f_\tau(t)$ is the density of default time τ, e.g. $f_\tau(t) = \frac{dF(t)}{dt}$.

Herbertsson (2016) (see also in Lando (2009)) makes two additional assumptions which help simplify the equation further:

1. The accrued premium term is dropped, meaning company A does not pay for the protection between time t_n and default time τ.

2. If the credit event τ happens in the interval $\left[\frac{n-1}{4}, \frac{n}{4}\right]$ the loss is paid at $t_n = \frac{n}{4}$, and not immediately at τ.

We can now simplify Equation (9) as follows:

$$S_T = \frac{(1-\phi)\sum_{n=1}^{4T} D(t_n)\left(F(t_n) - F(t_{n-1})\right)}{\sum_{n=1}^{4T} D(t_n)\frac{1}{4}(1-F(t_n))}. \tag{10}$$

Remember that the discount factor $D(t)$ is a function of the risk free rate and is therefore deterministic due to the assumption we made earlier about r_t being a deterministic function of t. We now have a simplified equation which we can use to derive the probability of default given the CDS spread in the market (Herbertsson, 2016). This process is explained in detail in Section 3.

2.7 Credit Value Adjustment

As explained in BIS (2015), credit value adjustment (CVA) has different definitions depending on in what context it is used. We therefore need to describe two measures for CVA: *accounting* CVA and *regulatory* CVA.

2.7.1 Accounting CVA

In the context of accounting, CVA is a measure to adjust an instruments risk free value when counterparty credit risk exists. Accounting CVA is illustrated as either a positive or a negative number, depending on which party is most likely to default and is calculated as the difference between the risk free and the true value of the portfolio. In other words, CVA is expressed as an expected value that includes expected exposure (EE) and probability of loss given default in order to achieve fair pricing. Alternatively, accounting CVA can be defined as the market value of the cost of the credit spread volatility.Accounting CVA is closely related to debit value adjustment (DVA) which is covered in Subsection 2.8 (Gregory, 2012).

2.7.2 Regulatory CVA

Regulatory CVA is a measure that specifies the amount of capital needed to cover losses on volatilities relating to the counterparty credit spread (BIS, 2015). CVA is analogous to a loan loss reserve, aiming to absorb the future potential credit risk losses on a loan. According to the regulation, it is not enough that the capital simply covers the expected losses; instead it should cover the expected losses with a very high probability (99%). This means that CVA is a measure of Value at Risk and is always a positive number (Gregory, 2012).

CVA as a capital requirement is needed since the CCR is volatile, which creates uncertainty regarding the expected value of the accounting CVA (BIS, 2015). The uncertainty brings risk of losses on mark-to-market (MTM), i.e. the unrealised loss resulting from a decrease in the asset market price.

As mentioned above, two thirds of all the credit losses during the 2007-2008 financial crisis were derived from CVA counterparty risk, and the CCR of financial actors mostly consists of OTC derivatives (BIS, 2015). The complex nature of OTC derivatives makes the calculation of CCR more difficult compared to other forms of risks. Firstly, it is difficult to calculate the relevant EE since the uncertain future value of the instrument is a function of the underlying asset. The *Net Present Value* (NPV) of an OTC derivative is at any point in time either an asset or a debt depending on the sign of the derivative. This means that the risk is bilateral, i.e. both parties are exposed to risk. Since OTC derivatives are priced on the market, the volatility of both gains and losses increases (Brigo et al., 2013).

2.8 Debit Value Adjustment

When a bank computes CVA, it often considers itself to be default free or that its counterparty has a much higher default probability. This is likely an unrealistic assumption causing the CVA to be asymmetric (Brigo and Mercurio, 2006).

When calculating unilateral CVA, the entity assumes that only the counterparty may default, not the entity itself. It is more realistic to assume that either party might default. The debit value adjustment (DVA) is the PV of the expected gain to the entity from its own default. It is calculated similarly to CVA.

DVA is controversial mainly due to the fact that if the credit rating of a firm drops, the same firm will gain MTM profits. Accounting CVA is calculated as the unilateral accounting CVA towards the counterparty, minus DVA. Since the DVA is calculated based on a firm's own credit quality, firms are able to profit and thereby boost their equity from the deterioration of their own credit quality. It is debated whether or not this is reasonable, but it is clear that DVA is important when it comes to the further development of the CCR framework. In accounting, *International Financial Reporting Standards* (IFRS) and U.S. *Generally Accepted Accounting Principles* states that DVA should be calculated if it leads to a more fair value of the derivatives, according to the *International Financial Reporting Standard 13: Fair Value Measurement*. Many banks that follow IFRS do calculate DVA, but not everyone (EBA, 2015, p. 20).

In the Basel framework however, the DVA volatility is not captured under the CVA risk charge and the entire DVA amount is derecognised from the banks' equity (BIS, 2011, p. 23, §75). The Basel Committee motivates this by reasoning that this source of capital could not absorb losses nor could it be monetised.

There has been a lot of discussion around DVA since 2011, but as late as 2015 BIS reinforced the message that DVA was not to be included in the banks equity (BIS, 2015, p. 4), and in 2016 DVA was "eliminated by the U.S. body that sets bookkeeping standards" according to Onaran (2016). For more about problems regarding DVA, see e.g. in Section 10.5 in Brigo et al. (2013).

2.9 Netting & ISDA Master Agreement

Netting means allowing positive and negative values to cancel each other out into a single net sum to be paid or received. Netting sets are sets of trades that can be legally netted together in the event of default which reduces the counterparty credit risk (CCR) (FederalReserve, 2006).

Netting can take two forms. *Payment netting* arises when two solvent parties combine offsetting cash flows into a single net payable or receivable. *Close-out netting* is best explained by the following example:

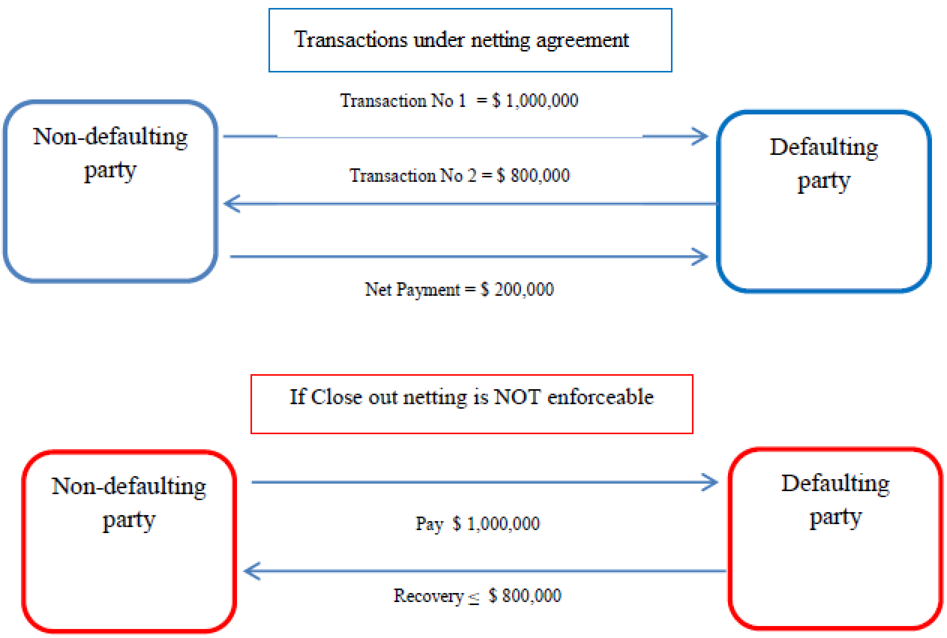


Figure 2: Close-out Netting (Perminov, 2016)

In Figure 2, a defaulting and a non-defaulting party engage into two swap transactions. In the first scenario, under a netting agreement, the non-defaulting party has an outflow of \$1 million in Transaction 1 while Transaction 2 brings an inflow of \$800,000. If close-out netting is enforceable, the non-defaulting party is compelled to pay the defaulting party the difference of \$200,000, illustrated in the top half of Figure 2. Without close-out netting, illustrated in the bottom half of Figure 2, the non-defaulting party would be compelled to immediately pay \$1 million to the defaulting party and then wait for the bankruptcy, which may take months or even years, for whatever fraction of the \$800,000 it recovers. Close-out netting reduces credit exposure from gross to net exposure.

According to research by *International Swaps and Derivatives Association* (ISDA), netting has reduced credit exposure on the OTC derivatives markets by more than 85 percent and without netting, total capital shortfall may exceed \$500 billion (Mengle, 2010).

An OTC derivatives trade is typically documented through a standard contract developed by the ISDA. This contract is called an ISDA master agreement and states the way the transactions between the two parties are to be netted and considered as a single transaction in the event that there is an early termination. The master agreement makes managing credit risk easier as it reduces the counterparty risk (Brigo et al., 2013).

Credit support annexes are included in the master agreements and used in documenting collateral arrangements and margin requirements between two parties that trade OTC derivatives. Collateral may take many forms but is usually made up out of cash or securities. Margin requirements for collateral are constantly monitored, ensuring that enough collateral is held per OTC derivative trading value. Consider the example of when firm A is required to post collateral. The threshold is the unsecured credit exposure to firm A that firm B is willing to bear. If the value of the derivatives portfolio to firm B is less than the threshold, firm A is not required to post collateral. If the value of the derivatives portfolio to firm B is greater than the threshold, then the required collateral is equal to the difference between the value and the threshold. If firm A fails to post the required collateral then firm B would be allowed to terminate its outstanding transactions with firm A (Hull and White, 2012).

Netting and ISDA master agreements significantly reduce the counterparty risk but also leave a net residual exposure, which may increase as the portfolio ages. However, since OTC derivatives are complex by nature, the counterparty credit risk can not be entirely eliminated (Brigo et al., 2013).

2.10 Central Counterparty Clearing

In order to further reduce the CCR firms may use so-called central counterparty (CCP) clearing, which is the process of entering an agreement with a central counterparty. A CCP acts as a neutral middleman during standard OTC transactions and assumes the responsibility of covering a counterparty in a bilateral contract if the counterparty defaults. The CCP manages all margin calls and steps in to cover the CDS seller if the seller fails to deliver liquid collateral.

Hull (2014) exemplifies CCP clearing with a forward contract transaction where A has agreed to buy an asset from B in one year for a certain price, the CCP agrees to:

1. Buy the asset from B in one year for the agreed price, and

2. Sell the asset to A in one year for the agreed price.

The CCP takes on the credit risk of both A and B. All members involved in transactions with the CCP has to provide initial margin. Transactions are valued on a daily basis so the member receives or makes margin payments every day. Only big market participants are clearing members and if an OTC market participant is not a member of a CCP, it can clear its trades via a CCP member who will provide margin to the CCP. This relationship between a non-member and a CCP member is similar to that of a broker and a futures exchange CCP member. (Hull, 2014)

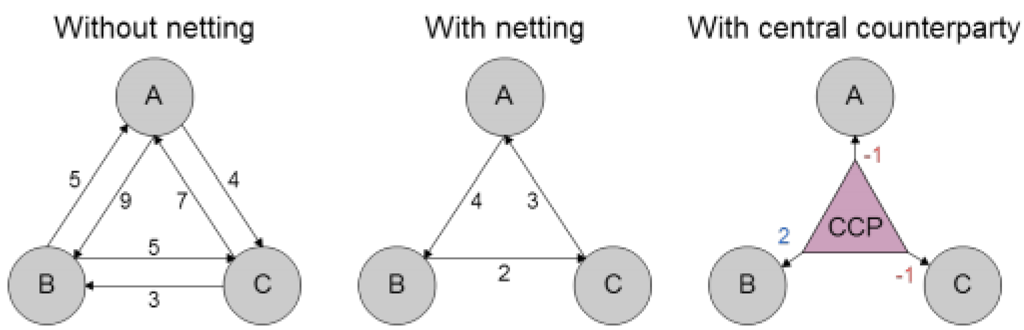


Figure 3: An illustration how different cash flows are netted against each other Franzén and Sjöholm (2014).

Figure 3 illustrates how the cash flows using CCP clearing are netted against each other. The total counterparty risk is reduced since the size of the clearing house enables it to net the counterparties. As can be seen in Figure 3, when there is no netting, all the cash flows are transferred between the counterparties. With netting, only the net between each counterparty is transferred, for example as A owes B 9 units and B owes A 5 units, it is enough to have A transfer 4 units to B. In the netting scenario, as A has a debt of 4 to B and a claim of 3 from C, A has a net claim of (3-4=) -1, i.e. a net debt of 1. Counterparty C also has a net debt of 1 (since 2-3=-1). Counterparty B has a net claim of (4-2=) 2, so the CCP covers A's and C's debt to B.

Also, the risk is further reduced since the CCP can easily monitor the creditworthiness of the counterparties and require that they post collateral. Monitoring and having overall information of all participants makes netting of collateral more efficient. Furthermore, the CCP can identify dangerous asymmetric positions and report this to regulators, thus increasing market transparency (Rehlon and Nixon, 2013).

Following the credit crisis in 2007-2008, regulators have become more concerned about systemic risk. One result of this has been legislation requiring that most standard OTC transactions between financial institutions be handled by CCP's (Hull, 2014).

The *European Markets and Infrastructure Regulation* (EMIR) is an European Union law aiming to reduce risks posed to the financial system by reporting derivative trades to an authorised trade repository and clearing derivatives trades above a certain threshold. The EMIR also mitigate the risks associated with derivatives trades by, for example, reconciling portfolios periodically and managing dispute resolution procedures between counterparties (Lannoo, 2011).

2.11 Risk Free Rate and Discount Rate

In this section we describe the term discount rate, which we use in our calculations. We begin by describing the risk free rate, what rate the market use and how it changed during and after the financial crisis.

2.11.1 Risk Free Rate

One might think that the rate of U.S. Treasury bills is the obvious way to derive the risk free rate. Treasury bills and bonds are issued by the U.S. government and are considered to be risk free investments. However, the Treasury bills and bonds rates are artificially low because of three points (Hull, 2014, p. 76-77):

1. Financial institutions are forced to buy Treasury bills and bonds to fulfil regulatory requirements, which creates a demand for these instruments. The price increases and the yield declines.

2. The capital that an institution has to hold to support the Treasury investment is much lower than the same value investment in any other low risk instrument.

3. In the U.S., there are tax advantages of buying Treasury instrument, since they are not taxed on state level.

Instead, institutions have used the LIBOR rate as the risk free rate. LIBOR stands for *London Interbank Offered Rate*, which is a reference rate on what rate banks pay when borrowing from each other and is calculated by the British Bankers' Association. LIBOR is stated in all major currencies and has maturities of up to 12 months. To be able to borrow at the LIBOR rate one has to be considered to have very low credit risk, typically an AA credit rating. Even if the LIBOR rate has a low risk, it is not totally risk free as we saw in the 2007-2008 financial crisis. Banks were not willing to lend to each other and the LIBOR rate increased drastically (Hull, 2014, p. 77).

Since the crisis, dealers have switched from the LIBOR rate to the *overnight indexed swap* (OIS) rate. In an OIS a bank receives a fixed rate for a period, which equals the geometric average of the overnight rates during the same period. The OIS rate is the fixed rate in the OIS. At the end of the day a bank can either have a surplus of cash or be short on cash to make all the transactions filed during the day. Therefore the bank is in need of overnight borrowing and the rate which they pay for that loan is the overnight rates in the OIS (Hull, 2014, p .77).

The spread between the LIBOR rate and the OIS rate can be a good indicator on how stable the financial economy is. If the market is uncertain, as it was in 2007-2009, the spread between the LIBOR and OIS rate will grow, and in times of stable markets the spread will shrink. This also helps to illustrate why the OIS rate is seen as a better choice for the risk free rate. The top half of Figure 4 below shows the 3-month LIBOR and the 3-month OIS rate in 2006-2017, where the LIBOR is the white line and OIS is the orange. The bottom half illustrates the difference between the LIBOR and the OIS in 2006-2017, i.e. the 3-month LIBOR-OIS spread.

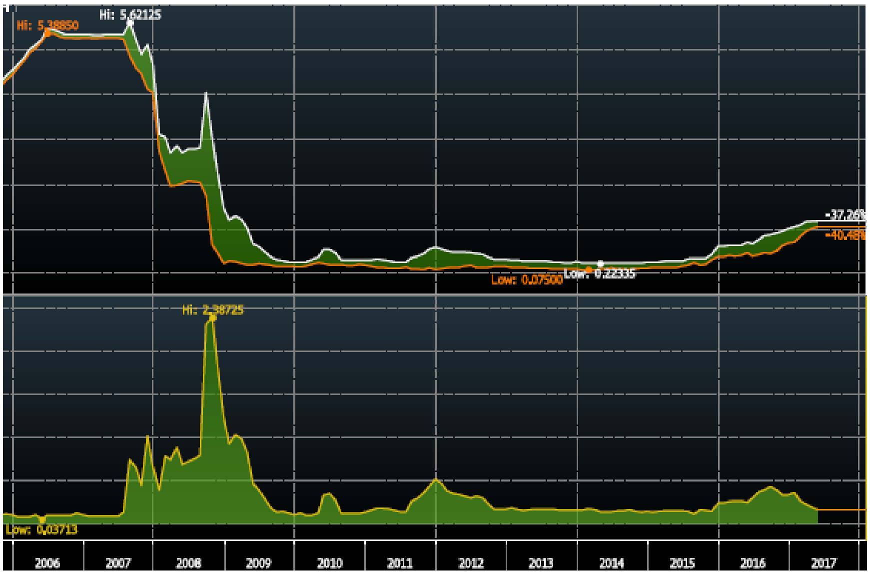


Figure 4: Monthly data of the 3-month LIBOR and the 3-month OIS rate (top) and the 3-month LIBOR-OIS spread (bottom) in 2006-2017, retrieved from Bloomberg

2.11.2 Discount Rate

Cash flows has to be discounted in order to take into account the time value of money. One dollar today is more valuable than one dollar in a year. One can assume that one dollar invested today would grow at the inflation rate, at least. Investments with similar risk should yield the same return and therefore be discounted by the same rate.

Brigo and Mercurio (2006, p. 3-4) express the discount factor $D(t, T)$, between time t and a future time T as:

$$D(t,T) = \frac{B(t)}{B(T)} = \exp\left(-\int_t^T r_s ds\right) \tag{11}$$

where $B(t)$ is the value of an investment at time t. We assumed earlier in Subsection 2.6 that the r_t is a deterministic function of time t, which would mean that the discount factor $D(t, T)$ also is deterministic, as we can see in Equation (11).

As explained by Hull (2014, p. 152-153), having the same rate, both as reference rate and as discount rate simplifies the calculation of the IRS. Although the floating reference rate has historically been based on the LIBOR, since the credit crisis of 2007-2008, most derivatives dealers now use OIS discount rates when valuing collateralised derivatives. This is based on the fact that collateralised derivatives are funded by collateral, and the OIS rate is usually paid on collateral (Hull, 2014, p. 207). Hull and White (2013) argues that the best proxy for the risk free rate should always be used when discounting and that the OIS zero curve is closest possible proxy to the risk free rate. Therefore we use simulated values of the OIS rate, both as discount rate and as reference rate in the calculation of the swap.

3 Modeling Default Intensities

As seen in Equation (10), we need to model the default time τ and its probability distribution $F(t) = \mathbb{P}[\tau \leq t]$. In Subsection 3.1 we explain intensity based models for τ, and in Subsection 3.2 we describe how we calibrate our default probabilities using piecewise constant CDS spreads.

3.1 Intensity Based Models

Here we introduce a so-called intensity based model for the default time τ. The intuition behind an intensity based model is the following:

Assuming that $\tau > t$ and given the information available on the market at time t, denoted by $\mathcal{F}_t$, the probability that a corporation's default time τ occurs in the time interval $(t, t + \Delta t)$ is approximately equal to $\lambda_t \Delta t$ for small values of Δt, i.e.:

$$\mathbb{P}[\tau \in [t, t + \Delta t) | \mathcal{F}_t] \approx \lambda_t \Delta t \quad \text{if} \quad \tau > t \tag{12}$$

where the stochastic process λ_t is positive for all values of t.

To give a rigorous construction of such a random variable τ, we proceed as follows: First, we let X_t be a d-dimensional stochastic process, which includes all the factors that drives the random variable τ. We then consider a function λ, which given X_t is the stochastic process $\lambda_t(\omega) = \lambda_t(X_t(\omega))$. Finally, we let E_1 be an exponentially distributed random variable with a mean of 1 and define τ as (Lando, 2009):

$$\tau = \inf \left\{ t \geq 0 : \int_0^t \lambda(X_s) ds \geq E_1 \right\} \tag{13}$$

which tells us that τ is the first time at which the positive function $\int_0^t \lambda(X_s) ds$ equals the random level E_1. One can show that if τ is constructed as in Equation (13), then τ will satisfy the relation in Equation (12).

Using the construction in Equation (13) we can derive the probability of survival up to time t as:

$$\mathbb{P}[\tau > t] = E \left[\exp \left(- \int_0^t \lambda(X_s) ds \right) \right]. \tag{14}$$

We use Equation (14) to calibrate our CDS spread function later. One can model λ_t in different ways depending on what version of X_t one uses, for example:

1. λ_t can be a deterministic constant, say λ.

2. λ_t can be a deterministic function that is dependent on time t.

3. λ_t can be a stochastic process, e.g. a CIR-process.

If the default intensity is a constant then we get $\mathbb{P}(\tau > t) = e^{-\lambda t}$ since:

$$\int_0^t \lambda dt = [\lambda \cdot s]_0^t = \lambda \cdot t - \lambda \cdot 0 = \lambda \cdot t$$

and $\mathbb{P}(\tau < t) = 1 - e^{-\lambda t}$. This expression is then used in place of $F(t)$ in Equation (10) to derive a simple expression for λ as a function of the CDS spread S_T:

$$S_T = 4(e^{\lambda} - 1)(1 - \phi) \Rightarrow \frac{S_T}{(1 - \phi)} = 4(e^{\lambda} - 1). \tag{15}$$

For small values of λ we can use so-called Taylor-expansion in Equation (15) to get the approximation:

$$\frac{S_T}{(1 - \phi)} = \lambda. \tag{16}$$

For a complete proof of Equation (15) and (16) see in Herbertsson (2016). If one would to use any of the two other approaches, where λ is either deterministically dependent on time or a stochastic process, then Equation (10) will in general not be possible to simplify to an easy formula, such as in Equation (15) or (16). In this thesis we assume so-called *piecewise constant* default intensities in order to mimic reality as well as possible without a too complicated equation. Usually, but not always, the spread of a CDS contract with maturity 3 years is lower than the spread of a CDS contract with a maturity of 10 years, which is not the case if one assumes constant default intensities. We can calibrate a more realistic function that is deterministic and time-dependent, if we base our calibration on the CDS spreads of different maturities.

3.2 Bootstrapping and Calibration

When calibrating the default intensities using bootstrapping, we consider a model where the default intensities $\lambda(t)$ for the default time τ, is piecewise constant between the time-points $T_1, T_2, ..., T_J$. Hence, $\lambda(t)$ is given by:

$$\lambda(t) = \begin{cases} \lambda_1 & \text{if } 0 \leq t < T_1 \\ \lambda_2 & \text{if } T_1 \leq t < T_2 \\ \vdots & \\ \lambda_J & \text{if } T_{J-1} \leq t < T_J \end{cases}$$

We defined the probability of τ being larger than time t in Equation (14), therefore we can also easily define the probability of τ being smaller than t as:

$$1 - \mathbb{P}[\tau > t] = 1 - \exp\left(-\int_0^t \lambda(s)ds\right) = F(t). \tag{17}$$

Using Equation 17 together with market CDS spreads for J maturities $T_1 \dots T_J$, we can calculate the probability of default in each time t:

$$F(t) = \begin{cases} 1 - e^{-\lambda_1 t} & \text{if } 0 \leq t < T_1 \\ 1 - e^{-T_1\lambda_1 - (t-T_1)\lambda_2} & \text{if } T_1 \leq t < T_2 \\ \vdots & \\ 1 - e^{-\sum_{j=0}^{J-1} \lambda_j (T_j - T_{j-1}) - \lambda_J (t - T_{J-1})} & \text{if } T_{J-1} \leq t < T_J \end{cases} \tag{18}$$

In order to calibrate the parameters $\lambda_1, \lambda_2, \dots \lambda_J$, we insert $F(t)$ in Equation (10) and set S_T equal to the market spread for that maturity, and then change λ_j so that the equality holds. Each calibration means that we solve an equation with one unknown parameter. The expression of $F(t)$ depends on what time period t we are in. If for example we want to calculate $F(t)$ quarterly, for each quarter up to the first maturity, when $0 \leq t < T_1$, we use the equation $1 - e^{-\lambda_1 t}$, and when $T_1 \leq t_j < T_2$ we use $1 - e^{-T_1\lambda_1 - (t-T_2)\lambda_2}$ and so on.

When we have calibrated our λ_j:s for each maturity T_j, we use them in Equation (18) above to compute the probability that the entity defaults in any given quarter. Remember that $F(t) = \mathbb{P}[\tau \leq t]$ is the probability of default up to time t, so the risk of default happening in one specific quarter j is the difference between the default probability for quarter j subtracted by the default probability for quarter $j - 1$, or $F(t_j) - F(t_{j-1})$ as was used in Equation (10) and will also be utilised in our CVA-calculation in the next section. The results of the derivation of the default probabilities are presented in Subsections 5.1 and 5.2.

4 CVA Formula

In this section we describe the credit valuation adjustment (CVA) formula, and its components, loss given default and expected exposure, and present the differences between the official CVA formula and the formula used in practice where the default intensities are assumed to be piecewise constant. We also present the method we use to calculate EE, the internal model method and we end the section by briefly discussing wrong-way risk.

Unilateral CVA is defined as the difference between the value of a portfolio assuming that the counterparty is default-free and the value of the portfolio including the risk of counterparty default (Brigo et al., 2013).

Consider a bilateral OTC derivative with maturity T between counterparties A and B. From the perspective of counterparty A, let $V(t,T)$ represent the risk-free value of the derivative at time t, $0 \leq t \leq T$, assuming that neither party can default. Moreover, let $V^D(t,T)$ represent the corresponding value of the defaultable version of the same contract, assuming that A is default free and B can default before time T. Then the CVA for the above contract at time t, is given by:

$$CVA(t,T) = V(t,T) - V^D(t,T).$$

We are only interested in calculating CVA at time $t = 0$, so:

$$CVA(0,T) = V(0,T) - V^D(0,T). \tag{19}$$

As proven by Brigo et al. (2013, p. 95), it is possible to rewrite Equation (19) as:

$$CVA(0,T) = \mathbb{E}\left[(1-\phi)\cdot 1_{\{\tau\leq T\}} D(0,\tau)(NPV(\tau))^+\right] \tag{20}$$

where ϕ is the recovery rate, which means that $(1-\phi)$ is the loss given default (LGD). Here $(x)^+$ denotes the positive part of (x), i.e. $(x)^+ = \max(x,0)$. The time of default for the party that can default is given by τ. Moreover, $NPV(\tau)$ is a shorthand notation for $NPV(\tau,T)$ representing the expected value of future cash flows between time τ and T (Brigo et al., 2013, pp. 94-96), where $NPV(t,T)$ is defined as:

$$NPV(t,T) = \mathbb{E}\left[\Pi(t,T)|\mathcal{F}_t\right].$$

Here, $\Pi(t,T)$ is the discounted net cash flows of the bilateral derivatives contract between the investor and the counterparty seen from the investors point of view at time t and $\mathcal{F}_t$ is the available market information at time t, see e.g. in Brigo et al. (2013). We make the following assumptions:

- τ is independent of $NPV(t,T)$, i.e. independent of $\Pi(t,T)$.
- The time period $[0,T]$ is divided into J intervals: $0 = t_0 < t_1 < ... < t_J = T$.
- The default time τ is replaced with the next t_j in the grid, so if $t_{j-1} < \tau < t_j$ then $NPV(\tau)$ is approximated by $NPV(t_j)$.
- LGD is constant, or equivalently, the recovery rate ϕ is constant.

These assumptions enables us to approximate Equation (20) by the following formula, (see e.g. in Brigo et al. (2013, p. 96)):

$$CVA_0 \approx (1-\phi) \sum_{j=1}^{J} (\mathbb{P}[\tau \le t_j] - \mathbb{P}[\tau \le t_{j-1}]) \cdot D_j \cdot EE_j$$

or, if $F(t) = \mathbb{P}[\tau \le t]$:

$$CVA_0 \approx (1-\phi) \sum_{j=1}^{J} (F(t_j) - F(t_{j-1})) \cdot D_j \cdot EE_j. \tag{21}$$

Furthermore, $D_j = \mathbb{E}[\exp(-\int_0^{t_j} r(X_s)ds)]$, i.e. the expected discount factor at time t_j. Finally, $EE_j = \mathbb{E}[\max(NPV(t_j), 0)]$ i.e. the expected exposure at time t_j, which can also be calculated using the internal model method described in Subsection 4.2.

In Equation (21) the term $(F(t_j) - F(t_{j-1}))$ equals the probability of a default occurring between time t_{j-1} and t_j. In Subsection 3.2 we determined that:

$$F(t) = 1 - \exp\left(-\int_0^t \lambda(s)ds\right)$$

and in Subsection 3.1 we saw that $\frac{S_T}{1-\phi} = \lambda$. Assuming a constant CDS spread transforms Equation (21) into:

$$CVA_0 \approx (1-\phi) \sum_{j=1}^{J} \left[\exp\left(\frac{-S \cdot t_{j-1}}{(1-\phi)}\right) - \exp\left(\frac{-S \cdot t_j}{(1-\phi)}\right)\right] \cdot D_j \cdot EE_j. \tag{22}$$

The official CVA formula in Basel III is derived in the same way, but the assumption of constant CDS spreads is dropped, giving:

$$CVA_{BIS} = (1-\phi) \sum_{j=1}^{J} max\left[0, \exp\left(\frac{-S_{j-1} \cdot t_{j-1}}{(1-\phi)}\right) - \exp\left(\frac{-S_j \cdot t_j}{(1-\phi)}\right)\right] \cdot D_j \cdot EE_j \tag{23}$$

where S_j is the CDS spread for the counterparty with a maturity t_j. For the piecewise model, $F(t_j)$ in Equation (21) is derived using the bootstrapping method which we explained thoroughly in Subsection 3.2.

The difference between Equations (21) and (23) is that the official CVA formula use a mathematically inconsistent assumption that the default intensity is not constant since it assumes that S_j might change in time. Since it is possible that $S_j > S_{j-1}$ and since the exposure cannot affect the CVA negatively, the official formula includes the "max" expression in Equation (23).

Both formulas are based on the assumption that the default probability and the market factor are independent. If these parameters are not independent, then so-called *wrong-way risk* (WWR) exists. WWR is the risk that arises when counterparty credit exposure during the life of the trade correlates to the credit quality of the counterparty (Herbertsson, 2016).

4.1 Loss Given Default

The loss given default (LGD) is the amount of capital a financial institution loses when a borrower defaults on a loan. In other words LGD equals one minus the recovery rate.

Theoretically, LGD can take any value from 0%, where the default does not lead to a loss, to 100%, where the entire exposure is lost. It is common, however not realistic, to assume a deterministic recovery rate. Banks set the value of LGD themselves. It is very difficult to determine a value, mainly because the sample data often is too small, and since it requires subjective estimations (Brigo et al., 2013).

The market LGD approach is a quantitative method that allows for an explicit estimation by looking at bond market prices immediately after default. These prices are then compared with the original par values. The LGD can be extracted from the company value, after discounting the observable recoveries and costs (Engelmann and Rauhmeier, 2011).

We estimate our value of the LGD based on data from Moody's. The data illustrates probability distribution of recoveries from 1970 to 2003 for all available bonds and loans. On average the LGD amounts to 60% which is the value we use in our calculation of CVA (Schuermann, 2004).

4.2 Internal Model Method

The internal model method (IMM) is a way to compute CVA for firms that have a regulator approved model for CCR, so-called IMM approval. An increasing number of banks all over the world are using IMM to calculate regulatory CVA capital as results obtained in practice from IMM is superior to other methods, according to a report by Thompson and Dahinden (2013). To calculate the exposure using the IMM, banks use a Monte Carlo model, which requires regulatory approval. The Monte Carlo model is presented in Appendix B. Historical data from at least the three previous years must be included in the Monte Carlo model, and one of those years has to be a so-called "stressed" scenario, meaning a particularly economically unstable period with increased credit spreads (Pykhtin, 2012).

Consider a bank with a portfolio of contracts towards a specific counterparty. The expected exposure at time t, $EE(t)$, is by definition the expected amount that the entity risk losing in an investment in the case of a counterparty default. This value will depend on $V(t)$, which is the mark-to-market (MTM) value of the portfolio at time t, and $C(t)$, the amount of collateral available at time t. Hence, $EE(t)$ is given by:

$$EE(t) = \mathbb{E}[\max\{V(t) - C(t), 0\}]. \tag{24}$$

In Equation (24), the exposure depends on whether the contract is an asset or a liability. The quantity $V(t)$ is the risk free value from the banks perspective. A negative value of $C_i(t)$ implies that the bank has posted collateral at time t, meaning an obligation to the counterparty. Should the counterparty default, this amount will still be due and has to be paid to the creditors of the defaulted company. A positive value implies that the bank holds collateral at time t, meaning it is an asset for the bank, and is expected to be received from the counterparty. If the counterparty defaults, this value will not be fully paid, so the exposure equals the present value (PV) of the asset. Therefore, the exposure is equal to the PV of the asset if $V_i(t) > C_i(t)$ and zero otherwise (Pykhtin, 2012). In our calculations we assume that no collateral has been posted, i.e. $C_i(t) = 0$.

4.3 Expected Exposure

In the CVA-formula given by Equation (21), the expected exposure (EE) is the value of the derivative and is the most difficult part of CVA to calculate since it is based on many different parameters. It may often require a massive amount of simulations depending on how many derivatives you hold. So in Equation (21), EE is the expected risk-neutral value of the exposure to the counterparty at future time t and is independent of counterparty default event, i.e. we assume there is no wrong-way risk.

These simulations of the distribution of counterparty-level potential future credit exposure are performed by using three main components:

1. *Scenario Generation.* Future market scenarios are simulated using evolution models of the risk factors for a fixed set of simulation dates.

2. *Instrument Valuation.* Valuation is performed for each trade in the counterparty portfolio for each simulation date and each realization of the underlying market risk factors.

3. *Portfolio Aggregation.* Counterparty-level exposure is obtained for each simulation date and each realization of the underlying market risk factors by applying Equation (25):

$$EE(t) = \mathbb{E}\left[\sum_k \max\left[\sum_{i\in NA_k} V_i(t), 0\right] + \sum_{i\notin\{NA\}} \max[V_i(t), 0]\right] \tag{25}$$

where, in the first term, the sums inside the brackets are only for the values of all trades covered by the k-th netting agreement (hence, the $i \in NA_k$ notation). The sum outside the brackets sums the exposures over all netting agreements. The second term in Equation (25) is the sum of contract-level exposures of all trades not belonging to a netting agreement (hence, the $i \notin NA_k$ notation) (Pykhtin and Zhu, 2007).

4.4 Calculating the Expected Exposure

Calculating the expected exposure (EE) for an interest rate swap (IRS) normally requires a stochastic interest rate model, such as e.g. the Cox–Ingersoll–Ross (CIR) model (Herbertsson, 2016). We will use a CIR-process to model the interest rate. According to the CIR model, the instantaneous interest rate follows the stochastic differential equation, also known as the CIR-process. The simplest version of this model describes the dynamics of the interest rate r_t as the solution of the following stochastic differential equation:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}\, dZ_t$$

where θ is the average of the 1-month OIS rates over the last year, κ represents the speed of adjustment to the long term mean θ. In other words, it is the continuous drift and is always positive. Here, Z_t is a standard Brownian motion, meaning it is a stochastic process where for each t, $Z_t \sim N(0,t)$, i.e. Z_t is normally distributed with mean zero and variance t, and dZ_t represents a normally scaled random number multiplied with the square root of the time step, at time t. The σ is the continuous volatility of the process and $dt = \frac{1}{360}$, i.e. a daily time step assuming a 360 day year. Continuous time variables have a particular value for only an infinitesimally short amount of time. Between any two points in time there are an infinite number of other points in time. The model has a condition that $2\theta\kappa > \sigma^2$, which puts a nonnegative restriction on r_t, hence $r_t \geq 0$ and θ is the equilibrium interest rate (Cox et al., 1985).

To value the EE we first have to simulate the path of the floating rate in the IRS and value the floating leg in the IRS.

To derive the EE, the following procedure is followed:

- The CIR method is used in order to simulate the interest rate path for a 10-year period.

- The interest rate simulation is used to value our instrument, the IRS.
- The IRS gives the exposure through Equation (24):

$$EE(t) = \mathbb{E}[\max\{V(t) - C(t), 0\}]$$

  where $V(t)$ is the value of the IRS contract at time t and as mentioned in Subsection 4.2, we set $C(t) = 0$.

4.4.1 Simulating the Interest Rate

In this subsection we describe the way we simulate the interest rate using the CIR model as described by Farid (2014). We first have to calibrate the CIR model parameters; the continuous drift denoted by κ, the continuous volatility σ, and average of the 1-month OIS rates θ, based on actual data.

We start by collecting data of the 1-month OIS rate from 5/5-16 to 5/5-17 from the archive of the Bank of England[1]. Each observation is denoted by r_{t^*}, i.e. 1-month OIS rate r, at time t^*.

As in Farid (2014), we now proceed as follows: To derive the values of κ and σ, we first need to calculate the value of the *residual sum of squares* (RSS). We want the RSS to be as small as possible. We need to transform the short rates r_t by subtracting the average of the short rates, denoted by θ, from each observation r_{t^*}. The transformed short rate is denoted by $\tilde{r}_t$ and equals $r_{t^*} - \theta$. The discrete drift term ψ, represents the values of the stochastic drift occurring at distinct, separate points in time, where the stochastic drift is the change of the average value of a stochastic (i.e. random) process. At first, ψ is taken as an arbitrary number and the altered in order to calibrate the minimum of RSS. RSS is given by:

$$RSS = \sum_{t=1}^{T} \frac{\tilde{r}_t - \psi \cdot \tilde{r}_{t-1})^2}{\tilde{r}_t + \theta}$$

where the sum includes all observations of $\tilde{r}_t$ we have, i.e. from a year back.

The next stage of the calibration process is to calculate the following terms:

- The discrete volatility parameter, σ_a is given by:

$$\sigma_a = \sqrt{\frac{RSS}{N-1}}$$

  where N is the number of residual terms.

[1]Retrieved from http://www.bankofengland.co.uk/statistics/Pages/yieldcurve/archive.aspx

- The continuous drift, κ is given by:

$$\kappa = \ln \frac{1}{\psi}.$$

- The continuous volatility, σ is given by:

$$\sigma = \sqrt{\frac{2\kappa\sigma_a^2}{1 - \exp(-2\kappa)}}.$$

Through this calibration we obtain the CIR model parameters we need in order to simulate the interest rate path over the next 10 years. The calibration gives $\kappa = 0.02$, $\sigma = 4.07\%$ and $\theta = 0.60\%$. We also need today's[2] 1-month OIS rate, which we retrieve from the Bank of England, $r_0 = 0.73\%$. We plug in these parameters into the built in CIR function in Matlab that simulates the interest rate paths. The simulation of the interest rate path is made 100 000 times. For simplicity we only illustrate 10 interest rate path simulations over 10 years in Figure 5 where the interest rate r_t follows a CIR process with $\kappa = 0.02$, $\sigma = 4.07\%$, $\theta = 0.60\%$ and $r_0 = 0.73\%$.

[2] At 5/5 -17

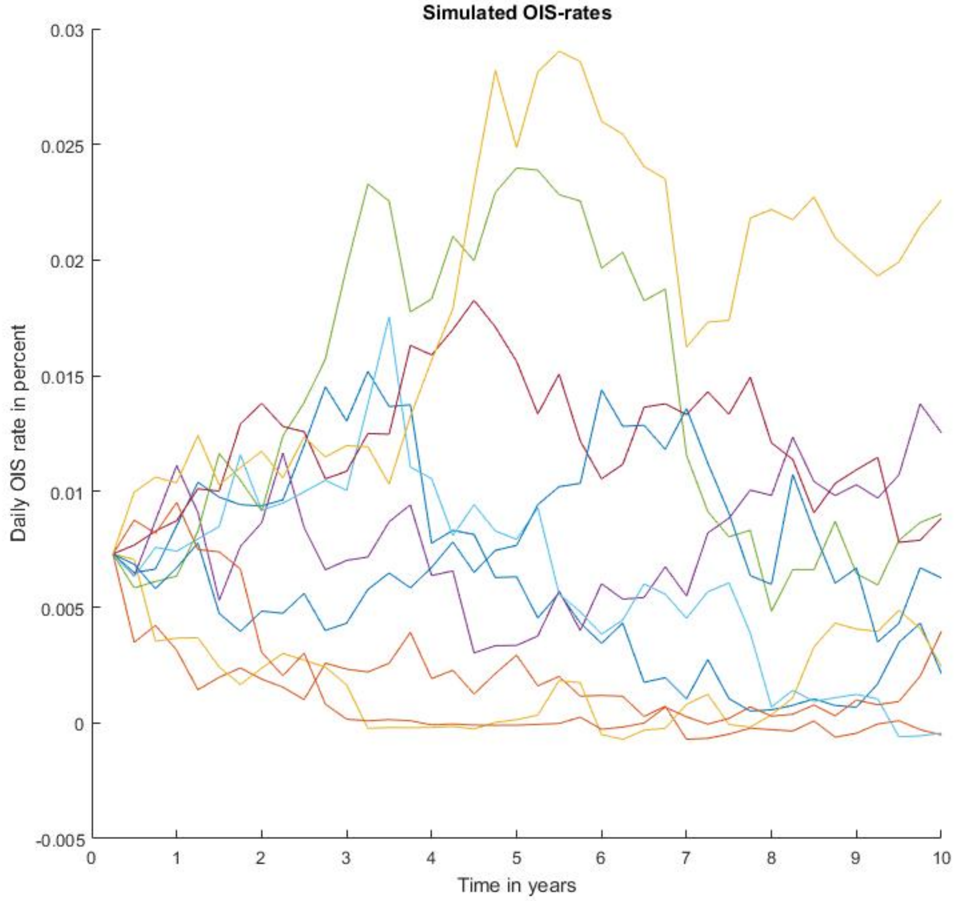


Figure 5: Simulation of 10 interest rate paths where the rate follows a CIR-process

This interest rate simulation enables us to now value the IRS.

4.4.2 Valuing the Interest Rate Swap

As mentioned in Subsection 2.5 the value of the IRS contract $\Pi_{\text{payer}}(t)$ is calculated using Equation (7) at all points in time t until maturity T_β, i.e. for all $t \leq T_\beta$. The calculation is based on the arbitrage-free price of a default-free zero coupon bond $P(t,T)$ which is given by (Björk, 2009):

$$P(t,T) = A(t,T) \cdot \exp(-B(t,T)) \tag{26}$$

$$A(t,T) = \left(\frac{\alpha \cdot \exp\left(\beta(T-t)\right)}{\beta(\exp\left(\alpha(T-t)\right) - 1) + \alpha}\right)^{\gamma}$$

$$B(t,T) = \frac{\alpha \cdot \exp(T-t) - 1}{\beta(\exp(\alpha(T-t)) - 1) + \alpha}$$

where $\alpha = \sqrt{k^2 + 2\sigma^2}$, $\beta = \frac{k+\alpha}{2}$ and $\gamma = \frac{2k\theta}{\sigma^2}$.

The value of the swap for the payer is given by Equation (7):

$$\Pi_{\text{payer}}(t) = N \cdot P(t, T_\alpha) - N \cdot P(t, T_\beta) - N \cdot \sum_{i=\alpha+1}^{\beta} \delta \cdot K_{\text{IRS}} \cdot P(t, T_i) \tag{27}$$

where $P(t, T)$ is given by Equation (26).

We let α depend on t, where t goes from 0 to T_β. For each time point t, $\alpha = \alpha(t)$ equals the integer such that the time point $T_{\alpha(t)}$ is the closest point in time to t, i.e. $T_{\alpha(t)-1} < t \leq T_{\alpha(t)}$.

In the formula for $\Pi_{\text{payer}}(t)$ above, N is the notional value and $N \cdot P(t, T_\alpha)$ is equivalent to a floating rate note. $N \cdot P(t, T_\alpha)$ equals N when t equals T_i for $i = \alpha, \alpha + 1, \alpha + 2, \ldots, \beta$ (Brigo and Mercurio, 2006).

The term $-N \cdot P(t, T_\beta) - N \cdot \sum_{i=\alpha+1}^{\beta} \delta \cdot K_{\text{IRS}} \cdot P(t, T_i)$ represents all remaining cash flows of the payment from the fixed leg, discounted at time t. Naturally, the sum decreases as the contract approaches maturity. $N \cdot P(t, T_\beta)$ is the discounted value of the fixed leg notional, which approaches N as the contract matures. Here K_{IRS} is the swap rate that makes the $PV_{\text{FIX}} = PV_{\text{FLOAT}}$ and is only calculated once, at $t = 0$, and δ represents the year fraction between T_{i-1} and T_i (Björk, 2009).

Using Equation (27), valuation of an IRS with t going from 0 to T_β, can be made at any time t since T_α is assumed to follow the coupon payments. Since α depends on t, the present value of the IRS is calculated by discounting all cash flows from $t \leq 0$ to $t = T_\beta$, We assume a contract length of $T_\beta = 10$ years, and $\delta \cdot (T_i, T_{i+1})$ is set to 0.25 since we assume quarterly coupon payments. After the payment of the first coupon is made at $0 < t \leq 0.25$ and $T_\alpha = 0.25$, the swap is calculated as a contract with a maturity of 9.75 years. This procedure is repeated until $t = T_\beta$. Since α is a function of the time, the calculations are made on shorter and shorter swaps as time passes.

When $t = 10 = T_\beta$, there are no coupons left and this means that there are no remaining cash flows of the fixed leg, given by $-N \cdot P(t, T_\beta) - N \cdot \sum_{i=\alpha+1}^{\beta} \delta \cdot K_{\text{IRS}} \cdot P(t, T_i)$. Since t is now equal to both T_α and T_β, both $P(t,\ T_\alpha)$ and $P(t,\ T_\beta)$ are equal to 1. Hence, at $t = 10$, the contract has a value of $N \cdot 1 - N \cdot 1 - 0 = 0$.

Simulations of the IRS value with $T_\beta = 10$ years are presented in the Figure 6 below.

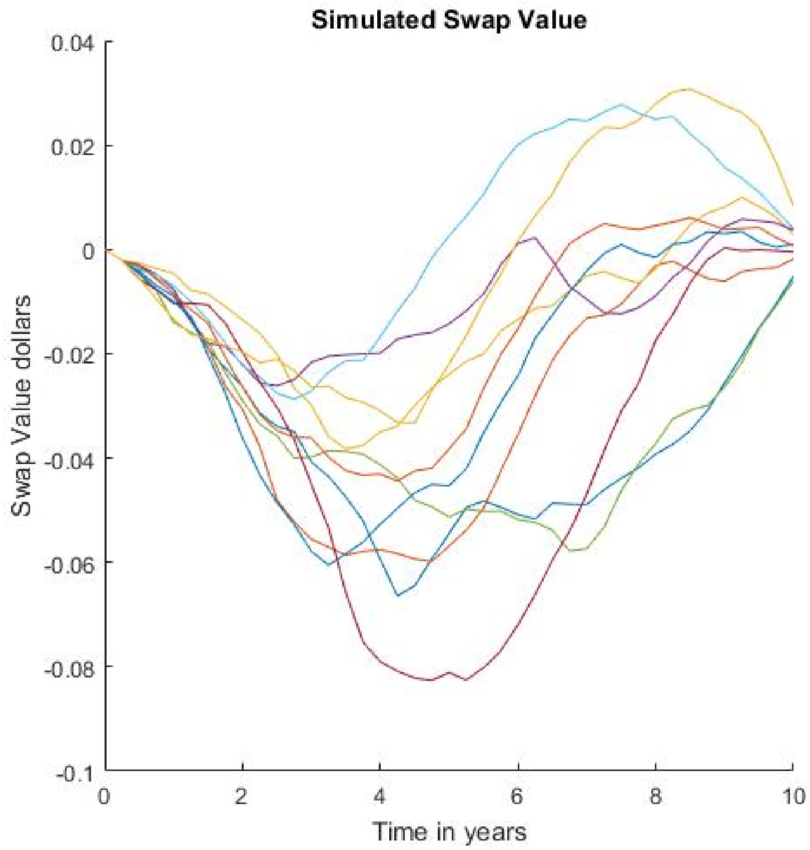


Figure 6: Simulation of 10 Interest Rate Swaps with a ten-year maturity

4.4.3 Valuing the Expected Exposure

Using the swap contract we can derive the expected exposure (EE) using the internal model method described in Subsection 4.2. We assume that there is no collateral posted, so $C(t)$ in Equation (24) will be 0 for all values of t. Hence the exposure is given by the value of the swap if it is positive and zero otherwise.

In Matlab we simulate the EE using the built in Euler method 100 000 times, where we use the CIR parameters estimated as in Subsection 4.4.1 as our inputs. The Euler method is complicated but in short it is a method in numerical analysis for solving stochastic differential equations with a given initial value. If we do not have small enough steps, then the function produces complex numbers, which means that EE also becomes a complex number. A complex number is an imaginary unit, for example the square root of -1. Because CIR contains the square root, the discrete approximation process becomes negative and thus undefined for real numbers.

As the discount rate we use the simulated interest from Subsection 4.4.1. We arbitrarily set the swap rate to 10% and the notional to 1, for simplicity. That gives us the total exposure. EE is obtained by dividing the total exposure by the number of simulations (Pykhtin and Zhu, 2007). The EE curve for our interest swap is shown in Figure 7 below.

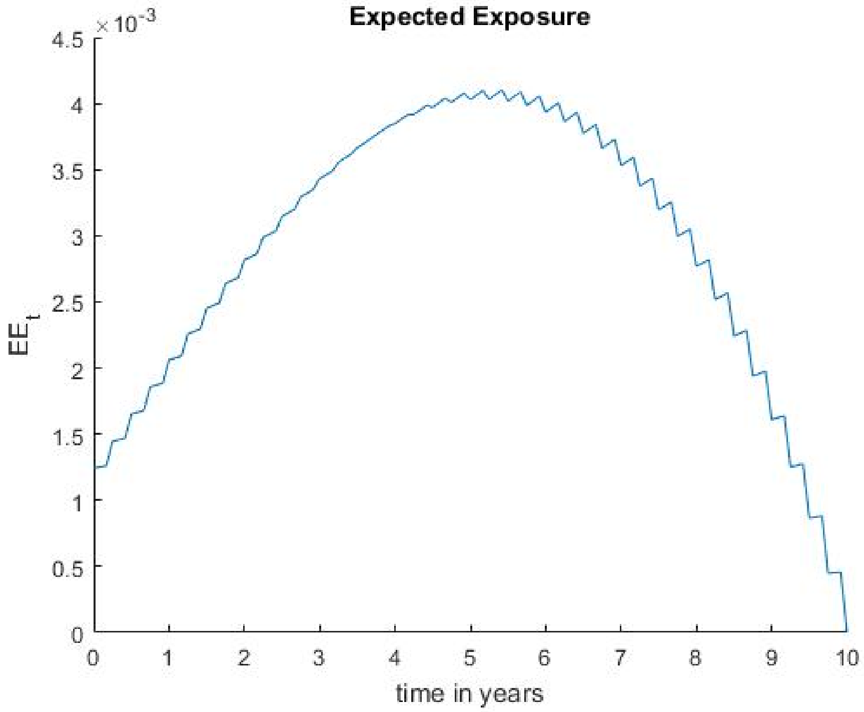


Figure 7: Simulation of the Expected Exposure

4.5 Wrong-way Risk

When calculating CVA, the probability of the counterparty defaulting is assumed to be independent of the dealer's exposure to the counterparty. *Wrong-way risk* (WWR) refers to the situation when there is a positive dependence, meaning the probability of counterparty default is high when counterparty exposure is high. When these two are negatively dependent, then so-called *right-way risk* exists (Rosen and Saunders, 2012).

Knowing the counterparty's business, with its surrounding risks, is vital when making a subjective estimation of the amount of wrong-way or right-way risk in a transaction. Knowledge of the transactions the counterparty has entered with others is also useful, but difficult to determine, although it has been made easier by the post-crisis legislation (Hull and White, 2012).

WWR tends to exist in situations where a company sells credit protection to the dealer, since the credit spreads are correlated. When the spreads are high, so is the value of the protection to the dealer. As a result the dealer has a large exposure to the company. Simultaneously, the credit spreads of the company are likely high, suggesting a relatively high default probability for the company (Rosen and Saunders, 2012).

Right-way risk tends to appear when a company buys credit protection from the dealer. Consider a situation where a company is speculating by conducting many similar trades with one or more dealers. This will likely lead to WWR for the dealers because the company's financial position and thereby its probability of default, will likely be affected unfavourably if the trades move against the company (Rosen and Saunders, 2012).

If a counterparty enters into a transaction as a way of hedging against an existing exposure, then right-way risk occurs since, if the transaction move against the counterparty, it will gain from the unhedged part of its exposure and the probability of default will be relatively low (Hull and White, 2012).

5 Results

In this section we present the results of our comparison between the Basel model and the piecewise constant model discussed in Section 4, where the computations are done for an interest rate swap, following a CIR process, as described in Subsection 4.4. The results of the comparison are then discussed in Section 6.

We compare the models using default probabilities implied from market CDS spreads in Subsection 5.1. The spreads are gathered from markets with low and high risk as well as when the CDS spread is inverted. In Subsection 5.2, we compare the models using fictive data where in the first scenario, the CDS spread is constant. In the second scenario, the spread changes drastically. We show the difference in CVA value between the models for all scenarios in Subsection 5.3.

Since the expected exposure does not depend on which default risk model we use, only on the CIR parameters, we do not present a comparison between the two models in this section. Instead we compute a sensitivity analysis in Subsection 5.4, where we alter the different inputs in the CIR model, namely the continuous drift κ, the continuous volatility σ and the average of the short term rates θ.

5.1 Default Intensity with Market Data

The default intensity or hazard rate is roughly defined as the rate of a default occurring in any time period, given no default up to a specific time. The probability of default between time t_{j-1} and t_j is calculated using three scenarios in which we gather our data of CDS spreads. Figure 8 displays time series of CDS spreads with maturities 3, 5 and 10 years for the Swedish bank Swedbank to illustrate the different levels of spreads. For the low risk scenario we use data from March 2017, when the

CDS spreads were relatively low, and for our high risk scenario we use data from November 2011. The CDS spreads from March 2009 are noteworthy since the CDS spreads for shorter maturities were higher than for longer maturities, see in Figure 8, which occurs during especially uncertain times. The interpretation of this is that the market believes that the main risk of a default is in the short term, and if the business survives this period the risk for default will decrease. When the shorter term instrument is yielding a higher rate of return than the longer term instrument it is said that the spread is inverted (Hull, 2015).

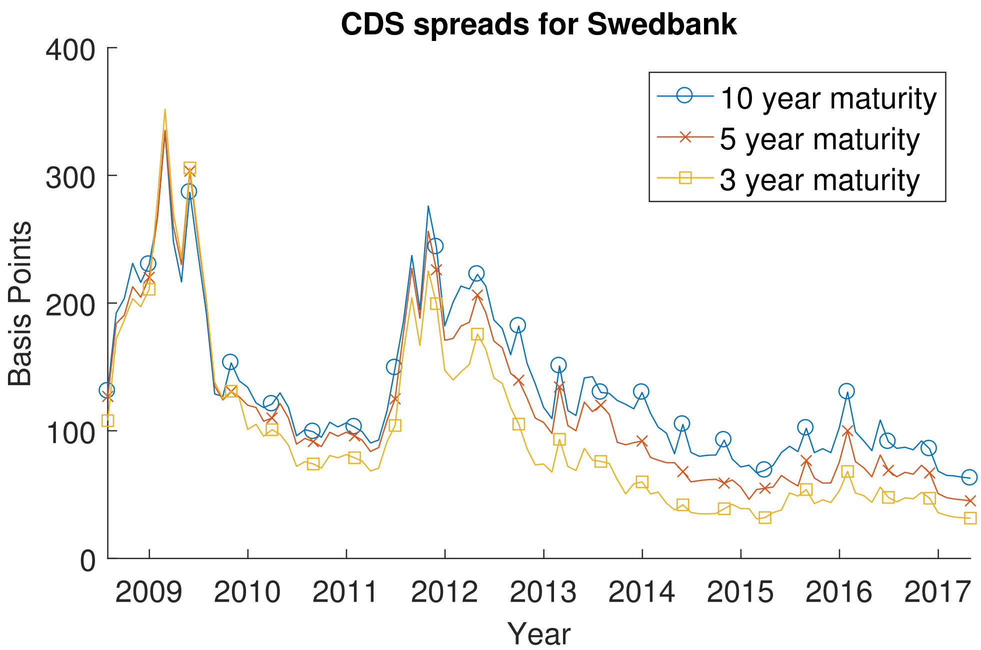


Figure 8: CDS spreads with maturities 3, 5 and 10 years for Swedbank during the period July 2009 - May 2017

When deriving our piecewise constant default intensities using bootstrapping we use data on Swedbank CDS spreads for five different maturities, 1, 3, 5, 7, and 10 years, see in Table 1 below, where the CDS spreads for the low risk scenario are from March 2017, the spreads used in the high risk scenario are from November 2011, and the inverted spreads are from March 2009.

Table 1: CDS spread data (in basis points)

Maturity	1 Year	3 Year	5 Year	7 Year	10 Year
Low risk	14.8	28	40.7	49.3	58
High risk	207.5	213	246.3	259.8	267.5
Inverted spreads	360.4	336.2	325.2	326.4	326.8
Constant spreads	200	200	200	200	200
Drastic change	10	50	150	300	450

In our calibration we apply Equation (10):

$$S_T = \frac{(1-\phi)\sum_{n=1}^{4T} D(t_n)\left(F(t_n) - F(t_{n-1})\right)}{\sum_{n=1}^{4T} D(t_n)\frac{1}{4}(1-F(t_n))}$$

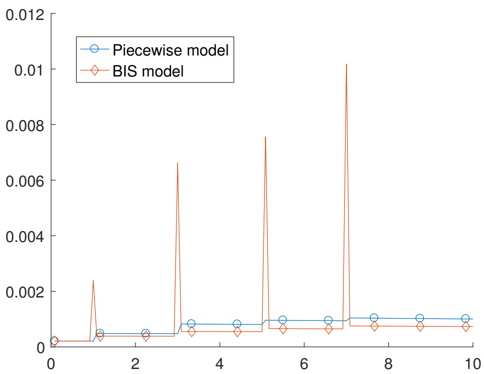
with $(1-\phi) = \text{LGD} = 0.6$.

As mentioned in Subsection 2.11.2, the reference rate is equal to the discount rate. Therefore, the discount factor is given by $D(t_n) = e^{-r \cdot t_n}$, where the discount rate r is derived from the simulation made in Subsection 4.4. Here, 10 years is the longest maturity available, so we end our calibration there. Our implied default intensities are shown in Table 2 together with the corresponding Basel model default intensities. The Basel model default intensities are calculated as $\frac{S_T}{LGD}$, where S_T is the market spread and LGD = 0.6.

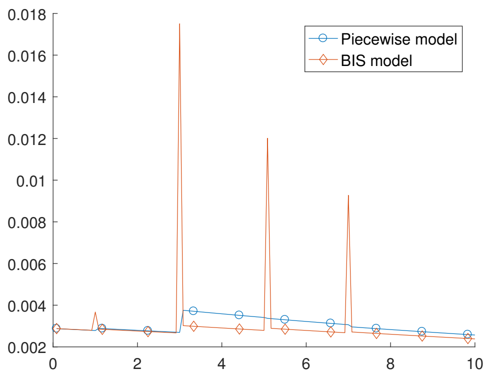
Table 2: Implied default intensities using Swedbank CDS market data

	Low risk scenario		High risk scenario		Inverted spreads scenario	
	Piecewise	Basel	Piecewise	Basel	Piecewise	Basel
$0 < t_j < 1$	0.0025	0.0025	0.0345	0.0346	0.0599	0.0601
$1 \leq t_j < 3$	0.0058	0.0047	0.0359	0.0355	0.0537	0.0560
$3 \leq t_j < 5$	0.0100	0.0068	0.0502	0.0411	0.0509	0.0542
$5 \leq t_j < 7$	0.0119	0.0082	0.0498	0.0433	0.0549	0.0544
$7 \leq t_j \leq 10$	0.0133	0.0097	0.0483	0.0446	0.0545	0.0545

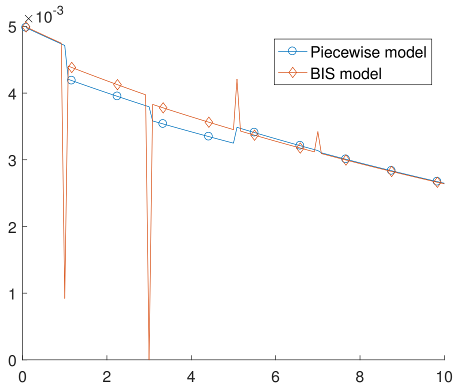
We calculate the probability of default occurring in any quarter by using the implied default intensities in Table 2. We explained how this is done for the piecewise model in Section 3. The Basel default probability in the period $[t_{j-1}, t_j]$ is calculated as $\max\left(0, e^{-\lambda_j \cdot t_j} - e^{-\lambda_{j-1} \cdot t_{j-1}}\right)$. Each scenario's default probabilities $F(t_j) - F(t_{j-1})$ are shown in Figure 9, where $F(t) = \mathbb{P}[\tau \leq t]$.



(a) Low risk scenario



(b) High risk scenario



(c) Inverted spreads scenario

Figure 9: Implied probability of default in the interval $[t_{j-1}, t_j]$ for the two models plotted on the y-axis for each market scenario with time in years on the x-axis.

When calculating the default probabilities in the low risk scenario we can see in Figure 9 that the Basel model greatly overestimates the default probabilities when t_j is close to the maturities. When t_j is not close to any maturity, the Basel formula underestimates the probabilities compared to the piecewise constant model.

The same holds for the high risk scenario. The difference between the first two scenarios is that in the low risk scenario the model difference increases with the maturity, whereas the opposite holds for the high risk scenario.

In the scenario when the spreads are inverted, the Basel model underestimates the default probability for swaps with short maturities, compared to the piecewise constant model.

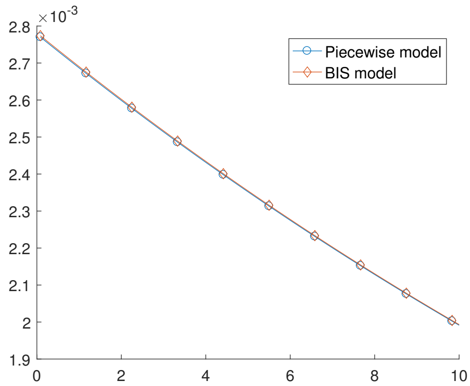
Both in the high risk scenario and in the inverted spreads scenario, both models appear to decline towards a long term value.

5.2 Default Intensity with Fictive Data

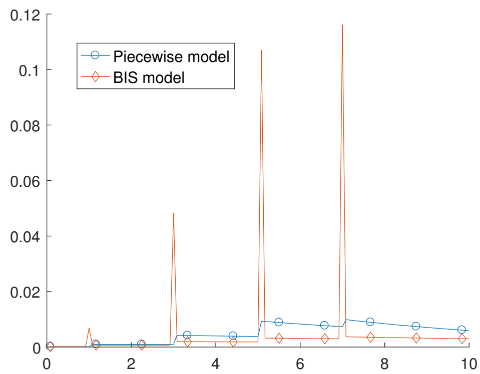
We will now use fictive data to examine the difference between the models in two extreme scenarios. In the first example, all maturities have the same spread of 200 basis points. In the other scenario we examine the difference between the models when the CDS spread changes drastically, meaning the CDS premium increases by more than it normally does, between two maturities. The default intensities are calculated the same way as in Subsection 5.1 and are presented in Table 3. The default probabilities for each time interval $t_{j-1} \leq t_j$ are plotted in Figure 10.

Table 3: Implied default intensities using fictive data

	Constant spread		Drastic change	
	Piecewise	Basel	Piecewise	Basel
$0 < t_j < 1$	0.0333	0.0333	0.0017	0.0017
$1 \leq t_j < 3$	0.0333	0.0333	0.0117	0.0083
$3 \leq t_j < 5$	0.0333	0.0333	0.0518	0.0250
$5 \leq t_j < 7$	0.0333	0.0333	0.1279	0.0500
$7 \leq t_j < 10$	0.0333	0.0333	0.1753	0.0750



(a) Constant spread



(b) Drastic spread

Figure 10: Implied probability of default in the interval $[t_{j-1}, t_j]$ for the two models plotted on the y-axis for each fictive scenario, with time in years on the x-axis.

As expected, the two models give approximately the same result when the spread is constant. When the spread changes drastically, the result is similar to the low risk scenario, except that default probabilities in the piecewise constant model are declining for long maturities.

5.3 Credit Value Adjustment

We now have all the values we need in order to compute the CVA value using both formulas.

As mentioned in Section 4, the official CVA formula in Basel III is given by:

$$CVA_{BIS} = (1-\phi)\sum_{j=1}^{J} max\left[0, \exp\left(\frac{-S_{j-1}\cdot t_{j-1}}{(1-\phi)}\right) - \exp\left(\frac{-S_j \cdot t_j}{(1-\phi)}\right)\right]\cdot D_j \cdot EE_j$$

where S_j is the CDS spread at the time t_j. The formula for calculating CVA with piecewise constant default probabilities is given by:

$$CVA_0 \approx (1-\phi)\sum_{j=1}^{J}[F(t_j) - F(t_{j-1})]\cdot D_j \cdot EE_j.$$

As mentioned in Subsection 2.11.2, the discount rate we use is the same rate as the simulated reference rate in the IRS. The IRS simulation is made 100 000 times whereas we only have one value of the EE_j for every point in time t_j. Since EE_j is the expected value of the exposure at t_j, it is also the average of all exposures derived from the simulated IRS. The IRS's are based on the simulated values of the reference rate. Therefore, for each time point, we use an average of our simulated reference rate as our discount factor, D_j.

Table 4: Comparison of CVA calculation between the two models (in 10^{-4})

	CVA		
	Piecewise	Basel	% Difference
Low risk	1.1167	1.1207	0.36%
High risk	3.8694	3.7786	2.40%
Inverted spreads	4.0274	4.0741	1.15%
Constant spreads	2.8236	2.8267	0.11%
Drastic change	8.1854	7.4403	10.01%

The fourth column in Table 4 illustrates the percentage difference between the Basel and the piecewise model, calculated as $\frac{|Piecewise - Basel|}{Basel}\cdot 100\%$. The values in the first two columns in Table 4 are presented in 10^{-4}, so for example the CVA in the low risk scenario calculated using the piecewise model amounts to 0.00011167.

By observing the two CVA formulas, it is clear that the bigger the exposure and the discount factor, the bigger the absolute difference between the two measures of CVA, since D_j and EE_j are multiplied with the default probability. The biggest relative difference between the models is observed for the scenario with drastic difference in CDS spreads for the different maturities. In practice, this scenario means that the market believes that an entity is not at all likely to default in the short run. However, in the long run default is a strong possibility. Even though the relative difference between the models is large in the drastic change scenario one should also consider the possibility of such a scenario occurring. In reality, the CDS spreads rise no matter the maturity in case of a drop in confidence of a corporation's survival. For example, by comparing the market CDS spreads to the drastic change scenario, we see that the difference between the first and the last maturity in the drastic change scenario is 440 basis points, which is several times larger than the difference in any other scenario.

Furthermore, looking at Table 4 we can see that the scenarios with high risk, inverted spreads and drastic change have the biggest relative differences. One could be inclined to believe that the higher the CDS spreads the bigger the difference. However, the constant spread scenario, which has substantially higher spreads than the low risk scenario, actually has the smallest relative difference. In addition to this, if the level of the CDS spreads was the only factor for the differences then we would have seen a constant difference in default intensities in Figures 9b, 9c and 10b.

It may be more relevant to look at the change in CDS spreads from maturity to maturity, since the larger the jump in spread level is, the larger the difference in default intensities seems to be. The largest changes in CDS spreads occurs in the high risk scenario and in the drastic change scenario of 60 basis points and 440 basis points, respectively. This is matched by the highest relative difference in CVA value. The low risk scenario actually has a higher change in spread level over the whole period of 10 years than the total change for the inverted spreads scenario (43.2 basis points versus 36.8 basis points), so this theory is not completely consistent with the results in Table 4.

However, one has to account for the discount factor in the calculations, which causes differences in default intensities between the models at an early time point to have a bigger impact than later differences. By comparing the default intensities and CDS spreads for the first half of the contract, i.e. up to 5 years, we find that the change in CDS spreads is consistent with the difference in CVA value. The difference in spreads for years 0-5 is 25.9 basis points in the low risk scenario, whereas the inverted spreads scenario has a spread difference of 35.2 basis points. The corresponding spread differences for the high risk scenario and the drastic change scenario are 41.8 basis points and 140 basis points, respectively.

5.4 Sensitivity Analysis

In this subsection, we compute a sensitivity analysis where we alter the different inputs in the CIR model, namely the continuous drift κ, the continuous volatility σ and the average of the 1-month rates θ. Sensitivity analysis of the percentage difference between the models is presented in this subsection, and in Appendix

A we present sensitivity analysis of the models, where the models are presented separately.

These CVA values are relatively low and are for simplicity counted on a notional of one. In reality the notional ranges up to billions of USD, so if $N = 10^8$, (100 million USD), then a CVA value of for example $3.8694 \cdot 10^{-4}$ means a CVA value of 38,694 USD.

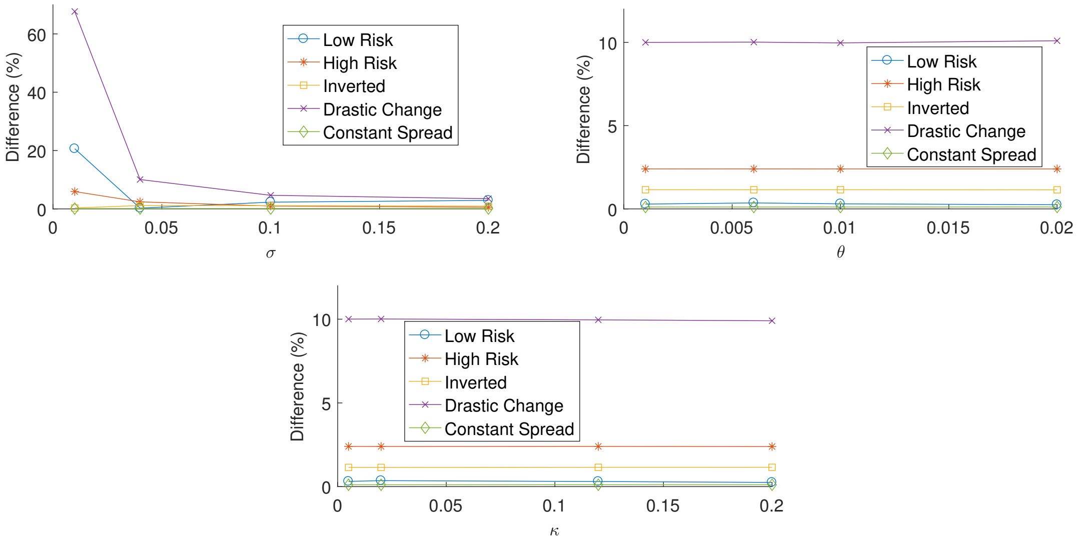


Figure 11: Sensitivity Analysis of σ (left), θ (right) and κ (bottom) on the percentage difference between CVA calculated using the Basel and the piecewise constant default intensity formula

The only factor that has any significant effect on the relative difference in CVA value between the models is σ. For low σ the difference is very large in the low risk scenario and the drastic change scenario.

In Appendix A we present the CVA values for each scenario in our sensitivity analysis. Neither θ nor κ, presented in Figures 13 and 14, has any large effect on the CVA values or the difference in CVA values between the models, as seen in Figure 11. Figure 12 shows that a larger σ yields a higher expected exposure. However, as seen in Figure 11, this has little effect on the difference in CVA values.

6 Discussion

In this section, we discuss the assumptions we make in calculating CVA, our findings from our numerical studies in Section 5 and provide a conclusion of our thesis.

6.1 Assumptions

We make the assumption that only one party is at risk of default, and the other party is risk-free, so-called *unilateral counterparty risk.* It is possible to calculate an entity's own CVA, as seen from its counterparty, called debit valuation adjustment (DVA). As mentioned in Section 2.8, Brigo et al. (2013) amongst others explain that this model leads to some controversial effects since it could lead to a positive change to the balance sheet of one party even though their risk of default is rising. Our calculations therefore do not contain DVA.

As discussed by Sokol (2012), the key problem with DVA is that it is only possible to monetise in theory but in practice it is impossible to unwind a complex derivatives portfolio prior to default, since DVA is the reduced value of a derivative payment obligation to a counterparty, which requires trades to be unwound and monetised.

We give our reasoning for using a recovery rate of 40% in Subsection 4.1. Having a lower recovery rate would lead to higher values of CVA. It is valid to assume that the LGD is not constant over all counterparties and can even change for the same counterparty over time. In reality LGD is a stochastic variable, so we could have simulated LGD values over time, but for this we need a model. Also, since our focus in this thesis is on the default probability we choose to assume a constant value of the LGD. Furthermore, it has proven by e.g. Altman et al. (2004) and Altman et al. (2005) that LGD is in fact positively correlated with the probability of default.

One issue with the Basel model is that it assumes that no WWR exists. As discussed by Rosen (2012), WWR can have a strong impact on CVA and capital calculations, but it is difficult and computationally intensive to estimate in practice since the exposure calculations are expensive and the systems in place often handle exposures and default simulations separately.

In this thesis we only research the case of CVA for one single interest rate swap, whereas in reality two counterparties usually have a portfolio of different swaps between them. As we briefly explain in Sections 2.9 and 4.3 the case is different when one has a portfolio of swaps with exposure to one counterparty. Our conclusions does not hold for a portfolio of swaps, but that would be an interesting subject to research in the future.

6.2 Scenario Analysis

Following the discussion in Section 5.3, we draw the conclusion that there exists a difference between the Basel model and the piecewise model, according to our data and results. We can also conclude that the size of this difference in our results is dependent on both the change in CDS spreads between maturities and the assumed discount rate.

6.3 Sensitivity Analysis

From our sensitivity analyses in Section 5.4 it is evident that changing the underlying variables of the expected exposure has little effect on the difference in CVA value. This is fairly intuitive since the same EE is used when calculating CVA for both models. The only factor that affects the relative differences of the models is low values of σ.

6.4 Conclusion

The main conclusion of our findings is that based on our data the default probabilities using the Basel model are overestimated, compared to the default probabilities calculated using the piecewise constant model, when t_j is close to the maturities. When t_j is not close to any maturity, the Basel formula underestimates the actual probabilities, compared to the piecewise constant model. Our results show that there is a difference in the CVA values between the Basel model and the piecewise model in our scenarios, and that the size of this difference is dependent on the size of the change in CDS spreads from one CDS maturity to another.

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Appendices

A Additional Figures

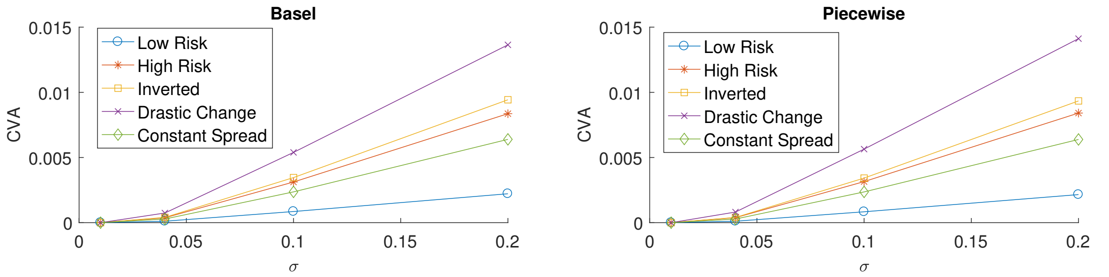

Figure 12: Sensitivity Analysis of σ on CVA calculated using the Basel formula (left) and the Piecewise formula (right)

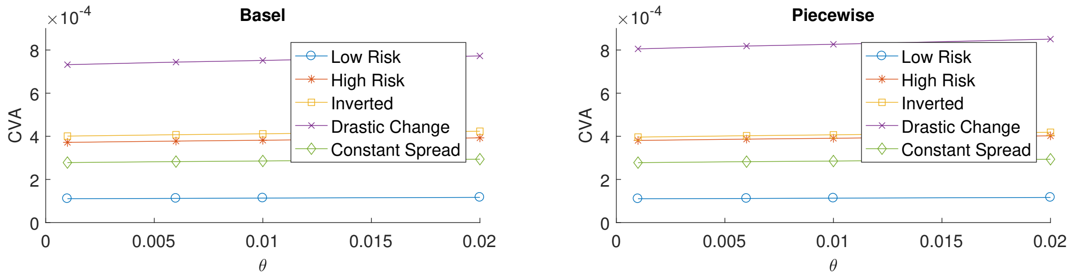

Figure 13: Sensitivity Analysis of θ on CVA calculated using the Basel formula (left) and the Piecewise formula (right)

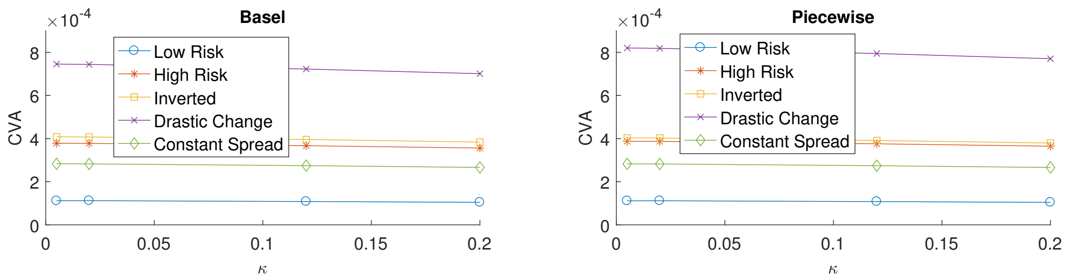

Figure 14: Sensitivity Analysis of κ on CVA calculated using the Basel formula (left) and the Piecewise formula (right)

B Monte Carlo Simulation

Let X be a stochastic variable with expected value $\mu = \mathbb{E}[X]$ and variance $\sigma^2 = Var(X)$. Assume that we want to calculate $\mu = \mathbb{E}[X]$ but there is no closed formula for $\mathbb{E}[X]$. A good way to find $\mu = \mathbb{E}[X]$ is through Monte Carlo simulations. Thanks to the law of large numbers, Monte Carlo simulations may provide an approximation of $\mathbb{E}[X]$ computed arbitrarily and is close to the true and unknown value of $\mathbb{E}[X]$.

Let us illustrate an example of Monte Carlo simulation:

1. Simulate n independent and equally distributed stochastic variables $X_1, X_2, \ldots, X_n$, with all having the same distribution as X, with the expected value $\mu = \mathbb{E}[X]$ and the variance $\sigma^2 = Var(X)$.
2. Calculate $\frac{1}{n}\sum_{i=1}^{n} X_i$
3. Let $\frac{1}{n}\sum_{i=1}^{n} X_i$ represent an approximation of the expected value $\mu = \mathbb{E}[X]$.

The fact that $\frac{1}{n}\sum_{i=1}^{n} X_i$ truly is a good approximation of the expected value $\mu = \mathbb{E}[X]$ follows the law of large numbers since it, for arbitrarily $\epsilon > 0$, holds that

$$\mathbb{P}\left[\left|\frac{1}{n}\sum_{i=1}^{n} X_i - \mu\right| \geq \varepsilon\right] \to 0 \text{ if } n \to \infty$$

i.e the stochastic variable $\frac{1}{n}\sum_{i=1}^{n} X_i$ likely converges towards the constant $\mu = \mathbb{E}[X]$ when n approaches infinity.

In practice, this means that for enough simulations n, the stochastic variable $\frac{1}{n}\sum_{i=1}^{n} X_i$ will be very close to the expected value $\mu = \mathbb{E}[X]$ (Herbertsson, 2016).