

Graduate School Master of Science in Finance

The Impact of Derivatives Trading on the volatility of S&P500 and its implied volatility

Abstract

Research on the relationships between spot volatility and trading exchange in the financial markets has been and still is the heart of great attention by scholars of econometrics, finance and statistics. The purpose of this thesis is to examine the movements of the underlying spot volatility and the CBOE Volatility Index, known as VIX Index, in the American Stock exchange market after the introduction of linear and non-linear derivatives trading activities on the Standard & Poor's 500. In order to state if derivatives trading affect the volatility of the indices, traditional measures and generalised autoregressive conditional heteroscedastic (GARCH) specification are settled on a index and asset framework.

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Academic Year: 2017/2018

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1. Introduction

Underlying spot volatility and trading exchange in the financial markets have been watched closely by scholars of econometrics, practitioners, quants and statisticians, regulators and policy makers worldwide. It has been often shown that an increase in market volatility lead to more intensive trades in derivative financial products. Most of those studies were based mainly on monthly or intra-day data, whereas in very few cases the analysis have been focuses on a daily basis. Moreover, no one tried to investigate it respect to a volatility index. Some empirical studies have confirmed the existence of this positive relationship between the volatility, volume and information flows of exchanges. Two main measures of activity are taken as reference in derivative markets: turnover (or average volume) refers to the number of trades of the various listed contracts during a certain period; time unit is the business day, and data on activities are normally expressed in number of negotiated contracts. Turnover is usually employed as a liquidity indicator of a contract. Open interest positions express the total number of contracts which have not been compensated by a transaction of opposite sign and extinguished by the delivery of the underlying. It is a stock data which represents the net result of transactions at a certain date and is often interpreted as an indicator of covering activity. Open positions are generally less than the turnover because many contracts purchased or sold within a trading day are compensated before the end of the stock exchange session. The aim of this project work is to examine the movements of the S&P500 and its Implied volatility Index, known as VIX Index, in the U.S Stock Market after the introduction of trading activities through Futures and Options, on the US Stock market. In order to state if derivatives trading affect the volatility of these two indices, traditional measures and generalized autoregressive conditional heteroscedastic (GARCH) specification are settled on an index and asset framework. The results of my analysis investigate possible changes in the spot market volatility of the SP500 components and the VIX Index by derivatives trading activities over the whole post-crisis age, after the introduction of trades. According to literature, speculators trading activities lead to significant movements of the volatility, while hedgers tend to reach the stability of the market. Firstly, my analysis will focus on the relevance of the CBOE Volatility Index and its properties, since it has become the main benchmark for stock market volatility, and the reasons why it is also known as the thermometer of fear. Section 2 goes through a briefly description of the S&P500 Stock Index and its evolution over time; then, the thesis will be structured around the investigation of the CBOE volatility Index, its features, the trading of VIX futures and VIX Options and correlation between stock volatility and VIX. Historical data suggest that VIX have had an inverse relation with the stock market in term of prices. Section 1 concludes on an examination of the data, obtained from DataStream, the American Stock Exchange Market and the methods used for the selection of a portfolio of companies in the analysis. Section 3 concerns the concept of volatility, classifying them into historical, implied and realized volatility, and I focused on review the very extensive literature conducted in this field. In section 4, I start out by exploring the statistical properties of the data on both a descriptive level and on basis of statistical tests. It illustrated the autocorrelation properties and heteroscedasticity of the financial historical time series taken into account. I concluded this section by selecting an econometric theory based on previous research and the investigation of the statistical patterns of the data. This theory lies behind the various components that make up the conditional volatility model a specification of the GARCH family. Section 6 presented the empirical results that tried to answer the research proposal, both on index and asset level. Finally, I summarized the most important limitations of my thesis in order to suggest possible topics for future research.

2. An analysis of the S&P's 500 Implied Volatility

2.1. Standard & Poor's 500 and its evolution

Standard & Poor 500, well known as S&P 500, was established in 1923. It is an index of 505 stocks traded on the American stock Exchange. The index was composed by 233 stocks but in 1957 it was expanded to 505 constituents. SP 500 associates the market performance of multi-sector companies. When it was first made, it consisted of 23 different sectors, but actually, more than 100 sectors are associated in the index. Component companies are selected according to several criteria like market size, liquidity and sector. The most represented sectors are Information technology (17.8%), Banking and Financial Services (15.1%), and Energy (12.7%).

Companies have different weight in the index, which means that firms with a higher market capitalization have a larger impact on the index behaviour. S&P 500 index represents the benchmark of how well the American economy is performing for many market participants (speculators, hedgers and arbitragers). Earlier the Dow Jones index was formerly served as the indicator of the performance of the American stock Exchange market. Nowadays, the S&P 500 is the most used by investors, since it represents 500 companies where the Dow Jones index only consists of the 30 largest companies in the same market. Empirical example of this default are Google and Apple, who do not show up in the Dow Jones due to their large stock prices. In table below, are reported the composition of index classified by sector (as August 2018):

Sector	Weight (%)
Technology	17.73
Financial Services	16.71
Health Care	12.97
Customer Discretionary	11.18
Energy	10.51
Industrial	10.28
Materials	3.31
Utilities	3.30
Telecommunication	2.58
Consumer Staples	10.41

Table 1: Index weight by segment - Thomson reuters

In order to be introduced in the index, Companies should have specific market capital-

ization levels (within the top 500 in all the US stock Exchange). Moreover, S&P admissions board will examine stringent requirements:

- Market capitalization greater or equal to 6.1 billion USD;
- Headquarters in the U.S. country;
- Annual dollar value traded to float-adjusted market capitalization greater than unit;
- Minimum monthly trading volume of 250,000 of its shares in each of the previous six months up to the evaluation date;
- Securities must be publicly listed on either the NYSE (including NYSE Arca or NYSE MKT) or NASDAQ (NASDAQ Global Select Market and NASDAQ Capital Market);
- At least half a year since its initial public offering;
- Four straight quarters of positive as-reported earnings.

The method used by the S&P500 to evaluate the weights is different from other weightingmethods, since the overall market capitalization, given by the product of price per shares and outstanding stock, is considered. Therefore, it gives a better measure of the overall size and market value of a company. The S&P 500s capitalization-weighted method is also float-weighted which means that it takes into account the number of shares available for public trading between institutional and retail investors, excluding any shares that are tied up in the company itself or any government holdings. The calculation of the index sum up all market capitalizations for each of the 500 companies and divides the sum by a measure set by Standard and Poors.

$$S\&P500 = \frac{\sum(stockprice * number pershares)}{SPX divisor}$$
(2.1)

This divisor is regulate to incorporate stock issuance, mergers and acquisitions, change in the financial structure because of corporate events to ensure sudden deviation of the index value.

Figure 1 shows the S&P 500 prices from 1999 to the start of June 2018.



Fig. 1. Historical evolution of S&P 500 (1999-2018) - Thomson Reuters

2.2. The CBOE Volatility Index VIX

2.2.1. Properties of VIX index

The volatility of financial assets is one of the most important risk indicators available to market participants and market observers. Volatility, changing stochastically over time with quite placate periods followed by unstable phase, is not only that. It is also a tradable market instrument for investors. The 1987 crash focused economists attention on volatility products. Many investors suffer from unexpected market fluctuations that can cause an increase in the risk exposure of their portfolios, even though there are the so-called risk lovers who pursue profits whatever the market conditions.

The benefits of volatility derivatives include:

1. Users are unaffected by directional moves in the underlying asset;

- 2. The contracts enable cross-index volatility arbitrage;
- 3. The risk exposure mirror volatility movements rather than delta hedging;
- 4. Participants trade their views on realized volatility levels against market implied volatility or trade their expectation in the volatility term structure.

Therefore, to know the nature of this movements is essential for their investment decisions. Many model-based and model-free volatility measures have been proposed in the academic literature.

In 1993, the Chicago Board Options Exchange (CBOE) launched the VXO on the market, the first volatility index, to be traded through linear and non-linear derivatives. This index was based on the prices of the weighted-average Standard Poor's 100 index as underlying asset, thus, it reflected the movements of the most traded and liquid 100 industries in different sectors in the US stock market. However, the realized volatility had been fully captured after the birth of a more responsive index, the VIX, in 2003, that has effectively become the standard measure of volatility risk. The VIX, which was recalculated by the CBOE to take into account investors behaviour and market fluctuations more clearly than the first volatility index VXO.

VXO was based on the Black-Scholes model, through which is possible estimating the market volatility of an underlying asset as a function of price and time without direct reference to specific investor characteristics like expected yield, risk aversion measures or utility functions. Although it has been considered the biggest innovation in financial theory by economist and critics, the model is built on unrealistic assumptions about the market. According to the model, volatility is constant over time; returns of underlying stock prices are normally distributed; the underlying stocks do not pay dividends during the entire life of the option; European-style options can be exercised on the expiration date; the model assumes that there are no fees for buying and selling options and stocks and no barriers for trading (Teneng, 2011). On the other hand, Vix is not based on the above formula but its payoff is linearly correlated to the variance of a swap. By definition, it is the square root of the risk-neutral expectation of the integrated variance of the SP500 over the next 30 calendar days, reported on an annualized basis. CBOE volatility index has become very suitable in the U.S. stock market due to its attitude to be used as a risk-hedging instrument, especially negative interest rate environment. Except its role as a risk measure, nowadays, it is possible directly to invest in volatility as an asset class. Specifically, on March 26, 2004, trading in futures on the VIX was introduced on the CBOE Futures Exchange (CFE). They are standard futures contracts on forward 30-day that cash settle to a special opening quotation (VRO) on the Wednesday that is 30 days prior to the 3rd Friday of the calendar month immediately following the expiration month. Almost Two years later, on February 24, 2006, European-type options on the VIX index started to be traded on the CBOE. Like VIX futures, they are cash settled according to the difference between the value of the VIX at the expiration day and their strike price. Moreover, they can also be seen as options on VIX futures. VIX derivatives are among the most traded financial instruments on CBOE and CFE, with an average daily volume close to 445.000 contracts in 2017. One of the main reasons for the high interest in these products is that VIX is used to trade market volatility of SP 500; it means that it has become the instrument to diversify the risk associated to the SP500 index, without having to delta hedge positions with the stock index (Szado 2009). As a result, for investors is cheaper to assume a long position in out-of-the-money call options on VIX than to buy out-of-the-money puts on the SP500. Fig. 2 displays the historical evolution of the VIX prices from 30th of June 1999 to the end of June 2018. The average closing price was 20.4 in the first ten years of our investigated sample. This time series is characterized by swings from low to high levels, with a finite behaviour that shows meanreversion over the long run but displays relevant persistent deviations from the mean during extended periods (Sentana, E.2013). In December 1993, it was recorder the lowest closing value (9.31). Moreover, volatility assumed law values in the interval February 2006-July 2007. This period, that recorded one of lowest value (9.89), had been called the calm before the storm by economists. In the last ten years (2008-2018), the largest historical closing price was 80.86, which took place on November 20, 2008. After this peak, VIX presented a decreasing trend until April 2010, period in which the Greek debt crisis deteriorated.



Fig. 2. Historical evolution of VIX Prices (1999-2018) - Thomson Reuters

2.2.2. Relation between S&P500 and VIX indexes

The main attraction of the VIX product lies in the negative correlation of this volatility index with the corresponding stock market. The evolution of S&P500 and VIX illustrated in Figure 2.2.2 supports the idea of a negative correlation, implying that adding VIX positions (via futures contracts) would help to reduce the risk of diversified portfolios. This connection helped the growth of volatility derivatives market to the extent that many investors perceive VIX, and other volatility indices, as an asset class of its own.

According to Chau(2012), "VIXs prices have moved in an almost perfect opposite direction to the SP 500 indexs for approximately 88% of the time". Investors ask for a higher return on stocks since they feel the market riskier, when the expected stock market volatility goes up, and VIX, negatively correlated, will assume higher values. Conversely, when the market volatility start to go up, reflecting lower values of VIX, investors feel the market riskless. The negative correlation between VIX and the US stock market is reported below:



Fig. 3. Daily time series of S&P500 and VIX between 30-06-1999 and 30-06-2018 - Thomson Reuters

Historical data shows that, as the price of the SP 500 goes down, the price of VIX assume an opposite direction. Specifically, between July and August 1998, during the Capital Management Crisis caused by the Russian debt default and the Asian financial crisis, VIX almost tripled while SPX tended to move upward. At the end of 1998, the volatility index moved back to the pre-crisis levels while SP 500 recovered all its losses recorded in that year. The Internet Bubble and the accounting scandals conducted by Enron, WorldCom and Global Crossings ,pillars in the US market, in 2002, recorded +25% in VIX correlated to a fall of US equity index to -36% (Alexander, Carol, and Andreza Barbosa,2006). Following the stock market crash caused by the disruption of the sub-prime segment and the bankruptcy of Lehman Brothers, the price of VIX has risen its most critical value, 80.86, on November 20, recorded a 240% rapid increase from the last ten months before. SP500 Equity index, on the other hand, recorded a fall of, approximately, -45%. The most recent European financial crisis in 2011 caused an increase of VIX prices up to 165% from the beginning of the year to end of August; at the same time, SPX goes down of 12%. Market participants tend to

purchase put options than call options on index since they are more risk-adverse during economic declines. Such reaction cause a rise of the implied market volatility. Thereupon, VIX changes much more energetically during deflation than during growing markets periods and investors are able to generate higher profits than they do during market improvements. "On average, when SPX drops by 100 basis points, then VIX increase by 4.493%. On the other hand, if SPX increase of 100 basis points, the VIX will drops by 2.99%" (Whaley, 2009). In Table 1.2.2 are reported the correlations between VIX index and SP 500 according to markets behaviour (bull market and bear market). The market was not very volatile itself in 1990-1998, until the occurring of the Long-Term Capital Management crisis in the last years of the considered period. In a similar way, between 1999 and 2002 VIX moved in an opposite direction to the market, due to the high volatility of the market caused by 2001 terrorist attack and the 2002 Internet crisis. The third considered period (2003-2007), even though the correlation was negative (-0.27), it was not as tough as the previous time interval. From 2008 to present, the market have been more volatile As a result, the negative correlation between these two indices have been quite high than the previous period. (-0.53)

	1990-1998	1999-2002	2003-2007	2008-2018
ρ	-0.12	-0.65	-0.27	-0.53

Table 2: correlations between SPX and VIX - Matlab analysis

2.2.3. VIX future

As implied volatility tends to be range-bounded and to follow a mean-reverting process, direct investments in VIX as underlying asset, were not yet feasible. "If it were predictable, then investors would be able to predict their returns from buying VIX when it is low and selling it when it assume higher values" (Merrill Lynch, 2). For that reason, on March 24, 2004, the CBOE launched the first VIX future on the CFE (Electronic CBOE Futures Exchange), in order to trade the VIX index as underlying asset. VIX futures are standard future agreements that constrain investors to buy or sell an asset at a predetermined future time and a predetermined price on forward 30-day implied volatilities of the SP 500 index. Studies have been conducted to analyse the correlation between VIX future prices and VIX. (Aramian F. 2014). "On the report of Jung (2016), VIX futures are negatively correlated with the SP 500 equity index but highly correlated to VIX movements. VIX future prices and returns are not normally distributed with positive skewness and excess kurtosis. There also exists a high positive correlation between VIX futures maturity. The longer the maturity of VIX futures, the higher their prices. This trend signifies that volatility tends to increase over time". The above result investigated by Jung (2016), did not hold anymore because of the global financial crisis in 2008: in particular, short-term volatility implied by the VIX reached its highest value during the catastrophe, leading to the term structure of VIX futures to be a decreasing function. Market participants can effectively utilize VIX futures for hedging strategies. They also allow speculators to trade volatility and have a profit. Future contracts make VIX tradable on the market through two common ways for investors: they can take a long position if they expect the VIX will go up and a bearing market in the future; on the other hand, they can take a short position or sell VIX futures if they believe the VIX will decrease and the stock market will move up in the future. The average daily volume has grown each year since the introduction of the first listed derivative based on expected market volatility. Particularly, in 2004, average daily volume was about 461 contracts; in 2017 was recorded an average daily volume for VIX futures of almost 250000 contracts per day. The figure below shows the average daily volume per year from 2004 to 2016.

VIX[®] Futures

\$1000 multiplier Volume on Aug. 10, 2017 hit a record 942, 109 contracts



Fig. 4. Average daily volume on VIX Futures - CBOE www.cboe.com/VIX (2017)

In conformity with Figure 4, the traded contracts on VIX in the U.S. stock market increased from less than 470 contracts in 2004 to almost 35.000 at the end of 2011. In 2012, the CBOE Futures Exchange, LLC (CFE), launched five different VIX futures: Weekly options on VIX futures (VOW), CBOE mini-VIX (VM), CBOE Gold ETF Volatility Index (GVZ) and CBOE SP 500 3-Month Variance (VT), and it is recorded that in 2011, daily volume reached the highest record in its trading history (more than 12 million contracts).

2.2.4. VIX Options

In 2006, CBOE launched non-equity type options, non-linear derivative products which allow market participants to trade the VIX index. Similar to VIX futures, VIX options do not need to physical delivery at the settlement. As claimed by Lin and Chang in 2009, "VIX options are European-style options, which can only be exercised at the predetermined expiration date. Therefore, investors are not allowed to trade the VIX index at an agreed upon price before the expiration date". Their prices are based on the less volatile forward price of VIX, respect to its spot value, even though spot prices tend to converge to forward one, gradually. Like VIX futures, liquidity on VIX options presented an increasing trend. The recorded average daily volume for put and call options has increased from 23,501 contracts per day in 2006 up to, approximately, 400,000 contracts per day at the end of 2011. Consequently, more and more market participants started to trade the option on VIX, as shown in Figure 1.2.4(Wien, 2010).

VIX® Options

\$100 Multiplier Volume on Aug. 10, 2017 hit a record 2,538,121 contracts



Fig. 5. Average daily volume on VIX Options - CBOE www.cboe.com/VIX (2017)

As reported in Table above, since 2006, it is clear that the most traded options on VIX are the call-type ones. Most of the speculators look for profit upside exposure while hedgers want to reduce risk caused by unexpected movements of the market, assuming only the risk to lose the premium paid for the option (Wien, 2010). As the existence of the hedging strategies, mentioned previously, investors, depending on their risk aversion, have developed several speculative strategies through VIX options. "If investors expect that the market

volatility implied by the VIX will go up or be bullish and the stock market will decline or be bearish in the future, they can consider three option strategies: taking a long position on VIX call options, establishing a VIX call bull spread, or combining a VIX put bull spread and long call" (CBOE White Paper, 2009).

2.2.5. VIX Estimation

Index values are given by the prices of their constituents, such as the SP 500. The VIX Index is a volatility index composed by options which price reflect the markets expectation of future volatility (CBOE white paper, 2017) VIX Index calculation is given by the follow relation:

$$VIX_t^2 = \frac{2}{T} \sum_{i=1}^N \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2$$
(2.2)

Calculation of the time to maturity is based on minutes, T, in calendar days and separates every day into minutes. Time to expiration is calculated as follow:

$$T = \frac{minutes from the current time to 8:30 hrson the set tl Ement day}{minutes in a year}$$
(2.3)

R, the risk-free interest rate, is the U.S. Treasury yields which has the same maturity as SPX option. The risk-free interest rates of near-term options and next-term options, with different maturities, are different. SP 500 options are of the European type with no dividends.

The put-call parity formula can be solved for R as follows:

$$R = \frac{-1}{T-t} ln \left[\frac{K}{S_t + P_t - C_t} \right]$$
(2.4)

Where K is the strike price, S_t is the underlying SP500 price, P_t is put option price and C_t is the call option price.

The Forward index price derived from index option prices, F is given by:

$$P = strikeprice * e^{RT}(C_t - P_t)$$
(2.5)

Then, K_0 is the first strike below the forward index level, F; K_i represents the Strike price of i^{th} out-of-the-money option (a call if $K_i \downarrow K_0$ and a put if $K_i \downarrow K_0$); both put and call if $K_i = K_0$; ΔK_i is the time interval between strike prices and it can be explained as $\Delta(K_i) = \frac{K_{i+1} - K_{i-1}}{2}$; finally, $Q(K_i)$ is the average of quoted bid and ask option prices, or mid-quote prices.

$$QK_i = \frac{Bid + Ask}{2} \tag{2.6}$$

3. Theoretical Framework

3.1. Introduction to Volatility

The concept of volatility has to be introduced in this work since different main volatility measures will be made known, to evaluate derivatives trading activities related to volatility of the US equity-volatility indices. Volatility is a measure of risk, which estimate how much the return of financial assets can swing with respect to its mean value; it is the standard deviation of the returns of each asset. Higher is the standard deviation, higher gains or losses can be recorded. It therefore provides a measure of the probability distribution of future returns. The term volatility can be explained by several definitions. In order to answer our research question, three main volatility measures are considered: the historical volatility, the implicit volatility and the stochastic volatility.

3.1.1. Historical Volatility

Historical volatility is a backward-looking measure; it is an estimate of the volatility based on the historical returns of the underlying asset. It is defined as:

$$\sigma_t^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \mu)^2 \tag{3.1}$$

Where N is the sample size, x_i is the observation and μ is the mean of the assets prices. However, pricing options must take into account the probabilities of future events: therefore, by definition, historical volatility cannot be an appropriate measure of expected risk.

3.1.2. Implied Volatility

According to Alexander Carol (2008), Implied Volatility is the volatility of underlying asset price process that is implicit in the market price of an option. Implied volatility of the price of the underlying (σ) can be derived from the option pricing formula, defined by the Black Scholes-Merton model.

$$c = S_0 N(d_1) - K e^{-rT} N(d_2)$$
(3.2)

$$p = Ke^{-rT}N(-d_2) - S_0N(-d_1)$$
(3.3)

where:

$$d_{1} = \frac{\ln(\frac{S_{0}}{K}) + (\frac{r+\sigma^{2}}{T})^{2}}{\sqrt{\sigma}T}$$
(3.4)

$$d_2 = \frac{\ln(\frac{S_0}{K}) + (\frac{r - \sigma^2}{T})^2}{\sqrt{\sigma}T}$$
(3.5)

Implied volatility is based on the current option prices containing all the forward expectations of market participants. These expectations are assumed rational and should include all the historical available information on the market.it present three relevant properties that need to be explained.

- 1. Volatility smile and skew: refers to the fact that implied volatility of an option is function of its strike price, which has a smile behavior; moreover, the feature that volatility smile tends to be asymmetric, is called volatility skew.
- 2. Volatility term structure: implied volatility is time to maturities dependent for a fixed strike price and it converges to a long- term average volatility level. Therefore, implied volatilities tends to decrease as time to maturity go up.
- 3. Volatility surface: is the three-dimensional combination of volatility smile or skew and volatility term structure. (as in Figure below)



Fig. 6. Volatility surface of options underlying on the S&P 500 index - J. Fonseca (2002)

3.1.3. Realized Volatility

Looking at volatility estimates, one of the most used measured by financial practitioners is the Realized volatility (RV) which uses high frequency intraday data to directly measures volatility through a semi-martingale and it produce a perfect estimate of volatility in the hypothetical situation where prices are observed over time continuously and without systemic errors. Among the most relevant properties of the realized volatility, I identify the long-term dependence, the presence of positive self-correlations at high lags, the leverage effect (since yields are negatively correlated to RV), the jumps (discontinuity in the process of prices that have a positive impact on future volatility), and the fact that RV provide a consistent ex-post non parametric estimate of the delta returns. Realized volatility, equal to the sum of all available intraday high-frequency squared returns, is defined as follow:

$$RV_t^2 = \sum_{i=1}^m (p_{i,t} - p_{i-1,t})^2 = \sum_{i=1}^m (y_{i,t})^2$$
(3.6)

3.2. Information flows and price volatility

The relation between trading volume and volatility has received considerable attention in the finance literature. Two hypotheses, the Mixture of Distributions hypothesis (MDH), advocated by Clark in 1973, and Sequential Information Arrival hypothesis (SIAH), backed by Copeland (1976) based on signalling and distribution theory justify the relation between stock prices changes and trading volume. Although both models provide support for the pragmatic relationship between price changes and trading volume, they diverge in terms of consistency. The information flow has a direct repercussion on the underlying spot markets for two reasons. Firstly, the commerce of futures engage speculators who bet on future expectation and future behaviour of assets. According to Ross theory (1989), the variance of price fluctuations are equal to the percentage of information flow, assuming that prices are martingales. Therefore, prices are do not depend on prices in t-1, and that prices reflect all the available information on the market, hence supporting the Efficient Market Hypothesis. As information flow vary, also the spot volatility take turns. The opponent theories have been developed by Robert J. Shiller (2005) and take the name of Irrational Exuberance, He observed excessive exuberance and found out an acute inefficiency of the market. The latter lead to a infringement of the martingale assumption. According to the mentioned information hypotheses investors are splitted into informed and uninformed entities.

3.2.1. The mixture of distribution hypothesis

The MDH theorem states that price variations and trading volume are driven by the same information stream (Khemiri,2012). An important characteristic of this framework concern the speed of information flow on the market. Specifically, trading volumes respond to changes in the speed with which new information are absorbed in the market and respond to changes in the distribution of traders expectations about the implication of this new information (Carroll 2015). Early studies of the MDH by Clark (1973) focused exclusively on the short-run relation between volatilities and volume, which he describes as an imperfect clock. He proved that that relationship between asset price volatility and trading volume is a function of a common latent variable, that is, the rate at which information arrives in the market. To put it simply, new information in the market is an underlying variable that directs a contemporaneous or simultaneous change in the price and the trading volume, thus creating a positive relationship without causation between price and volume (Sangram K. Jena1, 2016).

3.2.2. The sequential arrival of information hypothesis

According to Copeland (1976), information dissemination occurs sequentially to investors, causing a series of intermediate equilibrium prices, and thus leading to a final informational equilibrium price when all the investors are informed. Therefore, this hypothesis is constructed to be more consistent since it not only advocates contemporaneous change but also talks about the lagged relation between volume and volatility. Therefore, SIAH assumes that new information is disseminated to market participants sequentially with each participant reacting to the information as it is received resulting in a partial price equilibrium. The final price equilibrium is reached when the new information is fully disseminated to all market participants. The major implication of the SIAH is that lagged trading volume can be used to predict volatility (Epps and Epps, 1976)

3.2.3. The dispersion of believes hypothesis theorem

"The Dispersion of Believes hypothesis discriminate, opposed to above mentioned Hypothesis, investors into Informed traders with homogeneous believes (hedgers), and uninformed ones with heterogeneous believes (speculators), and try clarify the exuberance volatility". The latter concept is a fallout of how speculators view available information on the market. Two assumptions need to be explained:

1. Speculators and rational entities who pursue interests of various nature to engage to derivatives commerce. Under other conditions, rational backer trade in derivatives to mitigate market hazard, while speculators are attracted by the opportunity profit on their investments.

2. Kyle in 1985 defined that "speculators activities have to be significant relative to hedgers to affect the pricing process. An appropriate measure for this is the market depth, which is the order flow required moving prices by one unit".

3.2.4. The Effect of Noise Traders theorem

According to Black (1986), the noise traders are a type of investor who do not have access to inside information and irrationally act on noise. Fama and Friedman (1965) demonstrated that this "irrational investor affect the formation process of assets prices since, if noise traders today are pessimistic about the expectation of an asset movement on the market, and have driven down its price, an arbitrageur, buying this asset, must recognize that in the near future noise traders might become even more pessimistic and drive the price down even further. Conversely, an arbitrageur, selling an asset short when bullish noise traders have driven its price up, must remember that noise traders might become even more bullish tomorrow, and therefore take a position that accounts for the risk of a further price rise when he has to buy back the stock". Because of the unpredictability of noise traders future opinions, prices can diverge significantly from fundamental values even when there is no fundamental risk. Noise traders thus create their own space (De Long, 1989). Because noise trader risk limits the effectiveness of arbitrage, prices can become excessively volatile. If noise traders opinions follow a stationary process, there is a mean-reverting component in stock returns.

3.3. Literature review

Previous studies tried to identify a relationship between volatility and volume of trading activity of derivatives in the financial markets, focusing mainly on the dynamics of trades as the volatility changes over time. Karpoff (1987) noted that studies based on daily basis sets found a positive correlation between price volatility and the volume of trading in the stock and futures markets. In one of the few researches that consider the above analysis on a monthly basis, Martell and Wolf (1987) showed that volatility is the main variable explaining the monthly turnover in the markets of futures, although other macroeconomic factors also has to be incorporated in the model, such as interest rates and inflation. Rahman (2001) investigated the impact of futures trading on the volatility of the Dow Jones Industrial Average (DJIA) and its constituents . The study was developed through a simple GARCH (1, 1) model to estimate the conditional volatility before and after the introduction of future derivatives. Kim (2004) examined the relationship between the trading activities of the South Korea Stock Price Index 200 and its underlying stock market volatility, introducing openinterests, average daily volume and market prices; derivatives volume in linear derivatives increased the stock market standard deviation, while the latter in negatively correlated to open interest. Park T. (1999), investigated the affinity between option trding volume and a sample of 45 companies with the most actively traded equity options on the CBOE; his study has brought to light a high degree of integration between them .In particular, unexpected options trading activity contributes to increase volatility of the underlying equity returns and it this is consistent to the controversy that trading in the non-linear derivatives market does not systematically lead destabilization in the market. Chiang and Wang (2002) tested the introduction of futures on Taiwan spot index volatility. They applied an asymmetric timevarying GJR volatility model, in order to take into account the leverage effect, as discussed earlier. Their empirical results showed that the trading of futures on the Taiwan Index has do not diminish spot price volatility, while futures trading on MSCI Taiwan has no effects on price volatility. Kiymaz et Al. (2009) extended the empirical literature on the relation between the conditional volatility of stock returns and trading activity (volume and open interest), developing an empirical analysis in the rapidly developing emerging markets of Asia and Latin America. Focusing on the 30 most liquid stocks that constitute the Istanbul Stock Exchange (ISE) National-30 index, he found, through a specification of the GARCH family models, (the TGARCH) that conditional volatility is lower with the introduction of volume. Despite previous studies, effect of trading activity on volatility have been analyzed by separating activity toward expected and unexpected components and allowing them to have a isolated aftermath on market price variance. Some studies found out a negative relation between trading activity variables and the market price volatility. Bessimender and Segiun (1992), applying an estimation framework proposed by Schwert in 1990, observed that open interests and derivatives daily volume traded on S&P 500, are negatively correlated to equity volatility. These findings, consistent in all the eight financial markets they had investigated, led to the conclusion that trading activities improved liquidity expectations and rejected the destabilizing theory of price standard deviation. Similar to the previous theories, Pati (2008) examines the validity of the Mixture of distribution hypothesis to display time-to-maturity and trading volume as the sources of volatility in future on commodities. Through an ARMA-GJR-GARCH model, he found out that there exist a positive relation between volume, divided in unexpected variable and expected one, and volatility of futures prices. On the other hand, open interests seems to be negative related to volatility index movements. Similar conclusions has been developed by Victor Murinde (2001) focusing on the main six emerging market of Central and Eastern Europe. According to the above literature and many other studies, it is possible to state that the effect of trading activities is likewise enigmatic, since copious searches, based on the same reference period and on the same stock index, pre-post the introduction of derivatives trading activities, lead to different results. It means that these results are strictly related to each methodology and econometric model used in the analysis

3.4. Volatility before and after the introduction of trading activities

Many studies have investigated the impact of derivatives contracts on spot price volatility by comparing the return of the spot market prices before and after the introduction of these contracts. This particular issue is quite controversial since opposite results have been found out in various markets. One of the first paper proposed by Antoniou and Holmes (1995) examined the effect of trading in the FTSE-100 Stock Index Futures on daily basis on the volatility of the market. To examine relationship between information and volatility an Integrated GARCH specification had been adopted. The results suggest that the nature of volatility has not changed post-futures, but exhibit persistence in variance and volatility clustering phenomena. The results about price movements, implies that the introduction of derivatives activity had improved the speed and quality of information flowing to the spot market. In other words, the main cause of destabilization on the underlying spot market is caused by a high degree of leverage and the presence of speculative uniformed traders among market participants (Yilgor, Ayse, 2016). Shembagaraman (2003) explored the impact of derivative trading on volatility using aggregate data on stock index derivatives contracts traded on the Nifty Index. The empirical results of this investigation suggested no direct changes in the volatility of the underlying stock index, but the nature of volatility has changed after the introduction of futures on the market. He found no relationship between trading activity variables, such as volume and open interest in the futures market, and index volatility. Robanni and Bhuyan (2005), studied if spot market volatility and trading volume on the Dow Jones Industrial Average (DJIA); they found out, through a multivariate conditional volatility model, an increase in volatility, due to irrational investors trades. Reves (1996) uses an Exponential GARCH framework to scrutinize the impact of futures trading on the price volatility of French and Danish markets from August 1997 up to April 2005. His findings show that index futures trading has changed the distribution of stock returns in Denmark and France, leading to strong volatility persistence and asymmetry, especially after the introduction of trading activities. Hwang et Al. (2000) tried to answer the financial question whether derivative markets undermine asset markets. They examined the movements of fundamental volatility before and after the introduction of European options on the FTSE100 index (Hwang, 2000), comparing several Stochastic Volatility Models. They uncovered that the introduction of options trading activity on the FTSE 100 index do not destabilised both the underlying market and the existing derivative markets. According to Sorescu (2000) and Pok, Wee Ching and Sunil Poshakwale studies (2004), there exist a divergence between results gained though GARCH models and the ones resulting from the application of Stochastic Volatility models. In particular, the effect of futures trading on spot market volatility can be depicted by extraneous shifts events that affect variance and returns on the market.

4. Data and Econometric Models

4.1. Data Selection

The primary data used in this work are the daily closing prices of S&P500 and its constituents, and daily VIX levels and VIX Futures and Options tick information. The US Stock market index is composed by the 505 stocks traded on the American Stock market and it have documented a market capitalization of 23 billion \$ in 2018. The S&P 500 index is adequate to explore the effect of derivatives trading activities on the volatility of its constituents and to analyze the related impact on the CBOE index, due to the large use of derivatives on the index. Options and Futures are represented by Open interest and Volume. The selection of the sample starts from the most liquid 505 industries in the American Stock Exchange, which make up the S&P500 index. The selection of the companies is based on at least continuous five years of listed daily closing prices. Therefore, firms with less than five years of daily observations and the most illiquid ones will not be considered in my analysis. Some of the mentioned companies, subjected to mergers and acquisitions, are excluded because of the risk to obtain inconsistent results. Moreover, the sample is reduced, due to the number of trading days per year. This methodology ensures that the included companies have a consistent number of observations, which allow me to examine the statistical properties of the time series and to compare the trading activity among indices and markets. Liquidity parameter is also introduced in the selection process; specifically, trading-liquidity ratio has been considered, which is ratio between the actual number of days and the total number of days in the observation period for each asset. According to our assumptions, 9 companies are selected, ordered by market capitalization and to have a uniform proportion of industries in each sector.

Symbol	Security	GICS Sector	GICS Sub Industry	Founded
AAPL Apple Inc.		Information Technology Technology Hardware, Storage & Periph		1977
BAC	Bank of America Corp	Financials	Diversified Banks	1928
BA	Boeing Company	Industrials	Aerospace & Defense	1916
XOM	Exxon Mobil Corp.	Energy	Integrated Oil & Gas	1999
GE	General Electric	Industrials	Industrial Conglomerates	1892
INTC	Intel Corp.	Information Technology	Semiconductors	1968
JNJ	Johnson & Johnson	Health Care	Health Care Equipment	1886
MSFT	Microsoft Corp.	Information Technology	Systems Software	1975
PFE	Pfizer Inc.	Health Care	Pharmaceuticals	1849

Table 3: S&P 500 component stocks of my analysis - Thomson Reuters

4.2. Statistical properties of the Data

Statistical properties of financial time series have revealed a wealth of interesting stylized facts, which seem to be common to a wide variety of markets, financial products and time intervals. My investigation involves returns rather than prices, for two reasons: firstly, "for average investors, return of an asset is a complete and scale-free summary of the investment opportunity. Second, return series are easier to handle than price series because the former have more attractive statistical properties" (Tsay, 2005). Daily log returns and squared returns are considered for the S&P500 Index and its constituents, from June, 1999 to June, 2018, to highlight the spikes of the financial crisis. Since all the companies of the index exhibit same patterns, reflecting the same effects post-crisis, I just reported the plot of Apple stock while the others can be found in Appendix. Log returns are given by:



$$r_t = ln(\frac{P_t}{P_{t-1}}) \tag{4.1}$$

Fig. 7. Log returns S&P 500 (1999-2018) - R analysis



Fig. 8. Log returns Apple (1999-2018) - R analysis

4.2.1. Volatility clustering

Rama (2007) stated that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes". The analysis of the behaviour of returns, has shown that there are periods when variance tends to be low and other periods when it tends to remain high. Other studies such as Chou (1988) and Schwert (1989) have reported the empirical implications of this type of behaviour: shocks on volatility can impact on expected future volatility. Therefore, the existence of persistence increases the uncertainty of future investment. Let us define the persistency in volatility as follow, considering the expected value of the returns variance for N future periods:

$$\sigma_{t+N}^2 | t = E_t (r_{t+N} - \mu_{t+N})^2 \tag{4.2}$$

It is clear from the above equation that volatility forecasting depends on the set of information available. Statistically, the main consequence is the desertion of the hypothesis of a Gaussian distribution of returns in order to identify a model capable of describing the evolution of the conditional variance over time. Engle (2002) suggested a more formal measure of persistency, defined as the partial derivative from the conditioned variance on N periods with respect to the squared value assumed by the return, at time t.

$$\theta_{t+k}|t = \frac{\delta[\sigma_{t+N}^2|t]}{\delta r_t^2} \tag{4.3}$$

the above plots of log returns display some level of persistence, although it is no so evident; on the other hand, the plots of squared returns indicate persistency, which is a clear symptom of volatility clustering. It leads to two different findings. Firstly, the returns volatility is not constant over time. Secondly, the variance of time series roll out strong autocorrelation.



Fig. 9. Daily Squared log returns S&P500 (1999-2018) - R analysis



Fig. 10. Daily Squared log returns Apple (1999-2018) - R analysis

4.2.2. Leptokurtosis

Most of the statistical tests for symmetry are developed under the implicit null hypothesis of Gaussian (normal) distribution but many financial data exhibit fat tails, and therefore commonly used tests that assume symmetry are not valid to test the symmetry of leptokurtosis. It represents a measure of the 'taildness' of the probability distribution of a real-valued random variable. The kurtosis of any univariate normal distribution is equal to 3. It is widely used to compare the kurtosis of a distribution to this value. Positive or negative skewness indicate asymmetry in the series and less than or greater than 3 kurtosis coefficients suggest flatness and peakness, respectively, in the returns data. The most used procedure for the examination of the normality hypothesis is the Jarque Bera statistical test, which is based on the calculation of the difference between the symmetry and kurtosis indices of the observed series with respect to the values obtained from the normal distribution:

$$JB = (K^*)^2 + (S^*)^2 \sim N(0, 1)$$
(4.4)

$$S^* = \frac{S}{\sqrt{\frac{6}{T}}} \tag{4.5}$$

$$K^* = \frac{K - 3}{\sqrt{\frac{24}{T}}}$$
(4.6)

The Figures below show the distribution of the daily yields and descriptive statistics for S&P 500 and Apple stock, respectively. (In Appendix are reported distributions and descriptive statistics of all the considered sample)



Fig. 11. Distribution and Descriptive statistics of daily returns for S&P 500 - R analysis



Fig. 12. Distribution and Descriptive statistics of daily returns for Apple - R analysis

Figures 11-12 show the summary statistics of the S&P 500 returns and Apple. Means and volatilities are reported in annualized terms using 100 x returns. Skewness and kurtosis are estimated directly on the daily data and not scaled. The average daily return are 11.69% and 0.46%. The daily standard deviations are 1.72% and 0.50%, reflecting a high level of volatility in the market. The wide gap between the maximum 193.98 and minimum 0.93714 returns gives support to the high variability of price change in the market. Under the null hypothesis of normal distribution, J-B is 0. Thus, the Jarque-Bera values of 25.35 and 799.79 definitely rejects the normality hypothesis. Therefore, they deviated from normal distribution. The skewness coefficient of -0.03181 of SP 500 index is negatively skewed. Negative skewness implies that the distribution has a long left tail and a deviation from normality. The empirical distribution of the kurtosis is clearly not normal but peaked. Overall, the S&P500 return series do not conform to normal distribution but display negative skewness and leptokurtic distribution.

4.2.3. Stationarity

The main objective of the analysis of the historical series is to identify an appropriate stochastic process that has adaptable trajectories to the data, so that forecasts can be made. In order to make it, it is necessary to to restrict our attention to a class of stochastic processes that will allows:

• to univocally identify the process, that is, the model;

• to make inference on the moments of the same process, that is to say to obtain some correct and consistent estimates of the moments of the process.

A process is covariance-stationary or weakly stationary, if the main and the autocovariances, for various lags, vary over time. Time series tends to its mean over time. The speed of the mean reversion is strong-willed by the degree of autocovariances in the time series. According to the above reported plots, daily returns of S&P500 and Apple undulate around a mean value between 1999 and 2018. It was appropriate to examine unit root test to ensure stationarity. It was significant in order to avoid spurious results. The existence of a unit root is the null hypothesis for the test. According to our results returns series do not accept null hypothesis at the 1% significance level. Two different unit root tests have been developed: the Augmented Dickey Fuller and Phillips-Peron unit root tests. They are not statistically significant, indicating that they contain a unit root, and hence they are I (1).

4.3. Choice of the model

The GARCH (1,1), and the GJR-GARCH frameworks are used to capture the statistical properties of stock returns, volatility clustering, leptokurtosis and leverage effects, on the S&P 500 time series and VIX. Section 5 will focus on the investigation of the different components of the ARMA-GJR-GARCH, the chosen model to investigate the research proposal of this work. Firstly, the conditional mean equation can be explored from the autoregressive moving average (ARMA) processes while the conditional variance component is obtained by the autoregressive conditional heteroscedasticity (GARCH) processes. Then a dummy variable is interpolated in order to capture the size of prospective impact of positive or negative shocks on volatility, in other words the asymmetric leverage effect. Then, the model introduces an indicator function that assumes the value 0, when the conditional variance is positive and the value 1 if negative. In the latter model, open interest and volume are added in order to capture the effect of each financial instrument on the S&P500 index.

4.3.1. Arma - Garch models

Given the Information Set:

$$I_{t-1} = [\epsilon_{t-1}, \epsilon_{t-2}...\epsilon_{t-q}]$$

$$(4.7)$$

the disturbance term of a linear regression model follows an ARCH process if the following conditions are respected:

1. The Expected average of ϵ_t conditional to the Information Set, is equal to 0 over time.

$$E(\epsilon_t | I_{t-1}) = 0 \tag{4.8}$$

2. The idiosyncratic component, or innovation ϵ_t , is given by the following Equation:

$$\epsilon_t = u_t H_t^{1/2} \tag{4.9}$$

Where $u_t \sim N(0, 1)$ is the standardized process. From the above equation, it is clear that the conditional variance can fluctuate over time. This process is the formal instrument that relates the dynamics of the (conditional) of the volatility with the concept of leptokurtosis (not conditional). According to this contest, the following relation is validated:

$$\frac{E(\epsilon_t^4)}{[E(\epsilon_t^2)]^2} \ge 3 \tag{4.10}$$

Therefore, the value of the kurtosis index will be:

$$\frac{E(\epsilon_t^4)}{[E(\epsilon_t^2)^2]^2} = 3 + 3\frac{E(h_t^2) - 3E(h_t)^2}{[E(\epsilon_t^2)]^2} =$$
(4.11)

Introduced by Engle in 1982, the ARCH model specifies the conditional variance as a linear function of the squares of the past values of the innovations.

$$h_t = \omega + \sum_{i=1}^N \alpha_i \epsilon_{t-i}^2 \tag{4.12}$$

Where $\omega \geq 0$, and $\alpha_i \geq 0$ for i = 1, 2...N are the parameters to be estimated. Therefore, the ARCH is a process with zero mean and variance, which is constant over time, conditional linearly dependent on the squares of innovations. "This process is able to capture the phenomenon of fluctuations in the historical series related to the returns of the stocks, and then it can explain the volatility clustering" (Nelson, 1991). If the process is stationary as in our analysis, the unconditional variance of the innovation assumes the following value:

$$var(\epsilon_t) = \frac{\omega}{1 - \sum_{i=01}^{N} \alpha_i}$$
(4.13)

4.3.2. Garch process

Introduced by Bollerslev (1986), the Generalized ARCH (GARCH) represents a suitable tool to analyse the persistence of movements of the volatility without having to estimate the

high number of parameters present in the polynomial. Based on the information set I_{t-1} , the equation of a generic model GARCH (p,q) define the conditional variance h_t as follows:

$$h_{t} = \omega + \sum_{i=1}^{q} \alpha_{i} \epsilon_{t-i}^{2} + \sum_{i=1}^{p} \beta_{j} \sigma_{t-j}^{2}$$
(4.14)

Where $\omega \ge 0$, and $\alpha_i \ge 0$ for i = 1, 2...q and $\beta_j > 0$ for i = 1, 2...p According to Bollerslev (1986), the GARCH process is covariance-stationary if the following relation hold:

$$1 - \sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_j < 1 \tag{4.15}$$

In addition, the unconditional variance of the innovation turns out to be:

$$var(\epsilon_t) = \frac{\omega}{1 - \sum_{i=1}^q \alpha_i + \sum_{i=1}^p \beta_j}$$
(4.16)

The most used GARCH process is the GARCH (1,1), with q = p = 1, with the conditions $\omega \ge 0, \alpha_i \ge 0$, for i = 1, 2...q and $\beta_j > 0$ for i = 1, 2...p. The latter is enough to capture the existence of volatility clustering in the data (Brook and Burke, 2003).

4.3.3. Garch Specification

Glosten, Jangathann and Runkle (1993) incorporated the leverage effect into the GARCH model, through an indicator function, in order to capture the effect of positive and negative shocks on the conditional variance. The model is defined as follow:

$$\sigma_t^2 = \alpha_0 + \alpha_1 + \alpha_1^- I(y_{t-1} < 0)\epsilon_{t-i}^2 + \beta_1 h_{t-1}$$
(4.17)

With $\alpha_0 > 0$, and $\alpha_1, \beta_1 geq 0$ and $\alpha_1 > 0$

The indicator function, or dummy variable, $I(y_{t-1} < 0)$ can vary according to the value of the past return y in t - 1; in particular, it assumes a value equal to 1 in case of positive returns and 0 otherwise. The following conditional variance Equation in the GJR GARCH framework, can allow us to investigate the impact of derivatives trading on the volatility of SP 500 and its constituents, and the influence of these results on the volatility of VIX index:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \alpha_1 + \alpha_1^- I(y_{t-1} < 0) \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_j \beta_{t-j}^2$$
(4.18)

In this framework, continuously compounded returns represent the dependent variables, which are equal to the natural logarithms of return in t over the one in t-1. It is reasonable to introduce two explanatory variables, which will be described in the next section, in our GARCH specification for the conditional mean (first moment) and the conditional variance (second moment). Therefore, the previous Equation, given the Information set, become as follow:

$$E(y_t|I_{t-1} = \omega + \sum_{i=1}^q \alpha_q y_t - q + \sum_{i=1}^p \beta_j \epsilon_{t-p}^2$$
(4.19)

whith , $\epsilon_t \sim N(o, 1)$,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \alpha_1 + \alpha_1^{-I} (y_{t-1} < 0) \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_j \beta_{t-j}^2 + \delta_1 OI^{unexpected_t} + \delta_2 OI^{expected_t} + \delta_3 VOL^{unexpected_t} + \delta_4 VOL^{expected_t}$$

$$(4.20)$$

4.4. Volume and Open interest

In the previous sections, GARCH specifications are examined in order to identify the suitable model, which can be used to describe the effect on trading on the volatilitys indices. Trading activities can be represented by two variables: open interest and average daily volume. Open Interest is defined as the number of outstanding contracts at the end of any trading day that has not been settled. They reflect trading activity mainly used by market participants who pursue hedging strategies. Higher the open interest, higher the amount of money that are flown on the market. (KP. Paresh - 2017). As Ferris, Park H. and Park K. stated, "Open interest supplements the information provided by trading volume. It can proxy the potential for a price change while trading volume assesses the strength of a price level. The change in the level of open interest can also measure the direction of capital owns relative to that contract". Volume shows the amount of trading activity in a market in each specific trading day. An increase of daily volume means that more contracts were traded respect the previous day. In addition to this, volume gives a measure of speculative activities. According to Pati (2008), these two variables are splitted in expected component and unexpected one to identify which one of the two trading activities, based on hedging or speculative strategy, have a higher impact on the spot volatility of S&P 500 index. Total volume and Total open interest at time t, defined as follow, are the explanatory variables of the ARMA-GARCH specification:

$$VOL_{t}(t) = \lambda_0 + \lambda_1 VOL_{t-1} + \epsilon_{t-2} + \epsilon_t \tag{4.21}$$

$$OI_{t}(t) = \lambda_0 + \lambda_1 OI_{t-1} + \epsilon_{t-2} + \epsilon_t \tag{4.22}$$

The unexpected values are given by the difference between the total values and the expected volume and open interest.

5. Empirical Test and Results

5.1. Box-Jenkins framework

The Box-Jenkins framework is represented by a set of stages aimed to identify the statistical model to describe the historical series. by investigating the relationships between observations over time. The phase specifications are the following:

- 1. Identification of the parameter p and q of the ARMA process;
- 2. Estimation of the parameters;
- 3. Verification or diagnostic checking

5.1.1. Identification of parameters

The identification phase consists on the analysis of the estimated ACF and PACF of the historical series, in order to find the lags p and q of the ARMA-GJR-GARCH process. In particular. the construction of the ARMA model is based on the assumption that the process is stationary.

The autocorrelation function is given by:

$$p(h) = \frac{\gamma(h)}{\gamma(0)} = \frac{E(y_t - \mu)(y_{t-h} - \mu)}{\sqrt{E(y_t - \mu)^2}\sqrt{E(y_{t-h} - \mu)^2}}$$
(5.1)

Where p(h) is the k^{th} lag autocorrelation, y is the value of y at row t and x_t is the mean value of x.

The partial autocorrelation coefficient φ_{qq} measures the linear correlation between y_t and y_{t-k} . It is defined as follow:

$$\varphi_{qq} = \frac{p(h) - \sum_{j=1}^{k} -1\varphi_{q-1,j}p(h-j)}{1 - \sum_{j=1}^{k} -1\varphi_{q-1,j}p(j)}$$
(5.2)

The above formula is significant due to its attitude to relate the PACF and the changes in ACF, over time. The following plots report the correlograms, which represent the autocorrelation functions for S&p500 and Apple, respect to different length lags.



Fig. 13. ACF correlograms for returns of S&P 500 and APPLE stock - R analysis



Fig. 14. PACF correlograms for returns of S&P 500 and APPLE stock - R analysis

The horizontal bands above and below the zero value indicate the bounds of confidence interval, that is 5% in my analysis, within the correlations are statistically no different from zero. The sample ACF of the differenced series decays more quickly. The PACF cuts off after lag 1. This behaviour could be consistent with a first-degree autoregressive (AR(1)) process. The PACF are not able to capture the lags of the ARMA-GJR-GARCH (p.q) model. Therefore, frameworks that are more appropriate are applied to identify the order of the AR and MA terms. The assessment of the goodness of adaptability of a statistical model must take into account the long-established fact that the relation between estimated values and the corresponding observed values increases significantly, as the number of parameters goes up, without an increase in the predictive capacity of the model (Ljung, 1978). To embank this feature a number of criteria have been proposed which have as their common denominator the sum of two components, referable to distinct aspects of adaptation. The first part is the one derived from the sum of the squared observed residuals (almost in logarithmic scale), which concludes the identification procedure and model estimation. This component decreases with the increase of the goodness of adaptation. The second term is a quantity that is instead increasing with the number of unknown parameters estimated overall. The difference between the many measures is in the way these test balance the impact of the two addendums. Introduced by Akaike in 1973, the Akaike information criterion(AIC) is anestimator of a statistical model taking into account both the goodness of adaptation and the complexity of a given time series sample 8 (Tsay, 2010). AIC is computed for each of the approximating models in the set and they are ranked from the highest to the worst one. The AIC function is given by:

$$AIC = \frac{2k}{T} - \frac{2ln(L)}{T}$$
(5.3)

Where is the likelihood function and is the number of parameters to be estimated. The second component of the above function represent the goodness of fit to which the first component. representing the penalty, is added.

Similar to the above model, the Bayesian Information Criteria (BIC) is defined as follow:

$$BIC = kln(T) - 2ln(L)T$$
(5.4)

The first component represent the penalty term, which is double in the AIC model. It reflect the complexity of the model that is the number of parameters in the model. Moreover. it is useful to estimate the efficiency due to its attitude to predict the data. Several values of lags have been tested on index level (S&P 500 index) and asset level in the range 0-6. In Table below are reported the different combinations of p and q where the lowest AIC value is selected for each of them. For S&P 500, the suggested model from the autocorrelations functions is the GJR-GARCH (1,1).

	Asset	\mathbf{AR}	MA	Final Model
S&P500	S&P500			GJR(1.1)
Apple		2	1	GJR(1.1)
J&J		2	1	GJR(1.1)
Bank of Am	nerica	1	1	GJR(1.1)
Boeing		1	1	GJR(1.1)
General Ele	ctric	0	0	GJR(1.1)
Intel		0	0	GJR(1.1)
Microsoft	1	0	GJR(1.1)	
Pfizer	1	1	GJR(1.1)	
Exxon	1	0	GJR(1.1)	

Table 4: Lag process specifications - - R analysis

The models present different lags in the ARMA processes, which define the mean equation of the returns time series while the variance equation is described by the GJR term that is relevant to capture the effect of positive or negative shocks over time.

5.1.2. Estimation of the parameters

The estimation theory is divided in punctual, in case of evaluation of the value of a parameter, and in interval or interval estimation, when it assigns a range of values that includes the searched parameter with a prefixed level of confidence. The two most commonly used methods for the precise estimation of parameters are the maximum likelihood estimation and the ordinary least squared (OLS) one. The latter cannot be applied due to non-linearity features of parameter in the GJR specification. The Maximum Likelihood estimation, introduced by Fisher (1922), claim that the preferred values of the parameters of a likelihood function are those that maximize the probability to obtain the observed data; the estimations of the parameters with the maximum likelihood method are the solutions of the following system:

$$\begin{cases} \frac{dL}{d\lambda_1} = 0\\ \frac{dL}{d\lambda_2} = 0\\ \frac{dL}{d\lambda_m} = 0 \end{cases}$$
(5.5)

Where L is the likelihood function. It is defined as follow:

$$L = ln(L) = ln \prod_{i=1}^{N} f(x_i | \Delta) = \sum_{i=1}^{N} f(x_i | \Delta)$$
(5.6)

When the random variables are normally distributed, the OLS and the MLE methods are equivalent that is they lead to the same estimation of unknown parameters.

5.1.3. Verification or Diagnostic checking

The third stage of the Ljung-Box framework aims to analyze how the model fits the returns series and look for any track of an inappropriate fit using formal hypothesis tests. In the empirical analysis Breusch-Godfreys serial correlation LM test and Ljung-Box Q test are developed on standardized residuals to investigate the models ability to fit the data. The Breusch-Godfrey test (1978-1979) is useful to determine the existence of some serial dependence of the variables employee, within the dynamic linear models. The test is based on the following theory: if there is an autocorrelation not "captured" by the model, then the residuals should follow an AR process of order q > 0(Box, Pierce, 1970); let us consider the regression:

$$\epsilon_t = \alpha_0 + \alpha_1 \epsilon_{t-1} + \dots + \alpha_q \epsilon_{t-q} + v_t \tag{5.7}$$

in which the dependent variable is given by the historical series of the residues ϵ_t while the list of regressors is the same as the starting model to which all the delays of the residuals have been added up to the maximum order q. The GB test is obtained through an asymptotic approximation given by:

$$BG = TR \sim X_a^2 \tag{5.8}$$

Where the term R^2 is the auxiliary regression equation and T represent the sample magnitude. The limit of this test is applicable only in the case of linear dynamic models. The structure of the hypotheses to assess the presence of autocorrelation through the use of the BG test. According to our results on index and asset level, shown in Table 5, S&P500 with a BG test value of 3.66 and a corresponding probability of 59.9% percent, accept the null hypothesis; specifically, error terms do not present any autocorrelation. On the other hand, Apple displayed series correlation in the residuals, thus the white noise hypothesis is rejected. Based on the above mentioned information criteria, the model with the lowest one has been selected, which is found out to be for Apple stock, an AR(2)MA(1) process.

	Al	ople	S &2	P500
TEST	T-stat P-Value		T-stat	P-Value
LB(5)	6.5541	0.256	3.6626	0.599
LB(10)	11.527	0.318	9.7797	0.46
LB(15)	16.932	0.323	12.957	0.606
LB(20)	20.371	0.435	23.976	0.243
LB(25)	24.305	0.502	25.636	0.427

Table 5: Breusch-Godfreys serial correlation test - R analysis

The Ljung-Box Q (LB) test, similar to BG test is developed in the ARIMA family models as a diagnostic checking to assess the goodness of adaptation. In our case, it is applied to the squared standardized residuals $\epsilon_t^2 \sigma_t^2$ adopting p_k as the function of sample autocorrelation of observed residuals. The test statistic is defined as follow:

$$LB = n(n-2)\sum_{k=1}^{q} \frac{p_k^2}{n-k}$$
(5.9)

Where n is the sample size and q is the number of examined autocorrelations.

According to SAS Institute (2017), "the properties of Q-Q plots make it a useful tool of how well a specified theoretical distribution fits a set of measurements; in particular:

- If the quartiles of the theoretical and data distributions agree, the plotted points fall on or near the line y = x
- If the theoretical and data distributions differ only in their location or scale, the points on the plot fall on or near the line y = ax + b. The slope a and intercept b are visual estimates of the scale and location parameters of the theoretical distribution.

Q-Q plots are more convenient than probability plots for graphical estimation of the location and scale parameters because the x-axis of a Q-Q plot is scaled linearly. On the other hand, probability plots are more convenient for estimating percentiles or probabilities." (SAS Institute. 2017).

The extreme observations, which can be seen on the fat tails in both ends of the distribution curve indicate symmetric long tails. Therefore, we can conclude that the standardized residuals from our estimated ARMA(p,q)-GJR-GARCH(1.1) model are not normally distributed.

5.2. Empirical results

The impact of trading activity on the spot volatility of S&P 500 and its constituents has been examined through the application of GARCH family models. As mentioned above, on index level, the GARCH and the GJR-GARCH models have been conducted, where the latter introduce the Indicator function to capture the leverage effect. In Table below are reported our estimations on index level.

	GARCH(1,1)		GJR-GARCH(1.1)	
	Values	P-Stat	Values	P-Stat
Intercept	0.01	0.00	0.00	0.00
Arch1	0.09	0.00	-0.02	0.05
Garch1	0.90	0.00	0.91	0.00
GJR	-	-	0.18	0.00
Test LM	0.37	0.54	-	-
LB $Q(5)$	6.12	0.10	3.01	0.56

Table 6: GARCH and GJR-GARCH estimations for S&P 500 - R analysis

The value is equal to 0.099 while the $GARCH(\beta)$ is high and close to one (0.901). Persistence in volatility is suggested by the respect of the following relation:

$$\alpha_1 + \beta_1 < 1 \tag{5.10}$$

In our analysis, the above constraint is respected, justifying that current volatility of daily returns can be related to past volatility that turn have persistency over time. The presence of outliers in daily return time series is described by ARCH coefficients, which are found out to be significant in both the specification. Looking at the GJR-GARCH specification, asymmetry in our data is apprehended by the introduction of the indicator function, which imply that positive or negative information flows on the market can have a larger or more limited effect on volatility. From the results obtained we can concluded that returns series are leptokurtic and have a slight positive asymmetry. Moreover, for both the Chi squared test statistics, the values of the probability values are higher than the critical values, as is evident from the p-values. Therefore the hypothesis of normal distribution is rejected. In addition to this, my analysis of the return series have shown the presence of another characteristic typical of the financial series: the so-called clustering effect. I.e. the aggregation in groups of errors according to their order of magnitude (large errors tend to be followed by equally large errors and small errors by equally small errors); therefore the volatility of squared returns is auto correlated.

The second test has been conducted overall period on index level after the introduction of expected and unexpected components, which represent the explanatory variables of the

	ARMA-GJR-GARCH		
	Estimate	P-value	
Intercept	0.00	0.00	
Arch1	0.09	0.00	
Garch1	0.901	0.00	
GJR	0.05	0.34	
Unexp_OI	0.00	0.25	
Unexp_VOL	0.00	0.03	
Exp_OI	0.00	0.00	
Exp_Vol	0.00	0.00	
Test LM	7.50	0.01	
LB-Q(5)	365.21	0.00	

regression (open interest and volume). The regression, which assumed the Formula 4.20 form, give us the followinf results:

Table 7: ARMA-GJR-GARCH estimations with expected and unexpected OI and VOL - R analysis

According to the above results, the effect is found out to be significant for the whole time interval. The sum between ARCH effect and GARCH one, indicate persistency in the data suggesting that the conditional volatility in vary, related to past news of the market. The estimates of open interest and volume, decomposed into expected and unexpected nature, suggest that these trading activities cannot be fixed to explain the movements of the underlying spot volatility, except for the unexpected open interest, which has been significant up to 25% confidence interval. The sum of the expected and unexpected average daily volume is close to 0; it means that speculators trading activities do not destabilize the underlying spot market volatility. Our results do not hold, as in the literature, an increasing/decreasing effect from unexpected shocks from derivatives trading activities on the underlying volatility, so that we cannot establish if the shocks from hedgers dominate those from speculators. This results support the mixture distribution Hypothesis (MDH) and violate the dispersion of beliefs hypothesis, for which speculators create excess volatility and are the dominant investors when new information are incorporated by the market. The insignificance of the above regression variables suggest that the model could give different and more relevant results, if intraday data series are considered to predict the condition volatility from linear and nonlinear trading activities. Our results could be also affected by the geopolitical indirect effects of the financial crisis of 2008 on the American Stock Exchange. The third test has been conducted on the whole period on asset level where, once again, the expected and unexpected components represent the explanatory variables of the already mentioned regression. In the following Table are reported the results on asset level.

Asset	intercept	Aarch1	Garch1	Leverage	Unexp_OI	Unexp_VOL	Exp_OI	Exp_VOL	LM Test	LB(5)
GSPC	0.00	0.08	0.80	0.05	0.00	0.00	0.00	0.00	6,667,698.00	2.67
APPLE	0.00	0.02	0.87	0.17	0.00	0.00	0.00	0.00	0.43	61,222 .00
BAM	0.00	0.04	0.90	0.07	0.00	0.00	0.00	0.00	1.33	11,135.00
Boeing	0.00	0.15	0.60	0.05	0.00	0.00	0.00	0.00	311,628 .00	3.64
EXXON	0.00	0.02	0.86	0.13	0.00	0.00	0.00	0.00	0.02	3.03
GE	0.00	0.05	0.83	0.17	0.00	0.00	0.00	0.00	0.00	6.78
JnJ	0.00	0.03	0.85	0.14	0.00	0.00	0.00	0.00	0.04	2.29
INTEL	0.00	0.15	0.60	0.05	0.00	0.00	0.00	0.00	2.04	10,233 .00
MSFT	0.00	0.10	0.38	0.27	0.00	0.00	0.00	0.00	1.86	90,922 .00
PFIZER	0.00	0.04	0.90	0.08	0.00	0.00	0.00	0.00	0.02	2.29

Table 8: GJR-GARCH estimations with expected and unexpected OI and VOL on asset level - R analysis

Our findings reveal a total absence of relation between trading activities, represented by open interest and average daily volume, and the volatility of the S&P 500 and its ingredients. The same analysis has been conducted on VIX time series, and the results, although statistically significant, were not different.

	GJR-GARCH
CONSTANT	0.04
ARCH(1)	0.18
GARCH(1)	0.78
GJR	(0.23)
EXP OI	0.00
EXP VOL	0.00
UNEXP OI	0.00
UNEXP VOL	0.00
LM	0.03
LB $Q(5)$	1.49

Table 9: VIX GJR-GARCH estimations with expected and unexpected OI and VOL - R analysis

6. Conslusion

My work investigated the impact of linear and non-linear derivatives trading activities on the S&P500 and its selected constituents, and the VIX Index in the post-crisis age, in order to not incorporate the anomalies in the data due to the financial crisis in the US Stock Market (2007-2008). The analysis was carried out starting from a Theoretical framework, to arrive to answer the study in the most consistent, accurate and empirical possible way. Firstly, I focused on the introduction of the S&P500 VIX Index, which is also is known as the "fear index". In order to state if derivatives trading alter the volatility of the underlying asset and the VIX Index, the GJR-GARCH specification has been established both on a index and asset level. The just mentioned econometric model, useful to solve our research question, has been able to capture the ARCH effect of each time series; it assumed a significant LB-Q test value, which indicated a adequate fit-integrity. It was found out to be suitable due to volatility clustering, leptokurtosis and mean-reversion of our squared return time series. Then a dummy variable is interpolated in order to capture the size of prospective impact of positive or negative shocks on volatility, that is the so-called asymmetric leverage effect. The empirical results on index level suggest that derivatives trades do not have any significant effect on the volatility of the S&P500 and VIX Indices Although financial literature proposes an opposite conclusion, according to which speculator have a positive and greater effect on the underlying spot volatility than hedgers, whose shares have a balanced aftermath on the market, the same results do not hold in the American derivatives market. The analysis conducted on S&P 500 constituents is found out to be consistent with the index level results. In particular, unexpected volume and open interest, settled as explanatory variables of our GARCH specification model, do not influence the volatility of the stock Index. The main result of this work have been to have highlighted and represented with appropriate dynamic models the relationship between information and volume, and volatility of an implied volatility index that do not appear to be adequately taken into account in previous literature. It highlights a neglected aspect in the literature, according to which investor concern its total attention on the market variance. The ability to predict variations in volumes and new information arrivals assumed a relevant value; in fact, as well volumes exchanged and their changes as market news are important economic indicators of the investment sector. I would like to suggest to future researchers to undertake the latter analysis, in order to compare, before and after the financial crisis, the mutual effect between the S&P 500, VIX and derivatives activities. One more interesting angle of investigation could focus on the application of my analysis in other regulated and more liquid markets, different from the American one, and to consider other volatility indices,

that are becoming an increasingly common place for pursuing new hedging and speculative strategies. The adopted econometric model was a fundamental part of my thesis. It has been demonstrated that, in relation to the model chosen, the empirical results may differ. Despite their complexity of calculation, stochastic volatility models could be employed in future studies to investigate the effect of derivatives trading activities on the market volatility.

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8. Appendix



Fig. 15. Open Interest of VIX Future and Options (2004-2017) - Thomson Reuters







Fig. 16. Distribution and Descriptive statistics of daily returns for S&P500 - R analysis



Fig. 17. Distribution and Descriptive statistics of daily returns for Apple - R analysis



Fig. 18. Distribution and Descriptive statistics of daily returns for J&J - R analysis





Fig. 19. Distribution and Descriptive statistics of daily returns for Boeing - R analysis



Fig. 20. Distribution and Descriptive statistics of daily returns for Bank of America - R analysis



Fig. 21. Distribution and Descriptive statistics of daily returns for Exxon - R analysis







Fig. 22. Distribution and Descriptive statistics of daily returns for Intel - R analysis



Fig. 23. Distribution and Descriptive statistics of daily returns for General Electric - R analysis



Fig. 24. Distribution and Descriptive statistics of daily returns for Pfizer - R analysis