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Spectral properties of elliptic operators in singular settings and applications

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Abstract

The present thesis is focused on the investigation of the spectral properties of the linear elliptic operators in the presence of singularities. It is divided into three chapters.

In the first chapter, we consider geometric singularities. We construct the heat kernel on surfaces with corners for Dirichlet, Neumann, and Robin boundary conditions as well as mixed problems. We compute the short time asymptotic expansion of the heat trace and apply this expansion to demonstrate a collection of results showing that corners are spectral invariants.

The second chapter deals with linear elliptic second-order partial differential operators with bounded real-valued measurable coefficients. We emphasize that no smoothness assumptions are made on the coefficients. In the first half of this chapter, we study a time-harmonic electromagnetic and acoustic waveguide, modeled by an infinite cylinder with a non-smooth cross section. We introduce an infinitesimal generator for the wave evolution along the cylinder and prove estimates of the functional calculi of these first order non-self adjoint differential operators with non-smooth coefficients. Applying our new functional calculus, we obtain a one-to-one correspondence between polynomially bounded time-harmonic waves and functions in appropriate spectral subspaces. In the second half, we derive Weyl's law for the weighted Laplace equation on Riemannian manifolds with rough metric. Key ingredients in the proofs were demonstrated by Birman and Solomyak nearly fifty years ago in their seminal work on eigenvalue asymptotics.

In the last chapter, we investigate spectral properties of Sturm-Liouville operators with singular potentials. We consider different types of singularities. We find asymptotic formulas for the eigenvalues of the Sturm-Liouville operator on the finite interval, with potentials having a strong negative singularity at one endpoint. We establish that, unlike the case of an infinite interval, the asymptotics for positive eigenvalues does not depend on the potential, and it is the same as in the regular case. The asymptotics of the negative eigenvalues may depend on the potential quite strongly. Next, we study the perturbation of the generalized anharmonic oscillator. We consider a piecewise Hölder continuous perturbation and investigate how the Hölder constant can affect the eigenvalues. Finally, for the the Sturm-Liouville operator with δ-interactions, two-sided estimates of the distribution function of the eigenvalues and a criterion for the discreteness of the spectrum in terms of the Otelbaev function are obtained.

Keywords: Elliptic operators, spectrum, heat kernel, Sturm-Liouville operators, asymptotic of eigenvalues.