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Is there Spatial Dependence in Grading between Upper Secondary Schools in Sweden?

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Abstract

This paper suggests that there is spatial dependence in grade inflation between upper secondary schools in Sweden by providing evidence of grade inflation in the courses of Swedish and Mathematics. The findings indicate that if neighboring schools inflate their grades by roughly 10 percent, the own schools grade inflation will increase by roughly 1.3 percent in Mathematics and 0.8 percent in Swedish. No findings of spatial dependence in grade inflation are made in the course of English. The quality of the Swedish school market has long been under scrutiny and grade inflation has remained a subject of concern. Research has found evidence of grade inflation on the Swedish school market, but none have previously carried out the investigation by accounting for spatial dependence in grade inflation by neighboring schools. This study contributes to the literature by investigating the link between grade inflation and spatial competition using spatial econometrics. School specific data of the years 2012-2016 is used.

Keywords: Spatial dependence, Grade inflation, Spatial Durbin model, Spatial weight matrix

Acknowledgements

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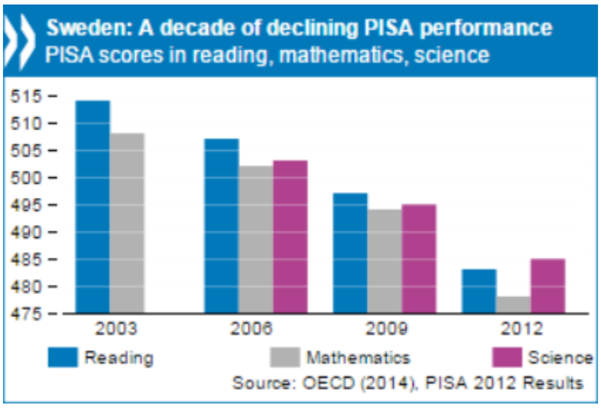
1 Introduction

This paper investigates if competition between neighboring schools in local school markets seems to be a driving cause of grade inflation in Sweden. The liberalization of the Swedish school market during the early nineties had the objective of promoting increased competition in order to yield higher quality within schools (Swedish National Agency of Education 2012). Sandström & Bergström (2005) show that increased competition in Sweden has resulted in higher grades. However, at the same time as the results in Swedish schools have increased, students have long shown deterioration in results of the international PISA tests. OECD states that the Swedish school system is in need of a new school reform to insure the quality and capacity of the schools (OECD 2015).



Graph 1, PISA statistics from OECD (2014)

The counterintuitive results between the developments of grades versus the results of the PISA tests might be due to the fact that grades have been exposed to grade inflation. This would imply that the liberalization of the Swedish school market might in fact have created an illusion of improved quality. Wikström and Wikström (2005) argue that a privatized school market ought to lead towards a more competitive environment. They argue that this could in turn create incentives for schools to inflate grades in order to attract future students. They further argue that the Swedish grading system is a fairly unique system due to the fact that it is decentralized. As the teachers set the grades locally, further incentives of grade inflation may be created. Research from Vlachos (2010), Wikström & Wikström (2005) and Pedersen (2016) finds selective evidence, particularly in independent schools, of competition or privatization leading to grade inflation in Sweden.

This paper aims to analyze whether Swedish upper secondary schools compete through grade inflation by accounting for spatial dependence between neighboring schools. Spatial dependence can be defined as a situation where the values of one location depend on the values of a neighboring location (LeSage and Pace 2009). The level of grade inflation chosen could thus act as a best reply function of a specific school. The best reply function would in turn determine what level of grade inflation that is optimal for a school; given the level neighboring schools have chosen. This paper contributes to the current literature by applying spatial econometrics in order to account for the potential of spatial dependence in grade inflation that may exist between schools. This is a new approach compared to previous work where both public and private schools are considered competitors and the aim is to account for local markets by allowing for spatial dependence between observations. LeSage (2008) argues that if one assumes that observations are independent from one another, which is common in regular econometrics, the estimates may become biased and inconsistent. This could occur when observations are not independent. This is due to the fact that if the spatial dependence in variables is ignored, the model may become incorrectly identified. Explanatory variables might become upwardly biased and misleading results could be generated. This paper thus claims that it is of essence to investigate the potential of spatial dependence on the Swedish school market. It is important in order to offer further knowledge and intuition of the structure of grade inflation and in attempt to help identify the model and find the true causes of grade inflation on the Swedish market.

A large sample of school level data from roughly 650 public and independent schools covering the years of 2012-2016 is used. The sample covers several explanatory variables and the grades received in the courses of Swedish, English and Mathematics in Swedish upper secondary schools. The samples differ slightly between courses and are based on availability. Parts of the data is collected through the database *SIRIS*, which is provided by the Swedish National Agency of Education. Pedersen (2016) furthermore provides additional parts of the data covering the years of 2012-2014. The measure of grade inflation is calculated by taking the difference between the grades received in a course as compared to the grade given on the Swedish national tests. The share of deflated grades is subtracted from the share of inflated grades given in the specific courses in order to yield the share of inflated grades given in a specific school.

Our findings suggest that spatial dependence in grading is present in the courses of Swedish and Mathematics on the Swedish upper secondary school market. No evidence of spatial dependence in grade inflation can be determined in the course of English. This might however be explained by the fact that the overall mean level of grade inflation is very low in the course of English suggesting that English may not be a subject to grade inflation overall.

1.1 Background

The Swedish school system is fairly unique since privately owned schools are allowed to operate on the same market as the public schools under the same conditions (Swedish National Agency of Education 2012). A key ingredient is the voucher system, where the number of students enrolled determines a school's budget. Both public and private schools get their funding from the voucher system, private schools are not allowed to charge any fees and they thus compete under the same conditions. This was intended to result in higher enrollment in schools with high quality and thus shift resources to these schools. Research from Black and Machin (2011) suggests that households value the quality of a school greatly and that parents have a willingness to pay additional fees to make sure that their children attends a high performing school. However, since the quality of a school is difficult to observe, many base the decision of their school of choice on the average level of grades in that school (Vlachos 2010). However, if grade inflation varies across schools it likely becomes more difficult to distinguish a high quality school from a poor quality school. Grade inflation has negative long-term effects for the students whom have not gotten their grades inflated. Diamond and Persson (2016) argue that students who get inflated grades at the age of 15 get higher grades in upper secondary school and higher wages at age 23. Furthermore, the selection into universities will benefit students who have gotten inflated grades, resulting in the acceptance of wrong individuals. Students thus have a major incentive to contend a well-functioning system; hence they ought to care that the correct individuals should be rewarded for the hard work put into receiving good grades. The wellbeing of the school system is thus important in many aspects, to insure the competence of individuals graduating in the country, to insure the quality of the schools on the market and the quality of grades between schools (Vlachos 2010).

2 Literature review

2.1 Literature on grade inflation in Sweden

Wikström and Wikström (2005) investigate if the implementation of allowing private schools to operate on the Swedish market seem to inflate grade levels. The authors use data from Swedish students graduating in 1997. The measure of grade inflation is composed by the difference in the grade given in a specific subject and the grade given in the Swedish Scholastic Aptitude Test, the so-called SWEsat, as a measure of grade inflation. The authors find very slim and selective evidence of grade inflation, but argue that their overall findings indicate that no grade inflation due to competition is present on the market. The main findings of Wikström and Wikström's research are however a strong evidence of grade inflation in private schools as opposed to public schools. A drawback of the study is that the investigation solely covers students graduating 1997 when the possibility of privatizing Swedish upper secondary schools was still in its initiation process. As of today the privatization of the school market is more widely spread and the market may be a more competitive one.

Vlachos (2010) makes a thorough investigation of the competitive structure in the Swedish school market and the potential effect of competition on grading. Vlachos argues that since the competition increased during the latter years, it is of essence to investigate if this increase seems to yield greater incentive of grade inflation. He uses data spanning the years 2003-2008 to investigate graduating students from upper secondary schools or middle schools. Vlachos measures competition by accounting for the share of students attending an independent school within a municipality. He considers the public schools as a non-competitive unit. Vlachos finds somewhat inconsistent evidence of grade inflation. He argues that during the last 10-15 years the increased level of competition has affected the grades, but that the latter years suggests a downturn in the level of grade inflation. He however argues that competition has a significant effect on grade inflation. Vlachos also finds that if a school's level of enrollment increases by 10 percent the grades seem to inflate by around 1 or 2 merit points. Thus arguing that schools with larger market share tends to be more willing to inflate grades. He does in line with Wikström and Wikström (2005) argue that private schools seem to be somewhat more generous when it comes to grades overall. He however argues that this difference is only slight, specifically during the earlier years of the time span.

Both studies displayed above used the number of students enrolled at independent schools per municipality as a measure of competition; this is a measure of some problematic features. Misra et al. (2012) claim that the more common measures of competition, as the Herfindahl-Hirschman index or the number of students enrolled in private schools for instance is insufficient in the market of education. They argue that number of students enrolled in a specific school (i.e. the market share of a school) is solely one of the conditions of school efficiency and that the school size likely correlates with many aspects other than competition. This other aspects could for instance be characteristics of municipalities etc. Vlachos (2010) and Wikström and Wikström (2005) furthermore ignore the fact that public schools also compete with each other in the voucher system. A municipality with many independent schools doesn't necessarily mean that competition is higher compared to municipalities with few independent schools. Their measure of competition thus likely becomes underestimated since it seems fairly unlikely that public schools are not affected by the competitive nature of the Swedish school market. These studies furthermore ignore the impact that distances between schools may have when it comes to the competitiveness between them. In other words, the fact that school markets may be smaller or greater than a municipality is not accounted for. Misra et al. (2012) argue that it is surely important to account for the distance between schools in order to define an educational market. This might be seemingly important in the Swedish school market since the distances and area size of the municipalities differ substantially depending on location. In some municipalities students are not able to commute through the entire municipality. Furthermore, schools that are close to one another should have higher incentives to compete with one another as compared to schools where the distance is large. As the commuting time to a school is a factor which students care about when selecting schools, this should preferably be accounted for.

Pedersen (2016) investigates if Swedish upper secondary schools compete through grade inflation. He makes use of the research from Misra et al. (2012) to create a similar competition index that takes distance into consideration. In line with Vlachos (2010) and Wikström and Wikström (2005), Pedersen solely finds selective evidence of grade inflation, but strong evidence of differences in grade inflation between private versus public schools. His measure of competition is likely a better fit as opposed to previous research since it accounts for distance between schools. The measure consists of number of students enrolled and he weights neighboring competitors by accounting for the distances between them in a range of 60 km. Nevertheless, as Pedersen puts his main emphasis on the number of students enrolled as a main driver of competition he ignores the possibility of spatial dependence in factors other than the measure of competition. He does not account for spatial dependence in grade inflation and other possible explanatories as he solely accounts for the distance in his index of competition. However, if grade inflation is not merely driven by competition due to school size and rather by other spatially dependent factors as well, his estimates may be biased and inconsistent. His model may suffer from spatially omitted variable bias and might not be fully identified. Pedersen's measure of competition may furthermore not be optimal if factors other than school size affect the competitive level on the school market. Misra et al. (2012) stated that (even though using the measure themselves) using the size of schools does not necessarily give a full explanation of the driving forces behind competition. This would likely apply even though the distance between schools have been incorporated. In other words, if school size is not a sole driver of competition, Pedersen's findings may be underestimated or inconsistent. His measure of competition may not tell the full story of competition and by that manner the grade inflation on the Swedish upper secondary school market. Even though his investigation likely is an improvement as compared to the previous research, this paper argues that it still is of essence to try to identify the model of grade inflation more thorough. It is seemingly important to investigate the potential effect of spatial dependence in grade inflation and other explanatory factors, as this has never been carried out before. This thus creates an incentive to make use of spatial econometrics in order to help find the true causes of grade inflation.

3 Theory and theoretical model

3.1 The theory of the Hotelling model

The theoretical framework used in this paper stems from the Hotelling's model of oligopolistic competition. Hotelling (1929) studies how price competition is affected by geographical differentiation. However, in the school market, producers don't compete through pricing, but rather through quality. Schools located nearby one another therefore presumable have higher incentives to compete with each other by offering high quality schooling, in order to attract potential students who are looking to attend any of the schools in the area.

3.2 A model of quality competition

Stennek (2017) provides an intuitive outlay of how Hotellings model can be used to study quality competition. To start describing the model in a simple manner one assumes that schools are competing in true quality (as opposed to grade inflation). A straight line represents a street in a town. At each end of the street, a firm or in this case a school, is offering services. Potential students are living along the street and have a cost of travelling. If the schools were to offer the same quality of schooling, they will gain half of the students living at the part of the street located closest to their school. If only two schools are present in a specific area, aspiring students living nearby are looking to maximize their utility by choosing school A or B. The utility facing one student located at x when choosing school thus becomes:

$$U_{A,x} = V_A - tx$$

And for the individual choosing school B:

$$U_{B,x} = V_B - t * (D - x)$$

Where V_i is valuation of the schooling service in the eyes of the students, D is the distance between the schools, and t is the cost of transportation. The potential student will choose school A if:

$$x \leq \frac{D}{2} * \frac{V_A - V_B}{2 * t}$$

Which is defined as the point of the street where the individual is indifferent between the two schools. This expression shows that any individual located below this point will go to school A. The demand facing school A is:

$$q_A = M * \left[\frac{1}{2} + \frac{V_A - V_B}{2 * t * D}\right],$$

where M is the total number of students located on the street. Each schools demand is therefore dependent on the quality of the other school and the distance each student has to each school. This leads to the profit function:

$$\pi_A = (p - c \cdot V_A) * M * \left[\frac{1}{2} + \frac{V_A - V_B}{2 * t * D}\right] - \frac{\lambda}{2} * V_B^2$$

Where p is the voucher price given by the government and $c \cdot V_A$ is the marginal cost of serving more students (which is increasing in quality) and the term in brackets is the demand. The last term illustrates the scale diseconomies. One can thus see that the profit of the school is dependent on the number of enrolled students and the cost of the production, i.e. the costs of producing high quality. The school's profit is also dependent on the other school's quality. We can derive the best reply, i.e. the optimal quality one school should choose, by differentiating the profit function and solve for the quality of the school:

$$V_A = \rho * \left(\frac{p}{c} - t * D\right) + \rho * V_B$$

Where:

$$\rho = \frac{1}{2} * \frac{\frac{M}{D}}{\frac{M}{D} + \lambda * \frac{t}{c}} \in \left(0, \frac{1}{2}\right)$$

Is a measure of the scale diseconomies. The second term of the best-reply function distinguish that given the higher quality of the competitor, a higher quality is thus also optimal for the own school. The first term ($t * D$) shows that the further away schools are apart from one another the lower the quality of the school will be, given the other schools quality. Applying this logic when considering three neighboring schools will result in the following best reply function:

$$V_A = \rho_A * \left(\frac{p}{c} - \overline{D_A} * t\right) + \rho_A * d_{AB} * V_B + \rho_A * d_{AC} * V_C$$

Where $\overline{D_A}$ is the average distance between school A and its neighboring schools. The d_{AB} and d_{AC} is the relative densities of students located between the schools. The term of ρ_A is the measure of diseconomies of scale:

$$\rho_A = \frac{1}{2} * \frac{\frac{M_{AB}}{D_{AB}} + \frac{M_{AC}}{D_{AC}}}{\frac{M_{AB}}{D_{AC}} + \frac{M_{AC}}{D_{AC}} + t * \frac{\lambda}{c}} \in \left(0, \frac{1}{2}\right)$$

The best reply function shows that the optimal level of quality is positively related to the quality level of the neighboring schools. When adding another neighbor, we can see that the relative densities of students and the spatial dependence, ρ, determine the coefficients, which is the importance of diseconomies of scale.

3.3 Competition in grade inflation

Since the quality is difficult to observe, the school's best way to distinguish themselves from other schools is then by grades (Wikström and Wikström 2005). The schools thus need to try to affect the student's valuation of the school, in order to gain competitive strength. Aspiring students may have an incentive to choose the school that has the record or reputation of giving out the highest grades since the high grades acts as an enticement. The theoretical model sketched above will thus be used to study competition in grade inflation in this paper. In this case one needs to interpret the cost of quality as the teachers own distaste for corrupting the grade system. In particular, we estimate the best reply function above using the spatial Durbin model to observe the spatial dependence in grade inflation.

4 Methodology and data

This section will illustrate the empirical methodology of the thesis and provide descriptions regarding the data used in the empirical analysis.

4.1 Empirical Method

As shown in the Hotelling model, if schools compete through grade inflation to appear to have higher quality compared to nearby schools, the inflated grades may be spatially dependent. In other words, one school's grading is dependent on the grading of other nearby schools. If observations are spatially dependent on one another, this dependence must be accounted for. Otherwise, the estimations might give biased and inconsistent results due to the fact that the model may become incorrectly identified (LeSage and Pace 2009). In spatial regression models, each observation has a corresponding location or region. By applying spatial econometrics, one can argue that smaller “submarkets” is created, where neighboring schools compete for the same students, who all live nearby. Easily explained, two schools located in the same city center is likely in greater competition to one another as opposed to two schools located 300 km apart. This is due to obvious commuting reasons and the fact that students often tends to pick schools located close to their home. It thus seems to be of importance to take distance between schools into account. We account for distance by creating a so-called spatial weight matrix, where the coordinates of all the schools are used. Anselin et al. (2008) explains the spatial weight matrix as an indicator of the presence and strength of a link between the observations. In other words, the strength of the spatial dependence in grade inflation between schools. The matrix consists of non-negative distances between

schools and is zero on the diagonal in order to prevent a school from becoming defined as a neighbor to oneself (LeSage 2009).
The full symmetric spatial weight matrix is determined by:

$$W_{i,j} = \begin{cases} \frac{1}{d_{i,j}}, & If\ j\ is\ a\ neighbor\ of\ i \\ 0, & Otherwise \end{cases}$$

Where $d_{i,j}$ is the distance between school i and j. The spatial weight matrix is row normalized, which implies that the sum of each row is one. The choice of normalizing the weight matrix is supported by the theory section, where the weights are assigned based on relative distance rather than the absolute distance. Forteringham and Rogerson (2008) argue that row normalization yields better characters of theoretical interpretation and the possibility to compare weights since their values span between 0 and 1. They however emphasize the fact that by performing a row normalization the weights are altered somewhat. Since the weights all sum to 1, the same distance may therefore yield different weights depending on how many neighbors that are operating in a certain area. In other words, a school that solely has 2 neighbors will have neighbors with larger weights than a school that have 10 neighbors. We however argue that this is a reasonable outcome in our research, since the number of students is limited and with more selectable units the overall demand facing a school should decrease. Row normalization is thus applied in our spatial weight matrices. This results in each school j being assigned a weight of its effect on school i based on its relative distance, in order to obtain a weighted average of the nearby schools. In the spatial weight matrix, one can determine either a fixed number of neighbors to consider, or the range in which schools are to be considered neighbors.

As mentioned earlier, we will estimate the best-reply function described in the theory section. We apply the spatial Durbin model in order to observe the spatial dependence in grade inflation. We will estimate three additional spatial regression models as robustness tests. In Appendix B, the difference between these models are provided, along with the motivation to prefer the spatial Durbin model. The spatial Durbin model can be displayed as:

$$g_{i,t} = \alpha + y_i + s_i + X_{i,t}\beta + \rho W_{i,j} g_t + \gamma W_{i,j} X_t + u_{i,t} \qquad \text{(Durbin)}$$

Where $g_{i,t}$ is the grade inflation for school i at time t, α is the constant, y_i and s_i are time and school fixed effects, respectively. Time fixed effects could for instance cover outside shocks such as changes in budgets, increased wages due to regulations etc. School fixed effects could be the specific neighborhood the school is located at for instance. $X_{i,t}$ is a matrix of time-variant control factors with its coefficient β, such as the number of students, students with foreign background etc. $u_{i,t}$ is the i.i.d error term and $W_{i,j}$ is the spatial weight matrix. The ρ estimator observes the spatial dependence in the dependent variable. The value of ρ can be interpreted as the weighted average of neighboring schools' grade inflation effect on one school's grade inflation. That is, based on the relative distance, how much a school will inflate their grades if its competitors inflate their grades by a certain percent. The γ estimator observes the spatial dependence in the control variables. It shows how neighboring schools control variables affect one school's grade inflation. For instance, if the number of students enrolled in the neighboring schools' increases, how does one school respond by using grade inflation? As the estimator includes the spatial weight matrix, this becomes the weighted average of the neighboring schools.

The range of distances is determined while creating the weight matrices and several different distances are demonstrated. Ranges between 30-120 kilometers are tested when trying to determine which schools that could be in competition with one and other. The range of 60 km is a commonly used distance since it is associated with a commuting time of roughly one hour, which is reasonable to be assumed the threshold of what a student at an upper secondary school is prepared to travel (Misra et al. 2011). However, the findings in the current paper seems to suggest that an upper threshold level of 90 km could be more fitting for this data. The benchmark in the spatial weight matrix will thus have a range of 90 km. Schools outside this distance will not be considered a neighbor and is given a weight of zero. As this threshold level cannot be guaranteed to be the most fitting, a robustness test covering the other mentioned distances is conducted.

Numbers of neighbors can also be used as a threshold instead of distances when creating the spatial weight matrices. Using this type of threshold can however be misleading when one wants to investigate large number of neighbors in Sweden, since the distances between schools can vary a lot. For example, when including many neighbors in the northern regions of Sweden, the distances can be so large that regular commuting would not be possible. Using the same number of neighbors in Stockholm could on the other hand result in a walking distance between schools. It might however be useful when investigating only a small number of neighbors, as the distance between schools are likely to stay reasonable. Therefore, this paper will as further robustness tests make use of this type of matrix for 1 to 10 neighbors to observe the spatial dependence for the most nearby neighbors. The results will be displayed in Appendix C.

The spatial lag is by definition endogenous in the sense that it allows for interdependence between neighbors (Anselin 1988). Simply put, school *i* is affected by a neighboring school *j*, but school *j* is in that sense also affected by school *i*. Due to the symmetric nature of a spatial lag, OLS estimates suffers from simultaneity bias of the spatial parameter $\hat{\rho}$, inconsistent standard errors and should therefore not be practiced in a spatial setting. Azomahou and Lahatte (2000) conclude that OLS will yield biased estimates when used together with a spatial lag. Several authors argue that the maximum likelihood (ML) estimation is consistent for the spatial regression models. Work by Ord (1975), Lee (2004), Anselin and Bera (1998) and Hsiao et al. (2002) demonstrates that ML estimation remains consistent under the normality assumption. Furthermore, Hsiao et al. (2002) uses findings from a Monte Carlo study to argue that the ML estimator should be used in favor over IV and GMM since the ML estimator appears to have attractive finite sample properties, even when both N and T are quite small. In line with the findings mentioned above, this paper applies the ML estimation method. Standard errors are clustered on municipality level, as the municipalities are responsible for the schooling in their municipality.

4.2 Data

The data is provided by the Swedish National Agency of Education and is gathered through the database *SIRIS*. Pedersen (2016) moreover provides most of the data covering the coordinates and data of the years spanning 2012-2014. Due to the overall nature of availability, school specific panel data spanning over the years 2012-2016 is used. In order for spatial estimations to be executed, the use of panel data requires perfectly balanced data. The data has therefore been “cleaned” in the sense of yearly gaps (De Hoyos et al. 2006). Even though the majority of the observations are present in all of the years, the restoration of the data will generate a loss of some observations. Furthermore, data for courses with fewer and 10 students are not provided by *SIRIS* and will therefore be excluded. The implications this missing data will have on the results is unclear.

A list of variables that are used and their descriptive statistics are provided in *Table 1* below.

Table 1: Descriptive statistics

	Swedish					Mathematics					English				
Variable	**Obs.**	**Mean**	**Std. Dev.**	**Min.**	**Max.**	**Obs.**	**Mean**	**Std. Dev.**	**Min.**	**Max.**	**Obs.**	**Mean**	**Std. Dev.**	**Min.**	**Max.**
Dependent variable															
Grade Inflation	3 270	.194	.167	-.533	.917	3 160	.298	.185	-.316	1	3 265	.053	.163	-.619	.889
Independent variable															
Parents' Education	3 263	.454	.162	.08	.93	3 155	.461	.159	.12	.93	3 262	.453	.162	.08	.96
Foreign Background	2 895	.245	.159	.01	.95	2 836	.244	.159	.02	.95	2 879	.243	.156	.02	.95
Teacher's Degree	3 228	.727	.176	.125	1	3 119	.734	.174	.116	1	3 223	.725	.177	.116	1
Number of Students	3 270	50.848	37.023	8	336	3 160	52.735	37.482	10	336	3 265	51.186	37.101	10	336
Number of Teachers	3 232	27.384	24.491	1.3	283.3	3 123	28.540	25.165	1.3	283.3	3 227	27.514	24.514	1.3	283.3
Students per Teacher	3 230	12.689	3.808	1.264	42.5	3 121	12.895	3.785	1.264	42.5	3 225	12.637	3.789	1.264	42.5
Independent Share	3 270	.393	.273	0	1	3 160	.379	.263	0	1	3 265	.391	.279	0	1

As can be seen in the table above there is a fairly large difference in the means of the variable grade inflation between the subjects. Grade inflation is calculated by subtracting the share of students who get a lower grade on the course compared to the national test from the share who gets a higher grade. In Swedish and Mathematics the mean is relatively large at around 20 and 30 percent respectively. In the subject of English on the other hand the mean is only around 5 percent. One can thus see that the level of inflated grades in the course of English is slim. Some of the control variables are included based on similarity of the variables included in literature from Wikström & Wikström (2005), Vlachos (2010) and Pedersen (2016). This is mainly due to the possibility to compare findings but also due to the nature of availability and explanatory relevance. Research has suggested that parent's education and foreign background pays a valid role when it comes to the future level of education for the child and thus possess reason to be included in the model. Research has also found that the size of a school and the school independency matters when it comes to grade inflation and for that matter these variables are included. Since a variable of independent schools is likely not to vary over time we use the variable *Independents share*, which is an interaction term of independent schools and number of students. The variable indicates the share of students in a municipality attending an independent school. By this mean we evade the dilemma of independent schools being time invariant since the number of students attending those schools will vary over time. Furthermore, the variable *Independents Share* is not included in the weight matrix. This is due to the fact that the weight matrix is a distance-based matrix where coordinates are used to arrange the area of 90 km. The matrix does not account for municipality borders, so by including it in the weight matrix the result might become misleading and is not applicable in our data. Students per teacher is included since it seems of interest to investigate if teachers with fewer number of students tend to inflate grades more or less. One could argue that a teacher with fewer students may have the possibility to give a more correct grade, but having fewer students may potentially also alter the level of grade inflation due to emotional reasons. The variable of Teacher's degree is included in order to catch any potential differences between teachers with a degree versus teachers without.

As stated earlier, due to the nature of spatial econometrics and its requirement for a perfectly balanced panel dataset, certain measures to insure this were made. Otherwise the spatial estimations cannot be executed due to dimensional issues when creating the weight matrices (Belotti et al. 2016). Some of the control variables possessed a few missing observations. For instance, the control variables *number of teachers, number of teachers with a university degree,* and *number of students per fully employed teacher* all involved roughly 1.20-1.60 percent missing observations. The variable for *number of students with highly educated parents* had a percentage of missing observations of 0.16 percent. The control variable of number of *students with a foreign background* however had around 10.25 percent missing observations. All of these variables were imputed with the MI-imputation method in Stata. Belotti et al. 2016 argue that the MI imputation method replaces the missing values by generating multiple plausible values by using Monte Carlo simulations. The process thus accounts the values of the already existing data in the dataset and simulates potential values to fill in the missing values. This creates a matrix of potential values, which is accounted for in all of the estimations. Rubin (1987) created the multiple imputation method in order to deal with non-responses in survey gathered data. The method is fairly common in applied spatial econometrics when a dataset is not perfectly balanced (Belotti et al. 2016). Since the amount of imputed values is mostly quite minor in this paper, one could likely assume the effect of the imputation to be fairly small. Regarding the variable *children with a foreign background* on the other hand, one should perhaps take into consideration that 10 percent of the observations are in fact imputed. This could potentially alter the result somewhat, and it is hard to predict exactly what effect this could have.

One drawback of the data used in this paper is that it does not cover teacher specific characteristics. This could perhaps include data over characteristics of morality, behavioral aspects and personality aspects. One could also imagine that pressure from superiors could be a factor of a teacher's willingness to inflate grades. Teacher specific data could potentially be harmful for specific individuals, which does not match the aim of this paper. Research of this kind might overall be sensitive from an ethical perspective, since it causes critique towards actors of the market. However, as this paper use school specific data and no specific school are mentioned in the findings, it's unlikely to cause harm to any specific school. Since individual data of the students is not used, no linking to a specific individual can be made either. This paper is merely looking to investigate the market as a whole, since the quality of the school market possesses great importance in society. By protecting the integrity of specific schools and individuals our research is likely not causing any major ethical discrepancies.

5 Empirical Results

5.1 Main results of the spatial estimators

Table 2 below displays the results of the variables effect on grade inflation from the spatial Durbin model yielded in the courses of Swedish, English and Mathematics. The results displayed are estimations from the use of a distance weight matrix of 90 km. A preliminary OLS is performed as a comparison measure; these results are displayed in Appendix C. The results of the other spatial models, i.e. the Mixed model, the SAR and the SEM estimations are also displayed in Appendix C.

Table 2: Spatial Durbin Fixed Effects Estimates

	Swedish	Mathematics	English
Main			
Number of Teachers	0.00485	-0.0192*	0.00830
	(0.0126)	(0.0113)	(0.0112)
Students per Teacher	-0.00315	-0.0310	0.0418**
	(0.0258)	(0.0301)	(0.0210)
Number of Students	-0.0237	0.0200*	-0.00000366
	(0.0146)	(0.0120)	(0.0131)
Parents' Education	0.0445	-0.0398	-0.0633
	(0.0932)	(0.112)	(0.0645)
Foreign Background	0.0690	0.115	0.0820
	(0.0653)	(0.0800)	(0.0558)
Teacher's Degree	-0.0563	-0.0118	0.0271
	(0.0429)	(0.0639)	(0.0397)
Independent Share	0.00771***	-0.00473*	0.000465
	(0.00185)	(0.00262)	(0.00174)
Time	0.0150***	0.0107***	-0.000814
	(0.00348)	(0.00348)	(0.00275)
Wx			
Number of Teachers	0.0521**	-0.0244	0.00661
	(0.0263)	(0.0278)	(0.0189)
Students per Teacher	0.0666*	0.0205	-0.0108
	(0.0360)	(0.0418)	(0.0402)
Number of Students	-0.0907***	0.108***	0.0509*
	(0.0235)	(0.0303)	(0.0268)
Parents' Education	0.234	0.0469	-0.122
	(0.219)	(0.236)	(0.185)
Foreign Background	0.142	-0.0303	-0.0658
	(0.146)	(0.123)	(0.116)
Teacher's Degree	-0.0341	0.0177	0.0474
	(0.117)	(0.134)	(0.106)
Spatial			
rho	0.0761***	0.133***	0.0110
	(0.0225)	(0.0232)	(0.0301)
Variance			
sigma2_e	0.0152***	0.0168***	0.0125***
	(0.000584)	(0.000690)	(0.000618)
Observations	3270	3160	3265

Standard errors in parentheses
Notes: Fixed effects estimates using data from 2012-2016.
Schools within range of 90km are considered neighbours.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01

In all subjects, $\hat{\rho}$ (the spatial dependence in grade inflation) is positive. However, statistical significance can only be seen in Mathematics and Swedish. In both Swedish and Mathematics, we find significance at a one percent level. The Durbin Model's estimates $\hat{\rho}$ of Mathematics and Swedish are 0.133 and 0.0799, respectively. The interpretation is that if the neighboring schools on average increase their grade inflation by 10 percent, the own grade inflation will increase with 1.3 percent in Mathematics and by around 0.8 percent in Swedish. Even though the magnitude is small, it is still economically significant as it affects many students. In the subject of English no significance is found in the spatial $\hat{\rho}$. This might be explained by the fact that the mean level of grade inflation is very low in this subject, as can be seen in table 1. In other words, since the mean level of grade inflation is low the course of English may perhaps not be a subject of grade inflation on the Swedish school market. The robustness tests displayed in Appendix C is coherent and supports the results from the Durbin model.

From the control variables, there seems to be a positive time trend in Mathematics and Swedish. This is indicated by the large significance of the variable of *Time* in both courses. This is however not confirmed in the course of English where the time trend variable is insignificant. It is however of essence to include the time variable in order to detect any potential time trends, in order to yield a more valid result. The spatial dependence of the variable *Number of students,* which may be interpreted as school size, is significant in all of the subjects. The significance indicate that the neighboring schools number of students affect the schools level of grade inflation. Our findings are however somewhat counterintuitive since in the courses of Mathematics and English suggest a positive effect on grade inflation, in other words that schools with a larger size tend to affect neighbors grade inflation more. The opposite findings are made in the course of Swedish, where larger schools affect neighbor's grade inflation less. One can see that the variable of *Independents Share* is significant in the course of Swedish. These findings are in line with Wikström and Wikström (2005) and Pedersen (2016), where they conclude that independent schools tend to inflate grades more compared to public schools. This is however not confirmed in the subjects of English and Mathematics. In the course of

English the variable of *Independents Share* remains positive but insignificant in all regressions. The findings in Mathematics are somewhat more disturbing since the effect is negative and is statistically significant at the ten percent level. The magnitude is small in the course of Mathematics, and one could argue they are not economically significant. In the subject of Swedish the variable *number of teachers* and *students per teacher* is significant on a 5 and 10 percent level respectively in the weight matrix. The variables thus seem to affect grade inflation. These findings are however not confirmed in the English and Mathematics, where the results of those variables are insignificant.[1]

Wald tests are performed in order to conclude the model of preference. All tests suggest that the SDM model is the preferred model in all of the courses, giving further evidence that the SDM model should be given the most emphasis.

6 Discussion and Conclusion

This paper investigates the effect of competition on grade inflation. By conducting the technique of spatial econometrics, we obtain the spatial dependence in grade inflation between schools. This offers help to distinguish how neighboring grade inflation affects a school's grade inflation. The results suggest that there is spatial dependence in grading in the courses of Swedish and Mathematics present on the Swedish upper secondary school market. That is, schools compete with each other by inflating grades and support the theory of quality competition. If a neighboring school inflates their grades by around 10 percent, the baseline school will inflate their grades by around 0.8-1.3 percent. We thus present findings suggesting that spatial dependence seems to be an

[1] When we compare the spatial Durbin model with the OLS regression we can see that statistical significance and magnitude of control variables change. In the OLS regressions, more control variables are statistically significant compared to the Durbin model. *Independent Share* and *Time* stays significant but the magnitude decreases in Swedish and Mathematics. In Swedish, *Teacher's Degree* is significant in the OLS but not the spatial Durbin model. In Mathematics, the variable *Number of Students* is only significant at the 10 percent level in the Durbin model, and the magnitude is decreased. The *Number of Teachers* becomes significant at the ten percent level. For English, the variables *Teacher's Degree*, *Students per Teacher* and *Number of Teachers* are significant at the ten, five and ten percent level, respectively. However, in the Durbin model, only the variable *Students per Teacher* stays significant at the five percent level. The significant variables in the Wx section of table 2 have a spatial effect on the dependent variable in the model and the dependence in grading between neighbors due to those variables can thus be concluded.

important aspect of grade inflation in Sweden and ignoring spatial dependence may result in biased results. As stated earlier no evidence of spatial dependence in grade inflation is found in the course of English. This is likely explained by the fact that schools don’t inflate grades in English to the same extent as in the subjects of Swedish and Mathematics. It could possibly have been an interesting addition to elaborate why English don’t seem to be a subject of grade inflation in Sweden, but due to time constraints this has not been further investigated in our paper.

In contrast to Misra et al. (2011), our findings further suggest that a distance of 90 -120 km seems to be preferred when constructing the spatial weight matrices, rather than 60 km. We find that the neighboring schools number of students enrolled has a significant effect on grade inflation. Though, the effect is not clear, as the effect is positive for Mathematics and English, and negative for Swedish. These findings contradict with Vlachos (2010), who found that schools own number of students’ affect grade inflation. They concluded that schools with more students enrolled inflated their grades more. We cannot add a clear reasoning to in what direction the number of students’ affect grades. It may however be the neighboring schools number of students’ who matters. We however argue that further research is needed to gather knowledge regarding in what direction this effect lies. In line with Wikström and Wikström (2005) and Pedersen we found that share of independent schools in a municipality positively affects grade inflation in Swedish. These findings are however not significant in English and Mathematics in the Spatial Durbin model and we suggest that further investigation of the matter is at hand.

An interesting addition to our study could have been the use of a spatial IV regression in order to help find the true causality, but due to lack of suitable data for the use of an instrument variable this could not be performed. However, given the normality assumption, Hsiao et al. (2002) argues from findings of a Monte Carlo study that the ML estimator should be used in favor over IV, GMM since the ML estimator appears to have attractive finite sample properties even when both N and T are quite small. The use of a spatial IV would thus potentially not have yielded more fitting results, but it would nevertheless have been a good addition for comparison measures. Some limitations of this study are the lack of teacher specific data, whereas one might argue that a teacher's individual characteristics may influence the teacher's willingness to inflate a grade. If teachers rather than schools in fact drives the grade inflation, our results will be biased. It would be of great interest for further research to investigate differences in grade inflation between teachers. Another drawback of our research is that the data is only available between the years of 2012-2016. ML estimation tends to be more consistent when a sample size is larger and more years added to the data would have been preferable. These are limitations whose potential effect is hard to determine in beforehand.

In conclusion, our results suggest that there is a spatial dependence in grade inflation on average and that schools thus compete by inflating grades. Our findings indicate that ignoring spatial dependence results in biased results. We also find that neighboring schools' number of students affects grade inflation rather than own schools number of students.

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Appendix A

Diagram 1: Distribution of Grade inflation in English

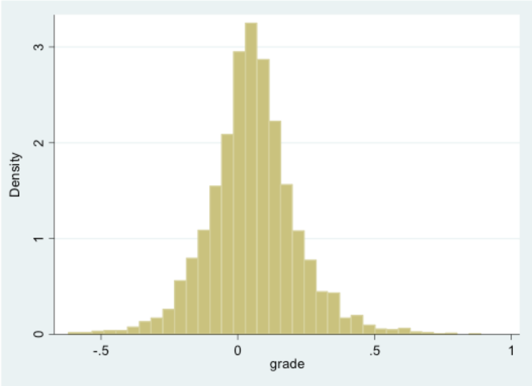


Diagram 2: Distribution of Grade inflation in Mathematics

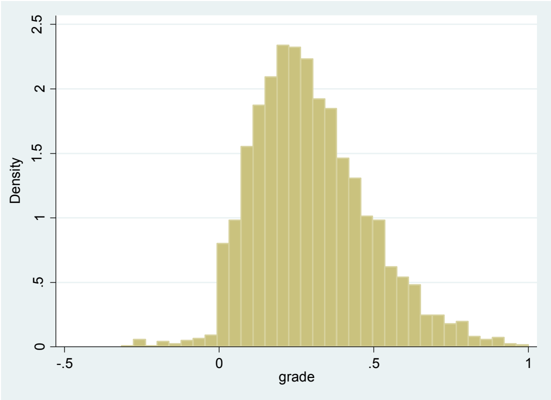
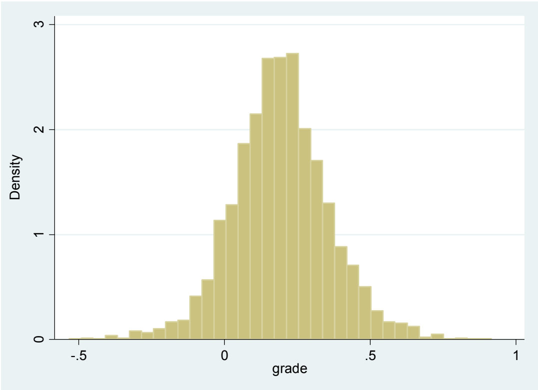


Diagram 3: Distribution of Grade inflation in Swedish



Appendix B

Appendix B describes each spatial model used and their differences. A motivation for the choice of using the spatial Durbin model as main model is also provided.

The simplest and most naive model of spatial econometrics is the spatial autoregressive model (SAR). SAR only assumes spatial dependence in the dependent variable:

$$g_{i,t} = \alpha + y_i + s_i + X_{i,t}\beta + \rho W_{i,j} g_t + u_{i,t} \qquad \text{(SAR)}$$

If the dependent variable is spatially dependent, other factors may also be spatially dependent such as the share of students with foreign background. Therefore, SAR might be biased and inconsistent. In contrast, the spatial error model (SEM) assumes spatial dependence in the error term. That is, SEM controls for the spatial dependence in the error term:

$$g_{i,t} = \alpha + y_i + s_i + X_{i,t}\beta + \lambda W_{i,j} \eta_t + u_{i,t} \qquad \text{(SEM)}$$

Where the difference from the SAR model is the $\lambda W_i \eta_t$ term. The λ estimator observes the spatial dependence in the error term. It thus observes the weighted average of neighboring schools' factors in the error terms effect on grade inflation. As SEM only observe the spatial dependence in the error term, one cannot know exactly which factors that affects grade inflation. As the SAR model is naive in its way, the SEM model is naive in the opposite way. As the error term affects the dependent variable and controls for spatial dependence in the error term, it is plausible there is spatial dependence in the dependent variable. The SEM model is then inconsistent and biased.

More sophisticated models which nests SAR and SEM has been developed in order to solve this dilemma. The spatial Mixed model (SAC) combines SAR and SEM in a straightforward way. The spatial Durbin model (SDM) nests SAR and SEM by including spatial dependence in the control variables:

$$g_{i,t} = \alpha + y_i + s_i + X_{i,t}\beta + \rho W_{i,j} g_t + \lambda W_{i,j} \eta_t + u_{i,t} \qquad \text{(Mixed)}$$

$$g_{i,t} = \alpha + y_i + s_i + X_{i,t}\beta + \rho W_{i,j} g_t + \gamma W_{i,j} X_t + u_{i,t} \qquad \text{(Durbin)}$$

As the Mixed model combines SAR and SEM, the spatial dependence in the dependent variable is observed in the ρ estimator and the spatial dependence in the error term in the λ estimator. If the explanatory variables do not make a material contribution in explaining the dependent variable, the Mixed model cannot identify the spatial dependence. The γ estimator in the Durbin model captures the spatial dependence in the explanatory variables. By including the spatial dependence in the explanatory variables, the spatial correlation in the omitted variables is accounted for. The assumption is that the spatial structure in the omitted variables is of the same structure as the explanatory variables (LeSage and Pace 2009). This further suggests that it ought to be reasonable to assume that if one is not completely certain that the model is perfectly identified, the Durbin model should be preferred. The model of choice thus depends on the true spatial structure, which is unknown. If the true spatial structure is that of the Durbin model, the Mixed model will produce biased and inefficient estimates of the spatial dependence due to the exclusion of the spatial dependence in the control variables. However, if the true spatial structure is that of the Mixed model, the Durbin model will still produce unbiased and consistent but inefficient estimates (LeSage and Pace 2009). All the findings mentioned above motivate the use of the Durbin model whenever omitted variables are suspected to be present.

Appendix C

Tests of Robustness

As a test of robustness the distances are altered between 30, 60 and 120 km in order to distinguish what distance seems to be the best fit for the model. Furthermore, we show results on a specific number of neighbors in order to investigate the spatial dependence when only accounting for few neighbors. Table 3 shows the results of the spatial $\hat{\rho}$ in Mathematics, Swedish and English, respectively.

Table 3: Results Spatial $\hat{\rho}$ Robustness tests

Km and number of neighbours	Spatial $\hat{\rho}$ Swedish	Spatial $\hat{\rho}$ Mathematics	Spatial $\hat{\rho}$ English
30 km	0,0519***	0,0866***	0,00732
60 km	0,0737***	0,118***	0,0184
120 km	0,0939***	0,133***	0,0133
Without Independet Share	0,0839***	0,137***	0,0094
1 Neighbor	0,0108	0,0329**	0,00235
5 Neighbors	0,0292*	0,0967***	0,00193
7 Neighbors	0,0406**	0,106***	0,00467
10 Neighbors	0,0481***	0,111***	0,00454

The robustness tests are in line with the benchmark Durbin model in all three subjects. In Swedish and Mathematics, the spatial $\hat{\rho}$ is statistically significant in all specifications. The signs are coherent but the magnitude differs somewhat. In English, all specifications are statistically insignificant and all signs are positive. The spatial $\hat{\rho}$ and the t-values tends to shrink when the distance are made smaller in all of the courses. It thus seems like a larger distance is to be preferred since the values of $\hat{\rho}$ and t increase.

Matrices with number of neighbors rather than distance are also used for comparison measures. We estimate the SDM model using 1, 5, 7 and 10 neighbors. From the table 5 one can determine that the spatial $\hat{\rho}$ becomes significant when including 5 neighbors in the course of Swedish. In Mathematics $\hat{\rho}$ is significant even for one neighbor and it is insignificant in all specification in English. When using "number of neighbor" matrices it is not certain that the distances stay within reason, one should therefore be somewhat careful when interpreting these results. What we can say is that all schools have at least one neighbor within the range 120 km. In line with the findings in the results from the distance-based matrices the $\hat{\rho}$ and the t-values (found in Appendix B) increases by number of neighbors. This further confirms the decision of using a larger distance in our main model. The same findings are shown in the Mathematics course, where the spatial dependence increases when number of neighbors is increased. In the subject of English the spatial dependence increases by numbers of neighbors but is as stated before insignificant. Comparing the specification without the *Independent Share* variable to the main model, we can see an effect on the spatial dependence. Ignoring the independent school's presence on grade inflation increases the spatial $\hat{\rho}$ and since it is significant in Swedish and parts of the regressions in Mathematics it seems reasonable to include it in the model. Leaving it out of the model may thus inflate the spatial $\hat{\rho}$ somewhat due to omitted variable bias. The results from the OLS, SAR, SEM and Mixed models are displayed below along with each regression for the different specifications in the weight matrix.

Table B1: English Fixed Effects Estimates

	OLS	Mixed	SAR	SEM
main				
Number of Teachers	0.0174*	0.00838	0.00841	0.00834
	(0.00975)	(0.0112)	(0.0112)	(0.0112)
Students per Teacher	0.0481**	0.0407**	0.0408**	0.0407**
	(0.0204)	(0.0207)	(0.0207)	(0.0207)
Number of Students	0.00111	0.00455	0.00417	0.00399
	(0.0120)	(0.0137)	(0.0135)	(0.0136)
Parents' Education	-0.0666	-0.0553	-0.0547	-0.0544
	(0.0777)	(0.0631)	(0.0634)	(0.0635)
Foreign Background	0.0947	0.0874	0.0875	0.0876
	(0.0615)	(0.0539)	(0.0539)	(0.0539)
Teacher's Degree	0.0643*	0.0312	0.0315	0.0313
	(0.0386)	(0.0412)	(0.0413)	(0.0412)
Independent Share	-0.000804	-0.000469	-0.000391	-0.000358
	(0.00291)	(0.00182)	(0.00177)	(0.00176)
Time	-0.00110	-0.00351*	-0.00348*	-0.00358*
	(0.00204)	(0.00198)	(0.00196)	(0.00202)
Spatial				
rho		0.0185	0.0240	
		(0.0330)	(0.0276)	
lambda		-0.00948		0.0223
		(0.0392)		(0.0288)
Variance				
sigma2_e		0.0157***	0.0126***	0.0126***
		(0.000627)	(0.000628)	(0.000628)
Observations	2836	3265	3265	3265

Standard errors in parentheses
Notes: Fixed effects estimates using data from 2012-2016.
Schools within range of 90km are considered neighbours.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01

The results of all spatial models are coherent. The spatial $\hat{\rho}$ is not significant in any of the models. Not surprisingly, the magnitude of the SAR model is larger than that of Durbin and Mixed, as it only assumes spatial dependence in the dependent variable.

Table B2: Mathematics Fixed Effects Estimates

	OLS	Mixed	SAR	SEM
main				
Number of Teachers	-0.0149	-0.0166	-0.0161	-0.0158
	(0.0108)	(0.0105)	(0.0105)	(0.0107)
Students per Teacher	-0.0291	-0.0248	-0.0243	-0.0256
	(0.0233)	(0.0302)	(0.0305)	(0.0309)
Number of Students	0.0313**	0.0229*	0.0224*	0.0197
	(0.0125)	(0.0117)	(0.0118)	(0.0120)
Parents' Education	0.0135	-0.0767	-0.0743	-0.0738
	(0.0896)	(0.108)	(0.108)	(0.107)
Foreign Background	0.0899	0.0569	0.0579	0.0590
	(0.0743)	(0.0680)	(0.0686)	(0.0697)
Teacher's Degree	0.0317	0.00300	0.000371	-0.000564
	(0.0458)	(0.0657)	(0.0658)	(0.0660)
Independent Share	-0.00748**	-0.00728***	-0.00643**	-0.00524*
	(0.00353)	(0.00250)	(0.00252)	(0.00286)
time	0.0104***	0.00885***	0.00854***	0.00988***
	(0.00236)	(0.00279)	(0.00288)	(0.00323)
Spatial				
rho		0.146***	0.155***	
		(0.0324)	(0.0230)	
lambda		-0.0390		0.153***
		(0.0261)		(0.0237)
Variance				
sigma2_e		0.0213***	0.0170***	0.0170***
		(0.000704)	(0.000709)	(0.000710)
Observations	2794	3160	3160	3160

Standard errors in parentheses
Notes: Fixed effects estimates using data from 2012-2016.
Schools within range of 90km are considered neighbours.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01

The spatial $\hat{\rho}$ is statistically significant in all models. The magnitude ranges from 0,133-0,155 where Durbin is the lower and SAR the highest. It is reasonable that Durbin has the lowest since it controls for more spatial dependence compared to the Mixed model and SAR.

Table B3: Swedish Fixed Effects Estimates

	OLS	Mixed	SAR	SEM
main				
Number of Teachers	0.00726	0.00575	0.00562	0.00474
	(0.0104)	(0.0128)	(0.0128)	(0.0129)
Students per Teacher	0.00961	-0.00203	-0.00192	-0.00349
	(0.0219)	(0.0256)	(0.0256)	(0.0257)
Number of Students	-0.0194	-0.0272*	-0.0268*	-0.0257*
	(0.0124)	(0.0145)	(0.0145)	(0.0145)
Parent's Education	0.00886	0.0247	0.0227	0.0196
	(0.0852)	(0.0948)	(0.0944)	(0.0946)
Foreign Background	0.0362	0.0610	0.0600	0.0592
	(0.0686)	(0.0622)	(0.0622)	(0.0625)
Teacher's Degree	-0.136***	-0.0606	-0.0603	-0.0590
	(0.0418)	(0.0445)	(0.0448)	(0.0452)
Independents Share	0.00880***	0.00882***	0.00867***	0.00838***
	(0.00327)	(0.00193)	(0.00191)	(0.00201)
Time	0.0207***	0.0165***	0.0164***	0.0184***
	(0.00228)	(0.00251)	(0.00264)	(0.00277)
Spatial				
rho		0.0921***	0.0982***	
		(0.0352)	(0.0221)	
lambda		-0.0149		0.0912***
		(0.0345)		(0.0228)
Variance				
sigma2_e		0.0191***	0.0153***	0.0153***
		(0.000593)	(0.000591)	(0.000591)
Observations	2847	3270	3270	3270

Standard errors in parentheses
Notes: Fixed effects estimates using data from 2012-2016.
Schools within range of 90km are considered neighbours.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01

We can see a similar pattern in the spatial $\hat{\rho}$ as in the Mathematics regressions. It is statistically significant in all models and SAR has the highest magnitude.

Table B4: English Robustness Test

	120km	60km	30km	Independent
Main				
Number of Teachers	0.00830	0.00805	0.00726	0.00866
	(0.0113)	(0.0112)	(0.0113)	(0.0113)
Students per Teacher	0.0406**	0.0399*	0.0401*	0.0408**
	(0.0204)	(0.0204)	(0.0206)	(0.0204)
Number of Students	-0.000628	0.000346	0.00211	0.000144
	(0.0135)	(0.0133)	(0.0132)	(0.0132)
Parents' Education	-0.0587	-0.0590	-0.0623	-0.0580
	(0.0650)	(0.0648)	(0.0643)	(0.0647)
Foreign Background	0.0886	0.0897*	0.0875	0.0880
	(0.0538)	(0.0536)	(0.0534)	(0.0537)
Teacher's Degree	0.0313	0.0324	0.0316	0.0318
	(0.0402)	(0.0402)	(0.0402)	(0.0401)
Independent Share	0.000569	0.000507	0.000186	
	(0.00170)	(0.00178)	(0.00178)	
Time	-0.000823	-0.00190	-0.00170	-0.00148
	(0.00298)	(0.00273)	(0.00240)	(0.00301)
Wx				
Number of Teachers	0.000810	0.0104	0.0263	0.00570
	(0.0200)	(0.0186)	(0.0170)	(0.0192)
Students per Teacher	-0.00204	-0.0111	-0.00663	-0.0140
	(0.0393)	(0.0350)	(0.0317)	(0.0397)
Number of Students	0.0664**	0.0516*	0.0210	0.0523*
	(0.0292)	(0.0267)	(0.0246)	(0.0272)
Parents' Education	-0.127	-0.0740	-0.0487	-0.103
	(0.196)	(0.182)	(0.150)	(0.191)
Foreign Background	-0.00735	0.0187	0.00744	-0.0151
	(0.156)	(0.132)	(0.108)	(0.150)
Teacher's Degree	0.00514	0.0529	-0.00196	0.0451
	(0.101)	(0.0986)	(0.0792)	(0.108)
Spatial				
rho	0.0133	0.0184	0.00732	0.00941
	(0.0315)	(0.0298)	(0.0262)	(0.0303)
Variance				
sigma2_e	0.0125***	0.0125***	0.0125***	0.0125***
	(0.000619)	(0.000617)	(0.000620)	(0.000619)
Observations	3265	3265	3265	3265

Standard errors in parentheses
Notes:Durbin model with fixed effects using data from 2012-2016.
In the Independent model 90km is used and excluding Independent Share variable.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01

Table B5: Mathematics Robustness Test

	120km	60km	30km	Independent
Main				
Number of Teachers	-0.0179	-0.0170	-0.0169	-0.0167
	(0.0109)	(0.0109)	(0.0108)	(0.0109)
Students per Teacher	-0.0265	-0.0246	-0.0255	-0.0261
	(0.0302)	(0.0306)	(0.0305)	(0.0307)
Number of Students	0.0167	0.0176	0.0200*	0.0179
	(0.0118)	(0.0118)	(0.0118)	(0.0118)
Parents' Education	-0.0738	-0.0804	-0.0790	-0.0727
	(0.112)	(0.110)	(0.110)	(0.110)
Foreign Background	0.0606	0.0597	0.0588	0.0627
	(0.0686)	(0.0690)	(0.0685)	(0.0690)
Teacher's Degree	-0.00769	-0.00830	-0.000564	-0.0103
	(0.0631)	(0.0636)	(0.0641)	(0.0631)
Independent Share	-0.00446*	-0.00518**	-0.00610**	
	(0.00257)	(0.00259)	(0.00267)	
Time	0.0119***	0.0126***	0.0111***	0.0108***
	(0.00336)	(0.00328)	(0.00296)	(0.00318)
Wx				
Number of Teachers	-0.0163	-0.0106	-0.00459	-0.0223
	(0.0303)	(0.0278)	(0.0244)	(0.0283)
Students per Teacher	0.0446	0.0650	0.0383	0.0200
	(0.0569)	(0.0636)	(0.0490)	(0.0429)
Number of Students	0.113***	0.0904***	0.0627**	0.109***
	(0.0328)	(0.0298)	(0.0261)	(0.0304)
Parents' Education	0.146	0.114	0.213	0.0499
	(0.228)	(0.222)	(0.180)	(0.228)
Foreign Background	-0.00101	-0.0348	-0.0241	-0.0127
	(0.129)	(0.124)	(0.117)	(0.120)
Teacher's Degree	0.00990	-0.0145	0.0533	0.0281
	(0.147)	(0.123)	(0.109)	(0.133)
Spatial				
rho	0.133***	0.118***	0.0862***	0.137***
	(0.0262)	(0.0218)	(0.0206)	(0.0227)
Variance				
sigma2_e	0.0169***	0.0169***	0.0170***	0.0169***
	(0.000690)	(0.000691)	(0.000696)	(0.000697)
Observations	3160	3160	3160	3160

Standard errors in parentheses
Notes:Durbin model with fixed effects using data from 2012-2016.
In the Independent model 90km is used and excluding Independent Share variable.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01

Table B6: Swedish Robustness Test

	120km	60km	30km	Independent
Main				
Number of Teachers	0.00576	0.00582	0.00617	0.00548
	(0.0127)	(0.0127)	(0.0125)	(0.0127)
Students per Teacher	-0.00186	-0.00230	-0.00113	-0.000989
	(0.0259)	(0.0256)	(0.0260)	(0.0264)
Number of Students	-0.0231	-0.0241*	-0.0248*	-0.0250*
	(0.0145)	(0.0145)	(0.0142)	(0.0145)
Parent's Education	0.0251	0.0252	0.0322	0.0228
	(0.0942)	(0.0939)	(0.0927)	(0.0942)
Foreign Background	0.0594	0.0592	0.0609	0.0551
	(0.0629)	(0.0626)	(0.0622)	(0.0630)
Teacher's Degree	-0.0546	-0.0563	-0.0548	-0.0480
	(0.0452)	(0.0446)	(0.0452)	(0.0448)
Independents Share	0.00733***	0.00763***	0.00747***	
	(0.00181)	(0.00181)	(0.00196)	
Time	0.0164***	0.0165***	0.0151***	0.0170***
	(0.00356)	(0.00322)	(0.00287)	(0.00353)
Wx				
Number of Teachers	0.0571**	0.0442*	0.0165	0.0476*
	(0.0277)	(0.0267)	(0.0191)	(0.0263)
Students per Teacher	0.0733	0.0234	0.0458	0.0680*
	(0.0539)	(0.0551)	(0.0418)	(0.0376)
Number of Students	-0.107***	-0.0923***	-0.0792***	-0.0984***
	(0.0238)	(0.0220)	(0.0212)	(0.0221)
Parent's Education	0.209	0.121	0.131	0.162
	(0.238)	(0.206)	(0.167)	(0.213)
Foreign Background	0.0405	0.000132	0.0775	0.0409
	(0.140)	(0.140)	(0.117)	(0.130)
Teacher's Degree	-0.0633	-0.0379	-0.0167	-0.0435
	(0.129)	(0.107)	(0.0929)	(0.113)
Spatial				
rho	0.0939***	0.0737***	0.0519***	0.0839***
	(0.0230)	(0.0199)	(0.0181)	(0.0214)
Variance				
sigma2_e	0.0152***	0.0152***	0.0152***	0.0153***
	(0.000588)	(0.000586)	(0.000589)	(0.000586)
Observations	3270	3270	3270	3270

Standard errors in parentheses
Notes:Durbin model with fixed effects using data from 2012-2016. Independent result is with range 90km.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01

Table B7: English with fixed number of neighbors

	1 Neighbor	5 Neighbors	7 Neighbors	10 Neighbors
Main				
Number of Teachers	0.00731	0.00726	0.00735	0.00739
	(0.0111)	(0.0112)	(0.0112)	(0.0112)
Students per Teacher	0.0400*	0.0405*	0.0407**	0.0405*
	(0.0208)	(0.0207)	(0.0206)	(0.0206)
Number of Students	0.00398	0.00126	0.000753	0.000430
	(0.0131)	(0.0132)	(0.0132)	(0.0133)
Parents' Education	-0.0517	-0.0540	-0.0553	-0.0572
	(0.0642)	(0.0643)	(0.0644)	(0.0647)
Foreign Background	0.0871	0.0900*	0.0903*	0.0897*
	(0.0529)	(0.0536)	(0.0537)	(0.0537)
Teacher's Degree	0.0337	0.0303	0.0306	0.0325
	(0.0415)	(0.0410)	(0.0410)	(0.0409)
Independent Share	0.0000380	0.000322	0.000442	0.000451
	(0.00165)	(0.00158)	(0.00158)	(0.00158)
Time	-0.0466***	-0.0468***	-0.0469***	-0.0470***
	(0.0148)	(0.0150)	(0.0150)	(0.0151)
Wx				
Number of Teachers	0.00493	-0.00291	-0.00520	0.00218
	(0.00979)	(0.0150)	(0.0162)	(0.0171)
Students per Teacher	0.0202	0.0170	0.0113	0.0132
	(0.0150)	(0.0303)	(0.0324)	(0.0330)
Number of Students	0.0145	0.0364	0.0426	0.0394
	(0.0144)	(0.0232)	(0.0273)	(0.0271)
Parents' Education	-0.150**	-0.0531	-0.0854	-0.0685
	(0.0754)	(0.122)	(0.129)	(0.145)
Foreign Background	-0.0106	0.0126	-0.00391	-0.0177
	(0.0551)	(0.0943)	(0.0981)	(0.103)
Teacher's Degree	-0.00192	-0.0164	-0.0141	0.00458
	(0.0357)	(0.0655)	(0.0731)	(0.0789)
Independent Share	-0.000441	-0.00651*	-0.00646	-0.00600
	(0.00228)	(0.00381)	(0.00421)	(0.00463)
Time	0.0444***	0.0457***	0.0460***	0.0463***
	(0.0149)	(0.0150)	(0.0149)	(0.0150)
Spatial				
rho	0.00235	0.00193	0.00467	0.00454
	(0.0133)	(0.0229)	(0.0247)	(0.0282)
Variance				
sigma2_e	0.0125***	0.0125***	0.0125***	0.0125***
	(0.000621)	(0.000620)	(0.000619)	(0.000620)
Observations	3265	3265	3265	3265

Standard errors in parentheses
Notes:Fixed effects estimates using data from 2012-2016.
Only considering neighbours within the 90km range.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01

Table B8: Mathematics with fixed number of neighbors

	1 Neighbor	5 Neighbors	7 Neighbors	10 Neighbors
Main				
Number of Teachers	-0.0170	-0.0177	-0.0178	-0.0180
	(0.0111)	(0.0112)	(0.0112)	(0.0112)
Students per Teacher	-0.0279	-0.0287	-0.0289	-0.0298
	(0.0297)	(0.0297)	(0.0298)	(0.0298)
Number of Students	0.0253**	0.0229*	0.0222*	0.0216*
	(0.0122)	(0.0121)	(0.0121)	(0.0121)
Parents' Education	-0.0388	-0.0366	-0.0389	-0.0392
	(0.111)	(0.110)	(0.111)	(0.112)
Foreign Background	0.109	0.116	0.115	0.114
	(0.0810)	(0.0803)	(0.0803)	(0.0804)
Teacher's Degree	-0.00555	-0.0130	-0.0117	-0.0116
	(0.0664)	(0.0646)	(0.0646)	(0.0643)
Independent Share	-0.00709***	-0.00589**	-0.00582**	-0.00552**
	(0.00261)	(0.00270)	(0.00267)	(0.00265)
time	0.0102***	0.0113***	0.0110***	0.0110***
	(0.00291)	(0.00321)	(0.00324)	(0.00330)
Wx				
Number of Teachers	-0.0143	-0.0128	-0.0147	-0.0208
	(0.0126)	(0.0231)	(0.0245)	(0.0261)
Students per Teacher	0.0197	0.0342	0.0333	0.0332
	(0.0207)	(0.0356)	(0.0360)	(0.0393)
Number of Students	0.0292**	0.0545**	0.0601**	0.0729***
	(0.0142)	(0.0242)	(0.0262)	(0.0277)
Parents' Education	0.00159	-0.106	-0.0851	-0.0465
	(0.0794)	(0.149)	(0.161)	(0.175)
Foreign Background	-0.0111	-0.0891	-0.0820	-0.0877
	(0.0501)	(0.0912)	(0.0971)	(0.0978)
Teacher's Degree	-0.00989	0.000602	0.0170	0.0192
	(0.0404)	(0.0821)	(0.0930)	(0.102)
Spatial				
rho	0.0331***	0.0975***	0.107***	0.112***
	(0.0128)	(0.0160)	(0.0168)	(0.0181)
Variance				
sigma2_e	0.0171***	0.0169***	0.0169***	0.0169***
	(0.000709)	(0.000696)	(0.000697)	(0.000697)
Observations	3160	3160	3160	3160

Standard errors in parentheses
Notes:Fixed effects estimates using data from 2012-2016.
Only considering neighbours within the 90km range.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01

Table B9: Swedish with fixed number of neighbors

	1 Neighbor	5 Neighbors	7 Neighbors	10 Neighbors
Main				
Number of Teachers	0.00584	0.00654	0.00645	0.00626
	(0.0128)	(0.0127)	(0.0125)	(0.0125)
Students per Teacher	-0.00111	-0.00179	-0.00206	-0.00213
	(0.0259)	(0.0260)	(0.0261)	(0.0263)
Number of Students	-0.0278*	-0.0263*	-0.0255*	-0.0249*
	(0.0145)	(0.0146)	(0.0145)	(0.0145)
Parent's Education	0.0255	0.0316	0.0307	0.0312
	(0.0942)	(0.0933)	(0.0938)	(0.0939)
Foreign Background	0.0606	0.0581	0.0577	0.0579
	(0.0621)	(0.0622)	(0.0627)	(0.0628)
Teacher's Degree	-0.0586	-0.0557	-0.0548	-0.0558
	(0.0451)	(0.0450)	(0.0451)	(0.0450)
Independents Share	0.00886***	0.00801***	0.00771***	0.00781***
	(0.00192)	(0.00191)	(0.00193)	(0.00189)
Time	0.0180***	0.0167***	0.0167***	0.0169***
	(0.00269)	(0.00295)	(0.00299)	(0.00300)
Wx				
Number of Teachers	0.0145	0.0243	0.0269	0.0286
	(0.0132)	(0.0194)	(0.0220)	(0.0220)
Students per Teacher	0.0285	0.0556*	0.0579*	0.0551
	(0.0241)	(0.0312)	(0.0351)	(0.0354)
Number of Students	-0.0243*	-0.0667***	-0.0749***	-0.0745***
	(0.0136)	(0.0177)	(0.0186)	(0.0191)
Parent's Education	-0.0112	0.142	0.106	0.122
	(0.0759)	(0.147)	(0.164)	(0.175)
Foreign Background	0.0447	0.0542	0.0500	0.0498
	(0.0496)	(0.0907)	(0.0960)	(0.104)
Teacher's Degree	-0.0236	-0.0285	-0.0569	-0.0666
	(0.0428)	(0.0789)	(0.0806)	(0.0876)
Spatial				
rho	0.0108	0.0292*	0.0406**	0.0481***
	(0.0111)	(0.0163)	(0.0175)	(0.0182)
Variance				
sigma2_e	0.0153***	0.0153***	0.0153***	0.0153***
	(0.000590)	(0.000592)	(0.000592)	(0.000590)
Observations	3270	3270	3270	3270

Standard errors in parentheses
Notes:Fixed effects estimates using data from 2012-2016.
Only considering neighbours within the 90km range.
Standard errors are clustered on the municipality level.
* p<0.1, ** p<0.05, *** p<0.01