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Internal and External Habits Formation: Policy Analysis Implications and Bayesian Estimation

Abstract

The Euler equation relies on strictly and rigid assumptions and it has been proved that consumers do not follow this equation in real data. One way to improve this consumption expression is introducing the hypothesis of *Habit formations*, which reflect the idea of non-time separable preferences. In this study, I show which are the consequences in policy analysis of working with habits persistence in a New Keynesian DSGE model, comparing the results coming from different ways of modelling habits formation. Moreover, I undertake an estimation of the new parameters through a Bayesian approach deploying Swedish aggregate data in order to provide a guide for future model calibration.

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1. Introduction

The Euler Equation is one of the most famous equation in the modern economy, and nowadays it is widely used in the macroeconomic models, like the *Dynamic Stochastic General Equilibrium* (DSGE) models, to carry on policy and forecasting analysis (Galí, 2007; Walsh, 2003). The Euler equation represents the intertemporal choice over the optimal consumption of the households between current and future periods and was firstly derived and tested by Robert Hall (1978) following the hypothesis of Permanent Income introduced by Milton Friedman in 1957, who stated that consumers decide the level of consumption at time t by accounting not only for the present but also for their expectations regarding the future. This behaviour adopted by the consumers is called *consumption smoothing* and implies that the economic agents smooth their spending over-time to maximize the utility throughout their entire life-cycle.

Several analysis, both micro and macroeconomic, have been conducted on this equation in the recent years in order to check if and how it successfully predict the consumption data, and yet, numerous studies have stressed the fact that the baseline Euler equation model does not work well when it comes to fit the aggregate consumption data dynamic (Ascari, 2016). Moreover, as expressed by Kiley (2010), there are also problems related to the theory of permanent income; for instance, the consumption growth is mostly predictable, and this is a basic violation of the permanent-income hypothesis proposed by Friedman. In fact, the main prediction of the life- cycle/permanent income model is that consumption changes follow a random walk, and that therefore they should be independent of the variables in the information set of the consumer, including current and lagged values of disposable income. For these reasons, various extensions to the baseline model have been proposed in the literature, in which some hypothesis (such as homogeneity of the consumers, time-separable preferences and rational expectations) have been relaxed. For instance, the literature has tried to overcome the idea of rational expectations introducing uncertainty about the future (De Grawe, 2012). Other studies focused on the assumption regarding heterogeneous agents, with and introduction of taste-shifters linked to the agents specific time-varying characteristics (Zeldes, 1989). Moreover, hypothesis on *rule-of-thumb consumers* have been introduced (Campbell and Mankiw, 1989; Galí, 2004; Kiley, 2010), an intuition that proposes an economic framework where a portion of the economic agent consumes immediately all the income owned at time t given the impossibility to enter in the financial market. Large part of the literature has analyzed also the idea of *Habits Persistence* in the consumption preferences of the consumers as a way to brake down the hypothesis of time-separable preferences.

Indeed, habits generate history dependence and inertia, thus lowering the responsiveness of consumption to the real interest rate (a similar effect obtained also with rule-of-thumb consumers). Moreover, it can account for the persistence and hump-shaped reaction of consumption to aggregate shocks as found in VAR studies (Fuhrer, 2000; Amato and Laubach, 2004).

The aim of this study is following and further analyze the idea and implication of adding habits formation in the Euler equation in the macroeconomic framework. Indeed, there are many ways to introduce habit persistence, and yet just few studies have gone through a one-to-one comparison among the different results obtained by various specification forms. Here, I firstly derive four different Euler equations following the paper proposed by Dennis (2009), and I specifically analyze the different specifications of multiplicative/additive combined with the cases of external/internal habits [1]. Then, I check the consequences of adding these different specifications of the Euler equation in a baseline New Keynesian model, comparing the different Impulse Response Functions resulting from both a productivity and monetary shocks and, therefore, checking what are the important policy implications caused by accounting for habits in the model. Moreover, I deploy Swedish aggregate data on output, short term nominal interest rate and inflation to estimate the new parameters entering in the model through bayesian approach, a method that is nowadays widely used to estimate the parameters of the DSGE models (An, Sungbae and Schorfheide, 2007). No one study before has estimated these parameters for Sweden, and, therefore, it can have important consequence for policy analysis in this country, with the new estimation results that can be deployed in a future model calibration.

The study is divided as follows: in section 2 I give to the reader an overview on what is behind both the Euler equation and the hypothesis of habit formation; in section 3 I present the New Keynesian DSGE model in which I operate; section 4 digs into the derivation of the Euler equations given the various hypothesis on habit formation; in section 5 I compare the results of the Impulse Response Function to given different shocks; section 6 is devoted the the estimation of the parameters; section 7 concludes.

[1] I will not cover the idea of *deep habits* proposed by Ravn et al. (2006), given that some papers have already shown the non-improvement of the models after the introduction of this kind of habit formation (e.g: Cantore et al, 2014)

2. An Overview on the Euler Equation and Habits formation

2.1. The Euler Equation

As expressed above, the Euler equation is a pillar in the modern DSGE models deployed for policy analysis. It represents the choice over the optimal consumption that the households undertake through time, a behaviour defined as *consumption smoothing*.

Formally, the intuition behind the Euler equation can be summarize by the following expression:

$$U'(C_{today}) = \beta E_{today}[(1+r)U'(C_{future})]$$

with $U'(C_{today,future})$ that denotes the marginal utilities of present and future consumption respectively, r the interest rate and β the discount factor. The idea is that in the optimum the economic agent is indifferent between the option of consuming one more unit at time t and saving the unit and consuming it in the future, so that the marginal utility lost from consuming a little less today (in order to save for future periods) and marginal utility gained from consuming more tomorrow are exactly the same (Browning and Crossley, 2001).

Figure 1 provides a graphical intuition of the subject. Here, consumption at time $t = 1$ and consumption at time $t = 2$ are drawn respectively on the x and y axis. The blue straight line represents the budget constraint before a change in r, with its slope given by -(1+r); U denotes the indifference curve, with the slope given by $-MRS_{c1,c2}$; E is the first endowment received at $t = 1$; $w1$ the level of consumption if everything is consumed at $t = 1$ and $w1(1+r)$ the amount of consumption if all the consumption is postponed at $t = 2$. The equilibrium is reached in the point where the two slopes are equal, namely $MRS_{c1,c2} = (1 + r)$. Given that $MRS_{c1,c2} = U'_{c1}/U'_{c2}$, we can easily derive the expression just described above. On the technical side, the Euler Equation can be seen as the first order condition of the optimization problem of the consumer that maximize the utility given an intertemporal budget constraint in which r represents the opportunity cost and β a parameter that signals the impatience rate of the consumer: the higher is r, the more profitable is to save and postpone consumption; the lower is β the more impatient is the consumer and, therefore, the higher is consumption at time t.

Fig. 1. Intertemporal Consumption

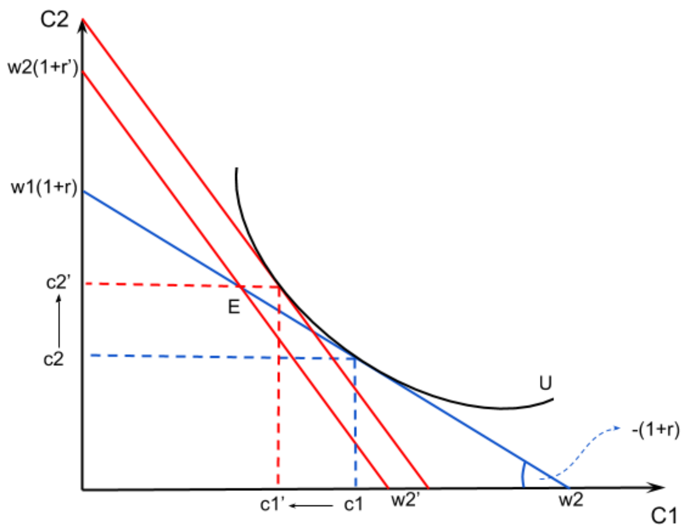


Figure 1 presents the case in which we observe an increase of the insterest rate r. The budget constraint pivotes around the endowment point E. As a consequence, the consumption at $t = 2$ will be relatively cheaper with respect to consumption at $t = 1$, and, therefore, the consumer prefers to save more, reducing the present consumption, in order to increase the future one [2].

In order to give some extra-basic intuitions to the reader, I present the log-linearized version [3] of the equation. Supposing that we are dealing with a consumer facing a Constant Relative Risk Aversion (CRRA) utility function and a generic intertemporal budget constraint, following Galí (2007) we can derive and express the baseline Euler in linear form as follows:

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \hat{\pi}_{t+1}) \tag{2.1}$$

or

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}\hat{r}_t \tag{2.2}$$

with $\hat{r}_t = \hat{i}_t - E_t\hat{\pi}_{t+1}$, and both $\hat{c}$ and $\hat{r}$ denoting the level of consumption and real interest rate log-linearized around the steady state. Here it is evident how the decision of

[2] In Figure 1 it is presented only the substitution effect

[3] What does it mean *log-linearization*? It will be explained later, in section 3.1.4. For now, it is enough for the reader to know that the log-linearization is a method that allows us to express non-linear equation (coming, for instance, from the solution of optimization problems) as linear equation, reducing the difficulties in the solution of system of equations.

consumption at time t is positively influenced by a forward-looking component (consumption at time $t+1$) and negatively affected by the response to change in the real interest rate. Regarding the former, it represents the expectations of the consumer on the future: if the economic agent expects the output, and consumption, to increase, she will be pushed to increase consumption in the present too. On the other side, the second component is related to changes of the current real interest rate, r_t, and its magnitude depends inversely on σ, with $\frac{1}{\sigma}$ that is also defined as the *Intertemporal Elasticity of Substitution*. How recently underlined also by McKay et al. (2017), these two elements are fundamental in the macroeconomic environment, and specifically for the dynamics of the DSGE models: the first component is important in the study of the effect of expected future policy changes, whereas the second one, linked to the interest rate, is crucial for the transmission of policies regarding monetary decisions.

Behind the Euler equation, however, there are strictly assumptions, such that homogeneity of the consumers, rational expectations and time-separable preferences. Basically, the introduction of habits formation help us to overcome relax the very last hypothesis, allowing the economic agents to be influenced in the present also by their decisions at previous time.

2.2. Habits Formation

The idea of habits formation in consumption choices has a long history in economics, as testified by the works of Thorstein Veblens in 1899, "The Theory of the Leisure Class", and James Duesenberrys 1949, "Income, Saving, and the Theory of Consumer Behavior". These studies have been followed by plenty research that have analyzed the existence of this specification in the consumption behaviour.

Generally, this hypothesis of Habit formation implies a specific preference specification in which the economic agent takes decision at time t considering also consumption at the previous lags. Therefore, an extra persistence is added in consumption decision through the inclusion of these levels of past consumption (defined also as *stock of habits*) in the utility function. The literature has focused on two main different categories of stock of habits upon which depends the new consumption decision : internal and external. The main difference relies on the fact that, while with the internal habits the consumer bases her decisions by looking at her own *internal* level of consumption, with the external ones the economic agent takes into account the *aggregate* past consumption. The latter category can be also seen as a way to analyze the "envy" variable in the consumption framework and it has also been renamed by the literature 'Catching up with the Joneses' (e.g: Abel, 1990; and Gali', 1994). Habits formation can be used to solve some problems arose both in finance and macroeconomic literature. Firstly, it may help to explain why consumers seem to adjust slowly to shocks to permanent income and why periods of high aggregate income growth are followed by periods of high aggregate saving, solving the problem that Campbell and Deaton (1989) documented as *excess smoothness* of aggregate consumption. Habits formation are also used in the macroeconomic DSGE models. Fuhrer (2000) showed how the introduction of habits significantly improves the short-run dynamic behavior of the model, both qualitatively and statistically, and the important consequences in the monetary policy once that the persistence in consumption is taken into account. The improvement relies in the ability of the model to better match the response of real spending to monetary policy shocks, given the improvement in the ability to fit with the hump-shaped response that the consumption data shows to output shock and monetary shock. Moreover, the idea behind habits persistence has been used in finance literature to solve anomalies such as the risk premium puzzle (Abel, 1990; Constantinides, 1990; Verdelhan, 2010) and poor performance of the Consumption CAPM (Campbell and Cochrane, 1999).

Plenty studies have been conducted on testing and quantifying the magnitude of internal and external Habits both in microeconomic and macroeconomic level. One of the pioneer in this framework was Dynan (2000) who tested for the presence of internal habits formation working on the Panel Study of Income Dynamics (PSID), a panel dataset containing information on U.S.A households. He deployed data on non-durable consumption, specifically on the expenditure in food, but he found no evidence on habits persistence. Many works have been conducted since, showing different results. Carrasco (2005) worked on Spanish panel dataset in which the household are observed for up to eight consecutive quarters, allowing him to take into account the time invariant unobserved heterogeneity across Households. His analysis showed how, once the account for the 'fixed effect' of the households, only food consumption shows habit formation, whereas no evidence is found regarding transports and services. Other authors have focused on checking the significance of both internal and external habits. Ravina (2007) deployed a panel dataset on credit card purchases and found evidence for both internal and external habits (with external habits specified as past aggregate consumption at city-level).

On the macroeconomic counterpart, Furher (2000) was among the first to introduce the idea of habit formation in a VAR framework, finding big evidence for external habit formation (introduced following a multiplicative approach). Same result was reached by Boldrin, Christiano, and Fisher (2001), that, instead focused on the internal counterpart. Since then, other authors have followed this intuition and have tried to estimate the importance of habits formation in the macroeconomic DSGE models (e.g: Christiano et al., 2005; Smets and Wouters, 2007; Rochelle, Kiley, and Laforte, 2008; Leduc and Liu, 2016). However, only Dennis (2009) has provided an analysis in which both the the cases of internal and external habits are compared, providing an overview on the implication of working either with one or the other specification. Finally, in the recent years, Cantore et al (2014) have gone through a bayesian estimation of a DSGE model in which they estimated and compared the hypothesis of superficial habits and deep habits. They demonstrated that, once superficial habits are taken into account, there is no reason for adding deep habits due to the non-improvement in the ability of the model to fitting the data.

3. The Model

3.1. DSGE New Keynesian Model

Here I present the baseline New Keynesian Dynamic Stochastic General Equilibrium (DSGE) Model in which I am going to introduce the different specifications of the Euler equation. This class of models are completely *micro-founded*, with the equations coming from the maximization problem of all different economic agents forming the economy. Thanks to the DSGE models, it is possible to understand the dynamic behaviour of key macroeconomic variables, and to study what are the effects of different shocks that hit the economy. Specifically, the New Keynesian model comprehends different actors: the consumer, who maximizes her utility given a budget constraint; the firms, which maximize the profit given a monopolistic market whit sticky prices; the central authority, that takes the decision of setting the nominal interest rate and, therefore, the monetary policy, and that is represented by the Taylor rule equation.

I will now dig into the technicality, describing the solution to the maximization problem of the agents forming the models and showing the passages that allow us to arrive to the basic system of equations.

3.1.1. The Households

The Euler equation is already been partially presented in section 1: *Introduction.* However, now I present it in the New Keynesian context.
Suppose there is one infinitely-lived representative household who wants to maximize her life-time utility at time *t=0*, with an utility function depending on consumption (C_t) and labour (N_t). The function to be maximized is expressed as follows:

$$E_t\Big[\sum_{t=0}^{\infty}\beta^t U(C_t, N_t)\Big] \tag{3.1}$$

with β_t equal to the discount factor. The budget constraint is, instead, defined as:

$$P_tC_t + Q_tB_t \leq W_tN_t + B_{t-1} + T_t \tag{3.2}$$

with P_t equal to the level of price, Q_t to the bond yield, B_t representing the amount of asset at time t, W_t, N_t and T_t the wage, labour supply and net transfers from the government at time t.
Given the following utility function

$$U_t(C_t, N_t) = \frac{{C_t}^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \tag{3.3}$$

with σ and ψ respectively equal to the coefficient of relative risk aversion and inverse of the Fisher elasticity. We then apply the lagrangian approach, obtaining:

$$\max_{C_t,N_t,B_t} E_0\Big[\sum_{t=0}^{\infty}\beta^t\Big(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi}\Big) - \lambda_t(P_tC_t + Q_tB_t - B_{t-1} - W_tN_t + T_t)\Big] \tag{3.4}$$

from which we derive the three first order conditions (FOC) and we obtain, respectively, the optimal labour supply and the non-linear form of the Euler Equation met in the introduction[4]:

$$\frac{W_t}{P_t} = N_t^{\psi}C_t^{\sigma} \tag{3.5}$$

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\pi_{t+1}}\right] = \frac{1}{1+i_t} = Q \tag{3.6}$$

3.1.2. The Firms

The second important component of the model represents the optimization problem that the firms face at time t. Let us suppose there is a continuum of firms, indexed by $i \in [0, 1]$, whit each firm producing a differentiated good in a monopolistic market, where, however,

[4]In the Introduction I presented directly the linear form obtained through the log-linearization.

the technology A_t is supposed to be equal for every firm, with a production function equal to

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{3.7}$$

with α equal to the output elasticity w.r.t. labor.
Moreover, the firms face an identical demand and take the aggregate demand and the aggregate price as given, a condition that give us the following expression:

$$C_t(i) = C_t\Big(\frac{P_t(i)}{P_t}\Big)^{-\epsilon} \tag{3.8}$$

with ϵ defined as the *constant elasticity of substitution between goods* and representing the power of the producers in the market: the higher it is, the lower is the market power of producers [5]. Therefore, the maximization problem can be summarize as:

$$\underset{P_t(i),N_t(i)}{max} P_t(i)Y_t(i) - W_t N_t(i) \tag{3.9}$$

given the two constraints

$$Y_t(i) = A_t N_t(i)^{1-\alpha}; C_t(i) = C_t\Big(\frac{P_t(i)}{P_t}\Big)^{-\epsilon} \tag{3.10}$$

Maximizing w.r.t N_t and P_t we obtain, respectively:

$$W_t = \underbrace{mc_t(i)}_{\substack{\text{nominal} \\ \text{marginal cost}}} (1-\alpha)A_t N_t(i)^{-\alpha} \tag{3.11}$$

and

$$P_t(i) = \underbrace{\frac{\epsilon}{\epsilon - 1}}_{\text{markup}} mc_t(i) \tag{3.12}$$

Then, following Gali' (2007), we obtain the equations:

$$MC_t(i) = \frac{W_t}{P_t(i)} \frac{1}{(1-\alpha)A_t N_t(i)^{-\alpha}} \tag{3.13}$$

[5] The limit case in which $\epsilon \to \infty$ corresponds to the *perfect competition* case

Where $MC_t(i)$ are equal to the real marginal cost [6]. The real milestone of the new keynesian model is the assumption of sticky prices, first introduced by Calvo (1983). Here the it is supposed that in each period each firm faces a probability θ of being locked with its old price, with (1-θ) representing the portion of firms that successfully readjust the price and, therefore, solve the problem of optimization. Therefore, the level of price at time t can be expressed as follows:

$$P_t = \Big(\theta P_{t-1}^{1-\epsilon} + (1-\theta)P_{t-1}^{*1-\epsilon}\Big)^{\frac{1}{1-\epsilon}} \tag{3.14}$$

from which one can derive the expression for the dynamic inflation rate:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\Big(\frac{P*_t}{P_{t-1}}\Big)^{1-\epsilon} \tag{3.15}$$

From here it is easy to understand how the inflation level depends on the number of firms that can and decide to re-optimize choosing a price level equal to $P^* > P_{t-1}$.

Thanks to the implication of the introduction of the *Calvo pricing*, we suppose that the firms now face an optimization problem at time t in which it decides the optimum price P_t* in order to maximaze the value of all the future profits actualized at time t. We can express it as:

$$\max_{P_t*} \sum_{k=0}^{\infty} \theta^k E_t\Big[\underbrace{Q_{t,t+k}}_{\text{discount factor}}\Big(\underbrace{P_t * Y_{t+k|t}}_{\text{revenues}} - \underbrace{\Psi_{t+k}(Y_{t+k|t})}_{\text{costs function}}\Big)\Big] \tag{3.16}$$

s.t:

$$Y_{t+k|t} = \Big(\frac{P_t*}{P_{t+k}}\Big)^{-\epsilon} Y_{t+k} \tag{3.17}$$

By plugging (3.17) into (3.16) and maximizing for P_t* we get:

$$\sum_{k=0}^{\infty} \theta^k E_t\Big[Q_{t,t+k} Y_{t+k|t}\Big(\frac{P^*}{P_{t-1}} - \mathcal{M} MC_{t+k|t}\Pi_{t+1,t+k}\Big)\Big] = 0 \tag{3.18}$$

Wit $\mathcal{M}$ equal to the *average mark-up*.

3.1.3. The Equilibrium Conditions

The DSGE model works, as stated before, in a *general equilibrium* framework. Generally, it means that the aggregate demand and aggregate supply are always set equal.

[6]Note that 3.13 comes from the expression $MC_t = \frac{mc_t(i)}{P_t(i)} = \frac{\epsilon-1}{\epsilon}$

The first condition is the so called *market clearing condition*, that implies:

$$Y_t = C_t \tag{3.19}$$

The other equilibrium, instead, is in the labour market. Given the aggregate labour equation equal to:

$$N_t = \int_0^1 N_t(i)di \tag{3.20}$$

we can rewrite the production function, solving it for $N_t(i)$, and, after some manipulation, obtaining:

$$N_t\Big(\frac{Y_t}{A_t}\Big)^{\frac{1}{1-\alpha}} \int_0^1 \Big(\frac{P_t(i)}{P_t}\Big)^{-\frac{\epsilon}{1-\alpha}} di \tag{3.21}$$

3.1.4. The Equations

To sum up, I present the non-linear system of equations composing the model. This is based on the following equations: labor supply (3.22), Euler equation (3.23), firms optimal price setting (3.24), firms cost minimization (3.25), price dynamics (3.26), inflation dynamics (3.27), goods market clearing (3.28), labor market clearing plus an exogenous law of motion for aggregate technology (3.29).

$$\frac{W_t}{P_t} = N_t^{\psi} C_t^{\sigma} \tag{3.22}$$

$$E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\frac{1}{\pi_{t+1}}\right] = \frac{1}{1+i_t} = Q \tag{3.23}$$

$$\sum \theta^k E_t\Big[Q_{t,t+k}Y_{t+k|t}\Big(\frac{P^*}{P_{t-1}} - \mathcal{M}MC_{t+k|t}\Pi_{t+1,t+k}\Big)\Big] = 0 \tag{3.24}$$

$$MC_t = \frac{W_t}{P_t}\frac{1}{(1-\alpha)A_t N_t^{-\alpha}} \tag{3.25}$$

$$P_t = \Big(\theta P_{t-1}^{1-\epsilon} + (1-\theta)P_t^{*1-\epsilon}\Big)^{\frac{1}{1-\epsilon}} \tag{3.26}$$

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\Big(\frac{P_t^*}{P_{t-1}}\Big)^{1-\epsilon} \tag{3.27}$$

$$Y_t = C_t \tag{3.28}$$

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \tag{3.29}$$

Moreover, an equation representing a Central Bank rule used to set the nominal rate of inflation is normally added to this system.

3.1.5. The Log-Linearized Equations

How stated above, the equations of the model just presented are expressed in non-linear form. However, this can bring some difficulties in the solution of the system of equations and, therefore, it may be helpful to work with linear equations. The log-linearization procedure is inserted in this specific context: it allows us to write the equations in linear form rather than non-linear, expressing them in term of log-deviation from the values in steady state and helping to simplify computational burden. The method employed in this case is the log-linearization based on the first order of the Taylor series approximation (Taylor, 1993). The technique is based on the fact that

$$f(X_t) \approx f(X) + f'(X)(X_t - X)$$

with $f'(X) = \frac{\partial f(X)}{\partial X}$ and assuming that X denotes the value of X_t at the steady-state level.

Dividing and multiplying by X, we obtain:

$$f(X_t) \approx f(X) + f'(X)X \quad \underbrace{\frac{(X_t - X)}{X}}_{\text{\% dev. from SS } X_t}$$

From here, we can express the log-deviation from the steady state $\hat{x}_t$ as:

$$\frac{(X_t - X)}{X} = \frac{X_t}{X} - 1 \approx ln\frac{X_t}{X} = log(X_t) - log(X) = (x_t - x) = \hat{x}_t$$

We then bring $f(X)$ to the right hand side, and, then, we divide both the right and the left sides for $f(X)$:

$$\frac{f(X_t) - f(X)}{f(X)} = \frac{f'(X)}{f(X)} X\hat{x}_t$$

The results is a set of linear equations from the system of non-linear equations presented above:

$$\hat{w}_t - \hat{p}_t = \psi \hat{n}_t + \sigma \hat{c}_t \tag{3.30}$$

$$\hat{c}_t = E_t \hat{c}_{t+1} - \sigma(\hat{i}_t - E_t \hat{pi}_{t+1}) \tag{3.31}$$

$$\hat{\pi}_t = (1-\theta)(\hat{p}^*_t - \hat{p}_{t-1}) \tag{3.32}$$

$$\sum_{k=0}^{\infty} \theta^k \beta^k (\hat{p} *_t - \hat{p}_{t-1}) = \sum_{k=0}^{\infty} \theta^k \beta^k E_t \big[\big(\hat{mc}_{t+k|t} + (\hat{p}_{t+k} - \hat{p}_{t-1}) \big) \big] \tag{3.33}$$

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t + \alpha \hat{n}_t \tag{3.34}$$

$$\hat{p}_t = (1-\theta)\hat{p}^*_t + \theta \hat{p}_{t-1} \tag{3.35}$$

$$\hat{c}_t = \hat{y}_t \tag{3.36}$$

$$\hat{n}_t = \frac{1}{(1-\alpha)}(\hat{y}_t - \hat{a}_t) \tag{3.37}$$

Following Galí(2007), we can rewrite the model in a closer and more intuitive form made of three equations. Specifically:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \sigma^{-1} \big(\hat{i}_t - E_t \hat{\pi}_{t+1} \big) \tag{3.38}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \tilde{y}_t \tag{3.39}$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + m_t \tag{3.40}$$

The first one is the Euler equation, and is defined also as *Dynamic IS*, whereas the second one is the *New-Keynesian Phillips* curve [7]. The variable $\tilde{y}_t$ is defined as *output gap*, and represents the difference between the actual output with rigidity in price and the potential

[7] $k = \lambda \frac{\sigma(1-\alpha)+\psi-\alpha}{1-\alpha}$

output at fully flexible prices [8]

$$\tilde{y}_t = (\hat{y}_t - \hat{y}_t^F) \tag{3.41}$$
$$\hat{y}^F = \Psi^n_{a,t} a_t \tag{3.42}$$

whereas Equation (3.40) denotes the *Taylor rule* and shows how the Central Bank sets short-term nominal interest rate in response to changes in output and, especially, inflation. a_t and m_t instead, denotes the productivity and monetary shock at time t.
Both the shocks are supposed to follow an Autoregressive Process of order 1, AR(1):

$$a_t = \rho_a a_{t-1} + \varepsilon_a \tag{3.43}$$

$$m_t = \rho_m m_{t-1} + \varepsilon_m \tag{3.44}$$

$$\text{with } \varepsilon_a, \varepsilon_m \sim N(0, \sigma_{a,m})$$

4. Euler specifications with Habit formation

From the previous section, it is intuitive that the introduction of habit formation requires us to work on equation the Euler Equation. However, the decision on how to insert the feature of habit persistence in the utility function is not straightforward, given the various forms in which this feature can be inserted in our model. Indeed, how partially explained, it is important to underline the fact there are two main approaches to introduce this feature: either the multiplicative or the additive approach.
With the former, utility depends on the ratio of consumption to the habit stock. Therefore, the utility function can be defined as follows:

$$U_t(c_t, H_t) = \frac{(\frac{c_t}{H_t})^{1-\sigma}}{1-\sigma} \tag{4.1}$$

[8]Recalling the optimization problem of the firms, if we solved it under the hypothesis of flexible price so that $P_t^* = P_t$, instead than rigidity in price, we would end up with a potential output equal to $Y^F = \left(\frac{1}{M}(1-\alpha)A_t^{\frac{1+\psi}{1-\alpha}}\right)^{\frac{1-\alpha}{\psi+\sigma(1-\alpha)+\alpha}}$. Its log-deviation expression is equal to the equation (3.42), with $\Psi^n_{a,t} = \frac{\psi+1}{\sigma(1-\alpha)+\psi+\alpha}$.

with c_t equal to the individual consumption of the consumer at time t, the parameter σ defined as the coefficient of relative risk aversion and H_t representing the stock of habits at time t. Authors that have implemented this type of specifications are, for instance, McCallum and Nelson (1999), Fuhrer (2000); Amato and Laubach (2004), Khvostova et al. (2014).

Conversely, in the additive approach it is the difference between consumption at time t and the stock of habits at time t that matters. In this sense, the utility function can be defined as:

$$U(c_t, H_t) = \frac{(c_t - H_t)^{1-\alpha_c}}{1-\alpha_c} \tag{4.2}$$

With c_t, H_t and α_c that respectively reflect the parameters found in the multiplicative case [9]. Authors implementing this approach to define habits formation are, for instance, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007).

We follow Dennis (2009) and Abe (1990) in the derivation of the new specifications of the Euler equation. However, here the maximization problem is not based on the choice of the consumer of consumption, labour and money [10], given my decision of not including money in the New Keynesian model just presented in section 3. Therefore, in order to be consistent with what explained above, the economic agent can choose the amount of consumption, the amount of labour and stock of Bonds.

4.1. Multiplicative Habits

In the multiplicative-habit case, the stock of habits H is defined by Abel (1990) as:

$$H_t = (c_{t-1}^{D} C_{t-1}^{1-D})^{\gamma} \tag{4.3}$$

The parameter γ represent the magnitude of the influence of the Habit stock in the utility function of the consumer. It can assume values between $[0, 1]$: if $\gamma = 0$, the utility function squishes to the one of the baseline model. The parameter D assumes a role of *shifter* between the importance of the individual consumption at $t-1$ (here defined as c_{t-1}) and the aggregate consumption at $t-1$ (defined as C_{t-1}). If $D = 1$, then H_t will depend only on internal habits, whereas if $D = 0$, the habit stock will depend only on external habits.

We now solve the new maximization problem defined as follows:

[9] Here we defined the coefficient of relative risk aversion with α_c instead that σ given that σ can assume value between [0,1], whereas α_c here can assume any positive values [0,∞], as suggested by Dennis (2009)

[10] Instead, this is the case presented in Dennis (2009)

$$E\sum\beta^t\frac{(\frac{c_t}{H_t})^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\psi}}{1+\psi} \tag{4.4}$$

Given the budget constraint equal to:

$$c_t+\frac{B_t}{P_t}=\frac{W_t}{P_t}N_t+\frac{(1+i_{t-1})B_{t-1}}{P_t}+\frac{T_t}{P_t} \tag{4.5}$$

with β^t equal to the discount factor, N_t representing the labour supply, ϕ defined as the inverse of the Fisher elasticity, B_{t-1} the amount of assets at time t owned by the consumer, T_t the net transfers received by the government, P_t prices at time t.
I apply the lagrangian approach, deriving the FOC. The first FOC can be expressed as:

$$\frac{\partial L}{\partial c_t}=[\frac{\partial U_t}{\partial c_t}+\beta\frac{\partial U(c_{t+1},H_{t+1})}{\partial H_{t+1}}\frac{\partial H_{t+1}}{\partial c_t}]-\lambda_t=0 \tag{4.6}$$

from which we derive:

$$c_t{}^{-\sigma}c_{t-1}^{\gamma(\sigma-1)}-\beta\gamma Dc_{t+1}^{1-\sigma}c_t^{\gamma(\sigma-1)-1}=\lambda_t \tag{4.7}$$

In the passage between the equation (4.6) and (4.7), I follow the idea expressed by Abel (1990) in which $c_t=C_t$ in the equilibrium.

The second FOC is obtained by maximizing for the amount of assets B_t:

$$\frac{\partial L}{\partial B_t}=-\frac{\lambda_t}{P_t}+\frac{\lambda_{t+1}(1+i_t)}{P_{t+1}}=0 \tag{4.8}$$

from which we derive:

$$\lambda_{t+1}\lambda_t^{-1}\pi_{t+1}^{-1}=\frac{1}{(1+i_t)} \tag{4.9}$$

Then, we log-linearize the equation (4.9) around the steady state, obtaining:

$$\hat{\lambda}_t=E_t\hat{\lambda}_{t+1}+(\hat{i}_t-\hat{\pi}_{t+1}) \tag{4.10}$$

and we log-linearize the (4.7) in order to get:

$$\frac{-(\sigma+(\gamma(\sigma-1)-1)\gamma\beta D)\hat{c}_t+\gamma(\sigma-1)\hat{c}_{t-1}-\gamma\beta D(1-\sigma)\hat{c}_{t+1}}{(1-\gamma\beta D)}=\hat{\lambda}_t \tag{4.11}$$

The next, and last, step is insert the (4.11) in the (4.10), through which we derive the generic log-linearized Euler equation accounting for internal and external habits persistence following the multiplicative habits. From here, it is possible to derive the two specific case of external and internal habits substituting respectively $[D = 0]$ and $[D = 1]$.

If D=0, the Euler equation is:

$$\hat{c}_t = \frac{\gamma(\sigma-1)}{\sigma+\gamma(\sigma-1)}\hat{c}_{t-1} + \frac{\sigma}{\sigma\gamma(\sigma-1)}E_t\hat{c}_{t+1} - \frac{1}{\sigma+\gamma(\sigma-1)}(\hat{i}_t - E_t\hat{\pi}_{t+1}) \tag{4.12}$$

Whereas, if D=1, the Euler equation is defined as follows:

$$E_t\Delta\hat{c}_{t+1} = \frac{\gamma(\sigma-1)}{\sigma+\gamma\beta(\sigma\gamma-1-\gamma)}E_t[\Delta\hat{c}_t + \beta\Delta\hat{c}_{t+2}] + \frac{(1-\gamma\beta)}{\sigma+\gamma\beta(\sigma\gamma-1-\gamma)}[\hat{i}_t - \hat{\pi}_{t+1}] \tag{4.13}$$

It is evident how one of the main difference between the Euler equation with habit persistence and the baseline one is in the response of the consumer in changes of the real interest rate (and, therefore, nominal interest rate decided by the Central Banks). Indeed, the coefficient of relative risk aversion and the elasticity of intertemporal substitution are not more strictly correlated, and the EIS is evidently lowered if compared with the one of the baseline version. This can have important implications when the policy-makers have to decide the level of nominal interest rate. Moreover, in the Habit-persistence version, consumption at time t does not depend only on consumption at t+1; instead, it is present also consumption at time t-1 and, for the internal case, consumption at time t+2.

4.2. Additive Habits

The derivation under the case of Additive Habits follows the one of Multiplicative Habits. The difference is how the Habit stock enters in the utility function. Indeed, in this specific case, $U(c_t, H_t, N_t)$ is defined as:

$$U(c_t, H_t, N_t) = \frac{(c_t - H_t)^{1-\alpha_c}}{1-\alpha_c} - \frac{N_t^{1+\psi}}{1+\psi} \tag{4.14}$$

with H_t expressed as:

$$H_t = \eta_c(c_{t-1}^D C_{t-1}^{1-D}) \tag{4.15}$$

The parameter η_c has a similar function of the previous parameter γ: the higher η_c, the higher the importance of the Habit stock for the economic agent in the decision of consumption at time t, whereas D has the same role seen before. There is no need to show the entire procedure of derivation, given its similarity to the one followed before.
From the maximization problem of the consumer, where we insert the new utility function (4.14), we obtain the following expression through the Lagrangian approach:

$$[(c_t - \eta_c c_{t_1})^{-\alpha_c} - \beta\eta_c D(c_{t+1} - \eta_c c_t)^{-\alpha_c}] = \lambda_t \tag{4.16}$$

Then, we log-linearize the expression, obtaining:

$$\frac{-\alpha_c(\hat{c}_t - \hat{c}_{t-1}) + \beta\eta_c\alpha_c D(\hat{c}_{t+1} - \eta_c\hat{c}_t)}{(1-\eta_c)(1-\beta\eta_c D)} = \hat{\lambda}_t \tag{4.17}$$

We insert the equation (4.17) in the F.O.C derived from the maximization of the stock of Bond B_t, and, by substituting respectively $D = 0$ and $D = 1$, it is easy to arrive at the external and internal forms of the Euler under the additive case.

With D=0, the Euler equation is:

$$\hat{c}_t = \frac{\eta_c}{1+\eta_c}\hat{c}_{t-1} + \frac{1}{1+\eta_c}E_t\hat{c}_{t+1} - \frac{(1-\eta)}{\alpha_c(1+\eta_c)}(\hat{i}_t - E_t\hat{\pi}_{t+1}) \tag{4.18}$$

Whereas, with D=1, we have:

$$E_t\Delta\hat{c}_{t+1} = \frac{\eta_c}{(1+\eta_c^2\beta)}E_t[\Delta\hat{c}_t + \beta\Delta\hat{c}_{t+2}] + \frac{(1-\eta_c)(1-\eta_c\beta)}{\alpha_c(1+\eta_c^2\beta)}(\hat{i}_t - E_t\pi_{t+1}) \tag{4.19}$$

Same considerations can be done here: the EIS changes with respect to the one of the baseline Euler version, and, moreover, the persistency is given by the presence of consumption at *t-1*, with possible important effects on the dynamic of consumption given a shock in the output and/or interest rate.
The next step will be simulating the model with the five different specifications of the Euler equation, comparing the results and the differences in the impulse response functions obtained.

5. IRF - Results

For every model we apply both a positive productivity shock and a monetary shock. Firstly, I will briefly show and describe the IRF resulting from the baseline model; then, I will compare the results obtained with the other Euler equations. The initial analysis will be carried out by calibrating the parameters of the model following the recent literature [11].

Table 1: DSGE Model - Baseline Calibration

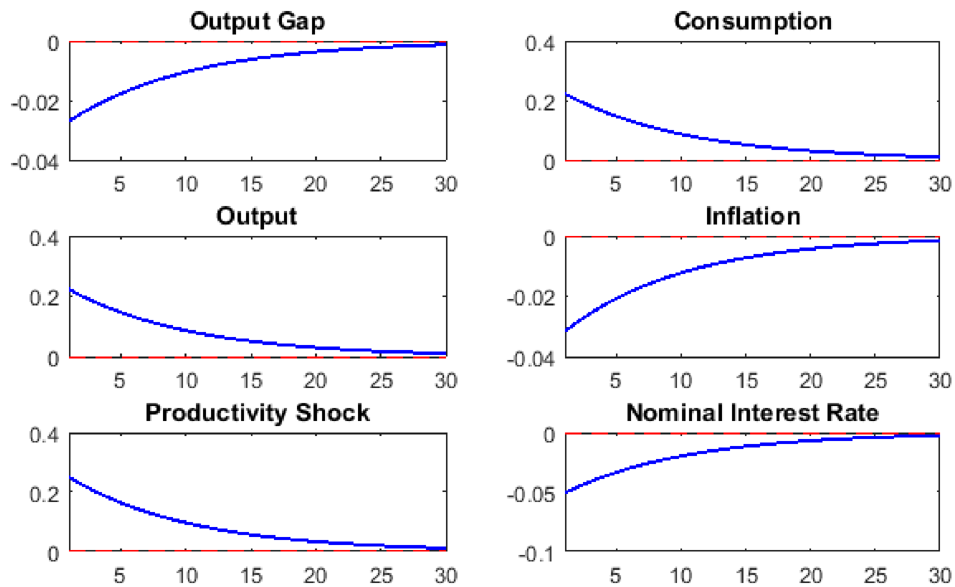
Parameters	**Description**	**Calibration**
σ	C.R.R.A	1
φ	Inverse of Frisch elasticity of labor supply	1
β	Discount Factor	0.99
ϕ_π	Reaction coefficient on Inflation	1.5
ϕ_y	Reaction coefficient on output	0.5/4
ϵ	Elasticity of Substitution between goods	6
α	Labor elasticity in the production function	1/3
θ	Probability of resetting price	2/3
ρ_a	Persistence of technological shock	0.9
ρ_m	Persistence of Interest rate Shock	0.5
σ_a	Volatility of technological shock	0.25
σ_m	Volatility of interest rate shock	0.25

5.1. Impulse Response Functions - Baseline Euler Equation

I initially analyze a positive productivity shock a_t that is present both in the Phillips curve and in the Taylor Rule equation (in the latter through the output gap relation). What we expect is a direct and positive effect on the output, that leads to a decrease of the output gap, and to a consequently reduction of the inflation. Moreover, given that changes in the outpgap $\tilde{y}_t$ affects the decision in the level of the interest rate set by the Central Bank, we can forecast a reduction in the nominal interest rate too. On the other side, due to a reduction in the interest rate, we expect the consumption at time t to increase because of the phenomenon of *consumption smoothing* expressed in the previous sections.
As visible from Figure 5.1, the impulse response functions give us exactly these results, with an increase of the output that leads to a decrease of the output gap and an interest rate that shows a reduction larger than the one shown by the inflation rate.

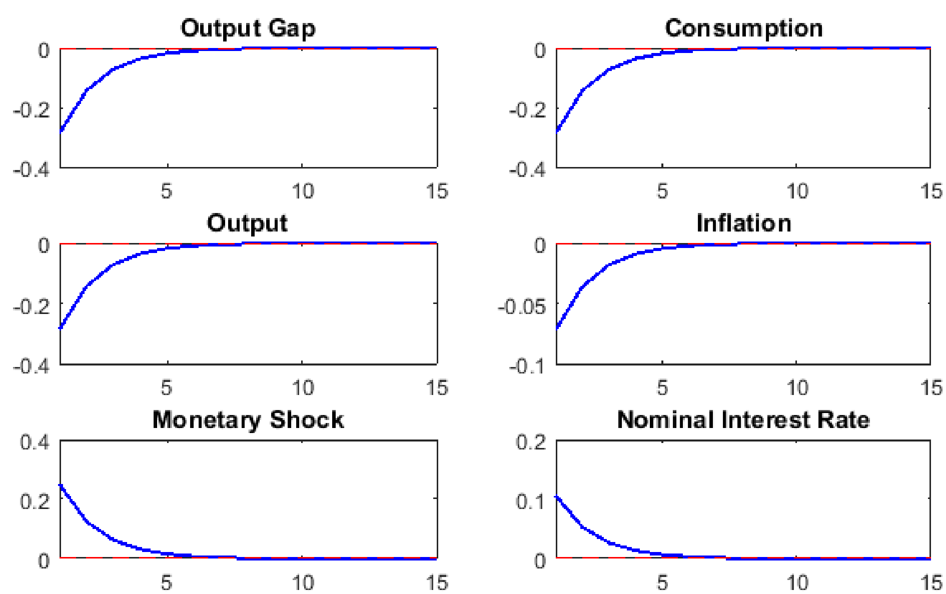
[11] In this specific case I follow the calibration given by Galí (2007) to be consistent with what presented in the previous section. In fact, I followed his derivations in order to construct the model presented above

Fig. 2. IRF in the baseline model - Productivity Shock



Here are plotted the Impulse Response Functions given a positive productivity shock, as the last graph bottom-right shows. As predicted by the theory, it causes not only a direct increase of the Output and, therefore, Consumption, but also of the Output Gap, which causes the Central Banks to answer with an increase in the Interest Rate.

Fig. 3. IRF in the baseline model - Monetary Shock



Here are plotted the Impulse Response Functions given a positive monetary shock, as the last graph bottom-right shows. As predicted by the theory,the consequences are a direct effect on the Nominal Interest Rate, which increases, which triggers the *consumption smoothing* phenomenon, leading to a decrease of the Consumption and Output. N.B: the persistence of the shock is lower compared to the productivity one given that ρ_m is lower than ρ_a

Furthermore, we can underline how the model fails to replicate the hump-shaped form in the IRF, with all the variables that follow a smooth path while returning back to the steady state level. Figure 5.1, instead, presents the IRF in case of a positive monetary shock, that directly affects only the nominal interest rate in the Taylor rule expression. An increase in the nominal interest rate, then, implies a corresponding increase in the real interest rate at the initial period. This effect depresses the demand in the economy as it leads households to delay their consumption through the intertemporal consumption smoothing, as reported in the Euler condition, causes a corresponding change in the output because of the market clearing condition, $c_t = y_t$, and, therefore, a drop in the inflation rate. The model fails to reproduce hump-shaped impulse response functions in this case too.

5.2. *Impulse Response Functions - Euler Equations with Habit Formation*

It is time to check the results obtained from the Euler equations derived in section 4. To sum up, in Table 2 I present the four new Euler equations compared to the baseline.

Table 2: Euler Equations - Comparison

Specification	**Cases**	**Equations**
Baseline		$c_t = c_{t+1} - \sigma^{-1}(\hat{r}_t)$
Multiplicative	Ext	$\hat{c}_t = \frac{\gamma(\sigma-1)}{\sigma+\gamma(\sigma-1)}\hat{c}_{t-1} + \frac{\sigma}{\sigma\gamma(\sigma-1)}E_t\hat{c}_{t+1} - \frac{1}{\sigma+\gamma(\sigma-1)}(\hat{r}_t)$
	Int	$E_t\Delta\hat{c}_{t+1} = \frac{\gamma(\sigma-1)}{\sigma+\gamma\beta(\sigma\gamma-1-\gamma)}E_t[\Delta\hat{c}_t + \beta\Delta\hat{c}_{t+2}] + \frac{(1-\gamma\beta)}{\sigma+\gamma\beta(\sigma\gamma-1-\gamma)}(\hat{r}_t)$
Additive	Ext	$\hat{c}_t = \frac{\eta_c}{1+\eta_c}\hat{c}_{t-1} + \frac{1}{1+\eta_c}E_t\hat{c}_{t+1} - \frac{(1-\eta)}{\alpha_c(1+\eta_c)}(\hat{r}_t)$
	Int	$E_t\Delta\hat{c}_{t+1} = \frac{\eta_c}{(1+\eta_c^2\beta)}E_t[\Delta\hat{c}_t + \beta\Delta\hat{c}_{t+2}] + \frac{(1-\eta_c)(1-\eta_c\beta)}{\alpha_c(1+\eta_c^2\beta)}(\hat{r}_t)$

Clearly, there are new parameters that enter the model depending on whether we are dealing with additive or multiplicative specification. For the calibration of these new coefficients I will deploy the results already obtained by the recent literature, whereas the estimation part will be devoted in the following section. Specifically, I follow the one of Fhurer (2000) who estimated the parameters of a multiplicative form equal to 0.8 [12]; in-

[12] His analysis was a VAR study focused just on internal habits, however I will apply, for now, the same results also for the external one

stead, I follow Dynan (2009) for the additive specification coefficients. Indeed, he conducted a thorough analysis on both the internal and external cases and, therefore, it perfectly fits with my study.

Table 3: DSGE Model - Habits Calibration

Specification	**Cases**	**Parameters**	**Calibration**
Multiplicative Habits	External	γ_c	0.8
	Internal	γ_c	0.8
Additive Habits	External	α_c	5.647
		η_c	0.832
	Internal	α_c	3.180
		η_c	0.824

One can immediately have a general clue of what to expect: as expressed in the previous sections, the presence of habits formation will lower the ability of the consumer to immediately adapt to changes in the consumption. Instead, she will try to smooth not only the consumption intertemporally, but also the change of consumption, and this will reflect in a lower response of the variable *consumption* at time $t = 0$. Furthermore, the persistence added in the utility function will intuitively cause the peak to be reached in the future leads rather than at time t, with consumption that will take more time to go back to the steady state level. The results are quite significant and fit with the recent literature only partially. In Figure 4, I firstly analyze the difference between the five blocks of DSGE models in case of a positive productivity shock. Here, all the Impulse response functions are consistent with the baseline version, and differ only in the path and magnitude. Indeed, given a shock in the technology, the output (and the consumption, given the market clearing condition) increases, the output gap decreases along with the nominal interest rate and the inflation rate. Surprisingly, the DSGE model having the multiplicative-external specification doesn't show any peculiarity and follows exactly the baseline version. Therefore we can assess that this specification fails to replicate the hump-shaped response function in consumption, a condition that holds also with regards to the monetary shock (Figure 5). However, the impulse response functions of the other cases strongly differ from the baseline version. The consumption path reaches its peak around the fifth period, even though the response is way lower if compared to the baseline version. Moreover, the path goes back to the steady state level smother in all the three new specification.

Fig. 4. IRF comparison of Productivity Shock

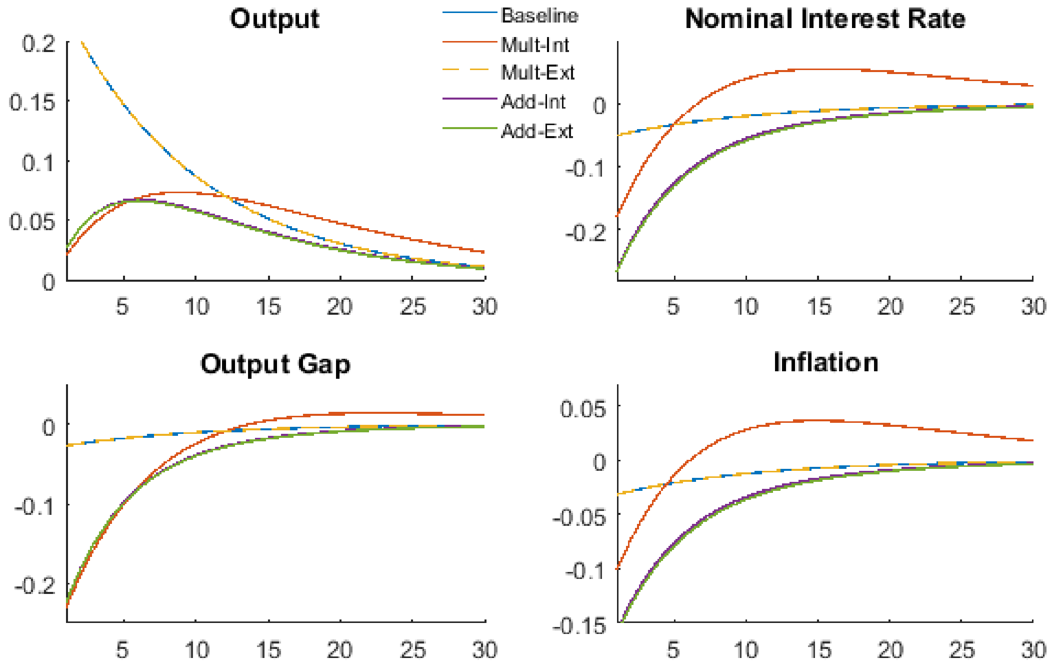
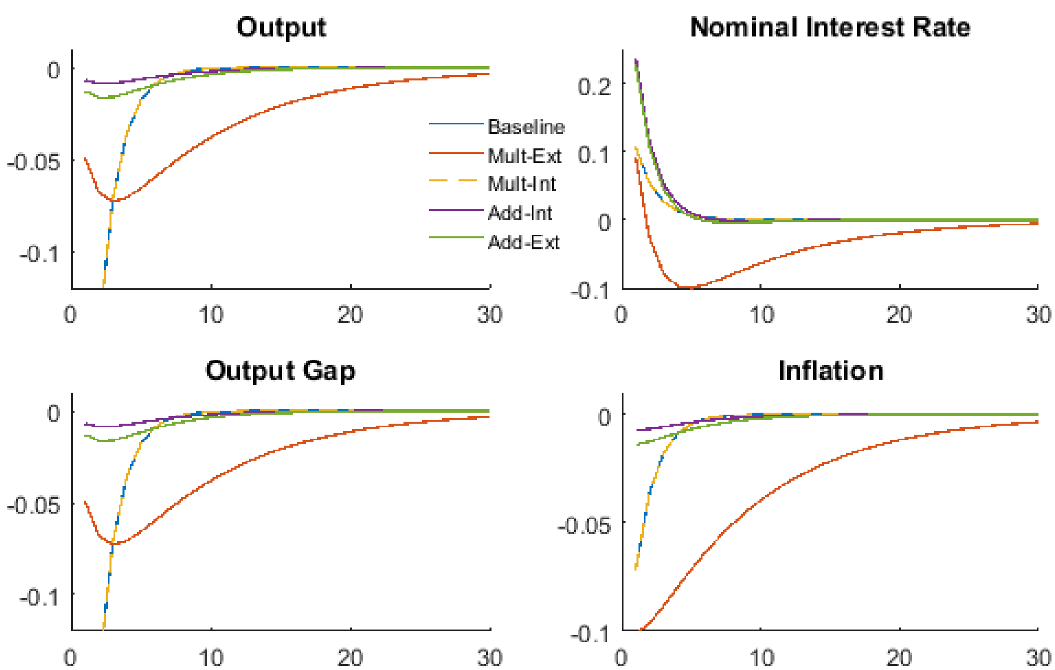


Fig. 5. IRF comparison of Monetary Shock



This behaviour follows exactly our beliefs: the consumer, in fact, wants to smooth not only the intertemporal consumption, but also the changes in consumption. As a consequence, she doesn't like marked changes over time and, therefore, we can observe lower reactions. Furthermore, whereas both the external/internal - additive specifications replicate each other pretty well, following the intuition of Dennis (2009) who showed that there were not consistent difference between the two cases, the choice for a DSGE having a multiplicative-internal Euler implies an increases of the *consumption smoothing* phenomenon: indeed, the consumption path reaches its peak slightly later than the other two Euler equations and shows a higher persistence of the shock, how testified by the longer time it takes to return to the steady state level. Conversely, the three specifications don't differ too much in the Output-gap graph: here it is important to underline how both the baseline and the multiplicative-external Euler equations show a weaker response to the shock with respect to the other cases. This can have important consequence in the policy choices, given the fundamental role of the output gap, for example, in the European framework [13]. The impulse response functions of Inflation and Nominal Interest Rate show the same characteristic of the ones observed in the Output gap: the path of the baseline is flatter and the response lower. This is not by chance, given the relation of these two variable with the one of the output gap, and, again, it demonstrates the importance of accounting for habit formation in policy analysis. Again, the two additive forms still have the same path, while, in the multiplicative internal case, it is important to underline how here the IRF has a positive peak around the tenth period, even though the shock initially implies a reduction of the prices.

Figure 5, conversely, represents the Impulse Response Functions in case of a positive monetary shock. A part from the multiplicative-external habits, that replicate the behaviour of the baseline version of the Euler, the results further stress the importance of considering habit formation and the different specifications in which they can enter the model. Indeed, a change in the nominal interest rate can be thought as one of the instrument held by the Central Authority to either cool down or stimulate the economy, and the decision of how to set the level of i_t is strictly correlated to changes in the output gap and inflation. In this sense, additive habits determine a quasi-zero response in consumption (an then output and output gap), but also lower change in the level of the inflation rate. On the other hand, the

[13] The estimation of the output gap has been increasing in importance in the last years. In fact, the difference between the *Potential Output* and *Actual Output* is fundamental in the framework of the *Fiscal Compact* agreement and in the decision of the levels of government expenditure and public debt. For a thoroughly reading on the subject: European Commission (2017), Recent developments in fiscal surveillance, Report on Public Finances in EMU 2017, 39-70; Schaechter, Andrea, et al. "Fiscal rules in response to the crisis-toward the'next-generation'rules: A new dataset." (2012).

multiplicative-internal form has important consequences both in the nominal interest rate and inflation level. Regarding the former, we can observe how the nominal interest rate reaches negative values and peak around the firth period at -0.1, returning back to the stead state level only at the thirtieth period. This can be related with the problem of the *zero-lower bound* which has recently affected all the European area [14]. Moreover, the persistence of the deflation level appears to be stronger and more persistent with respect to the baseline version and the additive cases.

6. Estimation

6.1. DSGE Models Estimation - The Bayesian Framework

In this section I estimate the structural parameters of the New Keynesian DSGE model following a bayesian technique. Given the advantages in using this methodology in the macroeconometric framework [15], this different approach has increased in popularity in the recent years as a better method for the estimation of DSGE models, and it has been applied in plenty analysis by several authors (e.g: An, Sungbae and Schorfheide, 2007; Smets and Rafael Wouters, 2007).

The basic difference between the *bayesian* approach and the *frequentistic* one is that in the former the parameters to be estimated are considered random variables instead than unknown constants (Koop, 2003). This leads us to a different estimation object: instead than looking for a *point estimation*, in the bayesian approach we compute a *density estimation*. A better intuition can be given by looking at the Bayes Theorem on two events A and B [16] first introduced by Bayes in 1763:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{6.1}$$

The probability of the event A given the event B depends on the marginal probabilities of A and B (defined as P(A) and P(B) respectively) and the probability of B given A; the

[14]The Zero Lower Bound (ZLB) occurs when the short-term nominal interest rate is at or near zero, causing a liquidity trap and limiting the capacity that the central bank has to stimulate economic growth. For further readings: Wu, Jing Cynthia, and Fan Dora Xia. "Measuring the macroeconomic impact of monetary policy at the zero lower bound." Journal of Money, Credit and Banking 48.2-3 (2016): 253-291.; Summers, Lawrence H. "US economic prospects: Secular stagnation, hysteresis, and the zero lower bound." Business Economics 49.2 (2014): 65-73.

[15]For further reading regarding the subject, I suggest: Fernández-Villaverde, Jesús (2010)Sims (2007), Canova(2011)

[16]The definition of *Bayesian Statistic* comes exactly from this theorem.

formula expressed in 6.1 basically shows that our view of event A should change given the new information provided by the event B. Let us now assume that we have a vector of unknown parameters θ such that $\theta = (\theta_1, ..., \theta_N)$ and that we observe the data y such that $y = (y_1, ..., y_N)$, we can rewrite the *posterior* distribution of θ given the data y following equation 6.1 as:

$$P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(y)} \tag{6.2}$$

Equation 6.2 perfectly explains what is behind this new approach. Indeed, the expression tells us that the *posterior* distribution (the left-hand side of the equation) is given by two main components (at the right-hand side of the equation): the likelihood function, $P(y|\theta)$, and *prior* density of the parameter $P(\theta)$ that represents our prior *beliefs* before we look at the data. Basically, our prior beliefs on the distribution of the parameter θ are *updating* by the information that we can extract from the new observation (the data), and the combinations of this two components gives us the updated density, defined as *posterior* density, of θ given y (Chan, 2017).
Therefore, by defining the symbol $\propto$ as *directly proportional to*, we can rewrite the previous expression as:

$$p(\theta|y) \propto p(y|\theta)p(\theta) \tag{6.3}$$

that is equivalent to write:

$$\text{Posterior density} \propto \text{Likelihood function}*\text{Prior density}$$
$$\text{Beliefs after data} \leftarrow \text{Influence of data}*\text{Beliefs before data}$$

Once obtained the posterior, one can calculate the moments by taking, for instance, the posterior mean

$$E(\theta|y) = \int \theta p(\theta|y)d\theta \tag{6.4}$$

However, the posterior distribution may not be always easily available. Indeed, there may be cases in which the product between the likelihood function and the prior density has a closed form solution, as in the case in which both the components have the same *shape* (for instance, both are normally distributed) [17], but also cases in which there is not an analytical

[17] This case is defined as *Conjugacy.*

solution between these two components.
To solve this problem, one can suppose to extract a number N of draws from the posterior $p(\theta|y)$ knowing that the sample mean $\hat{\theta} = N^{-1} \sum_{n=1}^{N} \theta^n$ converges in probability to $E(\theta|y)$ as $N \to \infty$ thanks to the weak law of large number. This approach is called *Monte Carlo Integration*.
The problem is, therefore, understanding how one can simulate directly from the posterior distribution, and, in many cases, this is not trivial at all.
Hoe can one solve this issue? Firstly, we can imagine to draw a density that covers the target density $p(\theta|y)$; it is not by chance that the name of this density is exactly *covering* density. Secondly, in order to extract the draws from this covering density, one possible solution is following the *Metropolis Hastings* (MH) algorithm, whose name comes from the studies of Metropolis et al (1953) and Hastings (1970). The MH algorithm is one class of the Monte Carlo Markov Chain (MCMC) algorithms, and thanks to it we can get draws that are going to be either *accepted* or *rejected* according to an acceptance function α. The draws are computed following the idea of a Markov Chain of order 1 in which the following draw θ^j depends on the previous θ^{j-1}:

if

$$\theta^j \text{ is accepted, } \to \theta^j \text{ is our new starting point}$$

whereas, if

$$\theta^j \text{ is rejected } \to \text{ we keep drawing from } \theta^{j-1} \text{ until we find an accepted draw}$$ [18]

6.2. *Estimation Methodology and Data*

Let us contextualize this bayesian technique in the framework of the estimation of the DSGE models. In this case, the likelihood function is distributed *Normally*. This is due to the fact that the likelihood function comes from the *Kalman Filter* algorithm, that provides normally distributed likelihood functions by definition. This algorithm was first introduced by Kalman in 1960, and nowadays it is used for estimation procedure of State Space models [19].
However, a closed solution between the likelihood function and the prior density of our parameters is not possible, and, therefore, it may be difficult to derive the posterior density.

[18] In the Appendix I provide a better explanation of how a MH and MCMC work.
[19] In the Appendix there will be a briefly explaination of what this power algotithm does. Moreover, for a deeper reading regarding State Space modelling and Kalman Filter: Durbin and Koopman (2012), Harvey (1993).

To this purpose, we apply the Metropolis Hasting algorithm explained above.

To construct the dataset I deploy quarterly data from 1994Q1 to 2017Q4 of Real GDP, CPI and Short Term Interest Rate [20] of Sweden, such that the observable variables match with the variables that are shocked in the DSGE model [21].
In order to fit the model variables, the observed series have been transformed in the following way:

$$\pi_t^{data} = log\Big(\frac{CPI_t}{CPI_{t-1}}\Big)\Big) - log\Big(mean\Big(\frac{CPI_t}{CPI_{t-1}}\Big) = \pi_t \tag{6.5}$$

$$i_t^{data} = log\Big(1 + \frac{i_t^{data}}{400}\Big) - log\Big(mean\Big(1 + \frac{i_t^{data}}{400}\Big)\Big) = i_t \tag{6.6}$$

$$y_t^{data} = y_t^{data} - y_t^{trend} = \tilde{y}_t \tag{6.7}$$

with y_t^{data} equal to the log of the Real GDP, and y_t^{trend} extracted using the Hodrick and Prescott filter with a smoothing value equal to 1600.

In accordance to the usual practice in the literature (Cantore et al, 2014), a number of structural parameters are kept fixed. In particular, the discount factor $\beta = 0.99$, the Cobb-Douglas parameter $\alpha = 0.36$, the Inverse of Frisch elasticity of labor supply $\psi = 1$, the elasticity of substitution between goods $\epsilon = 6$, the probability of resetting price $\theta = 0.66$.
Conversely, priors distributions are assigned to the rest of the parameters. For the parameters commonly found in DSGE models I use priors chosen in previous studies, such the one of Smets and Wouters (2007). With regards to the parameters of the Taylor rule, ϕ_π and ϕ_y, I assume a Normal distribution with mean and standard deviation respectively equal to 1.5 and 0.12, 0.25 and 0.05. The parameters linked to the persistence of shocks are, instead, distributed with a Beta pdf, given that they are bounded between [0,1]. All the three $\rho_{a,m,p}$ have mean 0.5 and standard deviation equal to 0.2. For the variance of the shocks ϵ_a, ϵ_m, ϵ_p, instead, we set the an Inverse Gamma distribution, with mean equal to 0.05 and standard deviation equal to 2. This choice is given by the fact that we need this parameter to assume only positive values, given that the variance assume only positive value by definition. The Inverse Gamma distribution provides us this condition given that it's support is $(0,\infty)$.

[20] All the data are available at the World Bank dataset and IMF Financial Statistics.

[21] For estimating need I have added an inflation AR(1) shock to the equation of the output gap with $\rho_p = 0.8$ and $\epsilon_p = 0.5^2$

Table 4: Bayesian Estimation - Priors

Parameters	**Priors**		
	Distribution	Mean	Std
Multiplicative			
$\sigma_{internal}$	*Normal*	1.5	0.3
$\gamma_{c,internal}$	*Beta*	0.8	0.1
Additive			
$\eta_{c,external}$	*Beta*	0.8	0.1
$\eta_{c,internal}$	*Beta*	0.8	0.1
$\alpha_{c,external}$	*Inverse Gamma*	5	0.3
$\alpha_{c,internal}$	*Inverse Gamma*	3	0.3

The focus goes now on our parameters of interest γ_c, η_c and the two coefficient of relative risk aversion: σ_c (in the multiplicative case) and α_c (in the additive case). Firstly, I decide not to estimate the parameters of the *multiplicative-external* case, given that it doesn't differ from the results given by the baseline version of the Euler equation. Regarding the parameter γ_c, it can assume value between [0,1], therefore I set a Beta prior distribution. Same hypothesis can be done on the parameter η_c. The values of the mean and standard deviation of these parameters are equal to 0.8 and 0.1 respectively. With σ_c I follow the study of Cantore et al (2014), where the prior distribution is a Normal pdf with mean and standard deviation equal to 1.5 and 0.3. For α_c I suppose an Inverse Gamma distribution given that is can assume any values between 0 and ∞. Following the previous literature, I set the its mean equal to 3 and 5 for the internal and external case respectively, and a standard deviation equal to 0.3 such that it fits with the one of σ_c. The choices regarding the priors of the parameters are summarized in Table 4

I run two parallel chains in the Metropolis-Hastings algorithm, each one with 50,000 draws and a burning period equal to the first 10% of the draws for each chain [22].

[22]The *burning period* indicates the total number of draws that are not stored and considered in the procedure of the MH algorithm. The utility of this option relies on the fact that the first draws are always correlated to the starting values; therefore, by discarding a portion of the first draws, one can solve this problem of correlation.

Table 5 shows the resulting posterior mean of the parameters along with the HPD (95% Highest Posterior Density) Interval. The posterior distributions of the parameters are, instead, plotted in the Appendix section.
Regarding $\sigma_{internal}$, its value is slightly below the level estimated by Cantore et al (2014), but still consistent with the theory.

Table 5: Bayesian Estimation - Results

Parameters	Mean	HPD Interval
Multiplicative		
$\sigma_{internal}$	1.61	[1.5403 ; 1.6863]
$\gamma_{c,internal}$	0.66	[0.6159 ; 0.6879]
Additive		
$\eta_{c,external}$	0.77	[0.7208 ; 0.8108]
$\eta_{c,internal}$	0.67	[0.6306 ; 0.7167]
$\alpha_{c,external}$	5.32	[4.8631 ; 5.7947]
$\alpha_{c,internal}$	2.38	[2.1086 ; 2.6346]

The parameters that are related to the magnitude of the persistence of habits formation, namely γ_c and η_c, show similar results. $\gamma_{c,internal}$ has a mean equal to 0.66, very close to the one of $\eta_{c,internal}$ which has a mean equal to 0.62. Instead, $\eta_{c,external}$ shows a value equal to 0.77. These results are in line with the previous literature, with the studies of Christiano, Eichenbaum and Evans (2005) and Gruber (2004) that respectively estimated η_c to be equal to 0.64 and 0.81, whereas Fuhrer (2000) estimated γ_c to be equal to 0.8. Finally, $\alpha_{c,internal}$ shows a posterior mean equal to 2.38, a value lower if compared to the estimation carried out by Dennis (2009) in which $\alpha_{c,internal}$ assumes a value equal to 3.180, while $\alpha_{c,external}$ matches with the finding of the same study conducted by Dennis, where this parameter is equal to 5.5.

7. Conclusion

In this study I have thoroughly analyzed what is behind the Euler equation and the importance of accounting for the hypothesis of Habits Persistence in a New Keynesian DSGE framework. In doing so, the work showed the various form in which it is possible to model the idea of habits formation, considering the combination between additive/multiplicative specifications and internal/external cases, where with *internal* we mean that consumption at time t depends on the own consumption at time $t-1$ of the consumer, whereas the *external* case is more related to the study of the envy component in the consumption behaviour, given that it is the aggregate consumption at $t-1$ that influences the consumption decision at time t.

In the first part, I have carried out a comparison of the impulse response functions (IRF) among five different DSGE models, each one having a specific way to model habits (i.e: no habit formation, multiplicative-internal habit, multiplicative-external habit, additive-internal habit, additive-external habit). Here, the results have shown the ability of the new Euler equations to replicate hump-shaped IRF (with the exception of the multiplicative-external case). Moreover, under the hypothesis of additive specification, there are only few difference between the internal and external cases, a result that is in linea with the previous finding of the literature (Dennis, 2009). However, within the multiplicative specification framework the difference in the choice of accounting for either internal or external habits becomes quite important. Indeed, the multiplicative-external case doesn't provide any improvement if compared to the baseline model, whereas the multiplicative-internal results strongly differs from the others, showing a peculiar path in the inflation and nominal interest rate both in the case of a productivity and monetary shock.

In the second part of the study, instead, I have gone through the estimation of the parameters of the different Euler equations using a Bayesian approach. In doing so, I have deployed aggregate Swedish data on Real GDP, short term interest rate and inflation, with the results are generally in line with the previous literature.

This study open the door for further analysis, for instance, in the comparison of the estimated parameters among different countries, such that it will be possible to understand the various dynamics behind habits formation among diverse cultures. Moreover, the estimated parameters can be compared also with estimation of the Euler equations on microdata, in order to try to reconciliate the habits evidence between the macroeconomic and microeconomic framework.

APPENDIX

The Metropolis Hasting algorithm

The Metropolis Hastings proceudre is based on the class of Monte Carlo Markov Chain algorithms. How the name suggests, the latter can be decomposed in two parts:
a) The Monte Carlo simulation, which the reader should be familiar to, that is related to the idea of randomly drawing observations from a specific density function with defined mean and variance;
b) The Markovian Chains, where, having a sequence of observations, each value is dependent only on the previous one. Basically, supposing we have an observation at time t, the only component from which it is dependent on (in all the information set S_{t-1}) is precisely the observation at time $t-1$.

The idea of the algorithm is, then, the following: given that, as explained in section 5, the product between the prior and the likelihood doesn't have analytical solution, we have to find another way to simulate from the posterior in order to find the sample moments. Therefore, we address a candidate density from which we will extract. This density has a variance and a mean, with the latter that depends only on the previous draw, so that θ^j is extracted by $q(\theta^{j-1}, \Sigma)$. However, it is just the mean that changes, whereas the shape of the candidate density will always be the same.
Finally, there is just a missing part, the *acceptance* and *rejection* procedure. Indeed, all the θ that are drawn following the method just explained are subjected to a *test*. We decide whether to accept or reject a specific observation just drawn according to this ratio:

$$\alpha = min\Big[1, r(\theta^j, \theta^{j-1})\Big] \tag{7.1}$$

with r that can be thought as a ratio between the posterior probabilities evaluated respectively with the new and old draws:

$$r(\theta^j, \theta^{j-1}) = \frac{\text{Posterior of } \theta^j}{\text{Posterior of } \theta^{j-1}} \tag{7.2}$$

$$r(\theta^j, \theta^{j-1}) = \frac{\text{Likelihood}(\theta^j)\text{Prior}(\theta^j)}{\text{Likelihood}(\theta^{j-1})\text{Prior}(\theta^{j-1})} \tag{7.3}$$

Equation 6.8 tells us that if the ratio is > 1, namely, the posterior probability of the new θ is higher than the one of the old θ, we accept the draw. Conversely, if the ratio is < 1, we extract a number from a Uniform density $U \sim [0, 1]$ and compare the draw with the value just simulated: if the draw is ¿ than the simulated number, we accept the draw; if the draw is ¡ than the simulated number, we reject the draw and repeat the all procedure starting again from θ^{j-1}.

The Kalman Filter algorithm

The Kalman filter provides the one-step-ahead predictions of our states component:

$$\alpha_{t+1|t} = Z\tilde{\alpha}_{t|t-1} + Kv_t \tag{7.4}$$

$$P_{t+1|t} = TP_{t|t-1}T' + RQR' + K_tF_tK_t \tag{7.5}$$

thanks to the following recursive algorithm:

$$v_t = y_t - Z\tilde{\alpha}_{t|t-1} \tag{7.6}$$

$$F_t = ZP_{t|t-1}Z' + H \tag{7.7}$$

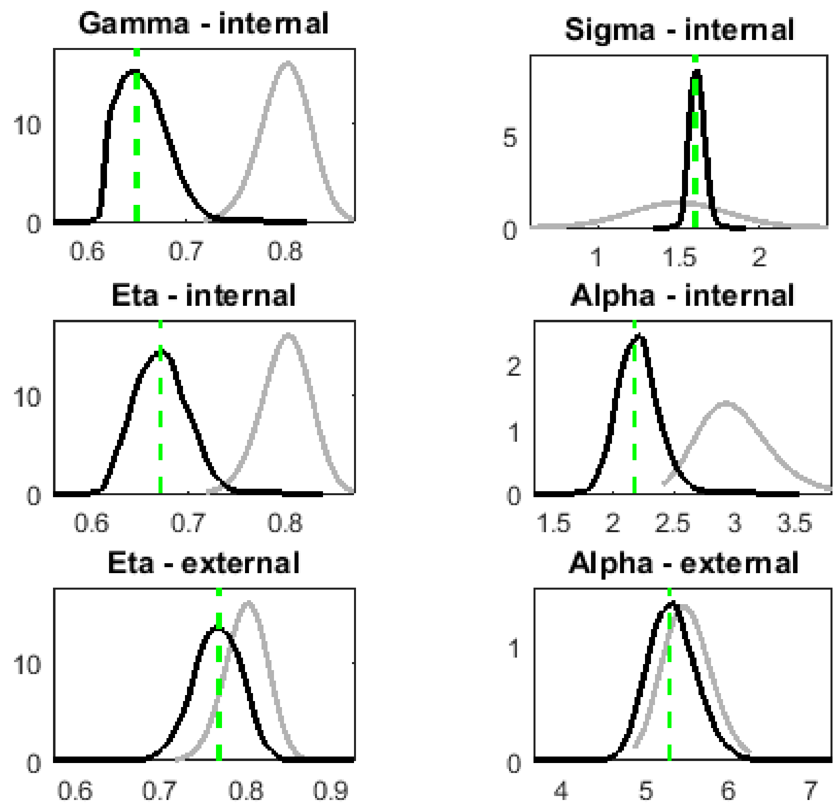
$$K_t = (TP_{t|t-1}Z' + R)F_t^{-1} \tag{7.8}$$

with v_t equal to the innovations, F_t the variance of y_t conditional on the information set at time $t-1$, K_t the *Kalman Gain* which is the key of the algorithm.
Through the *Prediction Error Decomposition*, we can obtain the Log-likelihood function, defined as:

$$L(Y_i;\theta) = -\frac{1}{2}\Big[log(2\pi) + \sum_{t=1}^{T} log(|F_t|) + \sum_{t=1}^{T} v_tF_tv_t'\Big] \tag{7.9}$$

Bayesian Estimation - Posterior Distributions

Fig. 6. Bayesian Estimation - Posterior Distributions



Impulse Response Function with Calibrated Parameters - Productivity Shock

Fig. 7. IRF - Internal Habits with Multiplicative specifications

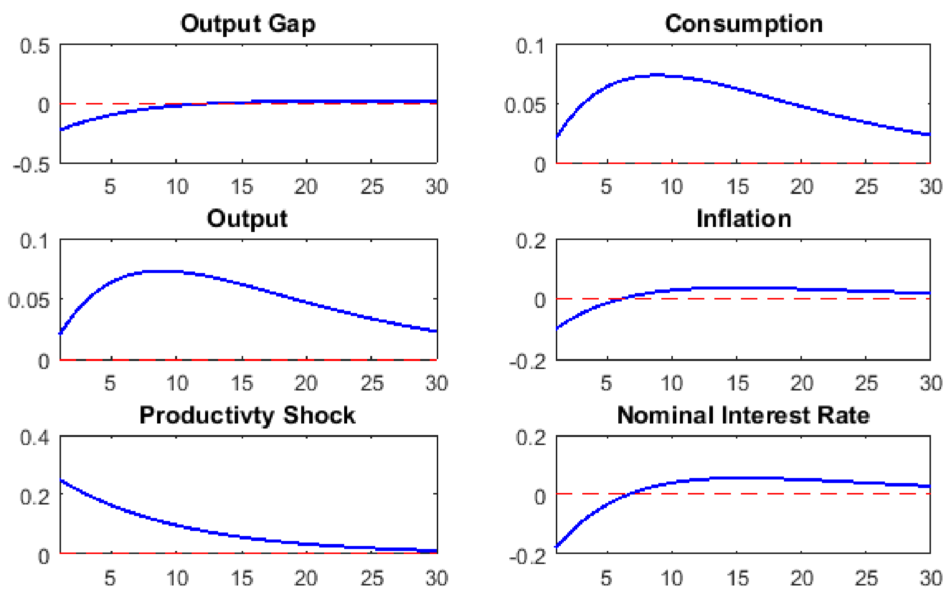


Fig. 8. IRF - Internal Habits with additive specifications

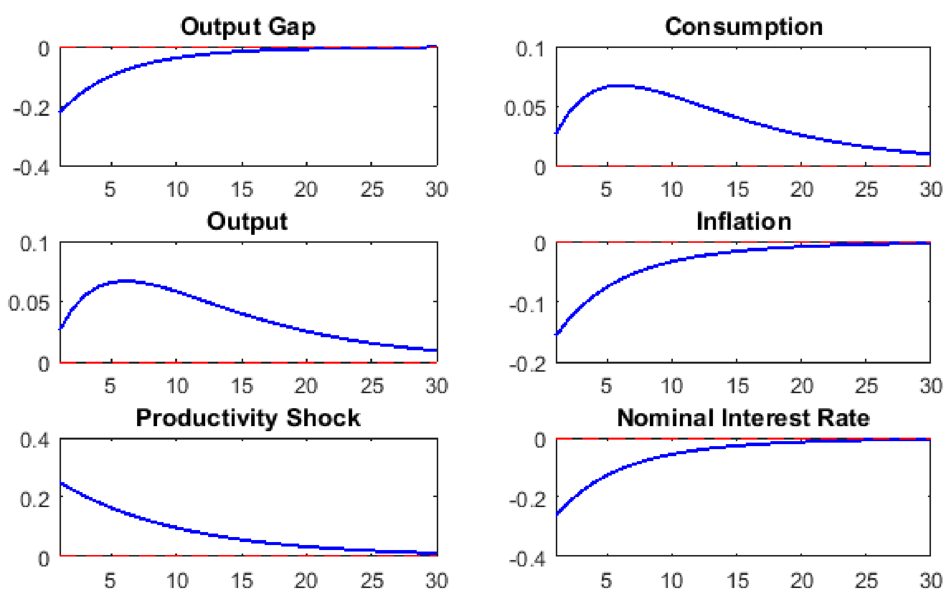


Fig. 9. IRF - External Habits with Multiplicative specifications

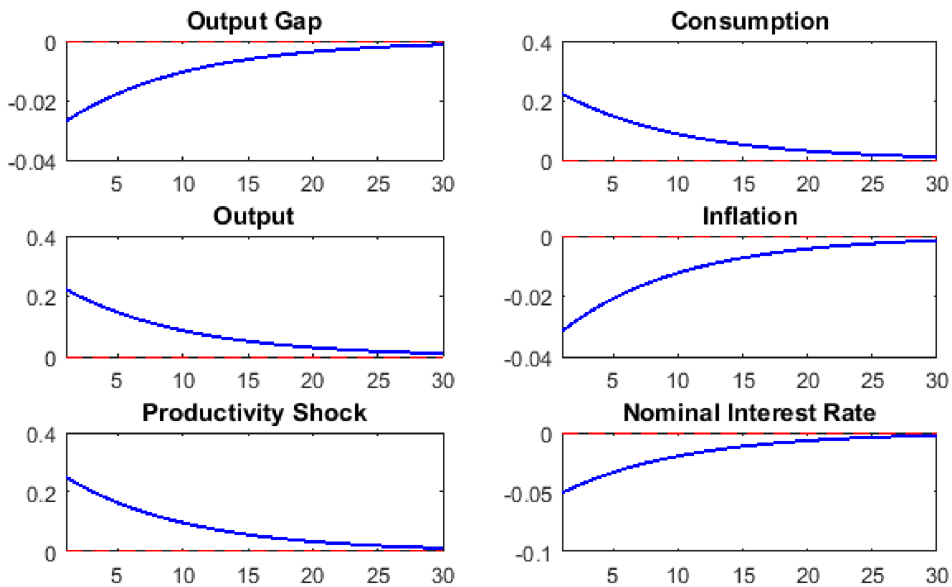
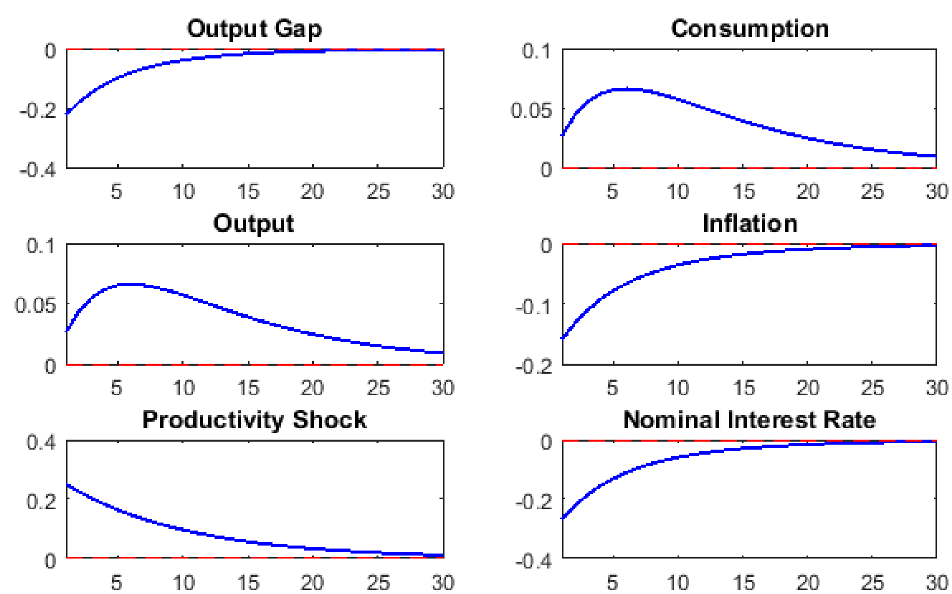


Fig. 10. IRF - External Habits with Additive specifications



Impulse Response Function with Calibrated Parameters - Monetary Shocks

Fig. 11. IRF - Internal Habits with Multiplicative specifications

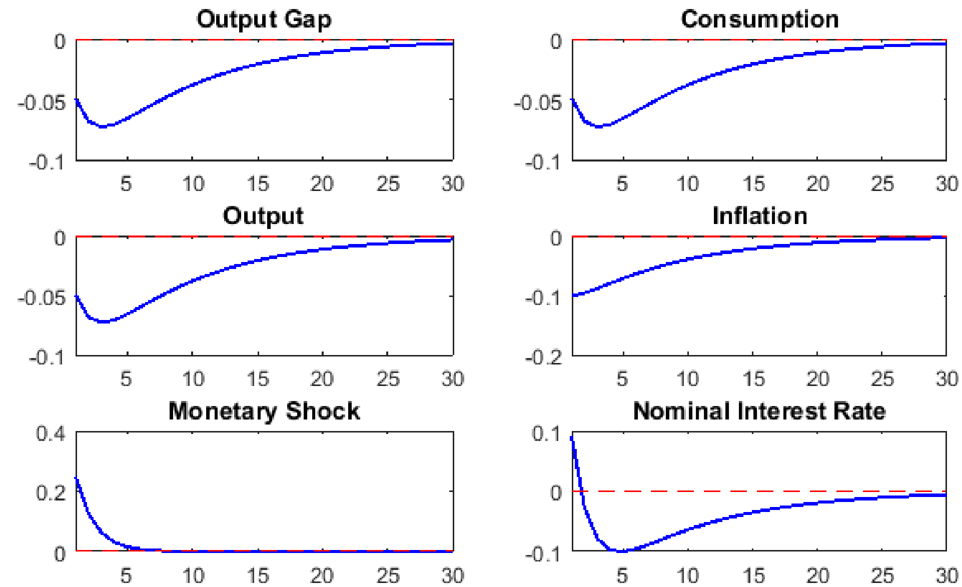


Fig. 12. IRF - Internal Habits with Additive specifications

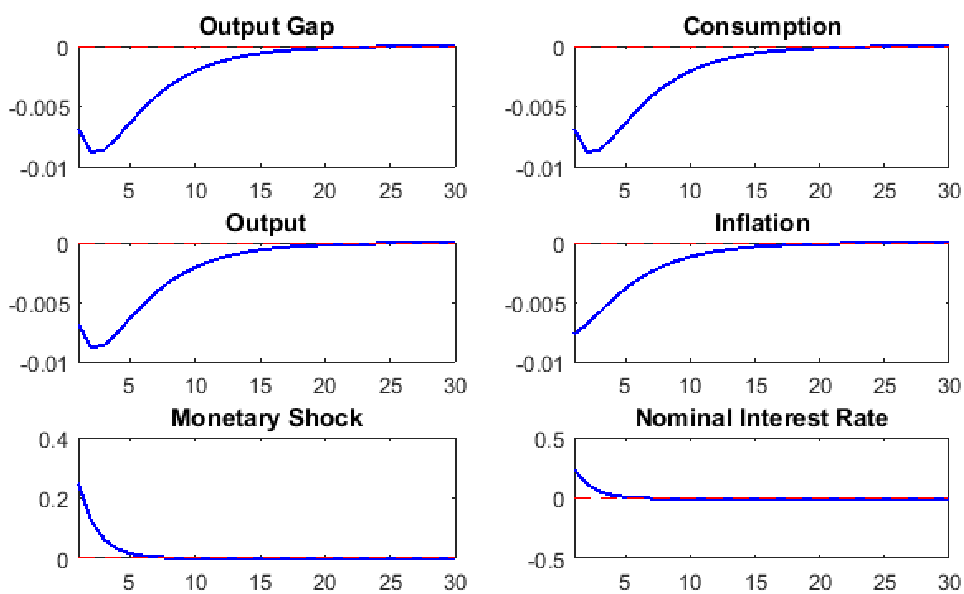


Fig. 13. IRF - External Habits with Multiplicative specifications

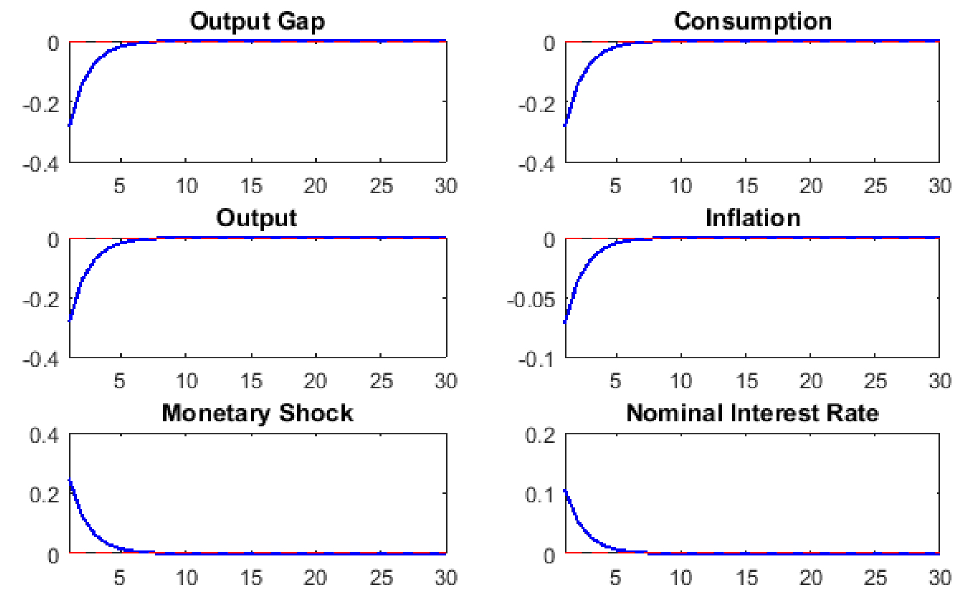
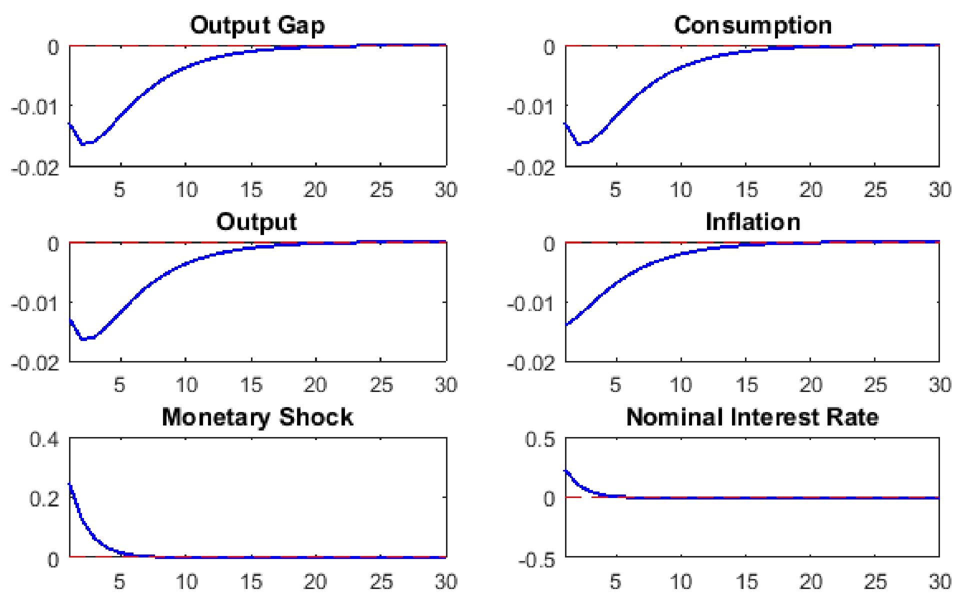


Fig. 14. IRF - External Habits with Additive specifications



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