



# UNIVERSITY OF GOTHENBURG

## SCHOOL OF BUSINESS, ECONOMICS AND LAW

Bachelor Thesis in Finance

### Application of the Kelly Criterion on a Self-Financing Trading Portfolio

*-An empirical study on the Swedish stock market from 2005-2015*

Supervisor: Dr. Marcin Zamojski

School of Business, Economics and Law at the University of Gothenburg

Institution: Financial Economics

Credits: 15 ECTS

**Authors:** Emil Ohlsson and Oskar Markusson

*Spring 2017, Gothenburg*

# Abstract

A Kelly strategy theoretically optimizes the growth rate of investor's capital. This paper evaluates its usefulness on the Swedish stock market between 2005 and 2015 by comparing returns to that of common portfolio strategies and a market index. We conclude that the Kelly strategy produces returns around five times that of the market for the same period. After conducting robustness tests, the results are less convincing.

## **Keywords:**

Kelly Strategy, Portfolio & Money Management, Abnormal Returns, Swedish Equities, Geometric Mean Maximization, Kelly Criterion

## Acknowledgement

*We would like to thank our supervisor Dr. Marcin Zamojski at the Department of Finance at the School of Business, Economics and Law at the University of Gothenburg for his great support, both in terms of knowledge and input. His responsiveness and generosity with time has been invaluable to this paper.*

## Table of Contents

<b>ABSTRACT</b> .....	<b>2</b>
<b>ACKNOWLEDGEMENT</b> .....	<b>3</b>
<b>1. INTRODUCTION</b> .....	<b>5</b>
<b>2. THEORETICAL FRAMEWORK</b> .....	<b>6</b>
2.1 KELLY STRATEGY .....	6
2.1.1 Kelly (1956) Formula for Discrete Probabilities .....	7
2.1.2 Thorp's (2006) continuous approximation .....	8
2.2 STRENGTHS AND LIMITATIONS WITH A KELLY STRATEGY.....	10
2.3 EMPIRICAL EVIDENCE .....	10
<b>3. METHODS</b> .....	<b>11</b>
3.1 MULTIPLE SCREENING CRITERIONS .....	12
3.1.1 Price-to-Book Ratio .....	12
3.1.2 Gross Margin.....	13
3.1.3 P/E ratio .....	14
3.2 ROBUSTNESS OF MULTIPLES .....	15
3.3 PORTFOLIOS.....	19
3.3.1 Kelly Portfolio .....	20
3.3.2 Equally Weighted Portfolio .....	21
3.3.3 Market Capitalization (Value) Portfolio.....	21
3.3.4 Mean-Variance Portfolio.....	21
3.3.5 High Beta Portfolio .....	21
3.3.6 SBX and Leveraging the SBX .....	21
3.4 KELLY PORTFOLIO INDUSTRIES .....	22
3.5 CARHART 4-FACTOR MODEL .....	23
3.6 ASSUMPTIONS .....	23
<b>4. RESULTS</b> .....	<b>24</b>
4.1 RETURN ANALYSIS .....	25
4.2 RETURN ANALYSIS WITH ROBUSTNESS TESTS .....	27
4.2.1 Impact on the Kelly Portfolio when Removing Outlier.....	27
4.2.2 Impact when Replicating Kelly Short Selling Restriction on Benchmark Portfolios .....	29
4.2.3 Impact when Replicating Kelly Short Selling Restriction on Benchmark Portfolios and Removing Outlier.....	30
4.2.4 Kelly Robust Portfolio Compared to SBX Leveraged.....	31
4.3 RISK ANALYSIS.....	32
4.4 CARHART 4-FACTOR REGRESSION .....	36
<b>5. DISCUSSION</b> .....	<b>38</b>
<b>6. CONCLUSION</b> .....	<b>41</b>
<b>7. SUGGESTIONS FOR FUTURE RESEARCH</b> .....	<b>42</b>
<b>8. REFERENCES:</b> .....	<b>43</b>
<b>9. APPENDIX</b> .....	<b>46</b>
A. LOGNORMAL PRICES (MERTON 1969 & MERTON 1992) .....	46
B. PROEBSTING'S PARADOX (THORP 2008).....	49
C. APPROXIMATION OF THE UNRESTRICTED KELLY PORTFOLIO .....	50

# 1. Introduction

A Kelly strategy as defined by Ziemba (2016) is a scheme that enables investors to find the growth optimal allocation of securities to maximize their final wealth. This strategy is based on Kelly's (1956) criterion, which defines the long-run growth optimal allocation size. Several studies examine the implementation of a Kelly strategy on financial markets but are divided in their conclusions. Roll (1973) shows that the Kelly strategy applied as a portfolio strategy is statistically indistinguishable from the market portfolio in terms of rate of returns, whereas Estrada (2010) argues it is superior in terms of long-term growth to traditional strategies. The theoretical framework on the properties of Kelly's criterion is commonly accepted (see Davis & Lleo, 2014; Rotando & Thorp, 1992; Thorp, 2006) meanwhile the usefulness in practice remains undetermined. With this in mind, the problem statement in this thesis is whether investors would benefit ("benefit" evaluated based on performance relative to benchmarks) from applying a Kelly strategy in practice and whether its previous findings are aligned with our findings from the Swedish stock market from 2005 to 2015.

Our purpose is to provide the reader with an objective evaluation of this strategy. We examine the applicability of a Kelly strategy in practice, i.e., apply a Kelly strategy to empirical data and compare it to other portfolio choice approaches. We use all data on Swedish listed stocks from 2005 to 2015. The intuition behind the choice of Swedish equities in our analysis is based on the fact that there is no previous evidence for implementations of a Kelly strategy on this asset class and market. In this paper, we show how the Kelly strategy differs in returns and in volatility of the returns. Estrada (2010) concludes that the Kelly strategy is, in fact, superior in terms of long-term growth to traditional strategies. In line with this, we illustrate that it is a relevant long-term allocation strategy in terms of wealth growth. However, there are potential biases in our results, hence we conduct a number of robustness analysis. We conclude that the robustness returns are in fact lower when adjusted. Since our adjustments are of major importance in order to evaluate the strategy, we build our analysis by looking at both the adjusted and the unadjusted Kelly portfolios when comparing to our benchmark strategies. Finally, we want to highlight how a rational and growth maximizing investor following a Kelly strategy might differ in their allocation relative to the market portfolio.

The remainder of this paper is organized as follows. Section 2 outlines the theoretical framework, which serves as a foundation on which this paper is built. First the Kelly strategy and formula are presented and discussed, whereby the existing literature is highlighted. In Section 3, we explain the method used throughout the paper and how we proceed by applying the Kelly strategy on the financial market. In the same section we also do a robustness test on our method. Section 4 presents our results, which also include robustness tests of the Kelly strategy. Our findings are later included in the final discussion in Section 5. Finally, in Section 6 we bring forward our conclusion.

## 2. Theoretical Framework

### 2.1 Kelly strategy

A Kelly strategy is the implementation of Kelly's (1956) criterion but to avoid confusion we only use the term Kelly strategy throughout this paper. To understand the intuition behind the usage of a Kelly strategy, consider a scenario where there are favourable (i.e., the probability to win is larger than the probability to lose) bets at hand available to choose from. Next, we proceed to determine what this bet should amount to, measured as a fraction of total wealth at hand. The Kelly strategy determines the optimal bet size for us in terms of growth maximization. Thorp, MacLean, & Ziemba (2011) demonstrate in practice how gambling houses can be beaten by individuals implementing a Kelly strategy, through betting the optimal fraction of their wealth.

The general formula used to implement the Kelly strategy for independent investments is to a high degree identical to the Sharpe ratio but uses variance instead of volatility, which is demonstrated below. The logic behind this is explained by, for example, Thorp (2006) who uses Tucker's theorem to transform the function into a Brownian motion. To use the Kelly formula one needs to estimate excess returns and return volatilities for securities. Following this, the optimal allocation of a portfolio consisting of these securities is constructed. Below we derive the Kelly formula. First we look at the discrete probabilities case, used in betting. Thereafter, we present two derivations of the formula used for continuous probabilities, one

illustrated in the next section, and one included in the appendix. Continuous probabilities are applicable when investing.

### 2.1.1 Kelly (1956) Formula for Discrete Probabilities

Assume a favourable bet with probability  $\frac{1}{2} < p \leq 1$  and outcome 1, a losing probability of  $q = 1 - p$  with outcome -1, and that the odds are even. The initial wealth is denominated by  $W_0$ . The wealth after  $n$  trials, betting a fraction  $f$  of the initial wealth, is given by

$$W_n = W_0(1 + f)^m(1 - f)^{n-m}$$

The exponential rate of asset growth per trial, equal to the logarithm of the geometric mean, can be restated by

$$\begin{aligned} G_n(f) &= \log \left( \frac{W_n}{W_0} \right)^{\frac{1}{n}} = \log \left[ (1 + f)^{\frac{m}{n}} (1 - f)^{\frac{n-m}{n}} \right] \\ &= \left( \frac{m}{n} \right) \log(1 + f) + \left( \frac{n-m}{n} \right) \log(1 - f) \end{aligned}$$

Our expected growth rate is given by

$$E[G_n(f)] = g(f) = p \cdot \log(1 + f) + q \cdot \log(1 - f) = E[\log(W)]$$

where  $p$  is defined as the winning probability and  $q$  the losing probability. Maximizing  $g(f)$  with respects to  $f$  results in

$$\begin{aligned} q'(f) &= \left( \frac{p}{1+f} \right) - \left( \frac{q}{1-f} \right) = \left[ \frac{p - q - f}{(1+f)(1-f)} \right] = 0 \\ &\Leftrightarrow f = f^* = p - q, \quad p \geq q > 0 \end{aligned}$$

The second derivative with regards to  $f$  shows that  $f = f^*$  is the unique maximum of the function

$$\begin{aligned} g(f^*) &= p \cdot \log(p) + q \cdot \log(q) + \log(2) > 0 \\ q''(f) &= - \left[ \frac{p}{(1+f)^2} \right] - \left[ \frac{q}{(1-f)^2} \right] < 0 \end{aligned}$$

Theorem 1 (Kelly): *the optimal fraction, under Bernoulli trials, which should be invested per trial, is  $f^* = p-q$ , the edge. This fixed fraction strategy maximizes the expected value of the logarithm of capital at each trial (Kelly 1956).*

Thorp (1971) points out that maximizing the expected logarithm of wealth  $E[\log(W_t)]$  is equivalent to maximizing the exponential rate of growth per time period  $g(f)$ .

Now, moreover, we assume that the odds are not even such that  $o \in \mathbb{R}^+$ ; thus, a game is favourable if  $po - q > 0$ , which results in a variation of the logarithm of the geometric growth rate

$$g(f, o) = p \log(1 + of) + q \log(1 - f)$$

Which is maximized by

$$f^* = \frac{op - q}{o} = \frac{\text{edge}}{\text{odds}}$$

The optimal fraction of wealth one uses to bet,  $f^*$ , is presented above. In the next section we derive it in another way and translate inputs into financial terms so that the formula is applicable when investing.

### 2.1.2 Thorp's (2006) continuous approximation

Since our goal is to apply the Kelly criterion on stocks, a similar result for a continuous distribution is relevant (Thorp 2006). The goal is still to maximize  $g(f) = E[\log(1 + fx)] = \int \log(1 + fx) dP(x)$  with  $P(x)$  being a probability measure and  $f$  a fraction of capital invested. Also we assume constraints  $1 + fx > 0$ , such that log can be defined, and  $\sum f_j = 1$ . If the outcomes of  $x$  are a symmetric random variable around  $E(x) = \mu$  with  $Var(x) = \sigma^2$  we can describe the wealth  $W$  as

$$W(f) = W_0[1 + (1 - f)r + fx] = V_0[1 + r + f(x - r)]$$

$r$  is the return on the risk free, thus  $g(f)$  is

$$g(f) = E[G(f)] = E \log \left[ \frac{W(f)}{W_0} \right] = E \log [1 + r + f(x - r)]$$

With subdivided time intervals with  $T$  independent steps

$$\frac{W_T(f)}{W_0} = \prod_{t=1}^T [1 + (1 - f)r + fx_t]$$

Taking the expectation and natural logarithm on both sides allows us to calculate  $g(f)$  from a second order Taylor-approximation as



$$g(f) = r + f(\mu - r) - \frac{\sigma^2 f^2}{2} + O\left(n^{-\frac{1}{2}}\right)$$

As  $t$  approaches  $\infty$ ,  $O\left(n^{-\frac{1}{2}}\right)$  approaches 0 resulting in

$$g_\infty(f) = r + f(\mu - r) - \frac{\sigma^2 f^2}{2}$$

Differentiating  $g(f)$  with respect to  $f$

$$\frac{\partial g_\infty(f)}{\partial f} = \mu - r - \sigma^2 f = 0 \Leftrightarrow f^* = \frac{\mu - r}{\sigma^2}$$

This is the Kelly formula, where  $\mu$  is the return,  $r$  is the risk-free rate, and  $\sigma^2$  is the return variance.  $f^*$  is the weight each security receives in the Kelly portfolio, or put in other words, the fraction of our wealth at hand we should invest in each security. Generally, in terms of risk and return, a portfolio based on a Kelly strategy differs significantly from other strategies. MacLean, Thorp, & Ziemba (2010) state that it is often prone to larger risk exposures due to the volatile and undiversified nature of an optimal Kelly strategy portfolio. This is because the strategy requires investors to frequently invest large fractions into few securities which common portfolio strategies, for example, equal weight or mean-variance, do not. Through estimating the inputs in the formula above, investors find the optimal long-term (long-term is defined based on frequency of trades in this case) allocation size for each security in the portfolio.

Kelly (1956) build on Bernoulli's (1738) utility theory, which states that marginal utility is a function of log wealth where increasing wealth induces lower marginal utility, where Kelly showed the maximization of the one period expected log of wealth. Latané (1959) independently implements Kelly's ideas as an investment criterion. This marked the first step in the usage of the Kelly criterion on financial markets and investing. Breiman (1961) shows that using the Kelly criterion is asymptotically optimal under two paradigms: first, it produces the maximal rate of increase of wealth and secondly it minimizes the expected time to achieve a fixed level of resources. Thorp (1969) concludes that the Kelly criterion should replace the Markowitz criterion (Markowitz 1959) as the guide to portfolio selection. Additionally, Thorp (2006) states that Kelly bettors maximize expected one-period log of wealth, and they are certain to win if the horizon is sufficiently long. Samuelson (1979), on the other hand, is critical towards the Kelly criterion and points out that 'when you lose – and you *sure can* lose – with  $N$  large, you can lose real big.'

## 2.2 Strengths and Limitations With a Kelly Strategy

MacLean, Thorp, & Ziemba (2010b) discuss a wide array of properties of the Kelly strategy. Amongst the positive characteristics discussed is, for example, the fact that the log growth of wealth bettor never risks ruin (Hakansson, & Miller 1975), maximizing log growth of wealth also asymptotically maximizes the rate of asset growth (Breiman 1961), and finally that the absolute amount bet is monotone increasing in wealth (MacLean, Thorp, and Ziemba, (2010b)). Markowitz (1959) states that even though Kelly's (1956) Criterion might not maximize the expected utility of wealth by maximizing the mean return it may still be a plausible and useful theory.

There are several key issues with a Kelly strategy when applied as an investment criterion. The most crucial is based on the fact that the market for financial instruments is characterized by uncertainty of future returns, which makes estimation of the probability of a certain outcome hard to determine. Since tomorrow's outcome is highly uncertain, and requires estimation, modeling uncertainty and forecasting errors can be high. This problem is likely to apply in this paper; to obtain the required inputs for the Kelly formula we are forced to rely on historical data and make predictions.

The problem mentioned above is also recognized in practice. Samuelson (1963) argues that if a rational individual rejects a single favourable bet, he would also reject a large number of such bets. In another paper, Samuelson (1971) agrees that aiming to maximize the geometric mean return would lead to a maximization of the terminal wealth given a timeframe which is long enough, but that this strategy would not necessarily maximize the expected utility unless the underlying utility function is logarithmic. He then concludes "the geometric-mean strategy is not optimal for any finite number of periods".

## 2.3 Empirical Evidence

Research papers on this topic and its implementation as a portfolio strategy in practice are rare. Estrada (2010) conducts a study similar to this thesis. He concludes that the Kelly criterion is, in fact, superior in terms of long-term growth to traditional strategies. Meanwhile, Estrada's analysis also shows that Kelly portfolios are less diversified, have a higher (arithmetic and geometric) mean return, and higher volatility than benchmark portfolios. Additionally, Evstigneev, Hens, and Schenk-Hoppé (2009) find that the Kelly rule

insures the survival of investors continuously applying this rule. Finally, MacLean, Thorp & Ziemba (2010) show the immense increase in return achieved when investing by Kelly's strategy.

In contrast to Estrada's findings, Roll (1973) concludes that the Kelly Strategy is statistically indistinguishable from the market portfolio. Worth to mention, though, is that Roll's findings emerge from studies on the NYSE and AMEX in the 1970s, whereas we will look at a different time era and market. Furthermore, Thorp (1971) shows the Kelly strategy is not mean-variance efficient *ex ante*. Cover (1991) illustrates how a universal portfolio with an equally weighted strategy outperforms a performance weighted (i.e., Kelly strategy based) portfolio. Finally, Markowitz (1976) concludes that the Kelly strategy was the limiting mean-variance portfolio under the assumption that an investor follows the strategy in the long run.

To conclude, Kelly's (1956) criterion is a high-risk portfolio strategy and prone to great uncertainty. Opinions on the effectiveness of a Kelly strategy vary; our goal is thus to test the strategy with data from the Swedish stock market to analyze how it performs in practice. An explanation of how we proceed with this implementation is presented in the Section 3.

### 3. Methods

To get the return component in the Kelly formula, we identify stocks that we believe have potential for positive abnormal returns, in order to find our edge. To identify such firms we define a ranking system. First, stocks are ranked based on a selection of multiples. Additionally, after the first screening process, the stocks in this universe are ranked again based on how they rank combined for the three multiples. Firms with low P/B-and P/E ratios and a high Gross Margin relative to all other listed Swedish equities get the best score. "Low" is defined as any value above 0 (if a firm has a multiple value of 0 it is excluded from our universe). "High" is defined as any value, which is positive; the higher the value of the Gross Margin the better. On average, during our 43 quarterly observations from 2005 to 2015, our universe consists of 109 stocks after they have been ranked. Once these stocks are identified we start building the portfolios, of which one allocation scheme is based on the Kelly strategy, and analyze their performance in relation to benchmark portfolio choices. In the next sections, we explain the intuition behind the choice of multiples and why multiples

should have a high or low value respectively, whereby we proceed by illustrating how and why we use selected benchmark portfolios to compare to our Kelly portfolio.

### 3.1 Multiple Screening Criteria

To find stocks with potential of abnormal returns relative to the market, SBX, we use a multiple approach. Three key ratios with theorized potential for outperformance relative to the market have been selected; the P/B-ratio with close similarities to the B/M-ratio from the Fama and French (1992), gross margin, and the P/E ratio. The choice behind these multiples is based on previous research on multiple strength and predictive power of returns. We provide details below.

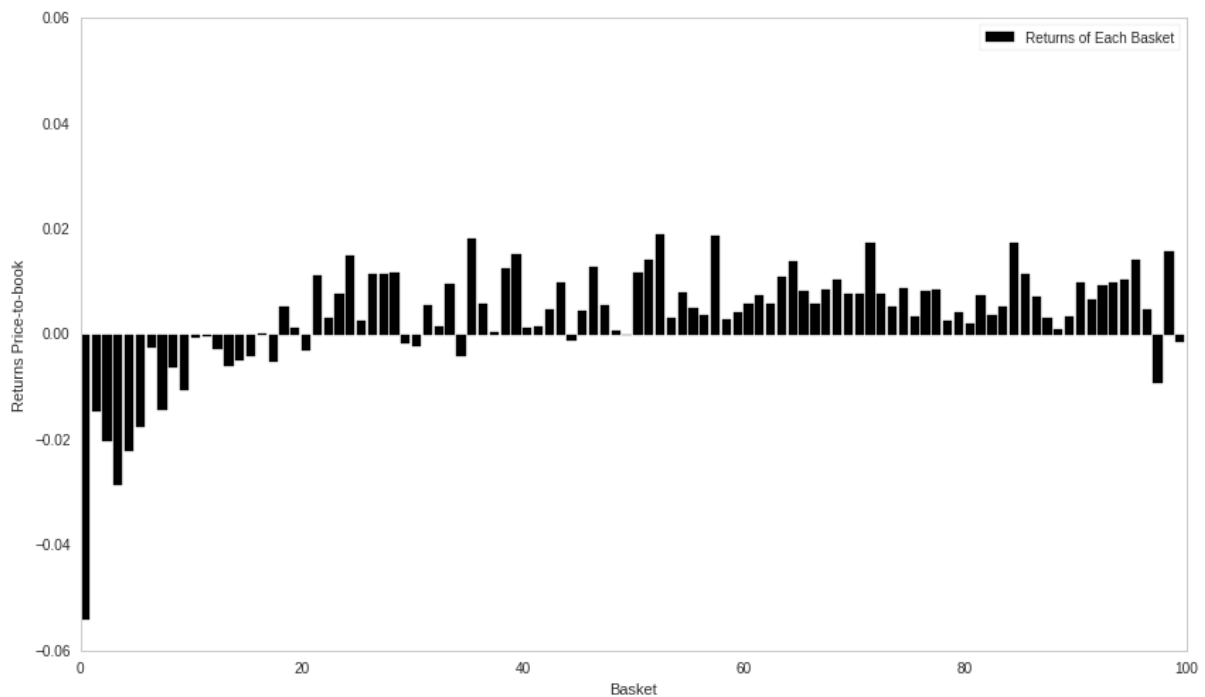
Finally, for each section where we explain our multiples, we also conduct our own analysis on the ability of our multiples to predict returns through a robustness test. This is made in order to support our choice with more evidence than that from previous studies only and also because the choice of multiples influence the data from which we conduct all calculations<sup>1</sup>.

#### 3.1.1 Price-to-Book Ratio

Banz (1981) shows that firms with high book-to-market ratio, i.e., value stocks, outperform those with a lower ratio. Put in other words, these stocks are undervalued and thus experience a value premium. Fama and French (1992) build on this idea and introduce the 3-factor model where one factor is the High Minus Low Book-to-Market ratio. In this paper, this factor is referred to as the Price-to-Book ratio; by looking at stocks with a low Price-to-Book ratio we obtain the equivalence of a high B/M-ratio (B/M is simply P/B inverted, hence we look for low P/B stocks), with the only difference being that the HML factor looks at portfolios whereas when using the P/B-ratio we look at individual securities. We do this to find stocks that theoretically could experience strong performance, which our P/B-multiple should be indicative of.

---

<sup>1</sup> This robustness test of multiples induces high exposure to errors, though, since we look at US equities, due to data availability. Also, due to the huge amount of observations we only look at the year of 2015. Yet, this enables us to get a general perception of how these multiples have performed empirically.

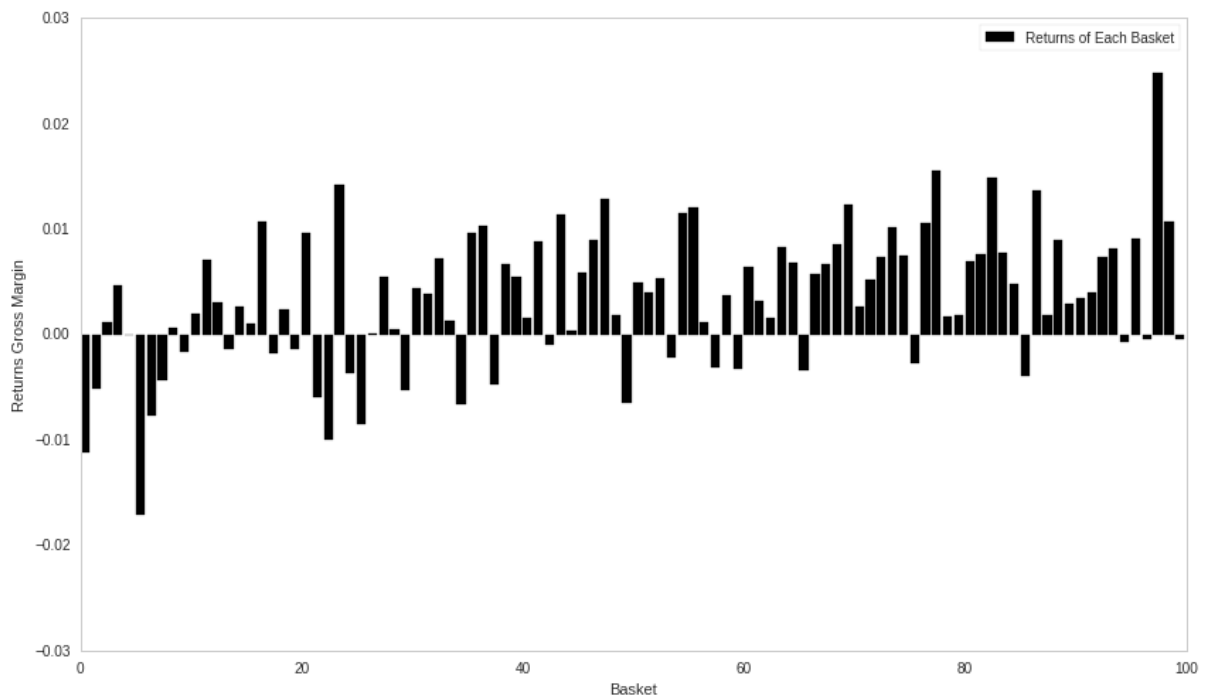


**Figure 1: Predictive Power of the P/B-ratio.** This figure shows the predictive power of the P/B-ratio. Data on US equities is retrieved from Morningstar.fundamentals API and Quantopian.USEquityPricing API. Sample period is 2015-2016 (12 months) and  $n=985\,559$ . The x-axis shows 100 portfolios, which are divided, based on the size of the multiple. From left to right we have the lowest to the highest value of the multiple applied as a screening criterion on the portfolios. The y-axis shows the multiple’s predictive power of realized return, measured daily.

Figure 1 reveals that portfolios based on a low multiple are ineffective at predicting returns; the first 10 portfolios, which are selected based on the lowest P/B-ratio relative to the other portfolios, only yield negative returns. Thus, using a low P/B-ratio to screen stocks would not predict future returns for US equities during 2015.

### 3.1.2 Gross Margin

We also include the gross margin to look for persistence in the Earnings-to-Cost-of-Goods-Sold (COGS) ratio. By identifying persistence in earnings amongst firms, this demonstrates strong performance and a solid market position. Novy-Marx (2013), for example, builds on this idea and states, “profitable firms generate significantly higher returns than unprofitable firms”.



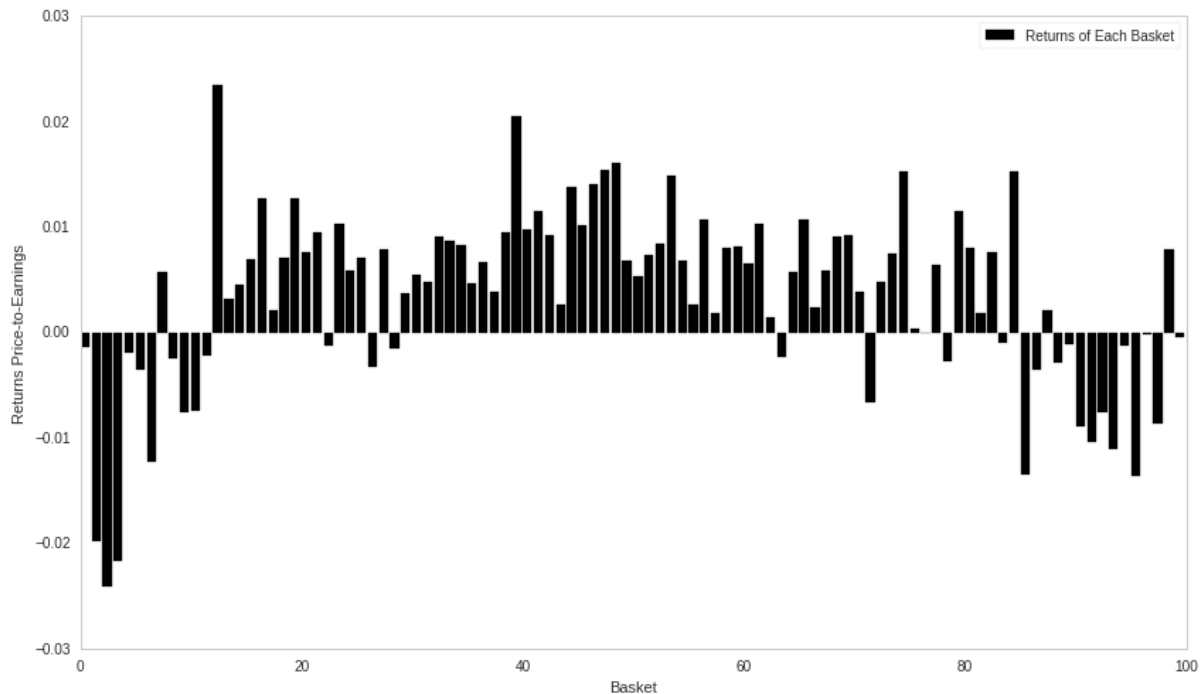
**Figure 2:** *Predictive Power of the Gross Margin.* This figure shows the predictive power of the Gross Margin. Data on US equities is retrieved from Morningstar.fundamentals API and Quantopian.USEquityPricing API. Sample period is 2015-2016 (12 months) and  $n=985\,559$ . The x-axis shows 100 portfolios, which are divided, based on the size of the multiple. From left to right we have the lowest to the highest value of the multiple applied as a screening criterion on the portfolios. The y-axis shows the multiple's predictive power of realized return, measured daily.

Figure 2 depicts that a high Gross Margin applied as a criterion on portfolio screening, has quite strong predictive power of returns, which is illustrated in the right section of the figure. This is in line with what we hope this multiple will do for us, but also with what the previous literature (see: Novy-Marx (2013)) states; firms with persistence in earnings experience higher returns. Hence, we can conclude that in this data set investors could somewhat predict future returns by identifying firms with a high Gross Margin, based on US equities during 2015.

### 3.1.3 P/E ratio

Finally, we look at the P/E ratio as an indicator of returns. This key ratio is widely used as a measure of valuation within finance. Furthermore, Basu (1977) shows this multiple has predictive power in measuring firm performance; he concludes that portfolios consisting of firms with a low P/E outperform portfolios where firms have relatively higher P/E-ratios.

Thus, we also use a low P/E-ratio when screening the Swedish stock market to find abnormal returns.



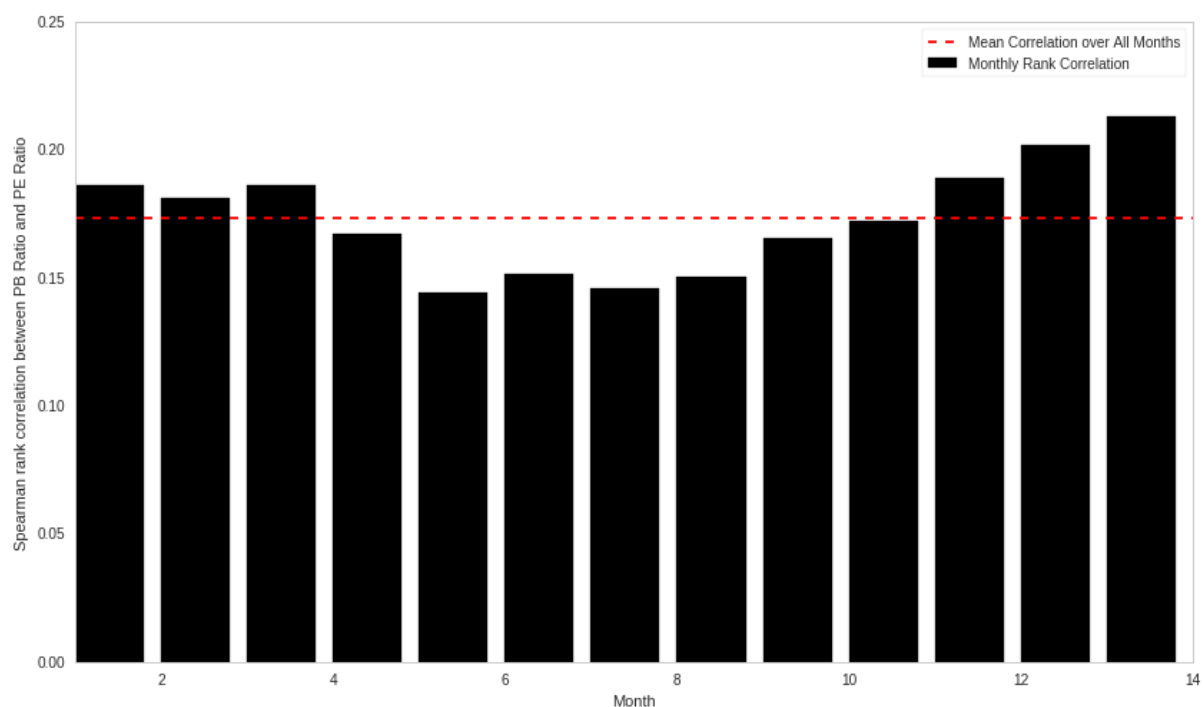
**Figure 3:** *Predictive Power of P/E-ratio.* This figure shows the predictive power of the P/E-ratio. Data on US equities is retrieved from Morningstar.fundamentals API and Quantopian.USEquityPricing API. Sample period is 2015-2016 (12 months) and  $n=985\,559$ . The x-axis shows 100 portfolios, which are divided, based on the size of the multiple. From left to right we have the lowest to the highest value of the multiple applied as a screening criterion on the portfolios. The y-axis shows the multiple's predictive power of realized return, measured daily.

Figure 3 illustrates the multiple is a relatively poor predictor of returns for US equities during 2015. Again, the first portfolio percentiles, which have been created based on stocks with a low P/E-ratio, experience negative returns. This implies that a low P/E-ratio does not have predictive power of returns based on US equities 2015.

### 3.2 Robustness of Multiples

The previous three figures show two of the multiples are weak in terms of predictive power of returns. Yet, in our study, all our portfolios created based on these criteria combined

outperform the SBX (illustrated in the results in Section 4). Perhaps this is due to the fact that our implementation of the multiples is during a longer time period, which strengthens the predictability of the multiples, but it could also be due to the fact that the three ratios are very general measurements of valuation. In other words, one can conclude that investors could have benefited, in terms of predicting returns, during the period 2005 to 2015 from using a ranking system on the Swedish stock market based on the combined multiples we explained above. To support this we conduct another robustness test for our multiples where we also look at sum of rank of monthly correlations between two key ratios at a time. Correlations imply one of the factors contains information about the other factor. Our goal is to have three key ratios, which are as uncorrelated as possible so that each multiple capture firm effects and consequently returns independent of the other two. This translates into extensive measures of returns<sup>2</sup>.



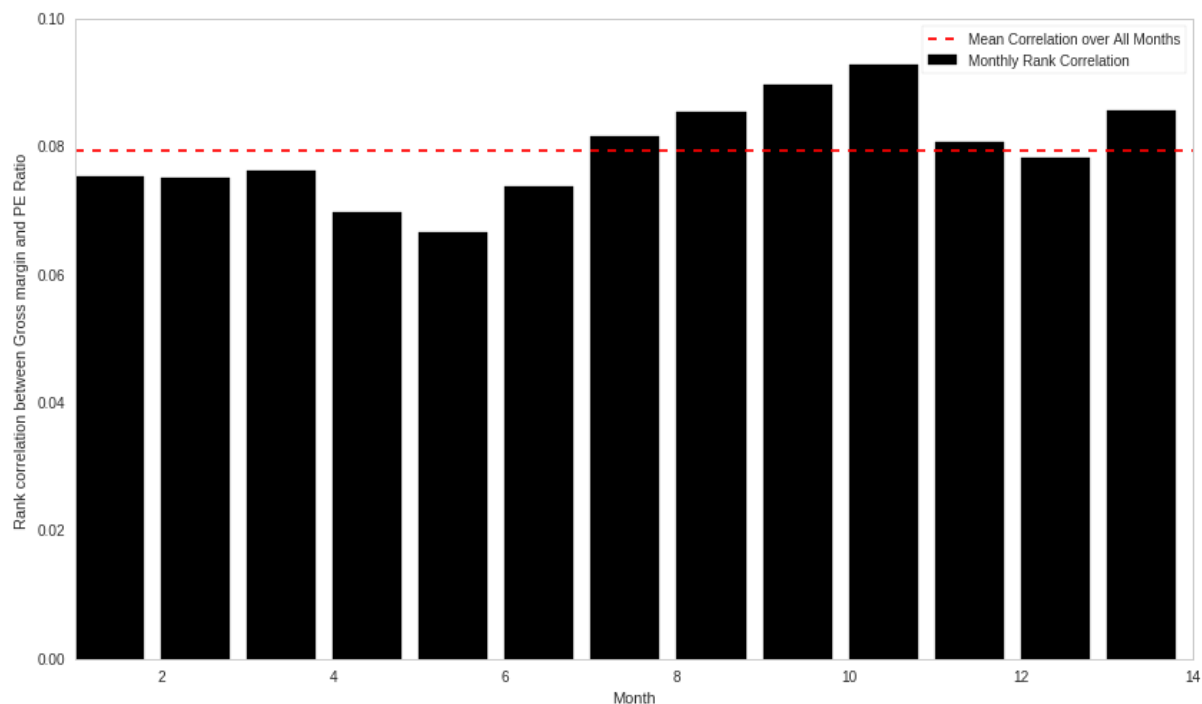
**Figure 4:** *Sum of Rank of Monthly Correlation between P/B-ratio and P/E-ratio.* This figure shows correlation between the P/B-ratio and the P/E-ratio. Data on US equities is retrieved from Morningstar.fundamentals API and Quantopian.USEquityPricing API. Sample period is 2015-2016 (12 months) and n= 985 559. The x-axis shows monthly portfolios. The y-axis

<sup>2</sup> Again, we look at US equities for the period 2015 below as well, hence the conclusions we draw are very general and not necessarily applicable to the Swedish data we use later in the paper. However, as we choose such general multiples to find predictive power we still believe the test is relevant.



shows the sum of rank of monthly correlations, measured by looking at the rank of one factor (i.e. multiple) versus the rank of the other factor, and returns in each factor rank.

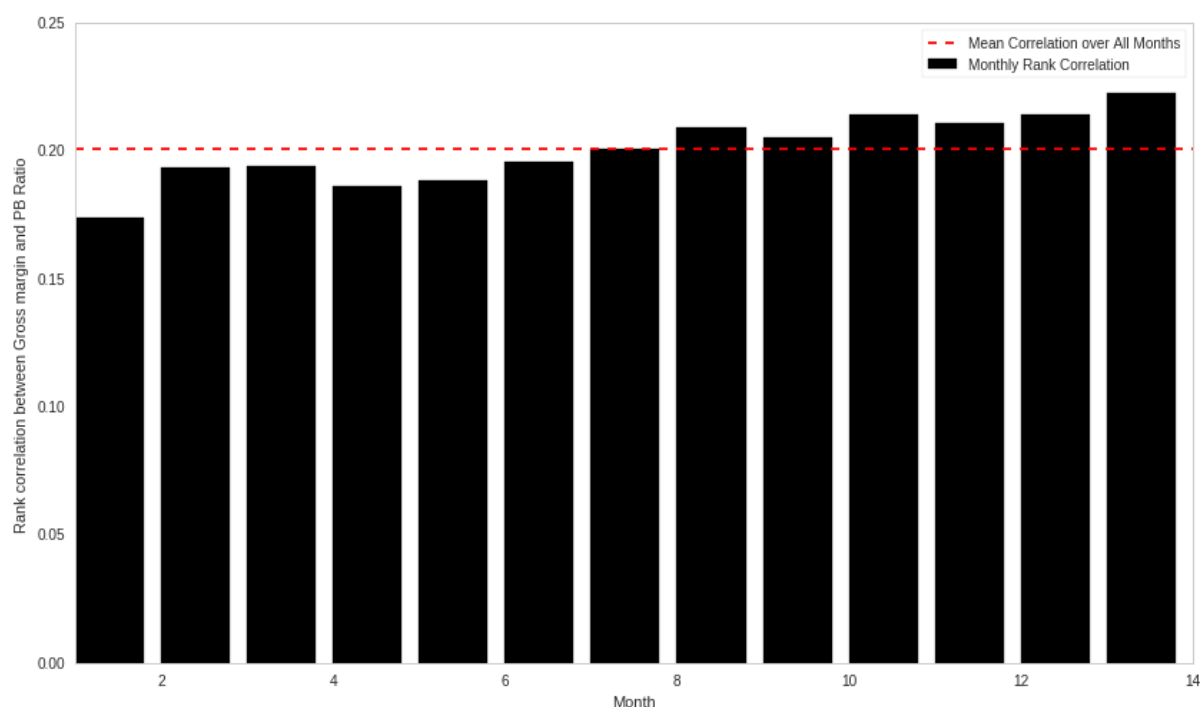
Figure 4 highlights that there is a tendency for weak positive correlation; the average monthly sum rank of correlation between the two key ratios is around 0,18. This implies that, together, these multiples are fairly extensive in measuring returns, meaning they capture returns separately from one another. This increases the probability that we do identify as many of the best performing firms as possible. In terms of having two independent measures, we can conclude that, based on US equities during 2015, these two multiples are relevant.



**Figure 5:** *Sum of Rank of Monthly Correlation between P/E-ratio and Gross Margin.* This figure shows correlation between the P/E-ratio and Gross Margin. Data on US equities is retrieved from Morningstar.fundamentals API and Quantopian.USEquityPricing API. Sample period is 2015-2016 (12 months) and n= 985 559. The x-axis shows monthly portfolios. The y-axis shows the sum of rank of monthly correlations, measured by looking at the rank of one factor (i.e. multiple) versus the rank of the other factor, and returns in each factor rank.

Compared to Figure 4, Figure 5 shows that more observations experience much weaker correlation; this means that the two key ratios are less correlated in their predictive power of return, and measure performance correspondingly to a smaller extent. We can conclude that

the P/E-ratio and Gross Margin are also relevant since they are independent of one another, with an average monthly sum of rank correlation of 0,08.



**Figure 6:** *Sum of Rank of Monthly Correlation between P/B-ratio and Gross Margin.* This figure shows correlation between the P/B-ratio and Gross Margin. Data on US equities is retrieved from Morningstar.fundamentals API and Quantopian.USEquityPricing API. Sample period is 2015-2016 (12 months) and  $n=985\,559$ . The x-axis shows monthly portfolios. The y-axis shows the sum of rank of monthly correlations, measured by looking at the rank of one factor (i.e. multiple) versus the rank of the other factor, and returns in each factor rank.

In line with previous reasoning, Figure 6 depicts that the P/B-ratio and Gross Margin share some similarities in the ability to predict returns for US equities during 2015, even though the correlation is low. This slightly stronger monthly sum of rank correlation of around 0.20 on average is something we want to avoid since our goal with the three key multiples is that they are as extensive as possible in predicting future returns. Meanwhile, as with Figure 4 and 5, Figure 6 experiences no significant dependence<sup>3</sup>.

<sup>3</sup> Finally, we again want to emphasize the fact that our robustness test of the multiples look at US equities due to data availability and simplicity, whereas our main test and results in this paper is based on Swedish equities. The idea is to conduct a robustness test on empirical data, independent of what previous literature says, which is as similar as possible to the Swedish data we use in this thesis, hence we believe the test is still relevant.

Our takeaway from the first robustness test on predictive power of return is less in line with the findings from the Swedish stock market; Figures 1 & 3 show that the multiples have weak predictive power of return. The second test, though, illustrated through Figures 4 to 6, show that the multiples are uncorrelated which is a sign of strength in terms of capturing as much of future return as possible. This leads us to the conclusion that the robustness tests have mixed results, but there is evidence for the strength of our selected multiple's ability to predict returns.

### 3.3 Portfolios

We form portfolios out of stocks that meet criteria in section 3.2. The portfolios are rebalanced on a quarterly basis; in this way we attempt to avoid Proebsting's Paradox, which states that the risk of over betting increases when string bets are involved (further explained in Appendix B). Furthermore, we choose to rebalance quarterly since there is new information on our multiples from the quarterly reports from the firms in our universe. Finally, the intuition behind our method of selecting stocks with a rolling window is to control for survivorship bias; if our model chooses a stock that later gets delisted, the contributing total return is calculated from the day of purchase until the last trading day's close.

When the Kelly strategy shows a high excess return-to-variance ratio, the number of stocks in our portfolio narrows, which increases weights (and thus decreases number of total securities) of our investments going forward. One portfolio allocation is based on the Kelly strategy, which is then compared to four other portfolio strategies with different weighting techniques; mean-variance, equally weighted, value weighted, and a high beta portfolio. Altogether, the five strategies are compared to one another in terms of risk and return. We also look at the allocation of stocks. Whereas traditional portfolio strategies often seek to maximize the return per unit of risk, the Kelly portfolio is designed to maximize return solely. Consequently, volatility can be extremely high relatively. For this reason, we include the relatively more aggressive high beta strategy portfolio as our fourth benchmark and weighting strategy (where "high" is defined as the higher the value the better), where Bloomberg creates a fourth rank and again rank all stocks combined with the beta criteria added. The other three portfolios are commonly used strategies. Finally, each individual portfolio is also compared

to the OMX Stockholm Benchmark Index (SBX). This is a market index of all Swedish equities listed on the Stockholm Exchange, which is weighted based on market cap and where dividends are reinvested. We use this primarily to compare the Kelly portfolio to the market, but also to look at the benchmark portfolio strategies in relation to the market index. Later, this index is also leveraged to the same volatility levels as Kelly to look at the performance when volatility levels are the same, in order to compare the risk-return tradeoff. All the benchmark portfolios are simulated through Bloomberg terminals, whereas Kelly is calculated manually.

### 3.3.1 Kelly Portfolio

A Kelly portfolio is often prone to large concentrated investments when the expected return is relatively high and has a high probability to be achieved. Therefore, it also tends to produce undiversified portfolios, which previous studies show (see for example: Nekrasov, 2014). We begin the construction of our Kelly portfolio through taking the average return for all selected stocks for each quarter between 2005 and 2015. From this return we then subtract the repo rate gathered from the Swedish Riksbank corresponding to that quarter, and finally we divide by the variance of the 3-month return to get the Kelly weight. For each quarter, the number of stocks included is based on the weight corresponding to that period. For example, let us assume we want to calculate the weight in the period Q2 2006. We retrieve average 3-month total return for all individual stocks for the preceding quarter, Q1 2006, which contains our universe that has been created through the ranking system, and proceed by calculating the total average for all stocks. This is our return component for Q2 2006. The value of  $\text{return}_{t-1}$  is thus of importance since it is used for our  $\text{return}_t$  component, whereby we assume that  $\text{return}_{t-1}$  will have predictive power for the following quarter. This is also why we earlier stressed the importance of predictive power of return. Thereafter, we adjust for the risk-free rate, whereby we divide by the variance of returns for the same universe of stocks. If the Kelly formula tells us the weight is 0.5, we include the top two ranking stocks within the universe since the formula indicates portfolio weights should be 50%. Additionally, we calculate the individual return for the two stocks and multiply by each stock's weight (in this case 0.5 for both stocks). We are left with the total return for our portfolio for Q2 2006 and proceed by doing this for all quarters. In this way we have 43 quarterly observations during the 11-year time period between 2005-2015, where our first Kelly weight is the 31<sup>st</sup> of March 2005.

### 3.3.2 Equally Weighted Portfolio

An equally weighted portfolio gives the same weight to all securities in the portfolio.

Finnerman and Kirchmann (2015) state, “the rationale behind the equal weighting technique is to avoid a large concentration of only a few stocks in the portfolio.” Additionally, Markowitz (1952) illustrates the strengths of this portfolio strategy.

### 3.3.3 Market Capitalization (Value) Portfolio

This strategy assigns weights based on size; larger firms receive larger portfolio weight. If implemented under CAPM assumptions it can be regarded as the market portfolio (Zhang, Shan, & Su 2009).

### 3.3.4 Mean-Variance Portfolio

The mean-variance portfolio, or Active Risk Minimized as defined in Bloomberg, is a portfolio simulated by Bloomberg. Active risk exists when managers seek to beat the market; they take on more risk to obtain excessive returns which gives rise to tracking errors, also defined as active risk. Bloomberg specifies that under the constraint that this portfolio will take long positions only, it behaves like Markowitz’s mean-variance portfolio. We impose the long only constraint to achieve this, meaning there is no short selling in this portfolio. Finally, this portfolio is very similar to the market portfolio.

### 3.3.5 High Beta Portfolio

The first three strategies compared to the Kelly strategy are, ex post, significantly less exposed to risk, measured as volatility of returns, and prone to diversification. Yet, by adding stocks with a beta larger than one our goal is to include firms which have more market risk than the other portfolio benchmarks, creating a portfolio which is more similar to Kelly in its’ risk characteristics. Consequently, we have one portfolio we hope is more similar, creating a more solid ground for comparison.

### 3.3.6 SBX and Leveraging the SBX

We also compare all strategies to the market portfolio (SBX). Finally, we create a portfolio, which is the market (SBX), with the exact same, ex post, volatility characteristics as the Kelly portfolio. We do this to look at whether the Kelly returns are solely due to higher volatility of returns and if this has explanatory power for the returns in our Kelly portfolio.

### 3.4 Kelly Portfolio Industries

<i>Industry</i>	Kelly (%)	SBX (%)
Financials	31,5	33,5
Industrials	16,1	29,8
Consumer Goods	13,7	9,5
Consumer Services	11,3	8,3
Technology	6,5	6,0
Health Care	14,5	4,6
Telecommunication	3,2	4,3
Oil & Gas	2,4	1,0
Basic Materials	0,8	3,0

**Table 1:** *Industry Breakdown for Kelly Portfolios and SBX*

Table 1 illustrates the largest industry holdings based on our Kelly portfolios and SBX. Data is retrieved from Bloomberg and Nasdaq for the period 2005 to 2015.

The inclusion of firms is based on if they have low P/B-and P/E-ratio and high gross margin, which favours some industries over others. Our take on why the average industry composite for Kelly looks the way it does in Table 1, is for example because firms investing in Real Estate have high book values, and thus a low P/B-ratio, scoring high in our ranking. Furthermore, firms in the financial sector have relatively low Cost Of Goods Sold in relation to revenue, which increases the gross margin. This partially explains why the most frequent industry in our portfolios is financials. Finally, the same applies for the industries at the bottom of the table, only reversed. For example firms in the basic materials sector have higher COGS and thus a low gross margin, scoring lower on our rankings, and are therefore not included as often in the Kelly portfolio.

SBX is also dominated by financials, which is reflected in the number of banks included, whereas our financials category involves more firms within Real Estate.

### 3.5 Carhart 4-factor Model

The final part of our theoretical framework is integrated in section 4.4, which is a regression analysis where we test if there is causality between the four factors included in Carhart's (1997) model, and the returns from our Kelly portfolios. Carhart (1997) illustrates return anomalies and the model is often used when running regressions on abnormal returns.

### 3.6 Assumptions

The Kelly strategy in theory only tells investors to invest when there is a potential edge for our risk-adjusted Kelly portfolio, which we seek to find through our multiple screening. If there is no edge, the wager in the particular equity should be 0 and the investor should instead turn to a market-portfolio, i.e., the index.

Furthermore, we assume no transaction costs or taxes. We also infer that stock returns are independent, which is not always the case in reality, but is assumed due to time constraints and computational constraints.

Finally, we assume no short selling and no leveraging in our portfolios, although our calculated Kelly weights for some years are larger than one (we should leverage) and, during the crisis, negative (we should short sell the portfolio). When this occurs we therefore turn to the alternative proposed to that of the Kelly strategy; if the weight is above 100% we simply invest 100% in the best ranking stock (instead of leveraging above this amount) and if our weights are negative we proceed with a risk-free investment instead. At this stage investors can choose between the risk-free rate and cash; in this paper we use the risk-free rate to keep our Kelly portfolio somewhat dynamic. We decide to impose constraints on leveraging and short selling since too many factors are unknown; short selling for example requires knowledge about parameters such as collateral, covenants, and the cost of borrowing. Although this restriction in leverage and short selling opts for misinterpreted returns, some researchers encourage avoiding leverage in a Kelly strategy. MacLean, Thorp, and Ziemba (2010) state that investors striving for long term growth maximization never gain from betting more than the Kelly strategy, arguing that risk increases (lower number of securities

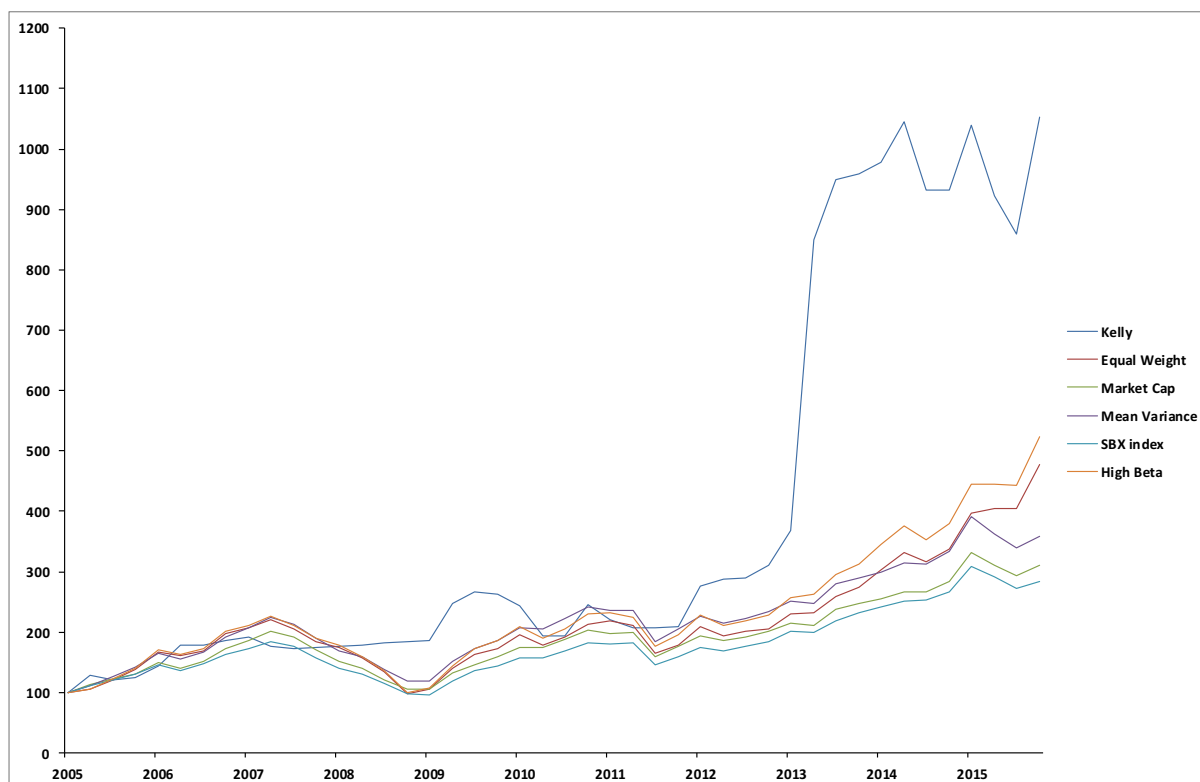
in the portfolio) and growth decreases. Finally, we provide an approximation of the Kelly portfolio without restrictions in Appendix C.

## 4. Results

In line with previous studies on Kelly, our expectation is that the portfolio based on a Kelly strategy differs significantly compared to other common portfolio strategies in terms of risk, return, and allocation. This translates into the Kelly portfolio being subject to, on the one hand, substantially higher volatility and larger drawdowns compared to benchmarks. But, on the other hand, produces (within our universe) a relevant growth strategy in the long run. This is very much in line with what our research finds. In the next sections we compare our portfolio strategies based on descriptive statistics. We also look at the general risk and return profile. Additionally, the Kelly portfolio is independently dissected and analyzed through a sensitivity analysis where we adjust for several factors to look at the robustness of the portfolio strategy. Finally, we proceed with a Carhart (1997) 4-factor regression, and compare the factors included to our portfolio strategies.



## 4.1 Return Analysis



**Figure 7: Cumulative Returns for All Strategies.** Figure 7 illustrates the performance of the Kelly portfolio and benchmark portfolios. Data is retrieved for the period 2005 to 2015 from Bloomberg.

Kelly clearly outperforms the market (SBX) and the benchmark portfolios over a 10-year period from 2005-2015. As underlined earlier in this paper we do not use leverage nor do we short sell securities, hence a graph where such constraints do not exist should demonstrate a smoother exponential character, in line with what previous studies show on the growth of wealth (see MacLean, Thorp, Zhao, & Ziemba 2010) but also most likely higher returns. When the Kelly portfolio returns pictured above are somewhat flat (for example during 2007-2009) we have invested in the risk-free asset, hence the linear sections in the graph, whereas the strategy encourages the investor to short the portfolio. Furthermore, leveraging would also have impacted returns for some periods. Yet, irrespective of the restriction we have imposed, the Kelly allocation strategy clearly outperforms all benchmarks. Cumulative Return for this period amounts to 953%, whereas the second best strategy (High Beta) produces 323%, and the market (SBX) 183%. Finally, we also look at the cumulative return for OMXS30 (not included in graph), in order to look at another market index, which during

this period amounts to 91%. By looking at the graph above we can see a huge increase in return around 2013. In the next sections we look at what drives this huge leap in the Kelly portfolio return, and analyze what happens if it is removed.

<i>Strategy</i>	Average Annualized Geometric Return	Average Number of Holdings	Maximum Drawdown
Kelly	<b>23.9%</b>	7	-20.0%
Equally Weight	15.9%	109	-28.2%
Market Cap	10.7%	109	-20.9%
Mean-Variance	12.0%	55	-21.9%
SBX	9.6%	<b>320<sup>4</sup></b>	-19.9%
High Beta	16.8%	109	-26.9%
Levered SBX	16.1%	<b>320</b>	<b>-57.2%</b>

**Table 2:** *Descriptive statistics (highest absolute value is highlighted in bold).* In Table 2, descriptive statistics for all portfolio strategies are depicted. Data is retrieved for the period 2005 to 2015 from Bloomberg.

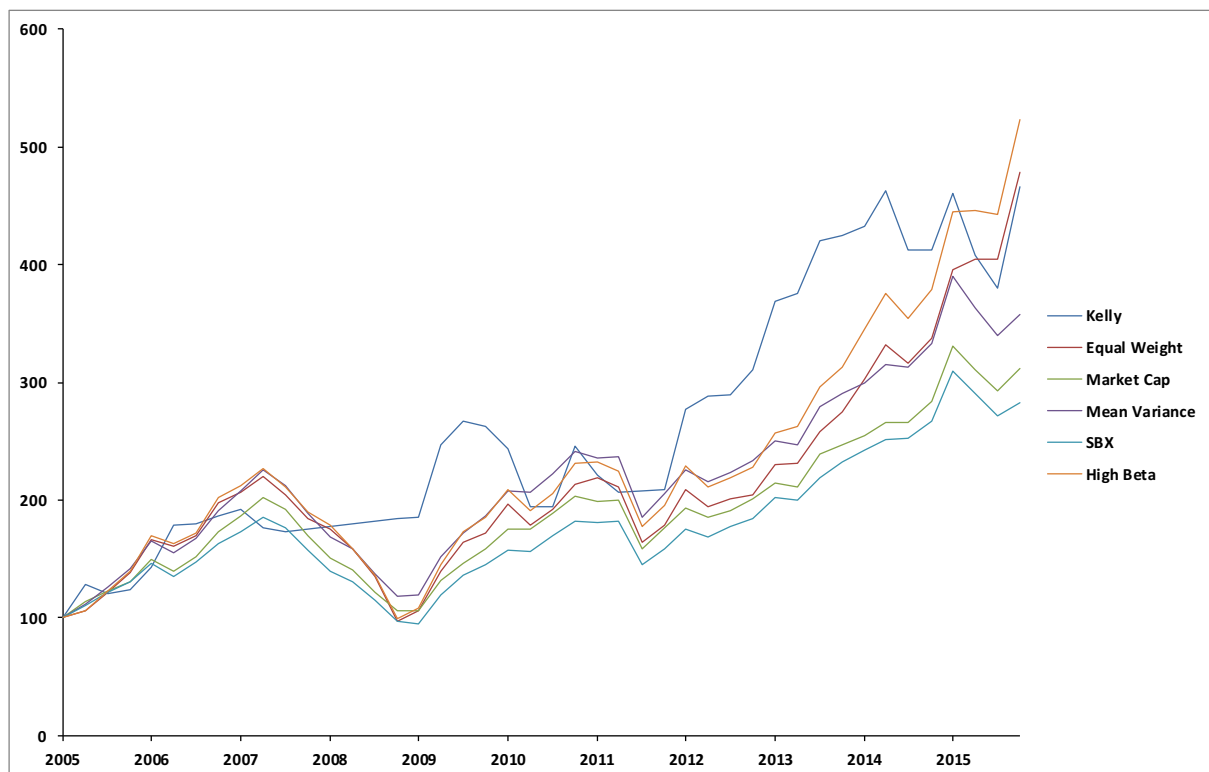
In line with previous statements on the characteristics of a Kelly portfolio (see for example the paper produced by MacLean, Thorp, and Ziemba, 2010) we get higher returns compared to benchmark portfolio strategies. The maximum drawdown for Kelly is relatively good given the previous statements on its risk nature; only the SBX is more attractive. This is largely explained by the fact that we hold the risk-free rate during the financial crisis, which our other portfolios do not, consequently punishing their downsides since the financial crisis saw returns suffer. Perhaps, though, advocates of the Kelly strategy want to see that the Kelly portfolio has the lowest maximum drawdown but in this case SBX is more stable in terms of

<sup>4</sup> We use data on holdings from 2017 for SBX due to the lack of data availability for 2005 to 2015.

downside losses, conceivably due to the more diversified nature of the market. Finally, the average number of holdings is much lower for the Kelly portfolio, which illustrates the undiversified nature of the Kelly strategy.

## 4.2 Return Analysis with Robustness Tests

### 4.2.1 Impact on the Kelly Portfolio when Removing Outlier



**Figure 8:** *Adjusted Cumulative Returns for the Kelly Portfolio During 2005-2015.* In Figure 8, we show results after Obducat AB is removed. Data is retrieved for the period 2005 to 2015 from Bloomberg.

To account for the fact that the boost in return during the second quarter of 2013 is largely due to one stock's huge increase, we look at what happens with the cumulative return if we exclude this stock. The stock we remove is Obducat AB, which during Q2 2013 returns 163% and receives a 78% weight in our portfolio, hence aggressively increases the accumulated return for that period. The reason for the increase in Obducat AB's stock price was mainly due to new contracts and orders, as well as lower costs due to a general restructuring of the firm structure. We act accordingly since this significant amount of increase of wealth is solely due to one stock during one quarter; hence we adjust for the risk that luck drives most

of the return. This results in an immense reduction of total return; after the adjustment our Kelly strategy returns 354% after 10 years, compared to the portfolio where Obducat AB is included, which produces a 953% increase in wealth. Also, the Kelly strategy is now only the third best performing strategy. From now on, when we refer to the adjusted Kelly in Figure 8 (where the outlier, Obducat AB, has been removed) we call it “Kelly Robust”. The Kelly Robust is of key interest since we believe the return in Figure 7 is mostly due to luck, as we have previously argued.

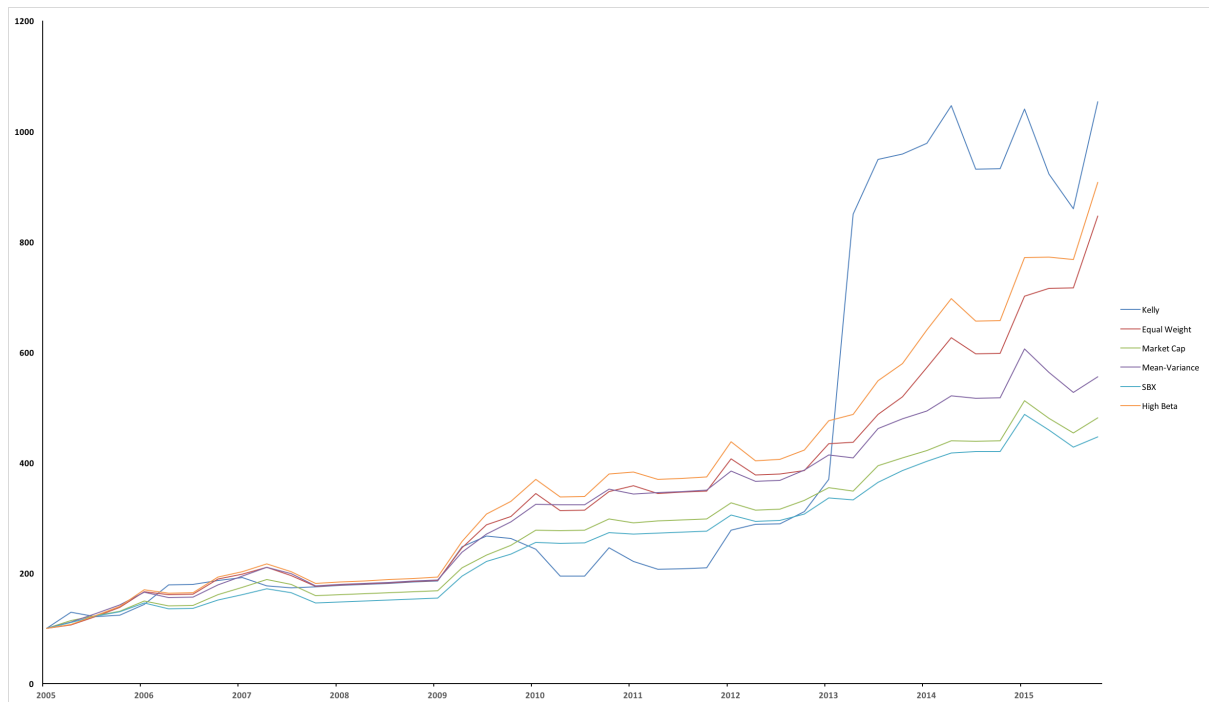
<i>Strategy</i>	Average Annualized Geometric Return	Average Number of Holdings	Maximum Drawdown
Kelly Robust	14.7%	7	-20.0%
Equally Weight	14.9%	109	-28.2%
Market Cap	10.7%	109	-20.9%
Mean-Variance	12.0%	55	-21.9%
SBX	9.6%	<b>320<sup>5</sup></b>	-19.9%
High Beta	<b>16.8%</b>	109	-26.9%
Levered SBX	16.1%	<b>320</b>	<b>-57.2%</b>

**Table 3:** *Descriptive statistics (highest absolute value is highlighted in bold).* Table 3 shows the descriptive statistics for all portfolio strategies, where Kelly is now replaced with Kelly Robust. Data is retrieved for the period 2005 to 2015 from Bloomberg.

All values are identical to that of Table 2, except the average annualized geometric return, which is now lower (14.7% vs 23.9%). The Kelly Robust portfolio is now less convincing when we adjust for the outlier.

<sup>5</sup> We use data on holdings from 2017 for SBX due to the lack of data availability for 2005 to 2015.

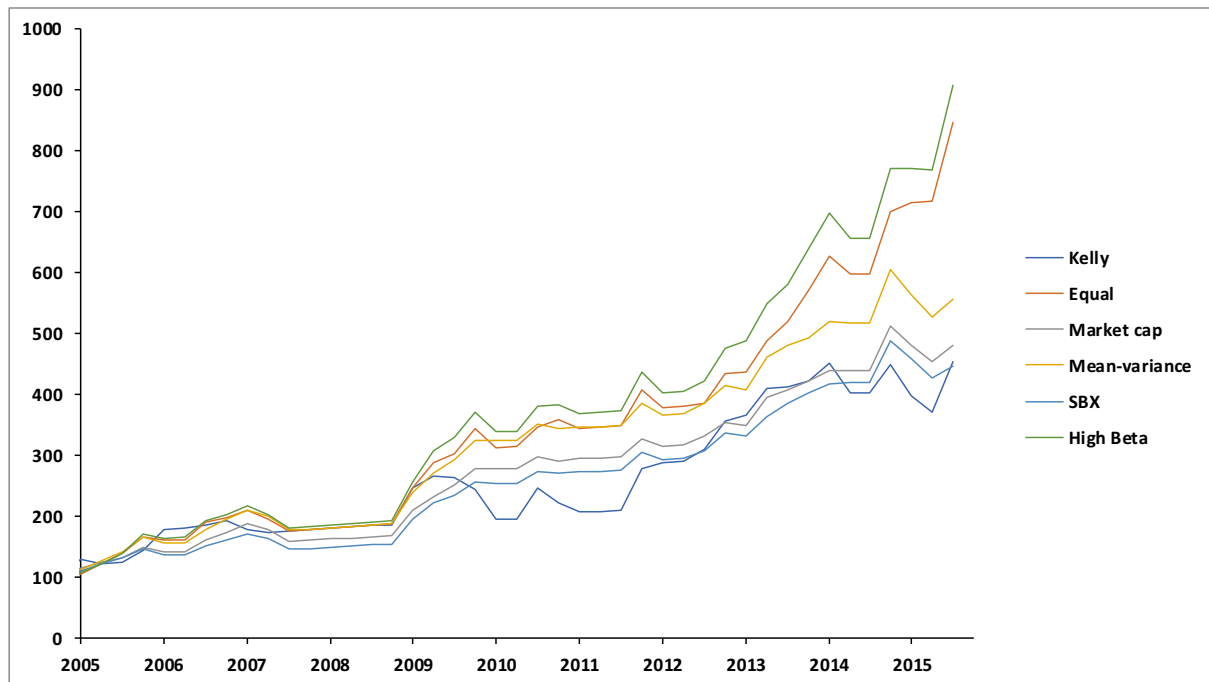
## 4.2.2 Impact when Replicating the Kelly Short Selling Restriction on Benchmark Portfolios



**Figure 9:** *Adjusted Cumulative Returns for All Strategies During 2005-2015.* Figure 9 depicts how during the period when the Kelly strategy tells us to short sell, i.e., when we hold the risk-free investment, the same holding is replicated in all other portfolios as well. Data is retrieved for the period 2005 to 2015 from Bloomberg.

Previously we explained that when weights are negative we should short sell according to our Kelly strategy, but instead we allocate into the risk-free rate, which gives the Kelly portfolio stability during the financial crisis and has a positive impact on our return. One take on the short selling constraint in this paper is done by looking at the outcome on return when Kelly is in the risk-free rate and at the same time all other strategies also invest in the risk-free rate. In this way, we are more “fair” in that the stability Kelly experiences for example during the financial crisis is also seen in the benchmark portfolios. The comparison above illustrates the effect on return when the benchmark portfolios hold the risk-free investment, i.e. Swedish repo-rate, during the same time as Kelly does. This time, Kelly underperforms compared to peer portfolios from the period around March 2010 to March 2013, but the cumulative return (953%) is still superior to that of all strategies, where the High Beta strategy is the second best (920%), and most importantly compared to the market index SBX (407%). We can therefore conclude that the strong performance of the unadjusted Kelly portfolio is not due to a bias in our short selling restriction.

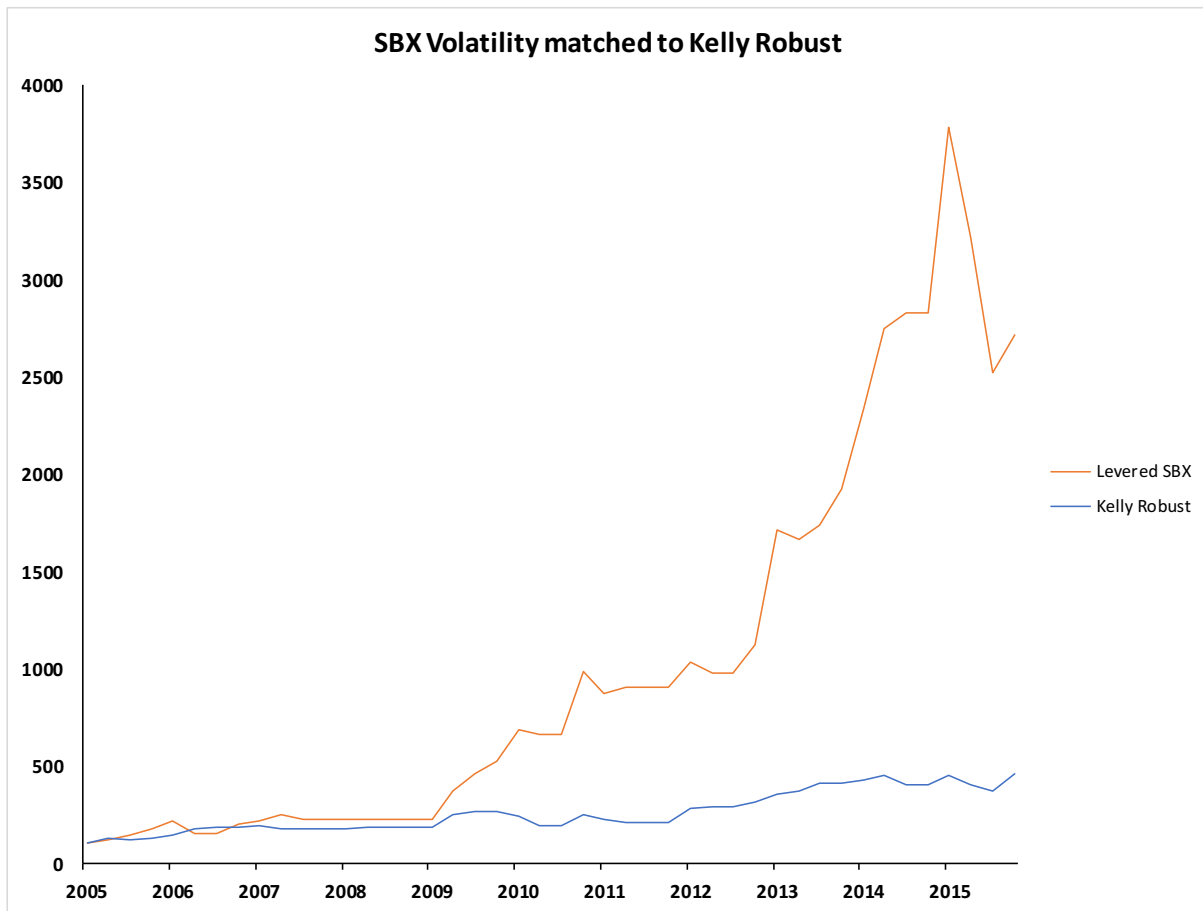
### 4.2.3 Impact when Replicating Kelly Short Selling Restriction on Benchmark Portfolios and Removing Outlier



**Figure 10:** *Adjusted Cumulative Returns for All Strategies During 2005-2015.* Figure 10 builds on Figures 8 & 9 and has the same premise regarding the risk-free rate holding for all strategies. Additionally, Figure 10 has also removed Obducat AB to adjust for potential luck as we previously argued could be the case. Data is retrieved for the period 2005 to 2015 from Bloomberg.

This time, the Robust Kelly portfolio has the second lowest return (354%), only superior to that of the market (346%). The best performing strategy is High Beta with 807% return. We can from this graph conclude that the Kelly strategy would be less relevant, or even irrelevant, for investors if Obducat AB would have been excluded from the portfolio and if the benchmark portfolios invested in the risk-free asset during the financial crisis. In the previous section we argued that our short selling restriction does not impose a bias; yet, if we look at the Kelly Robust strategy in Figure 10 this does not hold.

#### 4.2.4 Kelly Robust Portfolio Compared to SBX Leveraged

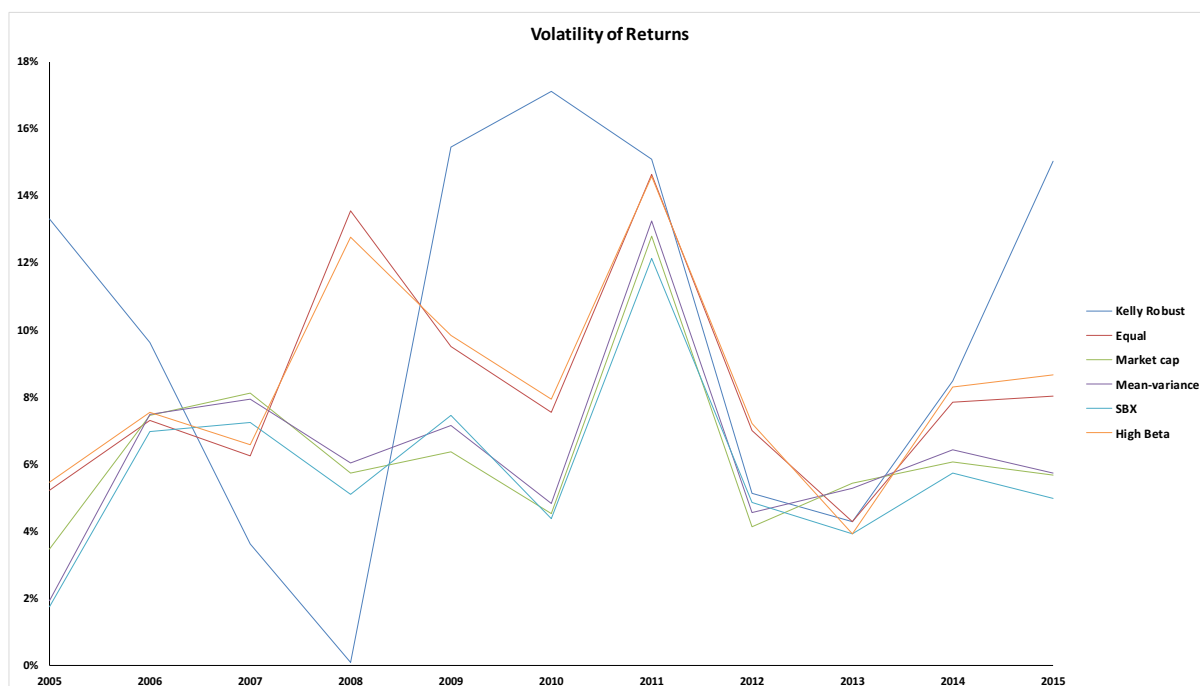


**Figure 11: Leveraging SBX to Match Kelly Robust Volatility.** In Figure 11 we leverage positions in SBX to match the ex post volatility of the Kelly Robust portfolio. Data is retrieved for the period 2005 to 2015 from Bloomberg.

By looking at Figure 11, we can conclude that the previous outperformance, of both Kelly and Kelly Robust versus SBX, is to a large extent attributable to higher risk. When we leverage SBX to the same level as that of the Kelly Robust portfolio, the cumulative return increases to 2615%, which is much higher than the Kelly Robust portfolio's return of 353%. This implies that the risk-return tradeoff, based on the adjusted Kelly portfolio, is not attractive. Often when we leverage the SBX, the factor by which we leverage is extremely high. This is due to the high underlying volatility of the stocks in our Kelly portfolio. Consequently, this implies the volatility of Kelly Robust is often much higher than for SBX, which we will show in the next sections.

### 4.3 Risk Analysis

An important feature of a Kelly strategy as highlighted by previous papers is the large risk exposure. In this paper, risk translates into volatility of returns. This section illustrates the risk characteristics of our Kelly portfolio; it is exposed to significantly more risk compared to other strategies, mainly due to poor diversification, which translates into the number of holdings of seven firms on average. The following figures illustrate the larger volatility for the adjusted Kelly portfolio.

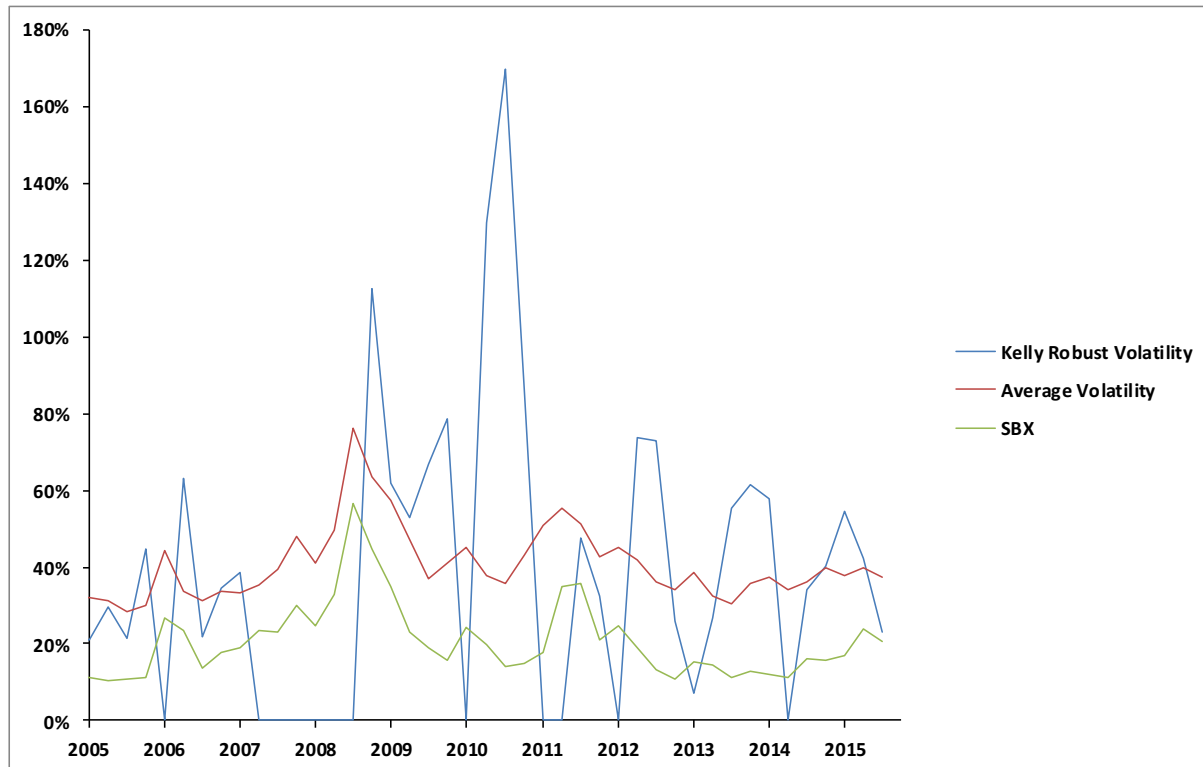


**Figure 12:** *Annual Volatility of Returns for all Portfolios.* Figure 12 shows the volatility characteristics of our Kelly Robust portfolio and the benchmark portfolio choices. Data is retrieved for the period 2005 to 2015 from Bloomberg.

From Figure 12 we can see that the volatility of returns moves in the opposite direction to that of our benchmarks. As one would expect the volatility of Kelly is relatively lower when we hold the risk-free asset, yet higher whenever our portfolio consists of positions in stocks. The graph could be of interest for investors with risk preferences (given that they have imposed the short selling restriction) who want to hedge risk; there is great stability in the adjusted Kelly portfolio in times of general uncertainty (e.g. the financial crisis) since we are in the risk-free. Investors could do risk budgeting, i.e., allocation based on individual



portfolio risk and return, and thereafter decrease exposure to the benchmark if its volatility goes up. Again, we should highlight the fact that the volatility would be different (most likely higher) had we not imposed our leverage and short selling restrictions.



**Figure 13: Comparison of Volatility of Returns.** Figure 13 shows the volatility between the underlying stocks in the Robust Kelly portfolio, the universal average after the multiple screening, and the Market. Data is retrieved for the period 2005 to 2015 from Bloomberg.

The Kelly volatility is the weighted sum of quarterly volatility for all firms in each Kelly portfolio for all periods. Hence, this is a measure of the volatility of the underlying stock rather than the portfolio as a whole; in this way we also illustrate how the underlying firms in the Kelly portfolio are volatile. We use the quarterly volatility since the portfolios are rebalanced quarterly. When our Kelly portfolio is based on a risk-free investment, we give this portfolio a volatility of 0 due to the stability of Swedish rates. The average volatility is the volatility for all the firms in our universe which have been selected based on our ranking criterion, and includes all firms irrespective if they are included in our Kelly portfolio or not. Finally, we also graph the SBX volatility. Again, we reach the same conclusion as above, but this time it is clearer; the volatility of returns for a Kelly strategy is significantly higher compared to benchmarks. This is most likely due to the Kelly portfolio holding fewer

securities on average, and in line with previous research (see for example MacLean, Thorp, & Ziemba (2010)) on the larger risk, relative to benchmarks, a Kelly strategy experiences.

The last part of the risk analysis is conducted through looking at correlations of returns.

<i>Strategy</i>	Kelly Robust	Equal Weight	Market Cap	Mean-Variance	SBX	High Beta
Kelly Robust	1.000					
Equal Weight	0.447 (0.003)	1.000				
Market Cap	0.410 (0.006)	0.890 (0.000)	1.000			
Mean-Variance	0.400 (0.008)	0.907 (0.000)	0.990 (0.000)	1.000		
SBX	0.404 (0.007)	0.904 (0.000)	0.983 (0.000)	0.984 (0.000)	1.000	
High Beta	0.473 (0.001)	0.996 (0.000)	0.904 (0.000)	0.921 (0.000)	0.916 (0.000)	1.000

**Table 4:** *Correlation Matrix of Quarterly Returns for all Strategies During 2005-2015 (p-value in brackets, where H0 states that correlation is insignificant).* Table 4 depicts correlations of returns. Data is retrieved for the period 2005 to 2015 from Bloomberg and Stata.

To further investigate matters concerning risk and consequently diversification opportunities for example, we create the correlations matrix in Table 4. In line with the principles which Kelly builds on, this strategy is high risk due to its unique characteristics, displayed in for example the low number of holdings, compared to benchmark strategies and indices. Correlation with Kelly is below 0.24 for all strategies. Again, as mentioned in previous tables in this section, due to Kelly's low correlation with the other strategies investors could turn to a Kelly strategy and include it in their overall portfolio for diversification purposes. Correlations are not significant though, most likely because the Kelly portfolio is dynamic

and because we have many holdings in the risk-free rate, whereas our benchmark portfolios are more static.

<i>Strategy</i>	Robust Kelly	Equal Weight	Market Cap	Mean-Variance	SBX	High Beta
Robust Kelly	1.000					
Equal Weight	0.557 (0.000)	1.000				
Market Cap	0.512 (0.000)	0.869 (0.000)	1.000			
Mean-Variance	0.486 (0.001)	0.893 (0.000)	0.984 (0.000)	1.000		
SBX	0.505 (0.000)	0.889 (0.000)	0.979 (0.000)	0.980 (0.000)	1.000	
High Beta	0.587 (0.000)	0.990 (0.000)	0.885 (0.000)	0.906 (0.000)	0.896 (0.000)	1.000

**Table 5:** *Correlations Matrix of Quarterly Returns for all Strategies During 2005-2015 with replicated risk-free holdings (p-value in brackets, where  $H_0$  states that correlation is insignificant).* Table 5 illustrates the correlations between all portfolio strategies when they replicate the risk-free holding. Data is retrieved for the period 2005 to 2015 from Bloomberg and Stata.

Finally, we look at correlations when the benchmark strategies replicate the short selling restriction. We do this to get the full picture of correlations with respect to Section 4.2. Logically, the correlations between Kelly and the benchmark portfolios are now larger since there are more identical holdings.

## 4.4 Carhart 4-factor Regression<sup>6</sup>

03/2005-12/2015	ReturnKelly	Return Kelly Robust	ReturnEW
Intercept	0.002** (2.22)	0.001** (1.87)	0.001*** (3.00)
SMB	0.139 (1.14)	0.169 (1.63)	0.280*** (6.91)
HML	0.438*** (3.46)	0.443*** (4.10)	0.065 (0.15)
Market Factor	0.262*** (4.63)	0.276*** (5.72)	0.857*** (45.49)
MOM	0.135 (1.35)	0.109 (1.28)	0.244*** (7.36)
$R^2$	0.014	0.022	0.521
Observations	2724	2724	2723

Note: T-statistics are presented in brackets. \*\*=p<0.05 \*\*\*=p<0.01.

**Table 6:** *Carhart 4-factor Regression Analysis*

Table 6 depicts a regression analysis on *daily returns* of the securities in our Kelly portfolios and includes Carhart's (1997) four factors as independent variables. We retrieve information from each factor from the database AQR where we get daily data for Swedish stocks. Data on the Kelly and benchmark portfolios is retrieved from Bloomberg, and the regression is conducted in Stata.

We choose to look at daily returns (instead of 3-month total return) this time to obtain more observations. Through the regression, we can see which factor each strategy is exposed to,

<sup>6</sup> For the benchmark strategies we only look at the Equal Weight portfolio. This is due to the fact that Bloomberg provides us with erroneous daily returns for the other strategies, appearing as sudden moves into risk-free and unexplained sporadic returns. This should not impose a severe problem, since the Equal Weight experiences correlation of at least 0.86 with the other strategies containing the same stocks.

which makes it easy to imitate the returns based on the beta with each factor. First, there is a significant, although extremely small, positive alpha on both the adjusted and unadjusted Kelly Portfolio, which implies that the Kelly strategy in fact outperforms relative to the four risk factors. This in turn supports the usage of the Kelly strategy. Secondly, HML and Market Factor are significant and have a positive beta. The significance of HML is explained by the fact that we use the P/B-ratio, which is closely related to HML as described in previous section. The market factor is most likely significant because of the high explanatory power observed in periods when the Kelly strategy is in the stock market. Interestingly, though, the correlations we looked at above were not significant for the market with our Kelly portfolio. Finally, the other factors are insignificant and based on this we can again conclude that the Kelly strategy have somewhat independent returns and can be regarded as a diversification strategy. However, our  $R^2$  are very low, which implies that modeling Kelly returns versus the other four factors has little explanatory power. Additionally, it implies that we should be critical towards what we have inferred from the regression analysis. Most importantly, there might not exist any alpha, at least not when Kelly is modeled towards the four factors. The low  $R^2$  in Table 6 is perhaps explained by the fact that we look at daily returns for the Kelly portfolio, which varies a lot from the daily returns from the other four factors. Although 1.4% and 2.2% are extremely low numbers irrespective of what we decide to compare with, previous papers (see for example Roll (1973)), also experience low explanatory power, illustrated in a low  $R^2$  of 5%, compared to benchmarks. This in turn implies the reader should be critical when analyzing the regression results, and generally we cannot draw accurate conclusions. We want to stress, though, that even though the significant alphas we find are extremely low, the strong performance of both the adjusted and unadjusted Kelly portfolios still remains, which is illustrated in our figures. Finally, we decide to run a regression on one of the benchmark portfolios. Since the Market Cap portfolio experiences very high correlation with the other benchmark portfolios (above 0.86 at all time) we decide to look at this strategy only, since it is similar to all others. This time, the benchmark strategy experiences a significant alpha of 0.01, which is around the same as for Kelly. Furthermore, SMB, Market Factor and Momentum have significant positive betas.  $R^2$  (52%) is also much higher than for Kelly. We can conclude that the benchmark portfolio(s) are to a larger extent explained by the four-factor model, and investors who are confident with portfolios which experience more explanatory power based on Carhart's (1997) model should replicate the Equally Weighted portfolio investment style.

## 5. Discussion

This paper illustrates how an investor can potentially benefit from trading based on a Kelly strategy. Based on historical returns, if one is willing to take on more risk and use a Kelly strategy this would have outperformed many traditional portfolio strategies during the 2005 to 2015 period. From a log growth perspective we have thus shown the strength of a Kelly strategy. Our results have strong resemblance to findings by Estrada (2010), who shows that Kelly portfolios are less diversified, have a higher (arithmetic and geometric) mean return, and higher volatility than benchmark portfolios. Also, when we conduct robustness tests the Kelly performs quite well relative our benchmark portfolio strategies. Yet, one test of Kelly Robust, where we exclude outliers and replicate the short selling restriction in benchmark portfolios, demonstrates weaker performance. Finally, if investors would have leveraged the market to volatility levels of the Kelly Robust instead, this would have induced more attractive returns.

There is still much left to discuss and comment on. Let us begin by emphasizing four approaches in this paper, which affect the final results. First, this paper shows full Kelly, whereas previous studies (see for example Davis & Léo (2014)) bring forward the fact that few investors use a full Kelly allocation of wealth in practice. Instead, they apply a fractional Kelly<sup>7</sup> due to the full strategy's significant risk exposure. This being said, readers who want to pursue with this strategy can simply choose to invest a fraction of the weights, which have been calculated, but should remember the fact that returns will decrease as a consequence. Secondly, we have imposed some constraints, which also affect our results. As mentioned throughout this paper, the results would most likely be different if we could leverage and short sell our Kelly portfolios since this would affect the return. In other words, this paper

---

<sup>7</sup> The Kelly strategy is perceived as risk-abundant and thus investors have turned to look for an alternative, namely "Fractional Kelly", introduced by MacLean, Thorp, & Ziemba (2010). This refinement seeks to adjust the initial Kelly (or "full Kelly") allocation fraction to a smaller amount. To illustrate this, let us assume our Kelly calculations on a certain stock results in an optimal allocation of 50% of total portfolio weight. Many investors find this number too high and use a fractional Kelly of  $\frac{1}{4}$  resulting in a new allocation of 12.5%. In this paper only the full Kelly strategy will be used and not a fraction of weights in order to enable us to look at as much risk budgeting of the Kelly strategy as possible.

illustrates our take on the Kelly strategy; meanwhile returns can vary greatly depending on the different approaches investors take. We provide an approximation of how Kelly returns could look without such restrictions in Appendix C. Our third point is that it would be interesting to look at the performance of the Kelly strategy whilst using another approach to identify the best performing stocks. In this paper we chose three multiples, which turned out to be rather successful in predicting returns, yet the usage of other multiples (or approaches) to identify predictive power of returns would most certainly generate other results. Previous studies, for example Roll (1973), rank stocks based on size of the expected risk premium. He concludes that the growth-optimal model (i.e. Kelly) share similarities to the market portfolio. In the end, whether the investor can successfully implement the Kelly strategy boils down to the ability in making predictions of stock movements; the Kelly strategy can be the optimal portfolio strategy only if the investor is successful in doing so. Finally, the Kelly (1956) criterion's main critique brought forward by Markowitz (1976) and Samuelson (1971) is the fact that it is most useful when implemented on an infinite time period. This being said, the strategy's main critics might regard the ten-year period we look at as inadequate in order to draw any robust conclusion on the usefulness. Yet, our choice of ten years is due to the fact that we believe an insufficient amount of stocks were listed before 2005, which would give us too little data to work with. Investors should bear these four premises in mind when analyzing the results.

Additionally, we have not taken transaction costs or taxes into account. In this paper, one can argue that transaction costs should be relatively lower for Kelly since there are fewer securities in the Kelly portfolio on average. Yet, in terms of accuracy, returns should be adjusted for transaction costs and taxes. We can, however, note that due to the concentrated nature of our strategy, we have fewer transactions (which translates into holdings) than the portfolio benchmark strategies.

Moreover, an important factor to note is that our strategy is highly dynamic in its positioning and risk taking with the most common holding being the risk-free rate; Patel and Lo (2008) discuss the difficulty of benchmarking dynamic strategies. They propose a solution to this problem, which is not applicable in this paper, i.e., we have not been able to create a suitable dynamic benchmark, resulting in us benchmarking our Kelly strategy against static indices, which limits the comparability. To adjust for this benchmark dilemma in the best way possible, we also included the high beta strategy, as well as looking at effects when

leveraging the SBX to the equivalent volatility levels as that of Kelly. Results are mixed, but generally not in favour of the Kelly strategy in terms of risk-return tradeoff. Moreover, the Kelly strategy beats the market in all our tests, which we have shown, though, is due to the larger values of volatility.

When removing the best performing stock the performance is more in line with the benchmarks, and even worse in one test. This could be explained by either the reduction of alpha or that the skewness of returns decreases. Hakansson (1971) discusses this effect and concludes that with low, to non-existing alpha, and normally distributed returns, a Kelly strategy is expected to achieve returns close to the mean-variance portfolio (see for example: Markowitz (1976); Roll (1973); Thorp (1971) for more on this). Meanwhile, the strongest and most typical effects of the Kelly strategy are illustrated in high alpha and highly skewed or fat-tailed distributions, which can be seen in our volatility of returns for example. This results in a recommendation to use the strategy in certain options trading systems or stock picking strategies with skewed returns.

Finally, the usage of this strategy in practice is highly dependable on the risk attitude of the investor. This paper shows that the returns of a Kelly portfolio are sometimes both significant and robust. Yet, due to the undiversified and volatile nature of the strategy, it would be difficult for institutional actors to implement this strategy, as they would face liquidity and volatility constraints. Retail investors should be aware of the dangers with this strategy; the risk of largely diminished wealth is significantly higher than alternative strategies and for unsophisticated investors the optimal bet size itself can be difficult to approximate. It could, however, prove to be a useful strategy for small institutions with a liberal investment philosophy, sophisticated (defined as being able to make accurate predictions of the inputs in the Kelly formula) retail investors, and day traders. This is because of the high frequency of trades each day, which serves as a ground on which day trader's predictions can be based, and which in turn makes predictions more reliable. Finally, we can conclude that some of the active Swedish day traders use similar allocation strategies to that of Kelly. Due to the large amount of predictions needed, many have created their own take on Kelly, with adaptations such as attempting to maintain a constant variance in their strategy. This results in a simplified method to optimize the potential edge in an investment.



## 6. Conclusion

This paper shows the implementation of the Kelly strategy on the Swedish stock market from 2005 to 2015. We have used a multiple approach to get all our inputs, whereby we proceed by comparing the Kelly strategy to benchmark portfolio choices. Although our multiple approach is subject to a great level of arbitrariness, by applying the Kelly model outlined in this paper an investor in the Swedish stock market between 2005 and 2015 would have achieved a superior return compared to investing passively in a market index or such. Our results share similarities with, for example, the portfolio simulated by Ziemba (2016) who shows that the Kelly strategy's superiority in producing long run maximum wealth from a sequence of favourable investments.

Yet, in order to be as neutral as possible, we do not use the unadjusted Kelly portfolio (Figure 7) as our foundation of comparison and evaluation; instead we use the Kelly Robust portfolio (Figure 8) to account for the potential lucky event with Obducat AB during our time interval. We do this in order to strengthen our general conclusions from the Kelly strategy. This in turn leads to a different perception of the strategy; the adjusted Kelly strategy is in fact not beneficial for investors from a risk-return tradeoff perspective.

Furthermore, contrary to previous findings (see for example MacLean, Thorp, Zhao, and Ziemba, 2010), returns do not clearly approach the theorized exponential shape of growth of wealth until after the financial crisis. Our model has limits such as no short selling and no leveraging which has an impact on this pattern. Surprisingly to us, we are faced with these constraints far more often than we had first anticipated. This results in applied allocation, which differs, from the "real" (i.e. where there are no constraints) allocations; a model without these limitations would during several periods, for example the financial crisis, leverage the positions as well as by to short sell stocks with surprising accuracy. Thus, as we have emphasized above, a Kelly portfolio without these limitations would be expected to perform differently compared our constrained strategy due to compounding.

Finally, if market participants are comfortable in their predictions, alternatively use the multiple approach presented in this paper, and are comfortable with an undiversified portfolio, investors could potentially benefit from applying the Kelly strategy in relation to

traditional portfolio choices. Investors should always bear in mind that the Kelly strategy is highly exposed to idiosyncratic risk, whether it is desirable or not.

## 7. Suggestions for future research

Generally speaking, the Kelly strategy deserves more attention in our opinion. Previous studies (see for example Estrada 2010) and also this paper show that a Kelly strategy would have produced strong long-term returns relatively for the period 2005 to 2015 in the Swedish market. This being said, its robustness should be tested on more markets and during longer and more time periods, and most importantly with leverage and short selling. Finally, earlier on we stated that we assume no correlation between stocks. A practical implementation of the Kelly strategy on this market and time era could be re-made with the inclusion of a correlation matrix to look at how results would change.

## 8. References:

Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of financial economics*, 9(1), 3-18.

Basu, S. (1977). Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis. *The journal of Finance*, 32(3), 663-682.

Bernoulli, D. (1738). Specimen theoriae novae de mensura sortis. *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, 5, 175–192.

Bodie, Z., Kane, A., & Marcus, A. J. (2011). *Investment and portfolio management*. McGraw-Hill Irwin.

Breiman, L. (1961). Optimal Gambling Systems for Favorable Games In: Jerzy NEYMAN, ed. *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume I*. Berkeley: University of California Press, pp. 65–78.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, 52(1), 57-82.

Cover, T. M. (1991). Universal portfolios. *Mathematical finance*, 1(1), 1-29.

Davis, M. H., & Lleo, S. (2014). *Risk-sensitive Investment Management*. World Scientific.

Estrada, J. (2010). Geometric mean maximization: an overlooked portfolio approach?. *The Journal of Investing*, 19(4), 134-147.

Evstigneev, I. V., Hens, T., & Schenk-Hoppé, K. R. (2009). Survival and evolutionary stability of the kelly rule.

Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. *The Journal of Finance*, 47(2), 427-465

Finnerman, E., & Kirchmann, C. R. (2015). Evaluation of Alternative Weighting Techniques on the Swedish Stock Market.

Hakansson, N. H. (1971). Multi-period mean-variance analysis: toward a general theory of portfolio choice. *The Journal of Finance*, 26(4), 857-884.

Hakansson, N. H., & Miller, B. L. (1975). Compound-return mean-variance efficient portfolios never risk ruin. *Management Science*, 22(4), 391-400.

Kelly, J. L. (1956). A new interpretation of information rate. *bell system technical journal*, 35(4), 917-926.

Latané H. A. (1959). Criteria for choice among risky ventures. *Journal of Political Economy*, 67(2), 144-155.

MacLean, L. C., Thorp, E. O., & Ziemba, W. T. (2011). *The Kelly capital growth investment criterion: Theory and practice* (Vol. 3). world scientific.

MacLean, L. C., Thorp, E. O., & Ziemba, W. T. (2010). Good and bad properties of the Kelly criterion. *Risk*, 20(2), 1.

MacLean, L. C., Thorp, E. O., Zhao, Y., & Ziemba, W. T. (2010). Medium term simulations of the full kelly and fractional kelly investment strategies. *The Kelly Capital Growth Investment Criterion: Theory and Practice*, 15, 543-562.

Markowitz, H. (1952). Portfolio selection. *The journal of finance*, 7(1), 77-91.

Markowitz, H. (1959). *Portfolio Selection, Efficient Diversification of Investments*. J. Wiley.

Markowitz, H. M. (1976). Investment for the long run: New evidence for an old rule. *The Journal of Finance*, 31(5), 1273-1286. JA

Merton, R. C. (1969). Lifetime portfolio selection under uncertainty: The continuous-time case. *The review of Economics and Statistics*, 247-257.

Merton, R. C. (1992). Continuous-time finance, revised edition. *Black w ell Publishers, Cambridge: MA*.

Nekrasov, V. (2014). Kelly criterion for multivariate portfolios: A model-free approach.

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of Financial Economics*, 108(1), 1-28.

Patel, P. N., & Lo, A. W. (2008). 130/30: The new long-only. *The Journal of Portfolio Management*, 34(2), 12-38.

Roll, R. (1973). Evidence on the “Growth Optimum” Model. *The Journal of Finance*, 28(3), 551-566.

Rotando, L. M., & Thorp, E. O. (1992). The Kelly criterion and the stock market. *American Mathematical Monthly*, 922-931.

Samuelson, P. A. (1963). Risk and uncertainty: A fallacy of large numbers.

Samuelson, P. A. (1971). The “fallacy” of maximizing the geometric mean in long sequences of investing or gambling. *Proceedings of the National Academy of Sciences*, 68(10), 2493-2496.

Samuelson, P. A. (1979). Why we should not make mean log of wealth big though years to act are long. *Journal of Banking & Finance*, 3(4), 305-307.

Thorp, E. O. (1969). Optimal Gambling Systems for Favorable Games. *Revue de l'Institut International de Statistique / Review of the International Statistical Institute*, 37(3), 273–293.

Thorp, E. O. (1971). Portfolio choice and the Kelly criterion. *Business and Economics Statistics Section, Proceedings of the American Statistical Association*. pp. 215–224.

Thorp, E. O. (2006). The Kelly criterion in blackjack, sports betting and the stock market. *Handbook of asset and liability management*, 1, 385-428.

Thorp, E. O. (2008). Understanding the Kelly criterion. *A mathematician on Wall Street in Wilmot Magazine*, May and September 2008.

Zhang, J., Shan, R., & Su, W. (2009). Applying time series analysis builds stock price forecast model. *Modern Applied Science*, 3(5), 152.

Ziemba, W. T. (2003). The stochastic programming approach to asset, liability, and wealth management.

Ziemba, W. T. (2005). Risk Measurement and Management. *Journal of Portfolio Management*, 32(1), 108-122.

Ziemba, W. T. (2016) Understanding the Kelly Capital Growth Investment Strategy.

## 9. Appendix

### A. Lognormal Prices (Merton 1969 & Merton 1992)

To derive a closed-form solution for the optimal fraction under lognormal prices  $P_j$  for assets  $j$  to  $k$ , Gaussian log-returns  $X_j$  with  $\mu_j$  and  $\sigma_j$ . The optimization problem is

$$\begin{aligned} \max E[G(f)] &= \max g(f) \\ \Leftrightarrow \max E\{\log[1 + r + f(X - r)]\} \end{aligned}$$

The crucial assumption for deriving the following results is that the logarithm of the price ratio follows a Geometric Brownian Motion, also referred to as a Itô-process. In other words the price of the risky asset  $j$  must satisfy the stochastic differential equation below

$$dP_{j,t} = \mu_{j,t}P_{j,t}dt + \sigma_{j,t}P_{j,t}dZ_{j,t}$$

Where  $Z_{j,t}$  are standard Brownian Motions which might be dependant. Also, a risk-free asset with price  $R$  and risk free return  $0 \leq r \leq \mu_j$  evolving according to

$$dR_t = rR_t dt$$

As with the Black-Scholes-Merton approach, the parameters  $\sigma_j$ ,  $\mu_j$  and  $r$  are supposed to be fixed over time to attain one-time constant solutions. The continuous wealth process, depending on the consumption factor  $C$  in period  $t$ , can be described as

$$dW_t = \left[ \sum_{j=1}^k f_{j,t} \mu_j W_t \right] dt + \sum_{j=1}^k f_{j,t} \sigma_j W_t dZ_{j,t}$$

In a univariate case, i.e. when there is one risky and one risk-free asset, the wealth dynamic can be written as:

$$dW_t = [(f \mu + (1 - f)r)W_t - C_t]dt + f\sigma W_t dZ_t$$

Merton (1992) defines the lifetime objective function, which is given by

$$I[W_t, t] = \max E \left[ \int_0^T e^{-\rho t} U(C_t) dt + B(W_T, T) \right]$$

With impatience factor  $\rho$  and the Bequest valuation function at time  $T$ , concave in wealth at  $T$ . Using a Taylor approximation at  $t$  and taking expectations

$$0 = \max \left[ e^{-\rho t} U(C_t) + \frac{\partial I(W_t, t)}{\partial t} + \frac{\partial I(W_t, t)}{\partial W} [f t ((\mu - r) + r) W_t - C_t] + \frac{1}{2} \frac{\partial^2 I(W_t, t)}{\partial W^2} f_t^2 \sigma^2 W_t^2 \right] \equiv \varphi$$

with first order conditions

$$\begin{aligned} \varphi_c &= e^{-\rho t} U'(C^*) - \frac{\partial I(W_t, t)}{\partial W} = 0 \\ \varphi_w &= (\mu - r)W \frac{\partial I(W_t, t)}{\partial W} + \frac{\partial^2 I(W_t, t)}{\partial W^2} f^* W^2 \sigma^2 = 0 \end{aligned}$$

The solution to  $\varphi$ , the life time objective function is not trivial we simplify it by assuming that

$$J(W_t, t) = e^{-\rho t} I(W_t, t)$$

Letting  $T \rightarrow \infty$  the Bequest function at  $T$ ,  $B(W_t, T)$ , falls out. We can now write the new objective function as

$$J[W_t] = \max E \left[ \int_0^\infty e^{-pt} U(C_t) dt \right], v \in [0, \infty]$$

Consequently, the Partial Differential Equation simplifies to the Ordinary Differential Equation

$$0 = \max \left\{ U(C_t) - pJ(W) + \frac{\partial J(W_t, t)}{\partial W} [f_t((\mu - r) + r)W_t - C_t] + \frac{1}{2} \frac{\partial^2 J(W_t, t)}{\partial W^2} f_t^2 \sigma^2 W_t^2 \right\}$$

Which is no longer a function of time, since  $d_t$  falls out

The goal is to produce optimal portfolio strategies under a log-utility function in a normative way. In the case of CRRA (Constant Relative Risk Aversion) the marginal utility is given by

$$U(C) = \frac{1}{\gamma} C^\gamma$$

The relative risk aversion (RRA) is

$$RRA = -\frac{U''(C)}{U'(C)} C = \frac{-(\gamma - 1)C^{\gamma-2}}{C^{\gamma-1}} C = 1 - \gamma$$

Notice that we assume that this is a constant, therefore if  $U(C) = \log(C)$ , then  $\gamma = 0$  and  $RRA = 1$ . Substituting the RRA into our first order condition  $\phi_c$  yields

$$e^{-pt(C^*)^{\gamma-1}} = I'(W) \Leftrightarrow C^* = [e^{-pt} I'(W)]^{\frac{1}{\gamma-1}}$$

$$f^* = -\left(\frac{\mu - r}{\sigma^2}\right) W \frac{J'(W)}{J''(W)}$$

As  $T \rightarrow \infty$ , we can now write our optimal decision rule as

$$C^* = J'(W)^{\frac{1}{\gamma-1}}$$

$$f^* = -\left(\frac{\mu - r}{\sigma^2}\right) W \frac{J'(W)}{J''(W)}$$

Merton (1969) shows that the solution of  $J(W)$  with  $\frac{v^{\gamma-1}}{\gamma} W^\gamma$  allows us to solve optimal consumption and investment rules in the infinite time case if price changes follows a Geometric Brownian Motion and the marginal utility is  $U(C) = \frac{1}{\gamma} C^\gamma$  with

$$C_{\infty, t}^* = \left[ \frac{p}{1 - \gamma} - \gamma \left( \frac{(\mu - r)^2}{2\sigma^2(1 - \gamma^2)} + \frac{r}{1 - \gamma} \right) \right]$$

$$f_{\infty}^* = \left( \frac{\mu - r}{\sigma^2} \right) [1 - \gamma]$$



Because we assume that  $\mu$ ,  $\sigma$  and  $r$  are constants our optimal fraction  $f_\infty^*$  only depends on the risk aversion parameter  $\gamma$ . As we assume logarithmic utility, such that  $\gamma = 0$  the formula is simplified to

$$C_{\infty,t}^* = (p + r)W_t$$
$$f_\infty^* = \left(\frac{\mu - r}{\sigma^2}\right)$$

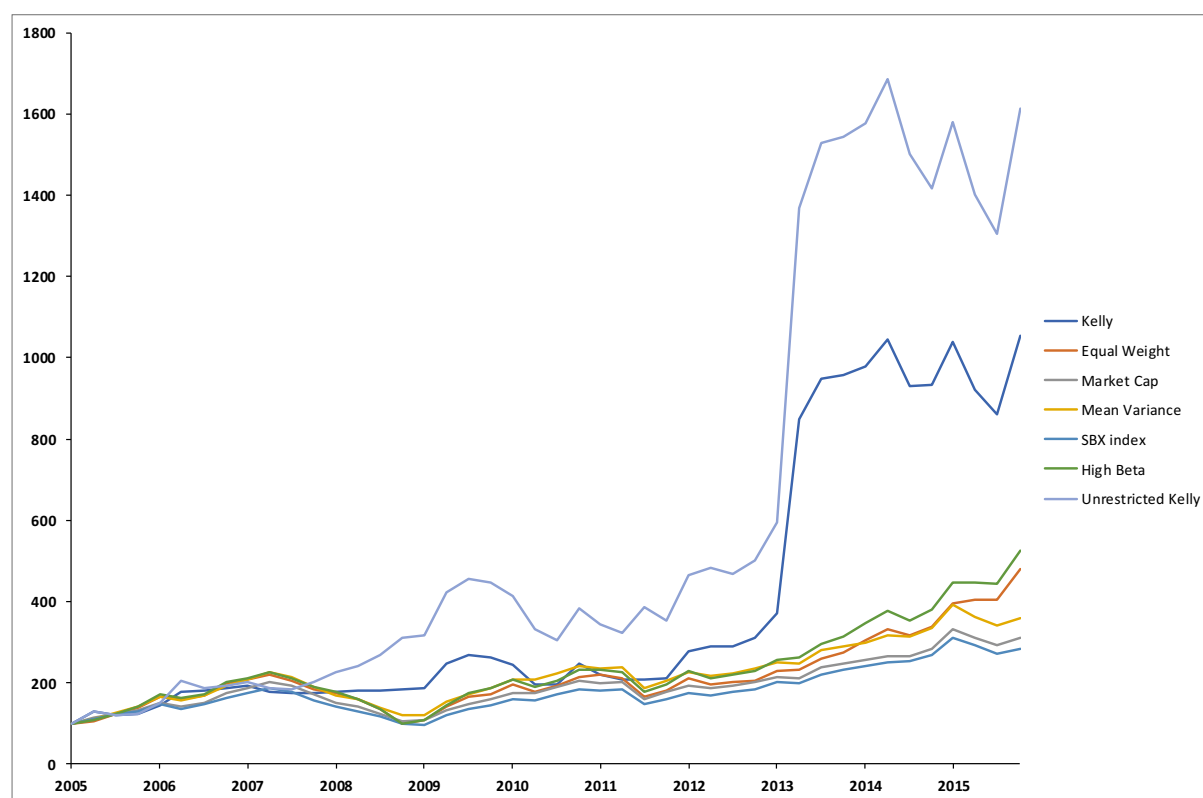
The formula above is equal to the formula we present in section 2.1.2; only this time we derive it another way.

## B. Proebsting's Paradox (Thorp 2008)

Due to the fact that we assume each of our bets are independent of one another, our method of rebalancing our portfolios is potentially subject to Proebsting's paradox (named after Todd Proebsting who discovered the paradox in a e-mail conversation with Thorp). To illustrate the intuition behind the paradox, suppose that you are offered a 2:1 bet with 50% probability. After this you are offered a second bet on the same premise but with 5:1 odds; the optimal Kelly allocation results in a 47,5% allocation of your bankroll when odds are 2:1, and 40% if you were offered the 5:1 bet. Hence, even though we are worse off in the first case, the allocation fraction is larger, which illustrates the paradox.

This problem arises due to the concave shape of the Kelly function and is the only potential way a Kelly bettor ever risks the entire wealth; normally one should include a correlation factor, which we do not, to solve this. Instead, as argued in the text, we solve it by avoiding string bets by rebalancing in discrete time intervals (i.e. quarterly).

## C. Approximation of the Unrestricted Kelly Portfolio



**Figure 14:** *The Kelly Portfolio Without Restrictions.* Figure 14 depicts our attempt to show Kelly returns for a portfolio, which does not have the leverage and short selling restriction we impose. Data is retrieved from Bloomberg for the period 2005 to 2015.

This time, when the Kelly formula indicates we should short sell the portfolio, we short SBX instead. We do this by taking the inverse return to that of the SBX due to simplicity. Additionally, during periods when we should leverage we simply multiply the factor by which we should leverage with the return for the Kelly portfolio that period (the strategy leverages six times, maximum 1,5 times the portfolio and minimum 1,04 times). The removal of our restrictions has a great positive effect on Kelly returns; cumulative return for the period 2005 to 2015 now amounts to 1512% (instead of the initial 953%). This underlines the relative strength of a Kelly strategy. We should stress, though, that the calculations in Figure 14 are *very* arbitrary since there are several factors, for example the cost of borrowing, which we do not consider. Also, we do not apply short selling on the Kelly portfolio; instead we use the SBX for simplicity, which most certainly does not reflect the same results. The intuition behind figure 14 is to give the reader a general picture of how the Kelly portfolio would perform without the restrictions.