

SELF-SIMILARITY IN THE FOUNDATIONS

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Abstract

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This thesis concerns embeddings and self-embeddings of foundational structures in both set theory and category theory.

The first part of the work on models of set theory consists in establishing a refined version of Friedman's theorem on the existence of embeddings between countable non-standard models of a fragment of ZF, and an analogue of a theorem of Gaifman to the effect that certain countable models of set theory can be elementarily end-extended to a model with many automorphisms whose sets of fixed points equal the original model. The second part of the work on set theory consists in combining these two results into a technical machinery, yielding several results about non-standard models of set theory relating such notions as self-embeddings, their sets of fixed points, strong rank-cuts, and set theories of different strengths.

The work in foundational category theory consists in the formulation of a novel algebraic set theory which is proved to be equiconsistent to New Foundations (NF), and which can be modulated to correspond to intuitionistic or classical NF, with or without atoms. A key axiom of this theory expresses that its structures have an endofunctor with natural properties.