Patent Licensing and Duplication in Cournot Structures

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An earlier version of this paper was presented at the Nordic Conference on Industrial Organization (NORIO) at Reykjavik University in Reykjavik, Iceland, May 30-31, 2016.

Abstract

This study examines and compares patent licensing through a fixed fee and a per-unit royalty under Cournot competition. We consider licensing by an incumbent patent owner to one or two other competing firms that can obtain the patented invention through a technology transfer or imitation. We assume that imitation is perfect (duplication), certain, instantaneous, and aims not to risk infringement. This study aims to determine the effect of licensing on firms' individual profits, consumers' surplus, duplication, and competition. The analysis suggests that licensing through a per-unit royalty might be preferable to licensing through a fixed fee for a patent owner, while fixed-fee licensing might be at least as good as royalty licensing for consumers. Additionally, patent owners might also use licensing to prevent duplication, but might not use it selectively to affect competition, at least before the patent expires and when one of the competing firms might duplicate.

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1 Introduction

The patent system emerged as a regime that grants privileges to inventors by mapping out territorial protection over a certain period. Essentially, a patent is a legal title that is enforceable in court and granted for an invention that has a new, non-obvious, and useful technical characteristic with potential industrial application (see, Taylor and Silberston, 1973). We consider the patent system as an institution of proprietorship for inventions and a source of scientific information. Patenting can thus serve as a mechanism that facilitates the diffusion of technology and fosters progress. Typically, there are two conventional sources of profit for a patent owner: working with the patent on its own and licensing the patent rights in return for a fee. Patent licensing is a significant source of profitability for innovating firms, and a valuable source of information for an industry (see, Nordhaus, 1969; Pepall et al., 2008, Chapter 13). Qualcomm, an industry leader in digital communication technology, generates more than half of its profit from licences.[1] In addition, firms use patent protection strategically in order to gain a competitive advantage over their rivals. For instance, a common reason that firms might license their patents is to prevent competitors from developing similar, if not superior, technologies. Typically, firms might use patent licensing to deter entry, prevent imitation, or select competition (Rockett, 1990; Denicolo and Franzoni, 2003). Overall, licensing is an important, inseparable extension of patents considering that most technology transfer agreements occur through the patent system.

In this study, we extend the analysis of Wang (1998), who studies patent licensing in a Cournot duopoly, by considering duplication and an industry composed of three firms with asymmetric per-unit costs.[2] In particular, we compare fixed-fee and per-unit royalty licensing of a cost-reducing technology in an industry of two and three incumbent firms that produce a homogeneous product and compete simultaneously in quantities, that is, a Cournot competition. Suppose that one of the firms develops and patents a cost-reducing technology and at least one of the other competing firms might obtain the patented invention through a technology transfer or imitation, which we assume is perfect and referred to hereafter as duplication. We also assume that duplication occurs with certainty and does not risk infringement. Then, a patent owner who aims to maximise his/her profit, might offer an exclusive licence to any competitor or both of them.[3] First,

[1]For more information on Qualcomm's patent licensing, see www.qualcomm.com.

[2]Licensing through a fixed-fee means that the patent owner will receive an up-front lump sum payment at the commencement of the licensing contract, while licensing through a per-unit royalty means that the patent owner will receive a continuous periodic fee per-unit of output produced with the patented invention.

[3]We assume that an exclusive licence does not lead to collusive behaviour.

we show when a technology transfer might occur in equilibrium. Second, we determine the optimal licensing choice from the perspectives of the patent owner and consumers. Last, we analyse the effect of licensing on duplication and competition.

The analysis suggests that royalty licensing is at least as attractive as fixed-fee licensing for a patent owner, while it is at most as good as fixed-fee licensing for the consumers. The analysis also suggests that licensing through a per-unit royalty, both exclusively and commonly, might occur in equilibrium in different situations. Additionally, we find that patent owners might use licensing to prevent duplication. These findings are consistent with propositions in the existing literature (Wang, 1998; Denicolo and Franzoni, 2003). Finally, licensing does not appear to be selective and, thus, patent owners might not use it, at least ex-ante patent expiration, to affect competition—offer an exclusive licence to the weak competitor aiming to weaken the strong competitor. This result might also arise due to the possibility of duplication.

The rest of this paper presents an analysis of the issues described above. Section 2 reviews selected literature on patent licensing. Section 3 presents the basic model. Section 4 extends the basic model to a three-firm industry with asymmetric per-unit costs. Section 5 concludes and proposes avenues for future research. All proofs are given in the Appendix.

2 Literature Review

Technology transfer through patent licensing has become a key ingredient for firms to gain a competitive advantage and stay at the frontier. Licensing enables a patent owner to retain control over an invention and stipulate conditions regarding its use. A patent owner has, in fact, an enforceable right to decide who shall and who shall not use a patented invention. One of the earliest studies that reviews the patent act is Smith (1890). The author argues that it is only through the patent system that the human passion to make improvements and to innovate can acquire a marketable value. However, Arrow (1962) is one of the first attempts to put patents into an economic perspective by formally analysing the profits a patent owner can realise from licensing a cost-reducing technology to a perfectly competitive industry. Following Arrow (1962), several studies focus on the licensing of innovating firms and the effects of technology transfer on the incentives to innovate (McGee, 1966; Scherer, 1967; Barzel, 1968; Kamien and Schwartz, 1972).

Taylor and Silberston (1973) are among the first to suggest that the patent system cannot grant firms perfect appropriability of returns from their investment in research and development for several reasons. First, patent protection is not absolute and, thus, cannot prevent imitation. Second, infringement might be difficult to prove; thus, patent enforcement might be weak. Third, patents have a probabilistic nature if we consider that in theory, all patents can be challenged in terms of their validity in a court of law. In turn, they study the economic impact of the patent system by focusing on patent owners that are incumbent firms in an industry. Like Taylor and Silberston (1973), we consider a single incumbent firm that competes with other firms in an industry that can duplicate the protected technology. However, note that innovating firms often specialise in developing new technologies with the main purpose of licensing these inventions rather than working with them (Kamien, 1992).

Wilson (1977) summarises the empirical evidence on domestic and international technology transfer. His analysis suggests that firms do not license domestically in order to preserve market barriers, while they are more willing to license to firms that they do not expect to compete with directly and that often operate in foreign markets. Two other influential empirical studies are by Mansfield et al. (1981) and Mansfield (1985). These studies focus on the effects of imitation speed and cost on the incentives to innovate and patent. Their findings indicate that the disclosure of valuable technical information and the ease of imitation might explain the increase in imitative activity and the concern about imperfect appropriability. In this study, we focus on the effects of the magnitudes of innovation and duplication cost on technology transfer to improve, in part, our understanding of these issues.

The most frequent modes of technology licensing are auction licensing, fixed-fee licensing, royalty licensing, and a combination of an upfront fixed fee and a per-unit royalty. In fact, the most common form in practice is licensing through a per-unit royalty (Rostoker, 1983; Jensen and Thursby, 2001).[4] In the 1980s, studies by Gilbert and Newbery (1982), Gallini (1984), Kamien and Tauman (1984, 1986), Shapiro (1985), Katz and Shapiro (1985, 1986), Shepard (1987), and Kamien et al. (1988) set the foundation for a game-theoretic approach to optimal patent licensing, which later became the basis for subsequent work on technology transfer in oligopolistic industries.[5]

[4]For an introductory review on patent licensing, refer to Pepall et al. (2008), Chapter 11.

[5]The most common issues prior studies on patent licensing address include the optimal patent breadth-length mix (Gallini, 1992; Denicolo, 1996; O'Donoghue et al., 1998; Hopenhayn and Mitchell, 2001; Koo and Wright, 2010), the welfare implications of patent licensing (De Laat, 1996; Fauli-Oller and Sandonis, 2002; Denicolo and Franzoni, 2003), the value of patent protection rights (Schankerman, 1998; Gallini, 2002; Denicolo, 2007), the trade-off between the social cost of temporary monopoly privileges granted to a patent owner and the gains from the technological progress of patented inventions (Denicolo and

These game-theoretic studies mainly consider the two most common modes of technology transfer. For instance, Kamien and Tauman (1986) compare the performance of fixed fees and royalties and find that a fixed-fee arrangement is preferable to a per-unit royalty from the perspective of both the patent owner and consumers. Marjit (1990) considers a technology transfer between two incumbent firms in a Cournot industry and suggests that the closer the technology gap between the two firms is, the more realisable a technology transfer is. Wang (1998) compares licensing modes in a homogeneous-good Cournot duopoly with an incumbent patent owner. He concludes that royalty licensing dominates fixed-fee licensing in equilibrium due to the cost advantage—a per-unit royalty always increases the licensee's marginal cost of production by the amount of the royalty. Extending his work to a differentiated Cournot duopoly, Wang (2002) reconfirms the superiority of royalty licensing. Wang and Yang (1999) study a differentiated-goods Bertrand duopoly and find that a per-unit royalty can be better than a fixed-fee arrangement from the perspective of a patent owner, despite the reduction in the production cost due to the patented technology. Wang and Yang (2004) consider a Stackelberg duopoly and find that royalty licensing always benefits a market follower more than it does a market leader. Replicating the analysis assuming Stackelberg competition can be a suitable extension of the basic model we develop in this study.

Mukhopadhyay et al. (1999) examine technology transfer through patents to a generalised oligopolistic industry. The authors suggest that cost asymmetry can be a key determinant of technology transfer. Kamien and Tauman (2002) extend Wang (1998)'s model to a general Cournot oligopoly market and find that an incumbent patent owner operating in an industry composed of a large number of firms is always better off licensing through a per-unit royalty than through a fixed fee. Gallini (2002) argues that patent licensing might facilitate the spread of technology transfer and, thus, encourage innovation. In addition, she suggests that licensing might reduce the number of costly infringement disputes because it discourages imitation. Similar to Gallini (2002), we focus on the effects of the magnitude of the innovation and the firms' choice to duplicate on the patent owner's licensing behaviour.

Fauli-Oller and Sandonis (2002) investigate two-part tariff contracts to license a cost-reducing invention to a rival firm and conclude that in a differentiated-goods Bertrand and Cournot duopoly, a positive royalty is always part of an optimal licensing contract.[6] Filippini (2002) performs the same analysis in a Stackelberg duopoly and confirms that

Franzoni, 2004; Hall, 2007; Hall and Harhoff, 2012), and the effect of the size of the innovation on the distribution of profits and welfare (Wang, 1998, 2002; Wang and Yang, 1999; Kamien et al., 1992; Kamien and Tauman, 2002; San Martin and Saracho, 2010, 2014).

[6] A two-part tariff contract is a licensing agreement composed of a fixed fee plus a royalty rate.

the patent owner's profit under royalty licensing exceeds that under fixed-fee licensing. Filippini (2005) studies licensing and its welfare implications assuming Stackelberg competition. He shows that the per-unit royalty still prevails in equilibrium. However, royalty licensing can reduce social welfare and does not benefit consumers. Sen (2002) proposes a simple royalty licensing contract that permits an incumbent firm with a cost-reducing technology to earn a monopoly profit in a Cournot oligopoly market with at least three firms. In a follow-up study, Sen (2005) shows that a royalty can dominate fixed-fee and auction arrangements in a general Cournot oligopoly with an outsider inventor.[7] Sen and Tauman (2007) extend the existing work on two-part tariff contracts by considering the licensing of a cost-reducing technology in a Cournot oligopoly of a general size for both incumbent and outside inventors. The authors show that relatively significant inventions are licensed through a per-unit royalty, whereas less significant inventions do not involve a per-unit royalty. A relatively recent study on two-part tariff licensing is by Kitagawa et al. (2014). The authors address the case when an incumbent firm can license a technology for a new product to a single rival in the industry, who has the option to imitate the protected invention without risking patent infringement. Unlike most of the existing research in this area, Kitagawa et al. (2014) acknowledge that imitation often yields imperfect substitutes for the new product. Furthermore, they point out the important influence of the cost of developing a technology on optimal patent licensing. In particular, they determine an optimal two-part tariff contract that depends on the cost to develop the technology and the degree of product differentiation; that is, the substitution effect.

San Martin and Saracho (2010) compare per-unit and ad valorem royalties and find that ad valorem royalty licensing is more attractive to an incumbent patent owner who competes in a homogeneous-good Cournot duopoly.[8] In a companion paper, San Martin and Saracho (2014) study two-part tariff licensing for an internal patent owner in a differentiated Cournot duopoly. Like Kitagawa et al. (2014), the authors find that the optimal two-part tariff contract depends on the degree of product differentiation. In this study, we also focus on the optimal licensing behaviour of a patent owner when the competing firms in the industry have a choice to duplicate the patented technology without risking infringement. However, we do not consider ad valorem royalties or two-part tariff contracts, which we leave for future work.

[7] See Kamien et al. (1992) for a general licensing policy with an outsider patent owner.

[8] An ad valorem royalty is a predetermined percentage of the licensee's total profit.

3 The Model

This section sets up the basic model, describes the firms' strategic behaviour, and determines the equilibrium actions and profits. We consider a Cournot industry where two firms compete in quantities of a homogeneous good (Cournot and Fisher, 1929). We assume that patent protection is not absolute, duplication does not risk infringement, and each firm aims to maximise its profit. Additionally, we assume a non-cooperative licensing game of complete information and a discount factor equal to one.[9]

3.1 The Nash-Cournot Equilibrium

Consider an industry composed of two identical incumbent firms; that is, Firm i for $i = \{1, 2\}$. Suppose that Firm 1 developed a new technology that reduces the per-unit production cost of a homogeneous good relative to the common per-unit cost c by an amount of ϵ, where $0 < \epsilon < c$.[10] In other words, ϵ reflects the exogenously determined magnitude of the cost reduction or innovation. The inverse demand function for the good is $P = a - Q$, where P is the price of the product, Q is the industry production output, and a is an industry parameter that characterises the demand for the product. Let q_i for $i = \{1, 2\}$ be the quantity of homogeneous goods produced by Firm i associated with a per-unit production cost of c_i for $i = \{1, 2\}$, where $0 < c_i < a$. Let π_i for $i = \{1, 2\}$ be the profit of Firm i. Then, the Cournot profits of Firms 1 and 2 are:

$$\begin{aligned} \pi_1 &= Pq_1 - c_1 q_1 \\ &= (a - c_1 - q_1 - q_2)q_1, \\ \pi_2 &= Pq_2 - c_2 q_2 \\ &= (a - c_2 - q_1 - q_2)q_2. \end{aligned} \tag{3.1}$$

Maximising π_1 with respect to q_1 yields $q_1 = \frac{1}{2}(a - c_1 - q_2)$, which we refer to as the reaction function of Firm 1. In turn, maximising π_2 with respect to q_2 gives the reaction function of Firm 2, which is $q_1 = \frac{1}{2}(a - c_2 - q_1)$. Let the superscript $*$ denote the value of a variable in equilibrium. Then, by solving the two reaction functions simultaneously,

[9] We assume that the costs, profits, actions, and type of invention are common knowledge.

[10] We indirectly assume that Firm 1's R&D expenditure to develop the new technology is a sunk cost, and is thus not relevant for decision-making (Scotchmer, 1991, 1999).

we obtain the following quantities in equilibrium:[11]

$$\begin{aligned} q_1^* &= \frac{1}{3}(a - 2c_1 + c_2), \\ q_2^* &= \frac{1}{3}(a - 2c_2 + c_1). \end{aligned} \tag{3.2}$$

It follows that the equilibrium price is $P^* = \frac{1}{3}(a - c_1 + c_2)$ and the equilibrium profits are:

$$\begin{aligned} \pi_1^* &= \frac{1}{9}(a - 2c_1 + c_2)^2, \\ \pi_2^* &= \frac{1}{9}(a - 2c_2 + c_1)^2. \end{aligned} \tag{3.3}$$

We now consider whether Firm 1 should offer a technology transfer to Firm 2 in return for a fee. In this case, Firm 2 should decide whether to accept the offer; that is, to produce using the existing technology or to duplicate.

3.2 The Equilibrium without a Technology Transfer

We consider the case when a technology transfer does not occur because Firm 1 does not offer a licence or Firm 2 does not accept the offer. In this case, Firm 1 uses the cost-reducing technology and has a per-unit cost equal to $c - \epsilon$. Firm 2, however, can choose between using the existing technology or duplication by incurring a lump-sum cost of C, where $C > 0$. First, suppose that Firm 2 chooses to use the existing technology and not to duplicate, and let π_i^{ND} for $i = \{1, 2\}$ be the profit of Firm i in this case. Then, a direct substitution of $c_1 = c - \epsilon$ and $c_2 = c$ into Equation 3.3 yields the following Nash-Cournot profits of Firms 1 and 2:

$$\begin{aligned} \pi_1^{ND} &= \frac{1}{9}(a - c + 2\epsilon)^2, \\ \pi_2^{ND} &= \frac{1}{9}(a - c - \epsilon)^2. \end{aligned} \tag{3.4}$$

The profit of Firm 1 is always positive according to Equations 3.2 and 3.4. However, Firm 2 produces a quantity larger than zero if $\epsilon < a - c$ and a quantity equal to zero

[11] Note that each solution has to be non-negative for the results to hold.

otherwise. In which case, Firm 1 will become a monopoly.

According to the existing literature, a cost-reducing invention is drastic if the fee set by a monopolist that uses the new technology is less than or equal to the per-unit production cost associated with the existing technology (Arrow, 1962). We will assume that a drastic invention corresponds to a magnitude of the innovation large enough to drive the competitors out of the industry. In particular, if $\epsilon < a - c$, then the invention is non-drastic, and if $\epsilon \geq a - c$, then the invention is drastic. Consequently, the firms' profits in equilibrium are:

$$\begin{aligned}
\pi_1^{ND*} &= \frac{1}{9}(a - c + 2\epsilon)^2 \quad \text{for } \epsilon < a - c, \\
&= \frac{1}{4}(a - c + \epsilon)^2 \quad \text{otherwise,} \\
\pi_2^{ND*} &= \frac{1}{9}(a - c - \epsilon)^2 \quad \text{for } \epsilon < a - c, \\
&= 0 \quad \text{otherwise.}
\end{aligned} \tag{3.5}$$

Now, suppose that Firm 2 chooses to duplicate and let π_i^D for $i = \{1, 2\}$ be the profit of Firm i in this case. We also assume that duplication, which is instantaneous and has a success probability of one, results in a cost reduction equal to ϵ. Then, the per-unit cost of Firm 2 is $c_2 = c - \epsilon$. Considering Firm 2's duplication cost and substituting for $c_1 = c_2 = c - \epsilon$ in Equation 3.3, we find the following profits in equilibrium:

$$\begin{aligned}
\pi_1^{D*} &= \frac{1}{9}(a - c + \epsilon)^2, \\
\pi_2^{D*} &= \frac{1}{9}(a - c + \epsilon)^2 - C.
\end{aligned} \tag{3.6}$$

Clearly, Firm 2 will choose to duplicate if it gains a profit at least as large as that gained by choosing not to duplicate. Comparing π_2^{ND*} and π_2^{D*}, we find that Firm 2 will duplicate a non-drastic invention if $C \leq \frac{4}{9}(a - c)\epsilon$ and a drastic one if $C \leq \frac{1}{9}(a - c + \epsilon)^2$. We assume that duplication is preferable to using the existing technology when Firm 2 is indifferent between these choices.

3.3 The Equilibrium with a Technology Transfer

We now consider the case when a technology transfer occurs. A technology transfer is an agreement that will occur if each firm is better off accepting the agreement than rejecting it. We examine the cases in which a technology transfer occurs through a fixed fee and through a per-unit royalty.

3.3.1 Fixed-Fee Licensing

Let F for $F > 0$ be the fixed licensing fee. In this case, both firms use the cost-reducing technology and have the same per-unit cost, namely, $c_1 = c_2 = c - \epsilon$. Additionally, let π_i^F for $i = \{1, 2\}$ be the profit of Firm i when a technology transfer through a fixed-fee occurs. Considering the fixed fee and substituting for the per-unit costs into Equation 3.3 yield the firms' profits:

$$\begin{aligned} \pi_1^F &= \frac{1}{9}(a - c + \epsilon)^2 + F, \\ \pi_2^F &= \frac{1}{9}(a - c + \epsilon)^2 - F. \end{aligned} \tag{3.7}$$

Clearly, Firm 2 will accept a licensing offer if the profit it will gain is equal to or larger than its profit from using the existing technology or duplication. In other words, if $\pi_2^F \geq \max\ (\pi_2^{D*}, \pi_2^{ND*})$, Firm 2 will be better off with a technology transfer. Therefore, the maximum licensing fee that Firm 1 can charge to Firm 2 in this case is:

$$F^* \quad = \begin{cases} \min\ (\frac{4}{9}[a - c]\epsilon,\ \mathrm{C}) & \text{for } \epsilon < a - c, \\ \min\ (\frac{1}{9}[a - c + \epsilon]^2,\ \mathrm{C}) & \text{otherwise.} \end{cases} \tag{3.8}$$

According to Equation 3.8, if the invention is non-drastic, the patent owner will charge a fixed fee that is equal to the minimum of $\frac{4}{9}(a-c)\epsilon$ and the duplication cost. If, however, the invention is drastic, the maximum fixed fee will be equal to the minimum of $\frac{1}{9}(a-c+\epsilon)^2$

and the duplication cost. Thus, the profits in equilibrium are:

$$\pi_1^{F*} = \frac{1}{9}(a - c + \epsilon)^2 + F^*,$$
$$\pi_2^{F*} = \frac{1}{9}(a - c + \epsilon)^2 - F^*, \tag{3.9}$$

where Equation 3.8 defines F^*.

We can now determine when a technology transfer might occur in equilibrium. First, suppose that using the existing technology is better than duplication for Firm 2. Comparing π_1^{F*} and π_1^{ND*}, we find that Firm 1 will always transfer the patented technology if $\epsilon < \frac{2}{3}(a-c)$. However, if $\frac{2}{3}(a-c) \leq \epsilon < a-c$, a technology transfer will not occur in equilibrium. In this case, both firms will produce a quantity larger than zero, as opposed to the case of a drastic invention where Firm 2 will become a monopoly. Now, suppose that duplication is at least as good as choosing not to duplicate for Firm 2. Then, a technology transfer will always occur in equilibrium, regardless of the type of invention. We can now establish the following result:

Theorem 1 *A technology transfer through a fixed-fee licence will occur if and only if*

1. $0 < \epsilon < \dfrac{2}{3}(a-c)$ *or*
2. $\dfrac{2}{3}(a-c) \leq \epsilon < a-c$ *and* $C \leq \dfrac{4}{9}(a-c)\epsilon$, *or*
3. $\epsilon \geq a-c$ *and* $C \leq \dfrac{1}{9}(a-c+\epsilon)^2$.

Figure 3.1 illustrates Theorem 1. The area above the ϵ-axis corresponds to the case when using the existing technology is better than duplication for Firm 2, while the area below the ϵ-axis corresponds to the case when duplication is at least as good as choosing not to duplicate. According to Figure 3.1, a technology transfer will always occur in the area above the ϵ-axis and on the left side of the dotted line, as well as in the area below the ϵ-axis.

3.3.2 Royalty Licensing

Let r for $r > 0$ denote the per-unit royalty rate that Firm 1 will charge Firm 2 in return for transferring the patented technology. In this case, the per-unit costs are $c_1 = c - \epsilon$ for Firm 1 and $c_2 = c - \epsilon + r$ for Firm 2. Additionally, let π_i^R for $i = \{1, 2\}$ be the profit of Firm i in this case. Then, considering that the patent owner's profit depends on the

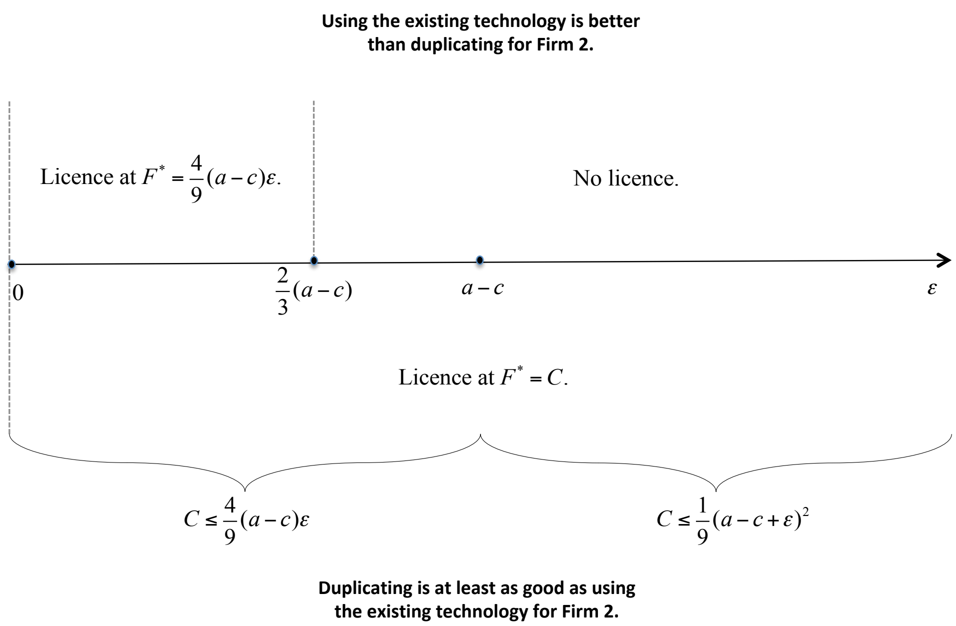

Figure 3.1: Illustration of the equilibrium under fixed-fee licensing.

quantity that Firm 2 will produce using the patented technology and substituting the per-unit costs into Equation 3.3, we obtain the firms' profits:

$$\begin{aligned} \pi_1^R &= \frac{1}{9}(a-c+\epsilon+r)^2 + \frac{1}{3}(a-c+\epsilon-2r)r, \\ \pi_2^R &= \frac{1}{9}(a-c+\epsilon-2r)^2. \end{aligned} \tag{3.10}$$

Clearly, the patent owner will charge Firm 2 a per-unit royalty that will maximise its profit, while Firm 2 will accept the offer if $\pi_2^R \geq \max\ (\pi_2^{ND*}, \pi_2^{D*})$. Solving the maximisation problem of Firm 1, we find that the royalty rate that satisfies these conditions is:

$$r^* = \begin{cases} \min\ (\epsilon,\ \frac{1}{2}[a-c+\epsilon] - \frac{1}{2}\sqrt{(a-c+\epsilon)^2 - 9C}) & \text{for } \epsilon < a-c, \\ \min\ (\frac{1}{2}[a-c+\epsilon],\ \frac{1}{2}[a-c+\epsilon] - \frac{1}{2}\sqrt{(a-c+\epsilon)^2 - 9C}) & \text{otherwise.} \end{cases} \tag{3.11}$$

Proof *See Appendix.*

According to Equation 3.11, if the invention is non-drastic, the optimal per-unit royalty will be equal to the minimum of the magnitude of the innovation and $\frac{1}{2}(a-c+\epsilon) - \frac{1}{2}\sqrt{(a-c+\epsilon)^2 - 9C}$. In the case of a drastic invention, however, the optimal royalty rate will be equal to the minimum of $\frac{1}{2}(a-c+\epsilon)$ and $\frac{1}{2}(a-c+\epsilon) - \frac{1}{2}\sqrt{(a-c+\epsilon)^2 - 9C}$. Note that when $C > \frac{1}{9}(a-c+\epsilon)^2$, duplication will not occur and the expression under the square root in Equation 3.11 will become negative. In this case, we need to consider only the licensing game without duplication. Therefore, we define $\frac{1}{2}(a-c+\epsilon) - \frac{1}{2}\sqrt{(a-c+\epsilon)^2 - 9C}$ only for non-negative values.[12]

Evidently, the optimal per-unit royalty cannot be larger than the magnitude of the innovation in any case. The firms' profits in equilibrium in this case are:

$$\begin{aligned} \pi_1^{R*} &= \frac{1}{9}(a-c+\epsilon+r^*)^2 + \frac{1}{3}(a-c+\epsilon-2r^*)r^*, \\ \pi_2^{R*} &= \frac{1}{9}(a-c+\epsilon-2r^*)^2, \end{aligned} \tag{3.12}$$

where Equation 3.11 defines r^*.

We now determine when a technology transfer might occur in equilibrium. Suppose that using the existing technology is better than duplication for Firm 2. Then, by comparing the equilibrium profits of the patent owner, we find that a technology transfer will always occur in equilibrium if the invention is non-drastic. In the case of a drastic invention, however, Firm 1 is indifferent between licensing and choosing not to license. In this case, Firm 1 will always earn the monopoly rent and Firm 2 will earn no profit with or without a technology transfer. We assume that Firm 1 will become a monopoly in this case.

Now, suppose that duplication is at least as good as choosing not to duplicate for Firm 2. Then, with either a drastic or non-drastic invention, a technology transfer will always occur in equilibrium. We can now establish the following result:

Theorem 2 *A technology transfer through a per-unit royalty licence will occur if and*

[12] We can find a more accurate optimal per-unit royalty using Equation 3.11 if $C \leq \frac{1}{9}(a-c+\epsilon)^2$ and with the following equation if $C > \frac{1}{9}(a-c+\epsilon)^2$:

$$r^* = \begin{cases} \epsilon & \text{for } \epsilon < a-c \\ & \text{and } C > \frac{1}{9}(a-c+\epsilon)^2, \\ \frac{1}{2}(a-c+\epsilon) & \text{for } \epsilon \geq a-c \\ & \text{and } C > \frac{1}{9}(a-c+\epsilon)^2. \end{cases}$$

only if:

1. $0 < \epsilon < a - c$ *or*
2. $\epsilon \geq a - c$ *and* $C \leq \dfrac{1}{9}(a - c + \epsilon)^2$.

Figure 3.2 illustrates Theorem 2. The area above the ϵ-axis corresponds to the case when the existing technology is better than duplication for Firm 2, while the area below the ϵ-axis corresponds to the case when duplication is at least as good as choosing not to duplicate. According to Figure 3.2, royalty licensing will occur in the area above the ϵ-axis and on the left side of the dotted line, as well as in the area below the ϵ-axis.

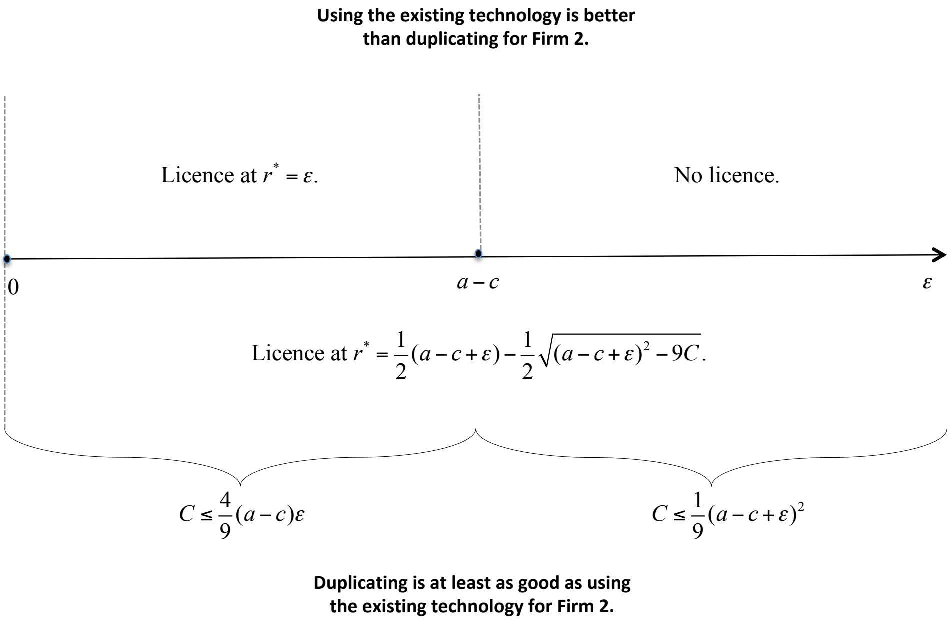

Figure 3.2: Illustration of the equilibrium under royalty licensing.

3.3.3 Licensing Choice Comparison

We can now compare the patent owner's equilibrium profits under the two licensing choices and determine the equilibrium of the fixed-fee licensing game. We will also analyse the effect of technology transfer on consumer surplus (Singh and Vives, 1984; Fauli-Oller and Sandonis, 2002; Tian, 2016). As before, we consider two cases.

Case I {*Using the existing technology is better than duplication for Firm 2; that is,* $\pi_2^{ND*} > \pi_2^{D*}$}. If $0 < \epsilon < \frac{2}{3}(a - c)$, a technology transfer might occur through both a fixed fee and a per-unit royalty. However, a comparison shows that royalty licensing is better than fixed-fee licensing from the patent owner's perspective because $\pi_1^{R*} > \pi_1^{F*}$. Additionally, let q_i^F and q_i^R be the quantity of Firm i for $i = \{1, 2\}$ under fixed-fee and royalty licensing, respectively. Then, licensing through a fixed fee is more attractive than licensing through a per-unit royalty from the consumer standpoint because $q_1^{F*} + q_2^{F*} > q_1^{R*} + q_2^{R*}$. However, if $\frac{2}{3}(a-c) \leq \epsilon < a-c$, a technology transfer might occur only through a per-unit royalty. Therefore, royalty licensing is also better than fixed-fee licensing for the patent owner in this interval of ϵ. Let q_i^{ND} be the quantity of Firm i for $i = \{1, 2\}$ when Firm 2 chooses not to duplicate. Then, the consumers are the same with or without technology transfer, given that $q_1^{ND*} + q_2^{ND*} = q_1^{R*} + q_2^{R*}$. Finally, if the invention is drastic, a technology transfer will not occur in equilibrium under either licensing choice.

Case II {*Duplication is at least as good as using the existing technology for Firm 2; that is,* $\pi_2^{D*} \geq \pi_2^{ND*}$}. In this case, a technology transfer might occur through both a fixed-fee and a per-unit royalty. However, comparing the patent owner's profits under the two licensing modes shows that regardless of the type of invention, royalty licensing is better than fixed-fee licensing because $\pi_1^{R*} > \pi_1^{F*}$ $\forall\epsilon$. Additionally, licensing through a fixed fee is always better than licensing through a per-unit royalty for consumers, given that $q_1^{F*} + q_2^{F*} > q_1^{R*} + q_2^{R*}$ $\forall\epsilon$.

We now summarise the results derived above as follows:

Theorem 3 *The equilibrium of the duopoly game when Firm 1 has a licensing choice between a fixed fee and a per-unit royalty is characterised as follows.*

1. *A technology transfer, whether the invention is drastic or not, through a per-unit royalty licence is at least as good as a technology transfer through a fixed-fee licence for the patent owner.*

2. *A technology transfer through a fixed-fee licence is at least as good as a technology transfer through a per-unit royalty for consumers.*

3. *A technology transfer will not occur, with either a fixed-fee or a per-unit royalty licence, if and only if the invention is drastic and the duplication cost is sufficiently large, particularly if* $\epsilon \geq a - c$ *and* $C \leq \frac{1}{9}(a - c + \epsilon)^2$. *In this case, Firm 1 will become a monopoly.*

Theorem 3 suggests that, in equilibrium, royalty licensing is at least as attractive as fixed-fee licensing for the patent owner, and at most, as attractive as fixed-fee licensing for consumers. The patent owner's cost-advantage explains the superiority of the per-unit royalty over a fixed fee. We showed that allowing for duplication does not alter

Wang (1998)'s propositions about fixed-fee and royalty licensing in a homogeneous-good Cournot duopoly.

4 Extension of the Model

In this section, we extend the basic model to a Cournot industry with three firms to determine if the superiority of royalties and the licensing properties continue to hold in a more complicated Cournot setting. Additionally, to analyse the effect of licensing behaviour on competition, we consider a non-cooperative licensing game consisting of three players with asymmetric costs: a patent owner, a strong competitor that can duplicate the patented technology, and a weak competitor that has the largest per-unit cost in the industry and cannot duplicate the invention. Note that most of the specifications in the basic model remain. For example, the game has complete information, and duplication is instantaneous and occurs with certainty. The findings from this alternate specification suggest that royalty licensing might be at least as good as fixed-fee licensing. Additionally, a patent owner might in fact use licensing to prevent duplication, but not to select his/her competition during the patent life term.

4.1 Equilibrium without Technology Transfer

Consider an industry composed of three incumbent firms. Let q_i for $i = \{1, 2, 3\}$ be the quantity of Firm i for $i = \{1, 2, 3\}$. Additionally, let the inverse demand function for a homogeneous good be $P = a - Q$, where a is an industry parameter that characterises the market demand for the product, and Q is the total quantity for the industry. Clearly, the industry output is equal to the sum of the quantities each firm in the industry produces; that is, $Q = \sum_{i=1}^{3} q_i$. Additionally, let c_i for $i = \{1, 2, 3\}$ and $0 < c_i < a$ be the per-unit cost for Firm i. We use the same procedure as in the duopoly game to obtain the Nash-Cournot quantities:

$$\begin{aligned} q_1^* &= \frac{1}{4}(a - 3c_1 + c_2 + c_3), \\ q_2^* &= \frac{1}{4}(a - 3c_2 + c_1 + c_3), \\ q_3^* &= \frac{1}{4}(a - 3c_3 + c_1 + c_2). \end{aligned} \tag{4.1}$$

Let π_i for $i = \{1, 2, 3\}$ be the Nash-Cournot profit of Firm i. Then, the firms' profits in equilibrium are:

$$\begin{aligned}
\pi_1^* &= \frac{1}{16}(a - 3c_1 + c_2 + c_3)^2, \\
\pi_2^* &= \frac{1}{16}(a - 3c_2 + c_1 + c_3)^2, \\
\pi_3^* &= \frac{1}{16}(a - 3c_3 + c_1 + c_2)^2.
\end{aligned} \tag{4.2}$$

First, suppose that the per-unit cost is the same for Firms 1 and 2, particularly $c_1 = c_2 = c$, while the per-unit cost of Firm 3 is $c_3 = c'$, where $c' > c > 0$. Clearly, Firm 3 has the largest per-unit cost in the industry and we can thus refer to it as the weak competitor. Now, suppose that Firm 1 develops and patents a new technology that reduces the per-unit cost by ϵ, where $0 < \epsilon < c$. Thus, we can refer to Firm 1 as the patent owner and Firm 2 as the strong competitor. We also assume that the weak competitor can only obtain the patented invention through a technology transfer. However, the strong competitor can obtain the invention either through a technology transfer or through duplication by incurring an upfront cost of C, where $C > 0$.[13]

We define π_i^{ND} for $i = \{1, 2, 3\}$ as the profit of Firm i when Firm 2 uses the existing technology. Then, a direct substitution of $c_1 = c - \epsilon$, $c_2 = c$, and $c_3 = c'$ into Equation 4.2 gives the firms' profits as follows:

$$\begin{aligned}
\pi_1^{ND} &= \frac{1}{16}(a - 2c + 3\epsilon + c')^2, \\
\pi_2^{ND} &= \frac{1}{16}(a - 2c - \epsilon + c')^2, \\
\pi_3^{ND} &= \frac{1}{16}(a + 2c - \epsilon - 3c')^2.
\end{aligned} \tag{4.3}$$

The quantity of Firm 2, and in turn its profit, is larger than zero only if $\epsilon < a - 2c + c'$. That is, if $\epsilon \geq a - 2c + c'$, the invention will drive the strong competitor, and evidently the weak competitor, out of the industry. In this case, the invention is drastic and Firm 1 will earn a monopoly profit, while Firms 2 and 3 will earn no profit. Furthermore, Firm

[13] We assume that Firm 3 cannot duplicate the patented technology because it lacks the appropriate know-how or necessary facilities, or for any other plausible reason.

3's profit is larger than zero if $\epsilon < a + 2c - 3c'$, given that $c' < \frac{1}{3}(a + 2c)$.[14] However, if $\epsilon \geq a + 2c - 3c'$, the weak competitor will earn no profit and thus the industry might become either a duopoly or a monopoly depending on the relative magnitudes of ϵ and $a-2c+c'$. Specifically, the industry will become a duopoly if $a+2c-3c' \leq \epsilon < a-2c+c'$ and a monopoly if $\epsilon \geq a - 2c + c'$. According to Equation 4.3 and acknowledging that the profits depend on ϵ, we obtain the firms' profits in equilibrium as follows:

$$\begin{aligned}
\pi_1^{ND*} &= \frac{1}{16}(a - 2c + 3\epsilon + c')^2 && \text{for } \epsilon < a + 2c - 3c', \\
&= \frac{1}{9}(a - c + 2\epsilon)^2 && \text{for } a + 2c - 3c' \leq \epsilon < a - 2c + c', \\
&= \frac{1}{4}(a - c + \epsilon)^2 && \text{otherwise}, \\
\pi_2^{ND*} &= \frac{1}{16}(a - 2c - \epsilon + c')^2 && \text{for } \epsilon < a + 2c - 3c', \\
&= \frac{1}{9}(a - c - \epsilon)^2 && \text{for } a + 2c - 3c' \leq \epsilon < a - 2c + c', \\
&= 0 && \text{otherwise}, \\
\pi_3^{ND*} &= \frac{1}{16}(a + 2c - \epsilon - 3c')^2 && \text{for } \epsilon < a + 2c - 3c', \\
&= 0 && \text{otherwise}.
\end{aligned} \tag{4.4}$$

We define π_i^D for $i = \{1, 2, 3\}$ as the profit of Firm i when Firm 2 chooses to duplicate. Similar to the duopoly game, duplication reduces the per-unit cost by ϵ. Considering the duplication cost of C that Firm 2 incurs and substituting for $c_1 = c_2 = c - \epsilon$ and $c_3 = c'$

[14] If $c' \geq \frac{1}{3}(a + 2c)$, then Firm 3 will earn no profit, regardless of the magnitude of the innovation, and in turn, the three-firm Cournot game will be equivalent to the duopoly game in the previous section.

in Equation 4.2, we find the firms' profits as follows:

$$\begin{aligned}
\pi_1^D &= \frac{1}{16}(a-2[c-\epsilon]+c')^2, \\
\pi_2^D &= \frac{1}{16}(a-2[c-\epsilon]+c')^2 - C, \\
\pi_3^D &= \frac{1}{16}(a+2[c-\epsilon]-3c')^2.
\end{aligned} \tag{4.5}$$

According to Equation 4.5, Firm 2 will consider duplication if $D < \frac{1}{16}(a-2[c-\epsilon]+c')^2$, while Firm 3 will earn a profit larger than zero if $\epsilon < \frac{1}{2}(a+2c-3c')$, given that $c' < \frac{1}{3}(a+2c)$. The equilibrium profits in this case are then:

$$\begin{aligned}
\pi_1^{D*} &= \frac{1}{16}(a-2[c-\epsilon]+c')^2 && \text{for } \epsilon < \frac{1}{2}(a+2c-3c'), \\
&= \frac{1}{9}(a-c+\epsilon)^2 && \text{otherwise}, \\
\pi_2^{D*} &= \frac{1}{16}(a-2[c-\epsilon]+c')^2 - C && \text{for } \epsilon < \frac{1}{2}(a+2c-3c'), \\
&= \frac{1}{9}(a-c+\epsilon)^2 - C && \text{otherwise}, \\
\pi_3^{D*} &= \frac{1}{16}(a+2[c-\epsilon]-3c')^2 && \text{for } \epsilon < \frac{1}{2}(a+2c-3c'), \\
&= 0 && \text{otherwise}.
\end{aligned} \tag{4.6}$$

We can now compare the equilibrium profits of Firm 2 and determine the equilibrium of the game without technology transfer. We find the following results: for $\epsilon < \frac{1}{2}(a+2c-3c')$, Firm 2 will choose to duplicate if $C \leq \frac{3}{16}(2a-4c+\epsilon+2c')\epsilon$; for $\frac{1}{2}(a+2c-3c') \leq \epsilon < a+2c-3c'$, duplication will occur if $C \leq \frac{1}{9}(a-c+\epsilon)^2 - \frac{1}{16}(a-2c-\epsilon+c')^2$; for $a+2c-3c' \leq \epsilon < a-2c+c'$, duplication will occur if $C \leq \frac{4}{9}(a-c)\epsilon$; and in the case of a drastic invention, Firm 2 will duplicate if $C \leq \frac{1}{9}(a-c+\epsilon)^2$. We assume that Firm 2 will prefer to duplicate if it is indifferent between its choices.

4.2 Equilibrium with Technology Transfer

Suppose that Firm 1's protected technology is transferable through a patent licensing agreement. In particular, Firm 1 might offer an exclusive licence to the weak competitor, an exclusive licence to the strong competitor, or licences to both competitors. Furthermore, Firm 1 might license through either a fixed fee or a per-unit royalty. We will first consider fixed-fee licensing and then turn to royalty licensing.

4.2.1 Fixed-Fee Licensing

Here, we have three situations to consider: an exclusive licence to Firm 2, an exclusive licence to Firm 3, or licences to both firms. We will consider each fixed-fee licensing choice in turn.

Licence to Firm 2: In this case, the per-unit costs are $c_1 = c_2 = c - \epsilon$ for Firms 1 and 2, and $c_3 = c'$ for Firm 3. Let F_2, where $F_2 > 0$ and π_i^F for $i = \{1, 2, 3\}$ is the fixed licensing fee and the profit of Firm i, respectively. Then, considering the licensing fee and substituting the per-unit costs into Equation 4.2, we obtain the firms' profits as follows:

$$\begin{aligned}
\pi_1^F &= \frac{1}{16}(a - 2[c - \epsilon] + c')^2 + F_2, \\
\pi_2^F &= \frac{1}{16}(a - 2[c - \epsilon] + c')^2 - F_2, \qquad (4.7) \\
\pi_3^F &= \frac{1}{16}(a + 2[c - \epsilon] - 3c')^2.
\end{aligned}$$

According to Equation 4.7, if $\epsilon \geq \frac{1}{2}(a + 2c - 3c')$, it is optimal for Firm 3 not to produce. We then obtain the firms' profits as follows:

$$
\begin{aligned}
\pi_1^F &= \frac{1}{16}(a-2[c-\epsilon]+c')^2+F_2 && \text{for } \epsilon<\frac{1}{2}(a+2c-3c'),\\
&= \frac{1}{9}(a-c+\epsilon)^2+F_2 && \text{otherwise},\\
\pi_2^F &= \frac{1}{16}(a-2[c-\epsilon]+c')^2-F_2 && \text{for } \epsilon<\frac{1}{2}(a+2c-3c'),\\
&= \frac{1}{9}(a-c+\epsilon)^2-F_2 && \text{otherwise},\\
\pi_3^F &= \frac{1}{16}(a+2[c-\epsilon]-3c')^2 && \text{for } \epsilon<\frac{1}{2}(a+2c-3c'),\\
&= 0 && \text{otherwise}.
\end{aligned} \tag{4.8}
$$

Firm 2 will accept a licensing offer only if its profit as a licensee will be equal to or larger than the maximum of its profit from using the existing technology and from duplication. In particular, considering that $\pi_2^F \geq \max\ (\pi_2^{ND*}, \pi_2^{D*})$ we find that the maximum licensing fee that Firm 1 can charge Firm 2 is:

$$
F_2^* = \begin{cases}
\min\ (\frac{3}{16}[2a-4c+\epsilon+2c']\epsilon,\ \text{C}) & \text{for } \epsilon<\frac{1}{2}(a+2c-3c'),\\
\min\ (\frac{1}{9}[a-c+\epsilon]^2-\frac{1}{16}[a-2c-\epsilon+c']^2,\ \text{C}) & \text{for } \frac{1}{2}(a+2c-3c')\leq\epsilon<a+2c-3c',\\
\min\ (\frac{4}{9}[a-c]\epsilon,\ \text{C}) & \text{for } a+2c-3c'\leq\epsilon<a-2c+c',\\
\min\ (\frac{1}{9}[a-c+\epsilon]^2,\ \text{C}) & \text{otherwise}.
\end{cases} \tag{4.9}
$$

According to Equation 4.9, if $\epsilon < \frac{1}{2}(a+2c-3c')$, the maximum fee that the patent owner can charge is the minimum of $\frac{3}{16}(2a-4c+\epsilon+2c')\epsilon$ and the duplication cost; if $\frac{1}{2}(a+2c-3c') \leq \epsilon < a+2c-3c'$, the maximum fee will be the minimum of $\frac{1}{9}(a-c+\epsilon)^2 - \frac{1}{16}(a-2c-\epsilon+c')^2$ and the duplication cost;[15] if $a+2c-3c' \leq \epsilon < a-2c+c'$, the maximum fee will be the minimum of $\frac{4}{9}(a-c)\epsilon$ and the duplication cost; and, in the case of a drastic invention, the maximum fixed fee that the patent owner can charge is the minimum of

[15] Note that we define $\frac{1}{9}(a-c+\epsilon)^2 - \frac{1}{16}(a-2c-\epsilon+c')^2$ only for non-negative values.

$\frac{1}{9}(a-c+\epsilon)^2$ and the duplication cost.

It follows that the firms' equilibrium profits are:

$$\begin{aligned}
\pi_1^{F*} &= \frac{1}{16}(a-2[c-\epsilon]+c')^2 + F_2^* \quad \text{for } \epsilon < \frac{1}{2}(a+2c-3c'), \\
&= \frac{1}{9}(a-c+\epsilon)^2 + F_2^* \quad \text{otherwise}, \\
\pi_2^{F*} &= \frac{1}{16}(a-2[c-\epsilon]+c')^2 - F_2^* \quad \text{for } \epsilon < \frac{1}{2}(a+2c-3c'), \\
&= \frac{1}{9}(a-c+\epsilon)^2 - F_2^* \quad \text{otherwise}, \\
\pi_3^{F*} &= \frac{1}{16}(a+2[c-\epsilon]-3c')^2 \quad \text{for } \epsilon < \frac{1}{2}(a+2c-3c'), \\
&= 0 \quad \text{otherwise},
\end{aligned} \tag{4.10}$$

where Equation 4.9 defines F_2^*.

We can now compare the profit of Firm 1 with and without technology transfer and determine the equilibrium of this case. Suppose that using the existing technology is better than duplication for Firm 2.[16] Then, a technology transfer will always occur in equilibrium if $\epsilon < \frac{2}{3}(a-c)$ because only in this interval of ϵ does licensing maximise the patent owner's profit and satisfy the strong competitor's participation constraints. Now, suppose that duplication is at least as good as using the existing technology for Firm 2. Then, the comparative statics show that technology transfer will always occur in equilibrium, regardless of the type of invention.

Licence to Firm 3: In this case, the per-unit costs are $c_1 = c-\epsilon$ for Firm 1, $c_2 = c$ or $c_2 = c-\epsilon$, and $c_3 = c'-\epsilon$ for Firm 3. Suppose that using the existing technology is better than duplication for Firm 2, and thus $c_2 = c$. Additionally, F_3^{ND}, where $F_3^{ND} > 0$, and $\pi_i^F(ND)$ for $i = \{1,2,3\}$ is the fixed licensing fee and the profit of Firm i. Considering the fixed fee and substituting the per-unit costs into Equation 4.2, we obtain the firms'

[16]Note that we already defined the critical values that trigger duplication while characterising the equilibrium of the game without technology transfer.

profits as follows:

$$\begin{aligned}\pi_1^F(ND) &= \frac{1}{16}(a-2[c-\epsilon]+c')^2+F_3^{ND},\\ \pi_2^F(ND) &= \frac{1}{16}(a-2[c+\epsilon]+c')^2,\\ \pi_3^F(ND) &= \frac{1}{16}(a+2[c+\epsilon]-3c')^2-F_3^{ND}.\end{aligned} \tag{4.11}$$

Firm 2 will produce an output larger than zero if $\epsilon < \frac{1}{2}(a-2c+c')$ and the industry will become a duopoly otherwise. We thus obtain the firms' profits as follows:

$$\begin{aligned}\pi_1^F(ND) &= \frac{1}{16}(a-2[c-\epsilon]+c')^2+F_3^{ND} && \text{for } \epsilon < \frac{1}{2}(a-2c+c'),\\ &= \frac{1}{9}(a-2c+\epsilon+c')^2+F_3^{ND} && \text{otherwise,}\\ \pi_2^F(ND) &= \frac{1}{16}(a-2[c+\epsilon]+c')^2 && \text{for } \epsilon < \frac{1}{2}(a-2c+c'),\\ &= 0 && \text{otherwise,}\\ \pi_3^F(ND) &= \frac{1}{16}(a+2[c+\epsilon]-3c')^2-F_3^{ND} && \text{for } \epsilon < \frac{1}{2}(a-2c+c'),\\ &= \frac{1}{9}(a+c+\epsilon-2c')^2-F_3^{ND} && \text{otherwise.}\end{aligned} \tag{4.12}$$

Firm 3 will accept a licensing offer if it gains a profit at least larger than that from using the existing technology. Considering that $\pi_3^F(ND) \geq \pi_3^{ND*}$, we find that the maximum fixed fee that Firm 1 can charge Firm 3 if $c' < \frac{1}{7}(a+6c)$ is:

$$F_3^{ND*} = \begin{cases} \frac{3}{16}(2a+4c+\epsilon-6c')\epsilon & \text{for } \epsilon < \frac{1}{2}(a-2c+c') \\ & \text{and } c' < \frac{1}{7}(a+6c), \\ \frac{1}{9}(a+c+\epsilon-2c')^2 - \frac{1}{16}(a+2c-\epsilon-3c')^2 & \text{for } \frac{1}{2}(a-2c+c') \leq \epsilon < a+2c-3c' \\ & \text{and } c' < \frac{1}{7}(a+6c), \\ \frac{1}{9}(a+c+\epsilon-2c')^2 & \text{for } \epsilon \geq a+2c-3c' \\ & \text{and } c' < \frac{1}{7}(a+6c), \end{cases} \tag{4.13}$$

and if $\frac{1}{7}(a+6c) \leq c' < \frac{1}{3}(a+2c)$, then the maximum fixed fee is:

$$F_3^{ND*} = \begin{cases} \frac{3}{16}(2a+4c+\epsilon-6c')\epsilon & \text{for } \epsilon < a+2c-3c' \\ & \text{and } c' \geq \frac{1}{7}(a+6c), \\ \frac{1}{16}(a+2[c+\epsilon]-3c')^2 & \text{for } a+2c-3c' \leq \epsilon < \frac{1}{2}(a-2c+c') \\ & \text{and } c' \geq \frac{1}{7}(a+6c), \\ \frac{1}{9}(a+c+\epsilon-2c')^2 & \text{for } \epsilon \geq \frac{1}{2}(a-2c+c') \\ & \text{and } c' \geq \frac{1}{7}(a+6c). \end{cases} \tag{4.14}$$

According to Equation 4.13, if $\epsilon < \frac{1}{2}(a-2c+c')$, the maximum fixed fee that the patent owner can charge Firm 3 will be $\frac{3}{16}(2a+4c+\epsilon-6c')\epsilon$; if $\frac{1}{2}(a-2c+c') \leq \epsilon < a+2c-3c'$, the maximum fixed fee will be $\frac{1}{9}(a+c+\epsilon-2c')^2 - \frac{1}{16}(a+2c-3c'-\epsilon)^2$; and if $\epsilon \geq a+2c-3c'$, the maximum fee will be $\frac{1}{9}(a+c+\epsilon-2c')^2$.

Likewise, Equation 4.14 shows that if $\epsilon < a+2c-3c'$, the maximum fixed fee that Firm 1 can charge will be $\frac{3}{16}(2a+4c+\epsilon-6c')\epsilon$; if $a+2c-3c' \leq \epsilon < \frac{1}{2}(a-2c+c')$, the maximum fixed fee will be $\frac{1}{16}(a+2c+2\epsilon-3c')^2$; and if $\epsilon \geq \frac{1}{2}(a-2c+c')$, the maximum fixed fee will be $\frac{1}{9}(a+c+\epsilon-2c')^2$.

We can now derive the firms' profits in equilibrium as follows:

$$
\begin{aligned}
\pi_1^{F*}(ND) &= \frac{1}{16}(a-2[c-\epsilon]+c')^2 + F_3^{ND*} && \text{for } \epsilon < \frac{1}{2}(a-2c+c'),\\
&= \frac{1}{9}(a-2c+\epsilon+c')^2 + F_3^{ND*} && \text{otherwise,}\\
\pi_2^{F*}(ND) &= \frac{1}{16}(a-2[c+\epsilon]+c')^2 && \text{for } \epsilon < \frac{1}{2}(a-2c+c'),\\
&= 0 && \text{otherwise,}\\
\pi_3^{F*}(ND) &= \frac{1}{16}(a+2[c+\epsilon]-3c')^2 - F_3^{ND*} && \text{for } \epsilon < \frac{1}{2}(a-2c+c'),\\
&= \frac{1}{9}(a+c+\epsilon-2c')^2 - F_3^{ND*} && \text{otherwise,}
\end{aligned}
\tag{4.15}
$$

where Equation 4.13 or Equation 4.14 define F_3^{ND*} if $c' < \frac{1}{7}(a+6c)$ or $c' \geq \frac{1}{7}(a+6c)$, respectively.

Now, suppose that duplication is at least as good as choosing not to do so for Firm 2. The per-unit costs in this case are then $c_1 = c_2 = c - \epsilon$ for Firms 1 and 2, and $c_3 = c' - \epsilon$ for Firm 3. Let F_3^D, where $F_3^D > 0$, and $\pi_i^F(D)$ for $i = \{1,2,3\}$ be the licensing fee for Firm 3 and the profit of Firm i in this case, respectively. The firms' profits are then:

$$
\begin{aligned}
\pi_1^F(D) &= \frac{1}{16}(a-2c+\epsilon+c')^2 + F_3^D,\\
\pi_2^F(D) &= \frac{1}{16}(a-2c+\epsilon+c')^2 - C,\\
\pi_3^F(D) &= \frac{1}{16}(a+2c+\epsilon-3c')^2 - F_3^D.
\end{aligned}
\tag{4.16}
$$

Considering that Firm 3 will accept a licensing offer only if it is better off with it than without it ($\pi_3^F(D) \geq \pi_3^{D*}$), we obtain the maximum fixed licensing fee as follows:

$$F_3^{D*} = \begin{cases} \frac{3}{16}(2a+4c-\epsilon-6c')\epsilon & \text{for } \epsilon < \frac{1}{2}(a+2c-3c'), \\ \\ \frac{1}{16}(a+2c+\epsilon-3c')^2 & \text{otherwise.} \end{cases} \tag{4.17}$$

Equation 4.17 shows that the maximum fixed fee that the patent owner can charge Firm 3 is $\frac{3}{16}(2a+4c-\epsilon-6c')\epsilon$ if $\epsilon < \frac{1}{2}(a+2c-3c')$ and $\frac{1}{16}(a+2c+\epsilon-3c')^2$ otherwise. Therefore, the firms' profits in equilibrium are:

$$\begin{aligned} \pi_1^{F*}(D) &= \frac{1}{16}(a-2c+\epsilon+c')^2 + F_3^{D*}, \\ \pi_2^{F*}(D) &= \frac{1}{16}(a-2c+\epsilon+c')^2 - C, \\ \pi_3^{F*}(D) &= \frac{1}{16}(a+2c+\epsilon-3c')^2 - F_3^{D*}, \end{aligned} \tag{4.18}$$

where Equation 4.17 defines F_3^{D*}.

We next determine when an exclusive licence to the weak competitor is optimal for the patent owner. Suppose that using the existing technology is better than duplication for Firm 2. Then, the comparative statics show that when $c' < \frac{1}{7}(a+6c)$, technology transfer will occur if $\epsilon < \frac{2}{3}(a-c)$, while when $c' \geq \frac{1}{7}(a+6c)$, technology transfer will occur only if $\epsilon < 2(a+4c-5c')$. Now, suppose that duplication is at least as good as choosing not to do so for Firm 2. When $c' < \frac{1}{11}(a+10c)$, technology transfer will always occur in equilibrium, while when $c' \geq \frac{1}{11}(a+10c)$, a technology transfer will occur if $\epsilon \leq \frac{2}{3}(a+4c-5c')$ or $\epsilon \geq 15c'-14c-a$.

Licences to Firms 2 and 3: The last case to consider is when the patent owner offers a licence to each competitor. The per-unit costs in this case are $c_1 = c_2 = c - \epsilon$ for Firms 1 and 2, and $c_3 = c' - \epsilon$ for Firm 3. Let $\mathbb{F}_i$ for $i = \{2,3\}$ and $\mathbb{F}_i > 0$ be the fixed licensing fee for Firm i when the patent owner offers a licence to each competitor. Additionally, let $\pi_i^{\mathbb{F}}$ for $i = \{1,2,3\}$ be the profit of Firm i. Then, considering the fixed fees and substituting the per-unit costs in Equation 4.2, we obtain the firms' profits as follows:

$$\begin{aligned}
\pi_1^{\mathbb{F}} &= \frac{1}{16}(a-2c+\epsilon+c')^2 + \mathbb{F}_2 + \mathbb{F}_3, \\
\pi_2^{\mathbb{F}} &= \frac{1}{16}(a-2c+\epsilon+c')^2 - \mathbb{F}_2, \qquad (4.19) \\
\pi_3^{\mathbb{F}} &= \frac{1}{16}(a+2c+\epsilon-3c')^2 - \mathbb{F}_3.
\end{aligned}$$

Firms 2 and 3 will accept the licences if each one is at least better off with a technology transfer than without it. Specifically, a technology transfer will occur if $\pi_2^{\mathbb{F}} \geq$ max $(\pi_2^{ND*}, \pi_2^{D*})$ and $\pi_3^{\mathbb{F}} \geq \pi_3^{ND*}$.[17] Thus, the maximum fixed fee for Firm 2 is:

$$\mathbb{F}_2^* = \begin{cases} \min\left(\frac{1}{4}[a-2c+c']\epsilon,\ A^{\mathbb{F}}\right) & \text{for } \epsilon < \frac{1}{2}(a+2c-3c'), \\ \min\left(\frac{1}{4}[a-2c+c']\epsilon,\ B^{\mathbb{F}}\right) & \text{for } \frac{1}{2}(a+2c-3c') \leq \epsilon < a+2c-3c', \\ \min\left(\frac{1}{16}[a-2c+\epsilon+c']^2 - \frac{1}{9}[a-c-\epsilon]^2,\ B^{\mathbb{F}}\right) & \text{for } a+2c-3c' \leq \epsilon < a-2c+c', \\ \min\left(\frac{1}{16}[a-2c+\epsilon+c']^2,\ B^{\mathbb{F}}\right) & \text{otherwise,} \end{cases} \qquad (4.20)$$

where $A^{\mathbb{F}} \equiv C - \frac{1}{16}(2a-4c+2c'+3\epsilon)\epsilon$ and $B^{\mathbb{F}} \equiv \frac{1}{16}(a-2c+\epsilon+c')^2 - \frac{1}{9}(a-c+\epsilon)^2 + C$,[18] while the maximum fixed fee for Firm 3 is:

$$\mathbb{F}_3^* = \begin{cases} \frac{1}{4}(a+2c-3c')\epsilon & \text{for } \epsilon < a+2c-3c', \\ \frac{1}{16}(a+2c+\epsilon-3c')^2 & \text{otherwise.} \end{cases} \qquad (4.21)$$

According to Equation 4.20, if $\epsilon < \frac{1}{2}(a+2c-3c')$, the maximum fee that Firm 1 can charge Firm 2 is the minimum of $\frac{1}{4}(a-2c+c')\epsilon$ and $A^{\mathbb{F}}$; if $\frac{1}{2}(a+2c-3c') \leq \epsilon < a+2c-3c'$, the maximum fee will be the minimum of $\frac{1}{2}(a+2c-3c')$ and $B^{\mathbb{F}}$; if $a+2c-3c' \leq \epsilon < a-2c+c'$, the maximum fixed fee will be the minimum of

[17] Note that $\pi_3^{ND*} \geq \pi_3^{D*}$; thus, $\pi_3^{\mathbb{F}} \geq \pi_3^{D*}$ will always hold if $\pi_3^{\mathbb{F}} \geq \pi_3^{ND*}$ is satisfied.

[18] Note that we define $A^{\mathbb{F}}$, $B^{\mathbb{F}}$, and $\frac{1}{16}[a-2c+\epsilon+c']^2 - \frac{1}{9}[a-c-\epsilon]^2$ only for non-negative values.

$\frac{1}{16}(a - 2c + \epsilon + c')^2 - \frac{1}{9}(a - c - \epsilon)^2$ and $B^{\mathbb{F}}$; and in the case of a drastic invention, Firm 1 will charge a minimum of $\frac{1}{16}(a - 2c + \epsilon + c')^2$ and $B^{\mathbb{F}}$.

Furthermore, Equation 4.21 shows that the maximum fee that the patent owner can charge Firm 3 is $\frac{1}{4}(a + 2c - 3c')\epsilon$ if $\epsilon < a + 2c - 3c'$ and $\frac{1}{16}(a + 2c + \epsilon - 3c')^2$ otherwise.

The firms' profits in equilibrium are therefore equal to:

$$\begin{aligned}
\pi_1^{\mathbb{F}} &= \frac{1}{16}(a - 2c + \epsilon + c')^2 + \mathbb{F}_2^* + \mathbb{F}_3^*, \\
\pi_2^{\mathbb{F}} &= \frac{1}{16}(a - 2c + \epsilon + c')^2 - \mathbb{F}_2^*, \qquad (4.22) \\
\pi_3^{\mathbb{F}} &= \frac{1}{16}(a + 2c + \epsilon - 3c')^2 - \mathbb{F}_3^*,
\end{aligned}$$

where Equations 4.20 and 4.21 define $\mathbb{F}_2^*$ and $\mathbb{F}_3^*$, respectively.

We can now determine the equilibrium of this last case. When using the existing technology is better than duplication for Firm 2, the patent owner will transfer the patented technology if $\epsilon < \frac{1}{2}(a + 2c - 3c')$ and will not do so otherwise. However, if duplication is at least as good as not doing so for Firm 2, it is always optimal for the patent owner to offer licences to both competitors, regardless of the type of invention.

We now compare the patent owner's fixed-fee licensing choices described above and characterise the equilibrium of the game. Suppose that using the existing technology is better than duplication for Firm 2. Then, the analysis suggests that offering an exclusive licence to the strong competitor is at least as good as the other two choices from the perspective of the patent owner. Additionally, a technology transfer will occur only if $\epsilon < \frac{2}{3}(a - c)$. However, if duplication is at least as good as using the existing technology for Firm 2, offering licences to both competitors is at least as good as the other two choices for the patent owner. These findings suggest that technology transfer exclusively to the weak competitor will not occur in equilibrium.

4.2.2 Royalty Licensing

Similar to the fixed-fee licensing game, we will now consider licensing through a per-unit royalty. The patent owner might offer an exclusive licence to Firm 2, an exclusive licence to Firm 3, or licences to Firms 2 and 3. We will consider each licensing choice in turn.

Licence to Firm 2: We define r_2, where $r_2 > 0$ is the per-unit royalty the patent owner charges Firm 2 in return for an exclusive licence and π_i^R for $i = \{1, 2, 3\}$ as profit of Firm i. Given that the per-unit costs are $c_1 = c - \epsilon$ for Firm 1, $c_2 = c - \epsilon + r_2$ for Firm 2, and $c_3 = c'$ for Firm 3, we derive the firms' profits as follows:

$$\begin{aligned}
\pi_1^R &= \frac{1}{16}(a - 2[c-\epsilon] + r_2 + c')^2 + \frac{1}{4}(a - 2[c-\epsilon] - 3r_2 + c')r_2, \\
\pi_2^R &= \frac{1}{16}(a - 2[c-\epsilon] - 3r_2 + c')^2, \\
\pi_3^R &= \frac{1}{16}(a + 2[c-\epsilon] + r_2 - 3c')^2.
\end{aligned} \tag{4.23}$$

Firm 3 will choose to produce nothing if $\epsilon \geq \frac{1}{2}(a + 2c + r_2 - 3c')$, in which case, the industry will become a duopoly. We showed that in a duopoly setting technology transfer will occur through a per-unit royalty only if the invention is non-drastic ($\epsilon < a - c$). We also showed that royalty licensing is at least as good as fixed-fee licensing in this case. Consequently, now we will only consider the case when $\epsilon < \frac{1}{2}(a + 2c + r_2^* - 3c')$. In turn, Firm 2 will accept a licensing offer if $\pi_2^R \geq \max\ (\pi_2^{ND*}, \pi_2^{D*})$ and reject it otherwise. Solving Firm 2's maximisation problem yields the optimal per-unit royalty, as follows:

$$r_2^* = \begin{cases} \min\ (\epsilon,\ A^R) & \text{for } \epsilon < \frac{1}{2}(a + 2c - 3c'), \\ \min\ (\epsilon,\ B^R) & \text{for } \frac{1}{2}(a + 2c - 3c') \leq \epsilon < \frac{3}{5}(a - 2c + c'), \\ \min\ (\frac{3}{11}[a - 2(c-\epsilon) + c'],\ B^R) & \text{otherwise,} \end{cases} \tag{4.24}$$

where $A^R \equiv \frac{1}{3}(a - 2[c-\epsilon] + c') - \frac{1}{3}\sqrt{(a - c + \epsilon)^2 - 16C}$ and $B^R \equiv \frac{1}{3}(a - 2[c-\epsilon] + c') - \frac{4}{3}\sqrt{(a - c + \epsilon)^2 - 9C}$.[19]

Proof *See Appendix.*

According to Equation 4.24, if $\epsilon < \frac{1}{2}(a + 2c - 3c')$, the optimal royalty rate will be the minimum of the magnitude of the innovation and A^R; if $\frac{1}{2}(a+2c-3c') \leq \epsilon < \frac{3}{5}(a-2c+c')$, the optimal royalty rate will be the minimum of the magnitude of the innovation and B^R; if $\epsilon \geq \frac{3}{5}(a - 2c + c')$, then Firm 1 will charge the minimum of $\frac{3}{11}(a - 2[c-\epsilon] + c')$

[19]Note that we define both A^R and B^R only for non-negative values.

and B^R.

We can now derive the firms' profits in equilibrium as follows:

$$\begin{aligned}
\pi_1^{R*} &= \frac{1}{16}(a-2[c-\epsilon]+r_2^*+c')^2+\frac{1}{4}(a-2[c-\epsilon]-3r_2^*+c')r_2^*, \\
\pi_2^{R*} &= \frac{1}{16}(a-2[c-\epsilon]-3r_2^*+c')^2, \\
\pi_3^{R*} &= \frac{1}{16}(a+2[c-\epsilon]+r_2^*-3c')^2,
\end{aligned} \tag{4.25}$$

where Equation 4.24 defines r_2^*.

Comparing the profits of the patent owner with and without technology transfer shows that licensing might be optimal for the patent owner. Suppose that using the existing technology is better than duplication for Firm 2. Then, technology transfer will always occur, given that we consider $\epsilon < \frac{1}{2}(a+2c+r_2^*-3c')$. However, if $\epsilon \geq \frac{1}{2}(a+2c+r_2^*-3c')$, the industry will become a duopoly and, thus, technology transfer will not occur only if the invention is drastic. Last, when duplication is at least as good as choosing not to do so for Firm 2, technology transfer will always occur in equilibrium, regardless of the type of invention.

Licence to Firm 3: In this case, the patent owner offers a royalty licence to the weak competitor. First, suppose that using the existing technology is better than duplication for Firm 2. Let r_3^{ND}, where $r_3^{ND} > 0$, and $\pi_i^R(ND)$ for $i = \{1,2,3\}$ is the royalty rate and profit of Firm i. Given that the per-unit costs are $c_1 = c - \epsilon$ for Firm 1, $c_2 = c$ for Firm 2, and $c_3 = c' - \epsilon + r_3^{ND}$ for Firm 3, we obtain the firms' profits as follows:

$$\begin{aligned}
\pi_1^R(ND) &= \frac{1}{16}(a-2[c-\epsilon]+r_3^{ND}+c')^2+\frac{1}{4}(a+2[c+\epsilon]-3[r_3^{ND}+c'])r_3^{ND}, \\
\pi_2^R(ND) &= \frac{1}{16}(a-2[c+\epsilon]+r_3^{ND}+c')^2, \\
\pi_3^R(ND) &= \frac{1}{16}(a+2[c+\epsilon]-3[r_3^{ND}+c'])^2.
\end{aligned} \tag{4.26}$$

In this case, it is optimal for Firm 2 to produce nothing if $\epsilon \geq \frac{1}{2}(a-2c+r_3^{ND}+c')$, in which case the industry will become a duopoly. This is similar to the royalty licensing case in the duopoly game, with only one difference: the per-unit cost of the less efficient incumbent firm is now c' instead of c, where $c' > c$. Therefore, the profit of a duopolist patent owner and the trigger value corresponding to a drastic invention will both be larger. Otherwise, the findings of the duopoly game hold, and thus we will consider only the case when $\epsilon < \frac{1}{2}(a - 2c + r_3^{ND} + c')$. In turn, Firm 3 will accept a licensing offer if $\pi_3^R(ND) \geq \pi_3^{ND*}$. Solving the patent owner's maximisation problem, we find that if $c' < \frac{1}{5}(a + 4c)$, the optimal royalty rate is:

$$r_3^{ND*} = \begin{cases} \epsilon & \text{for } \epsilon < \frac{1}{5}(3a + 2c - 5c') \\ & \text{and } c' < \frac{1}{5}(a + 4c), \\ & \\ \frac{1}{11}(3a + 2c + 6\epsilon - 5c') & \epsilon \geq \frac{1}{5}(3a + 2c - 5c') \\ & \text{and } c' < \frac{1}{5}(a + 4c). \end{cases} \tag{4.27}$$

Proof *See Appendix.*

Additionally, if $\frac{1}{5}(a + 4c) \leq c' < \frac{1}{3}(a + 2c)$, the optimal per-unit royalty is:

$$r_3^{ND*} = \begin{cases} \epsilon & \text{for } \epsilon < a + 2c - 3c' \\ & \text{and } \epsilon \geq \frac{1}{5}(a + 4c), \\ & \\ \frac{1}{3}(a + 2[c + \epsilon] - 3c') & \text{for } a + 2c - 3c' \leq \epsilon < \frac{1}{2}(9c' - 8c - a) \\ & \text{and } \epsilon \geq \frac{1}{5}(a + 4c), \\ & \\ \frac{1}{11}(3a + 2c + 6\epsilon - 5c') & \text{for } \epsilon \geq \frac{1}{2}(9c' - 8c - a) \\ & \text{and } \epsilon \geq \frac{1}{5}(a + 4c). \end{cases} \tag{4.28}$$

Proof *See Appendix.*

Equation 4.27, which corresponds to the case when $c' < \frac{1}{5}(a + 4c)$, shows that the optimal per-unit royalty will be equal to the magnitude of the innovation if $\epsilon < \frac{1}{5}(3a + 2c - 5c')$ and $\frac{1}{11}(3a + 2c + 6\epsilon - 5c')$ otherwise.

Furthermore, according to Equation 4.28, which corresponds to the case when $c' \geq \frac{1}{5}(a + 4c)$, if $\epsilon < a + 2c - 3c'$, the optimal per-unit royalty will be equal to the magnitude of the innovation; if $a + 2c - 3c' \leq \epsilon < \frac{1}{2}(9c' - 8c - a)$, the optimal per-unit royalty will be $\frac{1}{3}(a+2[c+\epsilon]-3c')$; and if $\epsilon \geq \frac{1}{2}(9c'-8c-a)$, the optimal royalty rate will be $\frac{1}{11}(3a+2c+6\epsilon-5c')$.

The firms' profits in equilibrium are:

$$\begin{aligned}
\pi_1^{R*}(ND) &= \frac{1}{16}(a-2[c-\epsilon]+r_3^{ND*}+c')^2+\frac{1}{4}(a+2[c+\epsilon]-3[r_3^{ND*}+c'])r_3^{ND*}, \\
\pi_2^{R*}(ND) &= \frac{1}{16}(a-2[c+\epsilon]+r_3^{ND*}+c')^2, \\
\pi_3^{R*}(ND) &= \frac{1}{16}(a+2[c+\epsilon]-3[r_3^{ND*}+c'])^2,
\end{aligned} \tag{4.29}$$

where Equation 4.27 or Equation 4.28 defines r_3^{ND*} if $c' < \frac{1}{5}(a+4c)$ or $c' \geq \frac{1}{5}(a+4c)$, respectively.

Now, suppose that duplication is at least as good as choosing not to do so for Firm 2. Additionally, r_3^D, where $r_3^D > 0$, and $\pi_i^R(D)$ for $i = \{1,2,3\}$ is the royalty rate and the profit of Firm i. Then, the per-unit costs are $c_1 = c_2 = c-\epsilon$ for Firms 1 and 2, and $c_3 = c'-\epsilon+r_3^D$ for Firm 3, which yield the firms' profits, as follows:

$$\begin{aligned}
\pi_1^R(D) &= \frac{1}{16}(a-2c+\epsilon+r_3^D+c')^2+\frac{1}{4}(a+2c+\epsilon-3[r_3^D+c'])r_3^D, \\
\pi_2^R(D) &= \frac{1}{16}(a-2c+\epsilon+r_3^D+c')^2-C, \\
\pi_3^R(D) &= \frac{1}{16}(a+2c+\epsilon-3[r_3^D+c'])^2.
\end{aligned} \tag{4.30}$$

Firm 3 will accept a licensing offer in this case if $\pi_3^R(D) \geq \pi_3^{D*}$. Solving the patent owner's maximisation problem, we find that if $c' < \frac{1}{7}(a+6c)$, the optimal royalty rate is:

$$r_3^{D*} = \begin{cases} \epsilon & \text{for } \epsilon < \frac{1}{8}(3a+2c-5c') \text{ and } c' < \frac{1}{7}(a+6c), \\ \frac{1}{11}(3[a+\epsilon]+2c-5c') & \text{for } \epsilon \geq \frac{1}{8}(3a+2c-5c') \text{ and } c' < \frac{1}{7}(a+6c). \end{cases} \tag{4.31}$$

Proof *See Appendix.*

In addition, if $\frac{1}{7}(a+6c) \leq c' < \frac{1}{3}(a+2c)$, the optimal per-unit royalty is:

$$r_3^{D*} = \begin{cases} \epsilon & \text{for } \epsilon < \frac{1}{2}(a+2c-3c') \\ & \text{and } c' \geq \frac{1}{7}(a+6c), \\ \frac{1}{3}(a+2c+\epsilon-3c') & \text{for } \frac{1}{2}(a+2c-3c') \leq \epsilon < 9c'-8c-a \\ & \text{and } c' \geq \frac{1}{7}(a+6c), \\ \frac{1}{11}(3[a+\epsilon]+2c-5c') & \text{for } \epsilon \geq 9c'-8c-a \\ & \text{and } c' \geq \frac{1}{7}(a+6c). \end{cases} \quad (4.32)$$

Proof *See Appendix.*

Equation 4.31, which corresponds to the case when $c' < \frac{1}{7}(a+6c)$, shows that the optimal royalty rate will be equal to the magnitude of the innovation if $\epsilon < \frac{1}{8}(3a+2c-5c')$ and equal $\frac{1}{11}(3[a+\epsilon]+2c-5c')$ otherwise.

Likewise, according to Equation 4.32, which corresponds to the case when $c' \geq \frac{1}{7}(a+6c)$, if $\epsilon < \frac{1}{2}(a+2c-3c')$, the optimal royalty rate will be equal to the magnitude of the innovation. In addition, the optimal royalty rate will be $\frac{1}{3}(a+2c+\epsilon-3c')$ if $\frac{1}{2}(a+2c-3c') \leq \epsilon < 9c'-8c-a$ and $\frac{1}{11}(3[a+\epsilon]+2c-5c')$ otherwise.[20]

Consequently, the firms' profits in equilibrium in this case are:

$$\begin{aligned} \pi_1^{R*}(D) &= \frac{1}{16}(a-2c+\epsilon+r_3^{D*}+c')^2 + \frac{1}{4}(a+2c+\epsilon-3[r_3^{D*}+c'])r_3^{D*}, \\ \pi_2^{R*}(D) &= \frac{1}{16}(a-2c+\epsilon+r_3^{D*}+c')^2 - C, \qquad (4.33) \\ \pi_3^{R*}(D) &= \frac{1}{16}(a+2c+\epsilon-3[r_3^{D*}+c'])^2, \end{aligned}$$

where Equation 4.31 defines r_3^{D*} if $c' < \frac{1}{7}(a+6c)$ and Equation 4.32 defines r_3^{D*} if

[20]Note that we define $9c'-8c-a$ only for non-negative values.

$c' \geq \frac{1}{7}(a + 6c)$.

Comparing the patent owner's profit with and without a technology transfer, we obtain the following results. When using the existing technology is better than duplication for Firm 2, technology transfer will always occur in equilibrium, given that $\epsilon < \frac{1}{2}(a - 2c + r_3^{ND} + c')$. Not surprisingly, when $\epsilon \geq \frac{1}{2}(a - 2c + r_3^{ND*} + c')$, the industry will become a duopoly, in which case technology transfer will not occur only if the invention is drastic. Last, when duplication is at least as good as choosing not to duplicate for Firm 2, licensing is at least as good as choosing not to license for the patent owner.

Licences to Firms 2 and 3: The last case to consider is when the patent owner offers a licence to each competitor. We define $\mathbb{r}_i$ for $i = \{2, 3\}$ and $\pi_i^{\mathbb{R}}$ for $i = \{1, 2, 3\}$ as the per-unit royalty and the profit of Firm i. Then, the per-unit costs are $c_1 = c - \epsilon$ for the patent owner, $c_2 = c - \epsilon + \mathbb{r}_2$ for Firm 2, and $c_3 = c' - \epsilon + \mathbb{r}_3$ for Firm 3. We can, thus, derive the firms' profits, as follows:

$$\begin{aligned} \pi_1^{\mathbb{R}} &= \frac{1}{16}(a - 2c + \epsilon + \mathbb{r}_2 + \mathbb{r}_3 + c')^2 + \frac{1}{4}(a - 2c + \epsilon - 3\mathbb{r}_2 + \mathbb{r}_3 + c')\mathbb{r}_2 \\ &\quad + \frac{1}{4}(a + 2c + \epsilon + \mathbb{r}_2 - 3[\mathbb{r}_3 + c'])\mathbb{r}_3, \\ \pi_2^{\mathbb{R}} &= \frac{1}{16}(a - 2c + \epsilon - 3\mathbb{r}_2 + \mathbb{r}_3 + c')^2, \\ \pi_3^{\mathbb{R}} &= \frac{1}{16}(a + 2c + \epsilon + \mathbb{r}_2 - 3[\mathbb{r}_3 + c'])^2. \end{aligned} \tag{4.34}$$

Clearly, technology transfer will occur in this case if $\pi_2^{\mathbb{R}} \geq \max\ (\pi_2^{ND*}, \pi_2^{D*})$ and $\pi_3^{\mathbb{R}} \geq \pi_3^{ND*}$.[21] Solving the patent owner's maximisation problem, we obtain the optimal per-unit royalty that corresponds to Firm 2:

[21]Note that $\pi_3^{ND*} \geq \pi_3^{D*}$; thus, $\pi_3^{\mathbb{R}} \geq \pi_3^{D*}$ will always hold if $\pi_3^{\mathbb{R}} \geq \pi_3^{ND*}$ is satisfied.

$$
\mathrm{r}_2^* = \begin{cases} \min\,(\epsilon,\ A^{\mathbb{R}}) & \text{for } \epsilon < \frac{1}{2}(a+2c-3c'), \\ \min\,(\epsilon,\ B^{\mathbb{R}}) & \text{for } \frac{1}{2}(a+2c-3c') \leq \epsilon < \frac{1}{6}(6a-7c+c'), \\ \min\,(\frac{1}{12}[6a-7c+6\epsilon+c'],\ B^{\mathbb{R}}) & \text{for } \frac{1}{6}(6a-7c+c') \leq \epsilon < a-2c+c', \\ \min\,(\frac{1}{2}[a-c+\epsilon],\ B^{\mathbb{R}}) & \text{for } \epsilon \geq a-2c+c', \end{cases} \tag{4.35}
$$

where $A^{\mathbb{R}} \equiv \frac{3}{8}(a-2[c-\epsilon]+c') - \frac{3}{8}\sqrt{(a-2[c-\epsilon]+c')^2-16C}$ and $B^{\mathbb{R}} \equiv \frac{1}{2}(a-c+\epsilon) - \frac{1}{2}\sqrt{(a-c+\epsilon)^2-9C}$.[22]

Proof *See Appendix.*

Furthermore, the optimal royalty rate for Firm 3 is:

$$
\mathrm{r}_3^* = \begin{cases} \epsilon & \text{for } \epsilon < \frac{1}{6}(6a-7c+c'), \\ \frac{1}{12}(6a-c+6\epsilon-5c') & \text{for } \frac{1}{6}(6a-7c+c') \leq \epsilon < a-2c+c', \\ \frac{1}{3}(a+c+\epsilon-2c') & \text{for } \epsilon \geq a-2c+c'. \end{cases} \tag{4.36}
$$

Proof *See Appendix.*

Equation 4.35 shows that if $\epsilon < \frac{1}{2}(a+2c-3c')$, the optimal per-unit royalty will be the minimum of the magnitude of the innovation and $A^{\mathbb{R}}$; if $\frac{1}{2}(a+2c-3c') \leq \epsilon < \frac{1}{6}(6a-7c+c')$, the optimal per-unit royalty will be the minimum of the magnitude of the innovation and $B^{\mathbb{R}}$; if $\frac{1}{6}(6a-7c+c') \leq \epsilon < a-2c+c'$, the optimal royalty rate will be the minimum of $\frac{1}{12}(6a-7c+6\epsilon+c')$ and $B^{\mathbb{R}}$; and in the case of a drastic invention, the optimal royalty rate will be equal to the minimum of $\frac{1}{2}(a-c+\epsilon)$ and $B^{\mathbb{R}}$.

Likewise, Equation 4.36 shows that if $\epsilon < \frac{1}{6}(6a-7c+c')$, the patent owner will charge Firm 3 a per-unit royalty equal to the magnitude of the innovation. Further, the optimal royalty rate will be equal to $\frac{1}{12}(6a-c+6\epsilon-5c')$ if $\frac{1}{6}(6a-7c+c') \leq \epsilon < a-2c+c'$ and $\frac{1}{3}(a+c+\epsilon-2c')$ if the invention is drastic.

[22]Note that we define $A^{\mathbb{R}}$ and $B^{\mathbb{R}}$, and $\frac{1}{12}(6a-7c+6\epsilon+c')$ and $\frac{1}{6}(6a-7c+c')$ only for non-negative values.

The firms' profits in equilibrium are then:

$$
\begin{aligned}
\pi_1^{\mathbb{R}*} &= \frac{1}{16}(a - 2c + \epsilon + \mathbb{r}_2^* + \mathbb{r}_3^* + c')^2 + \frac{1}{4}(a - 2c + \epsilon - 3\mathbb{r}_2^* + \mathbb{r}_3^* + c')\mathbb{r}_2^* \\
&\quad + \frac{1}{4}(a + 2c + \epsilon + \mathbb{r}_2^* - 3[\mathbb{r}_3^* + c'])\mathbb{r}_3^*, \\
\pi_2^{\mathbb{R}*} &= \frac{1}{16}(a - 2c + \epsilon - 3\mathbb{r}_2^* + \mathbb{r}_3^* + c')^2, \\
\pi_3^{\mathbb{R}*} &= \frac{1}{16}(a + 2c + \epsilon + \mathbb{r}_2^* - 3[\mathbb{r}_3^* + c'])^2,
\end{aligned}
\tag{4.37}
$$

where by Equations 4.35 and 4.36 define $\mathbb{r}_2^*$ and $\mathbb{r}_3^*$, respectively.

We can now determine the equilibrium of this case. When using the existing technology is better than duplication for Firm 2, technology transfer will always occur if $\epsilon < a + 2c - 3c'$. In addition, if $\epsilon \geq a + 2c - 3c'$, the industry will become a duopoly and thus, a technology transfer will not occur only if the invention is drastic. Finally, when duplication is at least as good as choosing not to duplicate for Firm 2, offering a licence to each competitor is at least as good as choosing not to license for the patent owner.

Now, we determine the equilibrium of the royalty licensing game. Suppose that using the existing technology is better than duplication for Firm 2. In this case, the analysis suggests that each licensing choice might occur in equilibrium. In particular, if $\epsilon < \frac{1}{2}(a + 2c + r_2^* - 3c')$, a technology transfer to both players is at least as good as the other two choices from the patent owner's perspective. If, however, $\frac{1}{2}(a + 2c + r_2^* - 3c') \leq \epsilon < \frac{1}{2}(a - 2c + r_3^{ND} + c')$, a technology transfer exclusively to Firm 2 is at least as good as the other choices for Firm 1. Additionally, if $\frac{1}{2}(a - 2c + r_3^{ND} + c' \leq \epsilon < a - c$, a technology transfer exclusively to Firm 3 is at least as good as the other choices for Firm 1. A technology transfer will not occur only in the case of a drastic invention. Finally, when duplication is at least as good as choosing not to duplicate for Firm 2, a technology transfer to both competitors is better than the other two choices.

4.2.3 Comparison of Fixed-Fee and Royalty Licensing

Finally, we can now compare licensing through a fixed fee to that through a per-unit royalty, and in turn, determine the equilibrium of the game. First, suppose that using the existing technology is better than duplication for Firm 2. The analysis suggests that royalty licensing is at least as good as fixed-fee licensing from the patent owner's perspective. Specifically, if $\epsilon < \frac{1}{2}(a + 2c + r_2^* - 3c')$, royalty licensing to both competitors is at least as good as all other equilibrium choices; if $\frac{1}{2}(a + 2c + r_2^* - 3c') \leq \epsilon < \frac{1}{2}(a - 2c + r_3^{ND*} + c')$, royalty licensing exclusively to the strong competitor is at least as good as all other equilibrium choices; and if $\frac{1}{2}(a - 2c + r_3^{ND*} + c') \leq \epsilon < a - c$, royalty licensing exclusively to the weak competitor is at least as good as all other equilibrium choices. Note that technology transfer will not occur in equilibrium if $\epsilon \geq a - c$.

Second, suppose that duplication is at least as good as choosing not to duplicate for Firm 2. The comparative statics show that royalty licensing is better than fixed-fee licensing from the patent owner's view. Specifically, royalty licensing to both competitors is better than all other equilibrium choices.

An explanation of the overall superiority of royalties might be the patent owner's cost advantage when a licensing agreement through a per-unit royalty occurs. However, the analysis suggests that the cost advantage due to royalties does not exceed the efficiency gain in the case of a drastic invention, and when duplication is not an equilibrium choice. In other words, the difference between the monopoly profit and the sum of the duopoly profits is more than the royalty cost advantage when duplication is not optimal.

In addition, comparing the quantity outputs corresponding to each equilibrium choice shows that that fixed-fee licensing might in fact be at least as attractive as royalty licensing for consumers. The analysis also suggests that patent owners might use licensing to prevent duplication, which is consistent with findings in the existing literature. Finally, the findings do not support the proposition that licensing is selective and that patent owners might, thus, use it to affect competition, which seems to be the case when we consider the effect of licensing on competition after patent expiration. Rockett (1990) suggests that patent owners might use licensing strategically to select competition aiming to preserve their dominant position in the market even after ex-post patent expiration. One reason that might explain this result is that, duplication might decrease the competitive advantage of the patent owner because of royalty licensing.

5 Conclusion

This study focuses on patent licensing, particularly the case in which the patent owner might license a cost-reducing technology to one or two competing firms in the industry. Patents and licences are of increasing importance in business, and a better understanding and appreciation of their strategic use might be beneficial for academics, practitioners, and policy-makers.

The analysis suggests that royalty licensing might be preferable to fixed-fee licensing from the patent owner's perspective, and that the opposite might be true from consumers' perspective. Specifically, licensing through a per-unit royalty, both exclusively or to all competitors, might occur in equilibrium in different situations. The cost advantage of royalty licensing is overwhelmed by the efficiency gain only in the case of a drastic invention and when duplication is not an optimal choice from the potential duplicator's view. This is consistent with Wang (1998), who argues that royalties are superior due to the cost advantage that arises from the patent owner's royalties. The analysis also suggests that a patent owner can use licensing to prevent duplication, but cannot use it strategically to select competition during the patent life term.

However, royalty licensing might be, at most, as good as fixed-fee licensing from the consumers' perspective. This might be seen as a sign of the necessity for more caution with respect to intellectual property licensing rights. Improving the understanding of the complex nature of patents and the licensing behaviour of innovating firms can enable policy-makers to increase the overall efficiency of the patent system.

Nevertheless, there are theoretical issues that we do not address in this study. For instance, we lack the dimension of random imitation or technical and commercial uncertainty, which can bring controversial insights to our findings for optimal licensing. Additionally, we leave the addition of an arbitrary number of firms and an uncertain and lengthy duplication process for future research. This study can be considered as a starting point for more explicit models on the strategic licensing of intellectual property rights.

Appendix

Proof of Equation 3.11. First, we solve Firm 1's maximisation problem subject to Firm 2's participation constraints:

$$\begin{aligned} \max_{r \geq 0} \quad & \pi_1^R, \\ \text{s.t} \quad & \pi_2^R \geq \pi_2^{ND}, \\ & \pi_2^R \geq \pi_2^D. \end{aligned} \tag{A.1}$$

Second, we form the Lagrangean equation according to the Lagrange Theorem and define the critical points. The first-order conditions of the Lagrangean are:

$$\begin{aligned} r &= \frac{1}{2}(a - c + \epsilon), \\ r &\leq \epsilon & \text{for } \epsilon < a - c, \\ r &\leq \frac{1}{2}(a - c + \epsilon) & \text{for } \epsilon \geq a - c, \\ r &\leq \frac{1}{2}(a - c + \epsilon) - \frac{1}{2}\sqrt{(a - c + \epsilon)^2 - 9C} & \forall \epsilon. \end{aligned} \tag{A.2}$$

Last, we compute the value of the objective function at each critical point and derive the solution.[23] The procedure is the same as applying the Theorem of Kuhn and Tucker to solve for an inequality constrained optimisation problem. Equation 3.11 follows from simple calculations.

Proof of Equation 4.24. We can apply the same basic analysis to this case. First, we solve Firm 1's maximisation problem subject to Firm 2's participation constraints:

$$\begin{aligned} \max_{r_2 \geq 0} \quad & \pi_1^R, \\ \text{s.t} \quad & \pi_2^R \geq \pi_2^{ND}, \\ & \pi_2^R \geq \pi_2^D. \end{aligned} \tag{A.3}$$

Second, we form the Lagrangean equation according to the Lagrange Theorem and

[23]Note that we also account for the non-negativity constraints and the Lagrangean multipliers, but do not include them here for computational ease. In addition, we also consider that the optimal per-unit royalty cannot be larger than the magnitude of the innovation in any case.

define the critical points. The first-order conditions of the Lagrangean are:

$$
\begin{aligned}
r_2 &= \frac{3}{11}(a - 2[c-\epsilon] + c'), \\
r_2 &\leq \epsilon && \text{for } \epsilon < a + 2c - 3c', \\
0 &\leq \frac{1}{16}(a - 2[c-\epsilon] - 3r_2 + c')^2 - \frac{1}{9}(a - c - \epsilon)^2 && \text{for } a + 2c - 3c' \leq \epsilon < a - 2c + c', \\
r_2 &\leq \frac{1}{3}(a - 2[c-\epsilon] + c') - \frac{1}{2}\sqrt{(a - c + \epsilon)^2 - 9C} && \text{for } \epsilon \geq a - 2c + c', \\
0 &\leq \frac{1}{16}(a - 2[c-\epsilon] - 3r_2 + c')^2 - \frac{1}{16}(a - 2c + c' + 2\epsilon)^2 - C && \text{for } \epsilon < \frac{1}{2}(a + 2c - 3c'), \\
0 &\leq \frac{1}{16}(a - 2[c-\epsilon] - 3r_2 + c')^2 - \frac{1}{9}(a - c + \epsilon)^2 - C && \text{for } \epsilon \geq \frac{1}{2}(a + 2c - 3c').
\end{aligned} \tag{A.4}
$$

Last, we compute the value of the objective function at each critical point and derive the solution.[24] By focusing specifically on the different intervals of the magnitude of the innovation and using simple algebra, the results shown in Equation 4.24 follow.

Proof of Equations 4.27 and 4.28. Similar to the optimisation problems above, we solve Firm 1's maximisation problem subject to Firm 3's participation constraint:

$$
\begin{aligned}
\max_{r_3^{ND} \geq 0} \quad & \pi_1^R(ND), \\
\text{s.t} \quad & \pi_3^R(ND) \geq \pi_3^{ND}.
\end{aligned} \tag{A.5}
$$

Second, we form the Lagrangean equation according to the Lagrange Theorem and

[24]Note that we also account for the non-negativity constraints and the Lagrangean multipliers, but do not include them here for computational ease. In addition, we also consider that the optimal per-unit royalty cannot be larger than the magnitude of the innovation in any case.

define the critical points. The first-order conditions of the Lagrangean are:

$$\begin{aligned} r_3^{ND} &= \frac{3}{11}(3a + 2c + 6\epsilon - 5c'), \\ r_3^{ND} &\leq \epsilon \qquad\qquad\qquad\qquad \text{for } \epsilon < a + 2c - 3c', \\ r_3^{ND} &\leq \frac{1}{3}(a + 2[c + \epsilon] - 3c') \qquad \text{for } \epsilon \geq a - 2c + c'. \end{aligned} \tag{A.6}$$

Last, we compute the value of the objective function at each critical point and derive the solution.[25] By focusing specifically on the different intervals of the magnitude of the innovation and using simple algebra, the results in Equations 4.27 and 4.28 follow. Note that the relationship between r_3^{ND} and ϵ changes for vales of c' smaller or larger than $\frac{1}{5}(a + 4c)$.

Proof of Equations 4.31 and 4.32. We first solve Firm 1's maximisation problem subject to Firm 3's participation constraint:

$$\begin{aligned} \max_{r_3^D \geq 0} \quad & \pi_1^R(D), \\ \text{s.t} \quad & \pi_3^R(D) \geq \pi_3^D. \end{aligned} \tag{A.7}$$

Second, we form the Lagrangean equation according to the Lagrange Theorem and define the critical points. The first-order conditions of the Lagrangean in this case are:

$$\begin{aligned} r_3^D &= \frac{1}{11}(3[a + \epsilon] + 2c - 5c'), \\ r_3^D &\leq \epsilon \qquad\qquad\qquad\qquad \text{for } \epsilon < \frac{1}{2}(a + 2c - 3c'), \\ r_3^D &\leq \frac{1}{3}(a + 2c + \epsilon - 3c') \qquad \text{for } \epsilon \geq \frac{1}{2}(a + 2c - 3c'). \end{aligned} \tag{A.8}$$

Last, we compute the value of the objective function at each critical point and derive the solution.[26] By focusing specifically on the different intervals of the magnitude of

[25] Note that we account for the non-negativity constraints and the Lagrangean multipliers, but do not include them here for computational ease. In addition, we also consider that the optimal per-unit royalty cannot be larger than the magnitude of the innovation in any case.

[26] Note that we also account for the non-negativity constraints and the Lagrangean multipliers, but do not include them here for computational ease. In addition, we do also consider that the optimal per-unit royalty cannot be larger than the magnitude of the innovation in any case.

the innovation and using simple algebra, the results in Equations 4.31 and 4.32 follow. Note that the relationship between r_3^D and ϵ changes for vales of c' smaller or larger than $\frac{1}{7}(a+6c)$.

Proof of Equations 4.35 and 4.36. We can apply the same basic analysis above to this last case. First, we solve Firm 1's maximisation problem subject to the participation constraints of Firms 2 and 3:

$$\begin{array}{ll} \max\limits_{\mathbb{r}_2,\mathbb{r}_3\geq 0} & \pi_1^{\mathbb{R}}, \\ \text{s.t} & \pi_2^{\mathbb{R}} \geq \pi_2^{ND}, \\ & \pi_2^{\mathbb{R}} \geq \pi_2^{D}, \\ & \pi_3^{\mathbb{R}} \geq \pi_3^{ND}. \end{array} \tag{A.9}$$

Second, we form the Lagrangean equation according to the Lagrange Theorem and define the critical points. The first-order conditions of the Lagrangean are:

$$\begin{array}{rcll}
\mathbb{r}_2 &=& \frac{1}{11}(3a-6c+3\epsilon+5\mathbb{r}_3+3c')=0, & \\
\mathbb{r}_3 &=& \frac{1}{11}(3a+2c+3\epsilon+5\mathbb{r}_2-5c')=0, & \\
\mathbb{r}_2 &\leq& \frac{1}{3}(2\epsilon+\mathbb{r}_3) & \text{for } \epsilon \leq a+2c-3c', \\
0 &\leq& \frac{1}{16}(a-2c+\epsilon-3\mathbb{r}_2+\mathbb{r}_3+c')-\frac{1}{9}(a-c-\epsilon)^2 & \text{for } a+2c-3c' \leq \epsilon < a-2c+c', \\
\mathbb{r}_2 &\leq& \frac{1}{3}(a-2c+\epsilon+\mathbb{r}_3+c') & \text{for } \epsilon \geq a-2c+c', \\
0 &\leq& \frac{1}{16}(a-2c+\epsilon-3\mathbb{r}_2+\mathbb{r}_3+c')^2-\frac{1}{16}(a-2c+c'+2\epsilon)^2+C & \text{for } \epsilon \leq \frac{1}{2}(a+2c-3c'), \\
0 &\leq& \frac{1}{16}(a-2c+\epsilon-3\mathbb{r}_2+\mathbb{r}_3+c')^2-\frac{1}{9}(a-c+\epsilon)^2+C & \text{for } \epsilon \geq \frac{1}{2}(a+2c-3c'), \\
\mathbb{r}_3 &\leq& \frac{1}{3}(2\epsilon+\mathbb{r}_2) & \text{for } \epsilon < a+2c-3c', \\
\mathbb{r}_3 &\leq& \frac{1}{3}(a+2c+\epsilon+\mathbb{r}_2-3c') & \text{for } \epsilon \geq a+2c-3c'.
\end{array} \tag{A.10}$$

Last, we compute the value of the objective function at each critical point and derive the solution.[27] Clearly, this case is relatively more complicated than the other cases are. Solving the first two conditions simultaneously, we obtain the two critical points as follows:

$$\begin{aligned} \mathtt{r}_2 &= \frac{1}{12}(6a - 7c + 6\epsilon + c'), \\ \mathtt{r}_3 &= \frac{1}{12}(6a - c + 6\epsilon - 5c'). \end{aligned} \tag{A.11}$$

Additionally, considering the fifth and last conditions, we obtain the following critical points:

$$\begin{aligned} \mathtt{r}_2 &= \frac{1}{2}(a - c + \epsilon), \\ \mathtt{r}_3 &= \frac{1}{3}(a + c + \epsilon - 2c'). \end{aligned} \tag{A.12}$$

Likewise, from the sixth and seventh conditions, we obtain another two critical points:

$$\begin{aligned} \mathtt{r}_2 &= \frac{3}{8}(a - 2[c - \epsilon] + c') - \frac{3}{8}\sqrt{(a - 2[c - \epsilon] + c')^2 - 16C}, \\ \mathtt{r}_2 &= \frac{1}{2}(a - c + \epsilon) - \frac{1}{2}\sqrt{(a - c + \epsilon)^2 - 9C}. \end{aligned} \tag{A.13}$$

It is clear that the magnitude of the innovation is also a critical point. In turn, by focusing specifically on the different intervals of the magnitude of the innovation and using simple algebra, the results in Equations 4.35 and 4.36 follow.

[27]Note that we account for the non-negativity constraints and the Lagrangean multipliers, but do not include them here for computational ease. In addition, we also consider that the optimal per-unit royalty cannot be larger than the magnitude of the innovation in any case.

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