UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Graduate School

ENHANCING MOMENTUM PROFITS THROUGH VOLATILITY TIMING AND COST MITIGATION TECHNIQUES

Nguyen Cao

Natalia Vdovina

June 2019

A thesis submitted for a degree of Master of Science in Finance

Supervisor: Adam Farago

Abstract

Despite the high expected returns of the momentum strategy, there are two main problems associated with it: (i) infrequent but severe losses known as momentum crashes, and (ii) high transaction costs. In this paper, we address the first problem with volatility timing strategies developed by Daniel and Moskowitz (2016) and Moreira and Muir (2017). Our results prove that not only are momentum crashes alleviated but returns on the WML (winner-minus-loser) portfolios formed with these strategies also go up remarkably compared to the simple buy-and-hold ones. However, like the simple momentum strategy, volatility timing strategies suffer from large trading costs. We, therefore, propose combining these momentum strategies with the buy/hold spread cost-mitigation strategy formed by Novy-Marx and Velikov (2015). The outcome is a noticeable reduction in turnover and transaction costs, together with an improvement in the portfolio returns.

Keywords: momentum, momentum strategy, momentum crash, volatility, transaction costs, turnover, return, volatility adjusted momentum, volatility timing

Acknowledgements

First and foremost, we would like to express our sincere gratitude to our supervisor, Adam Farago, Ph.D., for his inspiration, guidance, and enormous support. He not only gave us valuable advice on analyzing research data with MATLAB but also devoted his time and effort to assist us in overcoming difficulties arising from our master thesis project.

Second, we would like to express our appreciation to the Graduate School and the Centre for Finance for making it possible for us to pursue our Master in Finance and complete our master thesis. Coming from Russia (Natalia) and Vietnam (Nguyen) to Sweden and having the opportunity to study here was, probably, one of the most important and life changing experiences for both of us.

Last but not least, we would like to convey our deepest gratitude to our families and friends for their endless encouragement during this long journey.

Table of Contents

List of Figures

List of Tables

1. Introduction

Although the simple buy-and-hold momentum strategy – buying stocks that performed well in the past and selling stocks that performed poorly – produces positive returns on average, during periods of high market volatility, such as the 2008-2009 financial crisis, it can experience drastic losses, referred to as momentum crashes. In this paper, we investigate whether two volatility timing strategies developed by Daniel and Moskowitz (2016) and Moreira and Muir (2017) can address the momentum crash problem as claimed in these studies. Both Daniel and Moskowitz (2016)[1] and Moreira and Muir (2017)[2] share the idea that momentum crashes can be avoided by adjusting the momentum portfolio's exposure to market risk. In particular, the weights of the simple momentum portfolio can be scaled in order to keep its volatility unchanged.

Over and above, one prominent problem of using either the simple buy-and-hold momentum strategy or the volatility timing strategies is that they have high turnover due to the fact that the portfolios are rebalanced frequently (in the beginning of each month) based on past returns, which is a highly volatile indicator. Thus, there is a possibility that the big volume of trading costs incurred from implementing these momentum strategies can wipe a significant percentage off the portfolio gains. While Moreira and Muir (2017) did a robustness test on their strategy by checking the significance of the results after involving different (but generally low) levels of transaction costs, Daniel and Moskowitz (2016) did not even take transaction costs into account. Therefore, the feasibility of these volatility timing strategies, especially that of DM strategy, has come into question. In this study, we tackle this issue by taking the transaction costs into consideration. Notably, we investigate not only the effect of transaction costs on the portfolio performance but also the impact of their magnitude on the practicability of the above-mentioned volatility timing strategies. To do so, we include different trading cost components which are bid-ask spread estimates measures according to Abdi and Ranaldo (2017), commission fees and extra fees imposed on short-selling positions as recommended by Do et al. (2012).

The fact that trading costs have a detrimental impact on the returns on the momentum portfolios raises another question about how to alleviate such negative effects while still avoiding momentum

[1] Hereafter Daniel and Moskowitz (2016) strategy will be denoted DM
[2] Hereafter Moreira and Muir (2017) strategy will be denoted MM

crashes. Among the many cost-mitigation solutions proposed, the three strategies developed by Novy-Marx and Velikov (2015) are easy to understand and simple to implement: trading only stocks that have low transaction costs, rebalancing only a certain number of stocks in the portfolio, and using a buy/hold rule when rebalancing the portfolio. The idea of the buy/hold rule is that investors buy winner stocks if they are in the "buy" range but do not sell them until they fall out of the "hold" range, and similarly sell loser stocks if they are in the "sell" range and maintain these short positions as long as these stocks are in the "hold" range. According to Novy-Marx and Velikov (2015), this cost-mitigation strategy shows an apparent reduction in the trading costs of the momentum portfolios without compensating for portfolio returns too much compared to the first two strategies mentioned above. We, therefore, apply this buy/hold spread strategy to both the simple buy-and-hold momentum portfolio and the ones formed using two volatility timing strategies. Moreover, we develop different versions of this cost-mitigation strategy (simple 10%/20%, simple 10%/30%, and either 10%/20% or 10%/30% with a fixed number of stocks in the portfolio over time) and check which version produces the best trade-off between trading costs and portfolio returns. The main contribution of our paper is to study the **joint effect** of volatility timing strategies and cost mitigation strategies on the profitability of momentum trading, which, to our knowledge, has not been considered by previous studies.

Using US stock data for the period from 1980 to 2016, we obtain the following main findings. First, our results confirm that the DM and MM strategies not only help avoid the crashes experienced by the simple momentum strategy but also produce significantly higher returns on the momentum portfolios compared to the simple buy-and-hold one. Another important finding is that the DM strategy performs better than the MM strategy with the cumulative returns of the former by the end of December 2016 being 1.5 times higher than those of the latter. Second, as expected, taking into account transaction costs using bid-ask spread estimates leads to a striking decrease in the portfolio returns. In addition, the loss is more pronounced when transaction costs include additional components such as commission fees and extra fees on short positions. Third, when we apply cost-mitigation strategies to the momentum portfolios, both the portfolio turnovers and returns drop. However, among the four versions of the buy/hold spread strategy, the 10%/20% rule with a fixed number of stocks in the portfolio across time shows the best balance between costs and gains. Applying volatility timing and cost mitigation strategies jointly on momentum trading leads to highly profitable strategies, even after taking into account realistic transaction costs.

The rest of our paper is structured as follows. In section 2, we review literature on momentum crashes and prominent strategies used to tackle the problem so far, transaction costs associated with momentum portfolios and several cost-mitigation strategies. In section 3, we describe both the sample data and the methodologies used for our analysis. In section 4, we present the analysis results and discuss them in detail. Finally, in section 5, we summarize the key findings of our study and propose several further research directions.

2. Literature Review

2.1. Momentum strategy and its crashes

Jegadeesh and Titman (1993) were the first ones to suggest the momentum strategy: to buy stocks which have performed well in the past and sell stocks that have performed poorly. The authors managed to show that this strategy produced a compounded excess return of 12.01% per year on average over the 1965 - 1989 period.

Momentum is not specific only to the USA and to stock markets. Asness et al. (2013) demonstrated the ubiquity of momentum profits. The authors investigated these phenomena in eight different markets across different asset classes and found convincing evidence of momentum premia across all the markets. Fama and French (2012) also referred to the international evidence (developed markets in North America, Europe, Japan, and Asia Pacific regions) when they explored size, value and momentum patterns in average returns. They found strong momentum returns in all regions except Japan.

A series of other papers also addressed momentum universality, which is summarized by Barroso et al. (2015, p.111), who stated that "[this anomaly] has been shown in […] emerging markets, country stock indices, industry portfolios, currency markets, commodities […]". Rouwenhorst (1998) found that return continuation was present in all twelve countries from their sample for the years between 1980 and 1995. Desrosiers et al. (2007) examined 21 countries over the period 1988-2005 and found that 12-month past return did not produce good results for predicting expected relative country returns. However, Desrosiers et al. (2007) also found that a combination of fundamental and momentum-oriented variables could bring diversification benefits since these variables are not strongly correlated. Authors then suggested using a selection model (which consequently has higher returns and lower risk) based on ranking countries by both fundamental and momentum variables. Moskowitz and Grinblatt (1999) looked into industry momentum and concluded that (i) momentum returns become substantially less profitable if they control for industry momentum, but (ii) by buying stocks from past winning industries and selling from past losing industries, considerable additional profits are gained after after controlling for a number of factors. Okunev and White (2003) turned their attention to the currency markets, where the profitability of momentum strategy continued thorough 1990's according the authors. Commodity future markets were the center of Miffre and Rallis' (2007, p. 1863) paper, which found "13 profitable momentum strategies that generate 9.38% average return a year". Thomas and Ellis (2004) confirmed the existence of momentum profits in the period 1990-2003 for the FTSE 350 index.

However, a more recent wave of research pointed out that the momentum strategy can experience large crashes when the market is in a stressed situation. For example, Daniel et al (2012) showed that despite high positive returns and little systematic risk, the momentum strategy experiences infrequent but severe losses. Barroso and Santa-Clara (2015) showed that the winner-minus-loser (WML) portfolio delivered -91.59% return in 1932 and -73.42% in 2009. The authors also pointed out that these crashes, if not managed and hedged, would wipe away the positive returns gained earlier, and it would take years to recover.

Different hedging strategies were proposed in order to protect momentum from these infrequent large crashes. Grundy and Martin (2001) claimed that hedged portfolios based on forward-looking betas have a significantly improved performance. However, this approach was criticized by both Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016). The former paper (Barroso and Santa-Clara, 2015, p.117) showed that "the time-varying betas are not the main source of predictability in momentum risk," while the latter pointed out that the strategy of Grundy and Martin (2001) is basically not implementable, because it is based on forward-looking betas. Daniel et al. (2012) proposed using a Hidden Markov model to capture calm and turbulent states; in case of the latter, part of the portfolio should go to the risk-free asset. Barroso and Santa-Clara (2015) suggested managing the risk of the momentum strategy through the scaling of the portfolio. They argued that as the realized variance is highly predictable, investors can scale long-short portfolios by realized volatility in order to have a constant volatility. As a result, the crash risk goes down significantly, and the Sharpe ratio improves from 0.53 to 0.97 (Barroso and Santa-Clara, 2015). Daniel and Moskowitz (2016) contributed to the topic by developing a new approach, which they called the "dynamic momentum strategy." The authors revealed that crashes mostly result from the loser portfolio and that its behavior is similar to a written call option in the market (i.e. when the market falls, they gain little, but when the market rises, lose much). The authors suggested levering up and down the WML portfolio every month so that conditional volatility of the strategy is proportional to the conditional Sharpe ratio of the strategy. The weights for the momentum portfolio are calculated based on expected momentum returns and volatility. Daniel and Moskowitz affirmed that their strategy not only outperforms standard momentum strategy but is also superior to the constant volatility strategy proposed by Barroso and Santa-Clara (2015).

Zhuang (2018) explored the application of the Daniel and Moskowitz strategy in the foreign exchange market (9 prominent currencies were considered) and found evidence that the strategy's performance is considerably better than that of the simple momentum strategy. Another example of a simple momentum strategy alteration based on volatility was provided by Zaremba et al. (2018). In the strategy, called VARMOM (the volatility-adjusted residual momentum), the residual returns were normalized by volatility to isolate the risk effects. The strategy produces Sharpe ratios two to three times higher compared to the simple momentum strategy.

Moreira and Muir (2017) proposed a different volatility timing strategy by scaling factors (e.g. excess market returns, size, value, momentum and profitability factor) by the inverse of their conditional variance and comparing them with non-adjusted factors. The authors obtained very convincing results: alphas[3] are substantial for all factors and almost all alphas are statistically significant, while the largest alpha is for the momentum factor. Chen et al (2019) investigated the possibilities to improve the MM strategy by using a wide variety of ARMA specifications to forecast future variance over a sample period 1935 to 2016 (Moreira and Muir (2017) use the portfolio's realized variance in the previous period as a proxy for the future variance). They discovered that "only the fractionally integrated ARIMA model was able to produce an increase in the Sharpe ratio that is significantly above that produced by the simple historical average volatility" (Chen et al, 2019, p. 6).

2.2. Momentum strategy's transaction costs

Momentum portfolios are rebalanced monthly and, unlike other factors such as the value factor, the momentum factor is based on a more volatile indicator – past returns. This leads to a high turnover and could potentially lead to weighty transaction costs. The question whether the momentum strategy still generates positive excess returns after taking into account transaction costs has also been addressed in different papers with diverse results.

[3] Excess returns or abnormal returns

Jegadeesh and Titman (1993) accounted for the transaction costs in the amount of a 0.5% one-way transaction cost per trade and 9.29% per year. They stated that the risk-adjusted returns after transaction costs are significantly positive. Other authors came up with different estimates for the transaction costs, but their results were aligned with Jegadeesh and Titman (1993). To name a few, Desrosiers et al (2007) used a transaction costs estimate at 15bps and stated that portfolio returns (generated based on the both countries fundamental and momentum variables) remain statistically significantly different from zero even after introducing realistic transaction costs. Okunev and White (2003) did not include transaction costs in their analysis but stated that round-trip transaction costs are usually 10bps for trading in currency future markets, whereas mean monthly return varies from 45 to 60 basis points. These numbers imply that the strategies were profitable after transaction costs. Miffre and Rallis (2007) declared that it was unlikely that transaction costs would wipe away returns of the momentum strategies in futures markets. Thomas and Ellis (2004) investigated the momentum returns net of the transaction costs for the FTSE 350 index and included both direct and indirect costs in their analysis (bid-ask spread, commissions, stamp duty, short selling costs and price impact costs). Their conclusion is that even with a higher estimate of the transaction costs, momentum profits still exist.

Nevertheless, Lesmond et al. (2004) concluded that (for the momentum strategy) the impact of trading costs on price behavior is much larger than previously acknowledged and stocks that generate momentum returns are those with high trading costs. As a result, high momentum returns create an illusion of trading profit which does not really exist. It is important to state that Lesmond et al. (2004) used another metric for transaction costs. Unlike Jegadeesh and Titman (1993) who used trade-weighted mean commission and market impact of early 1985 NYSE trades to estimate trading costs, the authors appealed to different measures of spread (e.g. quoted spread estimate, direct effective spread estimate, roll effective spread estimate, quoted commission estimate). When the authors used spread estimates, other relevant costs such as commissions, short-sale constraints, and opportunity costs were excluded.

It is important to mention that authors who investigated volatility timing strategies did not generally put extra efforts on estimating transaction costs. For example, Daniel and Moskowitz (2016) did not address this question but instead state that transaction costs together with leverage (in case it gets too high) should be considered for practical implications. Barroso et al. (2015)

reasoned that the transaction costs for volatility timing momentum strategy should be even higher (by 38%) than for raw momentum in order to erode momentum returns. The authors do not make any direct estimates for the transaction costs. Moreira and Muir (2017) employed proportional trading costs at the levels of 1, 10, and 14 bps[4] (note that these numbers are considerably lower than those used by Lesmond et al. (2014)).

If we turn our attention to the elements of trading costs, the usual components are the bid-ask spread, brokerage cost, commission fees, price impact (which can be decomposed to permanent and temporary ones) and opportunity cost (Damodaran, 2019; Kociński, 2017). One of the main sources of transaction cost variation are bid-ask spreads determined by stock's liquidity, ownership structure and riskiness, to name a few (Damodaran, 2019).

There are several important observations about the transaction costs which were summarized by Frazzini et al (2014). As the authors have access to the internal database of a large institutional investor, this on one hand helped test different theories and explore transaction costs deeper; on the other hand it raised a question about the universality of the conclusions. Authors identified three groups of variables which determine transactions costs: (i) size of the trade, which can be expressed with the fraction of the daily trading volume, in other words, trading costs increase substantially with the trade size; (ii) the cost of the trading, which includes size of the firm, idiosyncratic volatility of the firm's equity (as expected, the large caps and more liquid stocks have lower transaction costs) and (iii) variation of trading costs over time (the downward trend over time). The authors stated that measuring transaction costs with effective bid-ask spread (referring the article of Novy-Marx and Velikov (2015)) was not a good estimate for a large arbitrageur, mainly because it did not take into account the size of the trade. The simple momentum portfolio's yearly transaction costs were estimated at the level of 4.78%, however optimization techniques can decrease them to 2.14% per year (based on 2000 US stocks and 2000 non-US stocks from 1980 to 2011). However, only US equity during 1998-2011 had an annual drag of 3.51% of transaction costs for simple momentum strategy (0.27% monthly trading costs).

These figures nicely correspond to the estimates made by Brandt et al (2009), who estimated (monthly) transaction costs of 0.6% for the smallest company and 0.35% for the largest one

---

[4] Basis points, 1 bps = 0.01%

(CRSP[5] data is used from January 1964 to December 2002). Grundy and Martin (2001) estimated that with the roundtrip transaction costs of 1.77% the momentum profits from 1926 to 1995 are driven to zero.

Considering what was said above, we believe that transaction costs were not addressed properly when estimating volatility adjusted momentum portfolios and that average costs (as in Moreira and Muir (2017)'s paper) can be used as a simple check, but it (i) neglects the fact that transaction costs vary substantially for different stocks, and (ii) does not allow to check Lesmond et al. (2004)'s statement that momentum profits are generated by the stocks with higher transaction costs and that a realistic estimate of the transaction costs will erode momentum profits. Therefore, we turn our attention to the papers which estimate different elements of the transaction costs.

Estimating transaction costs with the bid-ask spread (together with commission fees) is the most explicit way, but the bid-ask spread data is available in CRSP only from 1993 and onwards. The paper of Abdi and Ranaldo (2017) overcame this issue by suggesting a simple estimation of the spread from daily close, high, and low prices. The novelty of the paper is that it used close, high and low prices jointly and this bridges two approaches proposed earlier (based on close prices and more recent on high and low process). This estimate delivers the highest cross-sectional and average time-series correlation with the effective spread based on Trade and Quotes (TAQ) data, provides the lowest prediction errors and is not computationally difficult. All in all, the estimate suggested by Abdi and Ranaldo (2017) has a series of advantages compared to the counterparts such as HL (Corwin and Schultz' (2012) estimate of transaction costs based on high and low prices), Roll (estimate based on the close price), Gibbs (a Gibbs sampler Bayesian estimation of the Roll model), EffTick (measure based on the concept of price clustering), and FHT estimates (method which simplifies the estimate based on optimization of the maximum likelihood function for every single month to get the monthly estimates.).

Do et al (2012) provides transaction costs estimates for the U.S. equity market during the years 1962 to 2009 by explicitly estimating three components of direct trading costs: commissions, market impact, and short selling fees. When it comes to the commission fees, the authors continued a downward trend and received the average yearly one-way commission for the full sample equal

[5] Centre for Research in Security Prices

to 0.34% (starting from 0.87% in 1963 and going down to 0.09% in 2009). We decided to use the estimates of the authors for our research, however there is no data available in the article after 2009 and we fill the gap by using the same source as Do et al (2012), namely taking commission fees from the ITG global trading cost review for Q4/2018 (Investment Technology Group, 2019), as shown in Table 1.

Table 1. One-way commission fees in bps per transaction

The table contains one-way commission fees. Data from 1980 to 2009 is taken from Do et al (2012), 2010-2016 from ITG global trading cost review for Q4/2018 (Investment Technology Group, 2019). The sources complement each other as Do et al used the same report for the commission fees estimate.

Year	Commission fee, bps	Year	Commission fee, bps	Year	Commission fee, bps
1980	45	*1992*	19	*2004*	10
1981	41	*1993*	17	*2005*	10
1982	33	*1994*	17	*2006*	9
1983	28	*1995*	16	*2007*	7
1984	26	*1996*	14	*2008*	8
1985	25	*1997*	10	*2009*	9.1
1986	22	*1998*	10	*2010*	8.8
1987	20	*1999*	10	*2011*	7.5
1988	19	*2000*	10	*2012*	7.3
1989	20	*2001*	10	*2013*	6.6
1990	20	*2002*	10	*2014*	5.6
1991	21	*2003*	10	*2015*	5
				2016	5.3

Another important question is about any additional fees associated with different positions. Foltice and Langer (2015) investigated the applicability of the momentum strategy for an individual investor and stated that short positions are associated with “hard to borrow” fees, i.e. opening short positions for some stocks can be done only at a high cost. Do et al (2012) added that besides difficulty to open the position at the times desired (or at a higher price), there is a cost of shorting in a form of a loan fee and there can be a limited possibility to recall a borrowed stock prematurely. Li et al. (2009), who investigated transaction costs of the momentum strategy (for UK companies over the period of 31 December 1985 – 31 December 2005), got the striking result that the average round-trip commission for the winners is 1.06%, while for the losers it is 1.58%. Despite the fact that these figures seem a bit too high, (both Lesmond et al. (2014)’s and Li et al. (2009)’s estimates are upward shifted compared to the other authors’ estimates above), the main idea is that the losers have higher transaction costs. On the other hand, Frazzini et al (2014) were skeptical about the fact that short selling is more expensive, as the difference which they had found was not statistically significant. Do et al (2012) also mentioned that in recent years, the short sales did not lead to an increase in the substantial transaction costs. Nevertheless, Do et al (2012) controlled for the short sales constraints by including a constant loan fee of 1% per annum payable.

If we accept the premise that the transaction costs are high, another possible avenue of research is cost mitigation strategies. The natural technique on transaction costs mitigation is to leave the stocks with lower transaction costs. The paper "Low cost momentum strategies" (Li, Brooks, & Miffre, Low-cost momentum strategies.(Original Article), 2009) showed that this technique can save over 30 percent of the costs involved in the purchase and sale of winners and losers. However, this strategy is computationally complicated and thus not easy to implement. Another paper which investigated transaction costs reduction is "An empirical investigation of methods to reduce transaction costs" by Moorman (2014), which addressed the problem a bit differently. Moorman stated that "the decision to choose optimal weights is bundled with the objective of reducing transaction costs" (Moorman, 2014, p.231) and turned attention to the methods of portfolio constructing and choice of optimal weights. He claimed that the investor will get the best result if a levered-momentum portfolio and an equally-weighted market portfolio are used, whereas "it is more difficult to recover wealth for a zero-cost momentum portfolio" (Moorman, 2014, p.231). Novy-Marx and Velikov (2015) evaluated the most applicable cost mitigation strategies: limit trading to the universe of stocks that are expected to be relatively cheap to trade, rebalancing only a fraction of portfolio at each rebalance point and finally the buy/hold spread strategy. The last one is an elegant cost mitigation strategy that is based on the so called "sS rule", which does not require estimation of the transaction costs and described in more detail in the next section.

3. Data and Methodology

3.1. Data

Individual stock level data comes from the Centre for Research in Security Prices (CRSP). The data sample contains returns, stock prices and the number of shares outstanding for all stocks that are listed on the NYSE, Amex, and Nasdaq markets (CRSP exchange code 1, 2, and 3, respectively), and that are either US or non-US common shares (CRSP share code 10, 11, and 12). This sample consists of data from January 1980 to December 2018. On the other hand, data on daily and monthly market returns as well as daily momentum decile portfolios are obtained from Kenneth French's data library[6] and are available for the period from 1926 to December 2018.

The bid-ask spread estimates are used as a proxy for our trading costs according to Abdi and Ranaldo (2017)'s approach and are obtained directly from Abdi's website[7]. Since the authors' bid-ask spread estimates are available for the period of January 1926 till December 2016, in order to make our results comparable and ensure the consistency in our study, we decide to base our analysis on the period of January 1980 to December 2016. Noticeably, as the formation period of momentum portfolio is 12 months, the analysis results have the starting values in January 1981.

3.2. Methodology

As our study aims to compare the risk-adjusted momentum strategies with and without taking into account the transaction costs, our methodology includes the following steps.

3.2.1. Constructing momentum deciles portfolios

We follow Daniel and Moskowitz (2016) in forming the monthly momentum decile portfolios and use the same restrictions on the portfolios as they did in the paper. That is, all portfolios are value weighted and rebalanced at the end of each month. A firm is included in the portfolio formation procedure if there are no missing values in the stock price of month $t - 13$, in the stock return of month $t - 2$, and in the market capitalization of month $t - 1$. Moreover, the firm is required to have available data on its monthly returns for at least 8 months over the past 11 months ($t - 12$ to

[6] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[7] http://www.farshidabdi.net/index.html

$t - 2$). All the firms that satisfy the above requirements are then involved in the portfolio formation. Next, we calculate cumulative returns on each stock from month $t - 12$ to $t - 2$ using the formula:

$$R_{[t-12,t-2],i} = \left(\prod_{\tau=2}^{12} \left(1 + R_{t-\tau,i}\right) \right) - 1 \quad (1)$$

After that, we sort stocks based on their cumulative returns calculated above into ten decile portfolios, where the first decile portfolio represents the losers, i.e. stocks with the lowest past returns, and the last decile portfolio represents the winners, i.e. stocks with the highest past returns. Finally, the zero cost WML (winner minus loser) portfolio includes the two extreme deciles and has the average return equal to

$$R_{WML,t} = \sum_{i=1}^{w} w_{i,t} R_{i,t} \quad (2)$$

where $w_{i,t}$ are weights of the zero-investment WML portfolio at the beginning of month t and satisfy the following conditions.

$$\begin{cases} \sum_{i=1}^{w} w_{i,t} \mathbf{I}\left(w_{i,t} > 0\right) = 1\, for\ long\ positions \\ \sum_{i=1}^{w} w_{i,t} \mathbf{I}\left(w_{i,t} < 0\right) = -1\, for\ short\ positions \end{cases} \quad (3)$$

3.2.2. Cost-mitigation strategies

We apply the buy/hold strategies introduced by Novy-Marx and Velikov (2015) to the momentum portfolios to mitigate their high turnover/transaction costs. The idea of the strategy is (for the winners) to buy stocks if they are in buy range but not sell them until they fall out from the hold range. The course of action with the losers is identical. For example, a 10%/20% buy/hold rule applied to momentum portfolios suggests that investors should buy stocks when they enter the buy range, which is the highest 10% of the past 11-month cumulative returns (i.e., the top decile), and hold them as long as they are still in the hold range which is the highest 20% (i.e., the top two deciles). Similarly, on the short leg, traders should short sell stocks when they enter the lowest 10% (i.e., the bottom decile) and maintain this short position as long as they do not fall out of the lowest 20% (i.e., the bottom two deciles). We decide to conduct further experiments on this method by trying different versions of it. First, we simply use a 10%/30% buy/hold rule, where the hold range is the top three momentum deciles for the winner portfolio and the bottom three momentum deciles for the loser portfolio. Second, we modify the 10%/20% rule by adding a restriction to it. In particular, we suppose that in order to lower the monthly turnover, the number of stocks in the winner/loser portfolios in each month should stay the same as in the initial winner/loser portfolio. This is done by including all stocks that were in the past winner/loser portfolios and are still in the hold range and additionally only the best new winners or the worst new losers.

We assume that introducing the restrictions above to the original 10%/20% buy/hold rule will not only show a more significant drop in the trading costs of the WML portfolios but also produce higher returns compared with using the simple 10%/20% buy/hold rule alone.

3.2.3. Transaction costs

As mentioned in the Literature Review section, we take into account the transaction costs to check the feasibility of the simple momentum strategy as well as the volatility timing ones which will be discussed later in this section. Following Brandt et al. (2009), the cost-adjusted returns on the WML portfolio at month t can be calculated as follows:

$$R_{WML,t} = \sum_{i=1}^{N} \left(w_{i,t} R_{i,t} - c_{i,t} \left| w_{i,t} - w_{i,t}^{h} \right| \right) \tag{4}$$

with $w_{i,t}^{h}$ denoting the "hold portfolio weights" resulting from changes in the returns on the portfolio assets in the beginning of month t while the portfolio actually stays the same as in month $t-1$, and is defined by the following equation.

$$w_{i,t}^{h} = w_{i,t-1} \left(\frac{1 + R_{i,t-1}}{1 + R_{WML,t-1}} \right) \tag{5}$$

Noticeably, from Equation (4), $c_{i,t}$ denotes "proportional transaction cost for stock i at time t", and can be estimated (Brandt et al., 2009, p. 3424). However, the authors suggested that the estimates should represent one-way transaction costs, which is approximately half of the round-trip trading costs. In our analysis, $c_{i,t}$ is calculated using the three following formulas.

When trading costs include only (round-trip) bid-ask spread estimates:

$$c_{i,t} = 0.5 * bid - ask\ spread\ estimates \tag{6}$$

As mentioned earlier, we use the round-trip bid-ask spread estimates of Abdi and Ranaldo (2017) during our analysis. When trading costs consist of both (round-trip) bid-ask spread estimates and one-way commission fees:

$$c_{i,t} = 0.5 * bid - ask\ spread\ estimates + one - way\ commission\ fees \qquad (7)$$

The time-varying one-way commission fees used for our calculations are described in Table 1 in the Literature Review section. When transaction costs involve (round-trip) bid-ask spread estimates, one-way commission fees and short-selling fees:

$$c_{i,t} = 0.5 * bid - ask\ spread\ estimates + one - way\ commission\ fees + short - selling\ fees \qquad (8)$$

In line with Do et al (2012), short-selling fees are represented by a constant loan fee of 1% per annum.

3.2.4. Volatility timing strategies

We also apply the volatility DM an MM timing strategies and check whether these strategies can help alleviate momentum crashes. As mentioned in the Literature Review section, the main idea of volatility timing is to adjust the risk exposure of the WML portfolio by scaling up or down the portfolio weights. This idea can be demonstrated in the below equation,

$$\tilde{R}_{WML,t} = \sum_{i=1}^{N} \left(\mathcal{w}_t w_{i,t} R_{i,t} - c_{i,t} \left| \mathcal{w}_t w_{i,t} - \mathcal{w}_{t-1} w_{i,t}^{h} \right| \right) \qquad (9)$$

where $\mathcal{w}_t$ denotes the scaling factor applied to the portfolio weights at time t. It is noteworthy that $\mathcal{w}_t$ is calculated differently for the two above mentioned strategies.

Interestingly, when investigating the two volatility timing strategies, we notice that without restriction on $\mathcal{w}_t$, the returns on the scaled momentum portfolios, $\tilde{R}_{WML,t}$ become abnormally large due to high portfolio weights (Appendix F), questioning the practicability of these strategies. The only paper to our knowledge which addressed this question is Zhuang (2018) who imposed the restriction of [-1, 1] on the weights, which means that the portfolio is not leveraged. However, in order to leverage the portfolio, but not at the level that it can question the feasibility of the strategies, we decide to put an upper bound of 2 on scaling weights of the portfolios, i.e., an upper bound of 2 on the value of $\mathcal{w}_t$.

3.2.4.1. Daniel and Moskowitz (2016)'s dynamic volatility scaling approach

As shown in their paper, in order to maximize the in-sample unconditional Sharpe ratio, Daniel and Moskowitz (2016) suggested that the scaling factor optimal weight on the momentum portfolio at time t should be calculated as below,

$$w_t = \left(\frac{1}{2\lambda}\right)\frac{\mu_t}{\sigma_t^2} \tag{10}$$

where $\mu_t = E_t[R_{WML,t+1}]$ is the conditional expected return on the zero cost WML portfolio over the following month, $\sigma_t^2 = E_t\left[\left(R_{WML,t+1}^2 - \mu_t\right)^2\right]$ denotes the conditional variance of the WML return , and λ indicates a time-invariant parameter that controls the unconditional risk and return of the dynamic portfolio. In particular, λ is set so that the unconditional standard deviation of the dynamic scaled WML portfolio is the same as that of the simple buy-and-hold WML portfolio. Values of λ when we combine Daniel and Moskowitz (2016)'s dynamic volatility scaling approach with different versions of the buy/hold cost mitigation strategy are shown in Appendix A.

In order to forecast the above-mentioned conditional variance of the WML returns, similar to DM, we fit the following GJR-GARCH model (Glosten et al., 1993) to the WML returns,

$$R_{WML,t} = \mu + \epsilon_t \tag{11}$$

where $\epsilon_t \sim \mathcal{N}(0, \sigma_t^2)$ and σ_t^2 is defined by

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \left(\alpha + \gamma I(\epsilon_{t-1} < 0)\right)\epsilon_{t-1}^2 \tag{12}$$

However, in this step, Daniel and Moskowitz (2016) fitted the GJR-GARCH model to the full time series, which we suppose might lead to a "look-ahead bias". To avoid this problem, in order to predict the variance of the WML portfolio for month t, we estimate the coefficients of the above model using daily returns over 504 business days (approximately equivalent to 2 years) through the last day of month $t-1$.[8]

Meanwhile, the conditional expected return on the WML portfolio, μ_t, is predicted using the following regression model,

[8] Since we ran the GJR-GARCH model shown in Equation (11) and (12) 432 times in total, we cannot present all the coefficient estimates in the paper.

$$R_{WML,t} = \gamma_0 + \gamma_B I_{B,t-1} + \gamma_{\sigma_m^2} \hat{\sigma}_{m,t-1}^2 + \gamma_{int} I_{B,t-1} \hat{\sigma}_{m,t-1}^2 + \tilde{\epsilon}_t \quad (13)$$

where the dependent variable $R_{WML,t}$ denotes the monthly return on the zero-investment momentum portfolio; $I_{B,t-1}$ denotes "the ex-ante bear market indicator, which equals one if the cumulative CRSP value-weighted index returns in the past 24 months is negative and is zero otherwise"; $\hat{\sigma}_{m,t-1}^2$ denotes the lagged realized volatility, which is the variance of the daily market returns over 126 consecutive business days prior to the beginning of month t.

Here, it is notable that according to Daniel and Moskowitz (2016), the regression (13) should be run on the full time series starting from 1926 to 2016. However, since doing this also leads to "look-ahead bias", we decided to run the regression over the period from July 1926 to December 1979 and use the estimated coefficients to obtain the conditional expected return for the period from 1980 to 2016. The coefficient estimates from the above regression are shown in Appendix A.

3.2.4.2. Moreira and Muir (2017)'s time varying variance scaling approach

Moreira and Muir (2017) suggest that the scaling factor introduced in Equation (9) should be computed as

$$w_t = \frac{c}{\hat{\sigma}_t^2} \quad (14)$$

where $\hat{\sigma}_t^2$ is a proxy for the conditional variance of the portfolio and is calculated as the realized variance over the previous month

$$\hat{\sigma}_t^2(R_{WML}) = RV_t^2(R_{WML}) = \sum_{d=\frac{1}{22}}^{1} \left(R_{WML,t+d} - \frac{\sum_{d=\frac{1}{22}}^{1} R_{WML,t+d}}{22} \right) \quad (15)$$

Moreover, in Equation (14), c is a constant and satisfies the condition that the unconditional standard deviation of the volatility-managed momentum portfolio matches that of the buy-and-hold portfolio. According to Moreira and Muir (2017), "c controls the average exposure of the strategy". We report different values of c when we implement both Moreira and Muir (2017)'s time varying variance scaling approach and Novy-Marx and Velikov (2015)'s buy/hold cost mitigation strategy at the same time in Appendix A.

3.2.5. Performance measures

To evaluate the overall performance of all the above-mentioned strategies, we use the Sharpe ratio and the Maximum Drawdown as our key performance measures.

3.2.5.1. Sharpe ratio

In the 5th edition of "A Dictionary of Finance and Banking" (Law, 2014), the Sharpe ratio is defined as a "risk-adjusted measure of the performance of a portfolio" and is calculated as follows,

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma_{R_p}} \tag{14}$$

where R_p, R_f denote the portfolio returns and the risk-free rate respectively, and σ_{R_p} is the standard deviation of the portfolio returns. According to Pedersen (2015), the idea of using this ratio is to tell investors how rewarding the investment in the portfolio is compared to the risks that it takes.

3.2.5.2. Maximum Drawdown

Pedersen (2015) suggested that Maximum Drawdown, which is defined as the largest percentage loss of the investment over a fixed time period, can be seen as another key performance measure for a portfolio strategy,

$$Maximum\ Drawdown_T = max_{t \le T} Drawdown_t \tag{15}$$

where

$$Drawdown_t = \frac{max_{s \le t} R_s - R_t}{max_{s \le t} R_s} \tag{16}$$

with $max_{s \le t} R_s$ indicating the highest cumulative return over the past period and commonly known as the portfolio's high-water mark.

As stated by Pedersen (2015), Maximum Drawdown measures the largest loss, or peak-to-trough decrease in the portfolio's value over a specific time period. This measure has become an important tool for investors to control the downside risk of an investment, which is the risk of suffering from a loss in the portfolio value due to one or several changes in the market conditions (Mendes and Lavrado, 2017).

To get a better understanding of the measure, let us consider the following example. Assume that the initial value of a portfolio is one million dollars; over a period, the portfolio value rises to 1.25 million dollars before plummeting to $500,000. After that, the portfolio value bounces back to

$700,000 before diving to $300,000. Later, it rebounds to $600,000. The maximum drawdown of the investment portfolio is equal to $\frac{1\,250\,000-300\,000}{1\,250\,000} = 76\%$. This means that the largest percentage loss of the portfolio over the time period is 76%, implying a very high downside risk.

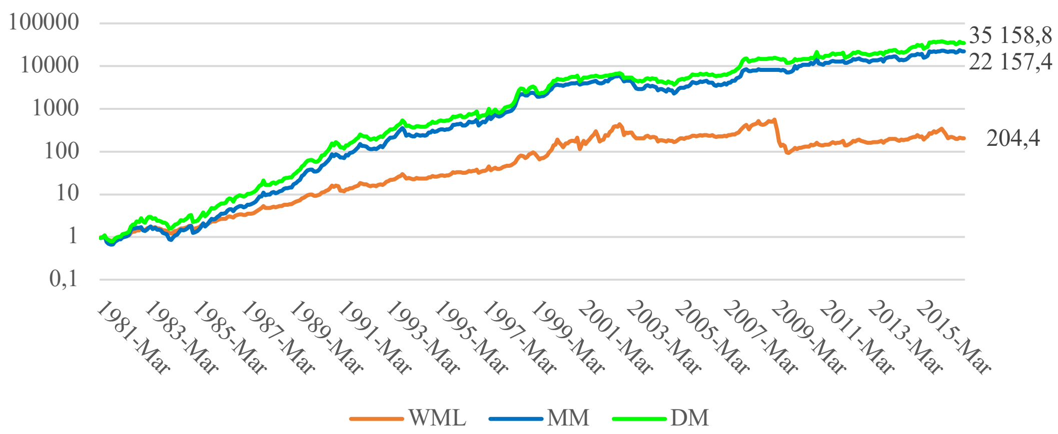
4. Results and Analysis

4.1. Momentum crashes and volatility timing strategies

As discussed in the Literature review section, the simple momentum strategy – buying stocks that performed well over the past year and selling stocks that had a poor performance during that period – brings positive returns on average. The overall gain can be significant, as it can be observed in Figure 1, which depicts cumulative returns on zero-investment WML portfolios. With a one-dollar investment in the beginning of 1981, an investor could have gained a cumulative return of 204.4 dollars on the simple momentum strategy by the end of December 2016. However, as documented in many previous studies, the simple buy-and-hold momentum strategy can experience infrequent but severe losses during the periods of high market volatility. Illustration of this phenomenon can also be found in Figure 1. There is a plunge in the cumulative returns on the plain WML portfolio during the financial crisis (2008-2009) from above 320 dollars in the beginning of 2009 to slightly above 90 dollars in September 2009. Moreover, referring to Table 2, the monthly Sharpe ratio of the simple buy-and-hold momentum portfolio for the analysis period (0.1486) is positive and just slightly higher than the Sharpe ratio of the US market (0.143). In addition, the distribution of the returns on this portfolio is highly kurtotic (8.5314) and left-skewed (-1.2279).

Figure 1. The cumulative returns on the zero-cost momentum portfolio before and after applying volatility timing strategies

The plot demonstrates the cumulative returns of the simple buy-and-hold, MM and DM momentum portfolios during the period 1981-2016.

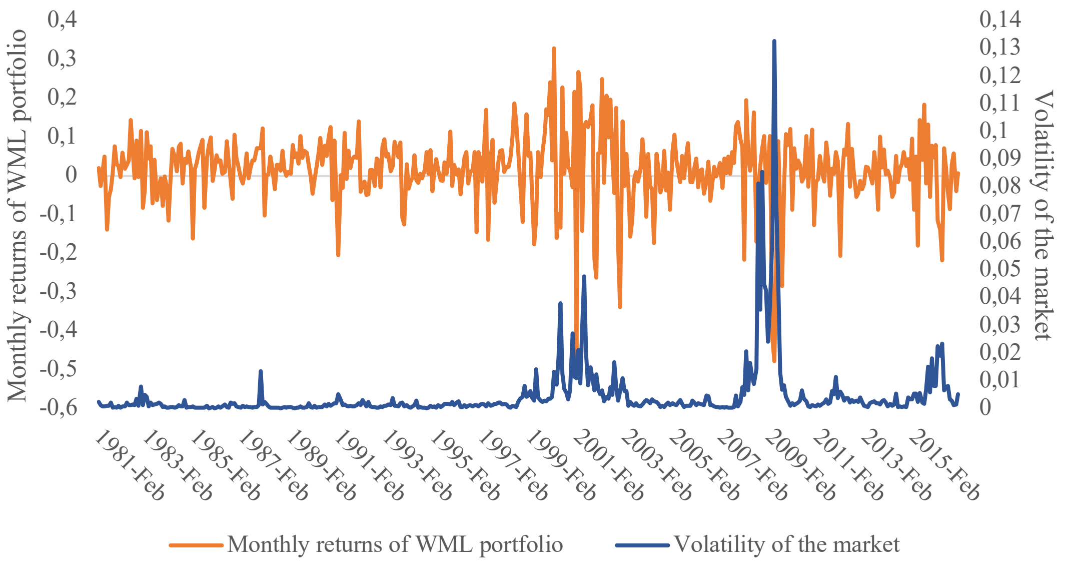


However, the momentum crashes can be avoided if the volatility timing strategies are implemented. Figure 1 shows that there is no drop in the cumulative returns during the financial crisis on the scaled momentum portfolios constructed using the DM and MM strategies. The contrast between simple and volatility adjusted momentum portfolio returns proves that volatility adjusted strategies successfully address the problem of crashes. It is also noteworthy that the WML portfolios scaled using volatility timing strategies show a remarkable increase in the cumulative returns compared to the plain buy-and-hold one (35,159 dollars and 22,157 dollars for DM and MM's WML portfolios respectively compared to the 204 dollar return on the simple buy-and-hold momentum portfolio). In addition, returns on the WML portfolios when applying DM strategy are above 1.5 times bigger than those when using MM strategy.

In order to get a deeper understanding of how volatility timing strategies improve returns on the momentum portfolios, we present a graph on monthly returns of the unscaled WML portfolio and the market volatility for the period 1981-2016 in Figure 2.

Figure 2. Monthly returns on WML portfolios and market volatility

The graph shows both the monthly returns of the simple buy-and-hold momentum portfolios and the market volatility over the period 1981-2016.

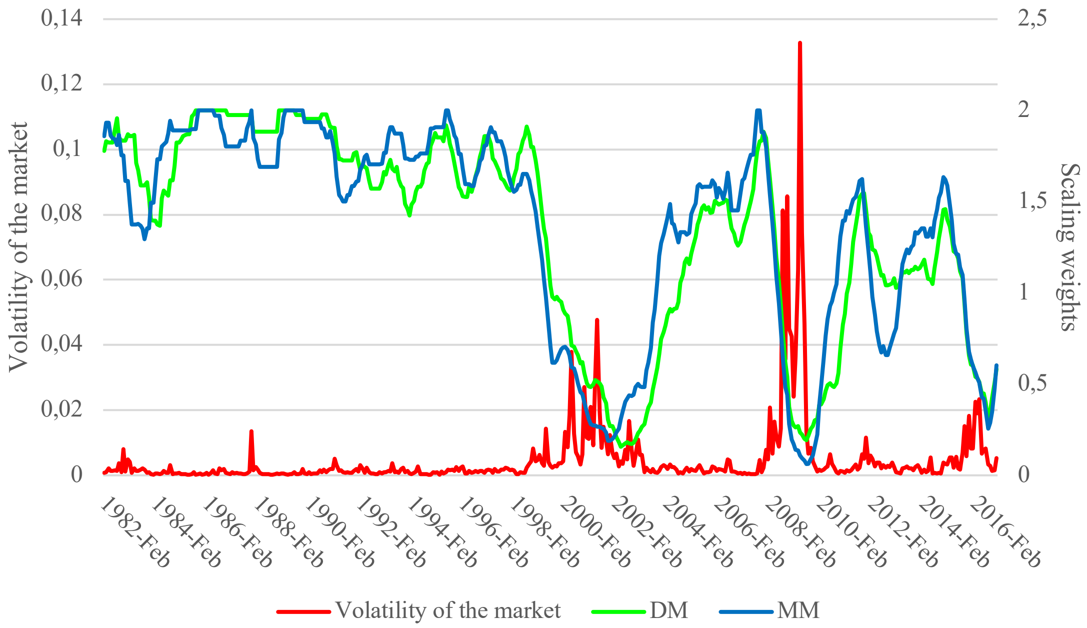


As it can be seen from Figure 2, during the turbulent times (2000 – 2002 and 2008 – 2009), there is a pronounced drop in the monthly returns of the WML portfolio. The volatility timing strategies handle such decrease by adjusting the portfolios' risk exposure through different weights.

Specifically, the weight of the WML portfolio goes up in the calm times and down in the volatile periods. To get a grasp of how volatility timing strategies work, we look at the relation between market volatility and the scaling weights for the WML portfolio as demonstrated in Figure 3.

Figure 3. Smoothed scaling weights of MM and DM strategies and volatility of the market

The plot depicts the scaling weights of MM and DM momentum portfolios along with the market volatility. It is noteworthy that an upper bound of 2 is set for the scaling weights of these volatility timing strategies.



There are several important points we can observe from the graph.[9] First, both volatility timing strategies share quite similar patterns. Second, scaling weights plummet right before periods with high volatility, and rocket right after those times. We can hypothesize that changes in the weights in turbulent and calm times serve different purposes: risk exposure in times of high market volatility is reduced to avoid crashes, whereas risk exposure in calm times is increased to gain more profits. Further discussion on this hypothesis can be found in section 4.5.

Implementation of different volatility timing strategies helps avoid crashes and thus improves distribution characteristics of the momentum returns. Measures in the Table 1 show that the returns on WML portfolios with the application of volatility timing strategies have a more “normal” distribution with lower kurtosis and less negative skewness. Not less importantly, returns on WML

[9] As it is pointed out in the Data and Methodology section, we set an upper bound of 2 for the scaling weights. Further discussions and outcomes of this decision are available in section 4.5.

portfolios formed from MM strategy are slightly more kurtotic but less left-skewed than those DM dynamic approach, implying that the former strategy might perform better than the latter upon dealing with momentum crashes.

Table 2. Different measures* for WML portfolio before and after implementing volatility timing strategies

The table includes monthly distributional characteristics (mean, volatility, kurtosis and skewness) of WML portfolios together with some performance measures (Sharpe ratio and Maximum drawdown) over the period 1981-2016.

	Buy&Hold	**MM**	**DM**
Mean	0.0171	0.0276	0.0288
Volatility[10]	0.0903	0.0903	0.0903
Sharpe ratio	0.1486	0.2663	0.2766
Maximum drawdown	0.9355	0.7699	0.6323
Kurtosis	8.5314	4.7696	4.7603
Skewness	-1.2279	-0.1164	-0.1492

**All values are in monthly terms*

As expected from the higher cumulative returns depicted in Figure 1 and higher mean returns from Table 2, the Sharpe ratios show considerable improvement after implementing volatility timing strategies. Both volatility timing strategies lead to an increase in Sharpe ratio by about two times (0.266 and 0.277 for volatility strategies vs 0.149 for simple momentum portfolio), but the DM strategy performs a bit better than the MM (with larger returns and a slightly higher Sharpe ratio). Observably, both the simple buy-and-hold WML portfolio and those formed using MM and DM strategies have the same volatility of 0.0914. This is already pointed out in the Methodology section, where we state that the unconditional standard deviation of the volatility-managed WML portfolio must match that of the simple buy-and-hold one. Regarding the Maximum drawdown indicator, which shows how much the downside risk of the portfolio goes down when implementing volatility timing, we can note that MM and DM volatility timing strategies made it possible to decrease maximum loss since the peak by 25% and 44% respectively.

In summary, despite positive returns on the WML portfolio, the simple buy-and-hold momentum strategy suffers from severe crashes. However, volatility timing strategies can help alleviate this problem, resulting in a significant increase in the returns and improvement in both distributional

[10] The volatility is calculated over the full period 1981-2018

characteristics (kurtosis, skewness) and performance measures (Sharpe ratio and maximum drawdown).

4.2. Transaction costs

Another problem with the momentum strategy is possibly higher transaction costs caused by frequent and considerable rebalancing. As we discussed in the Literature Review section, the momentum strategy requires monthly rebalancing, which can lead to high transaction costs. In this section, we investigate returns after transaction costs for the simple momentum portfolio as well as for the volatility managed momentum portfolios.

Figure 4 illustrates the effect of transaction costs on the portfolio returns. That is, all three graphs depict the cumulative returns with transaction costs on the momentum portfolios before and after applying volatility timing strategies; however, the transaction costs used for the first graph consist of only bid-ask spread estimates while the costs for the second graph involve additionally different commission fees for every year, and the trading costs used for the last graph include 1% of annual fees for short positions in addition to the bid-ask spread estimates and yearly commission fees.

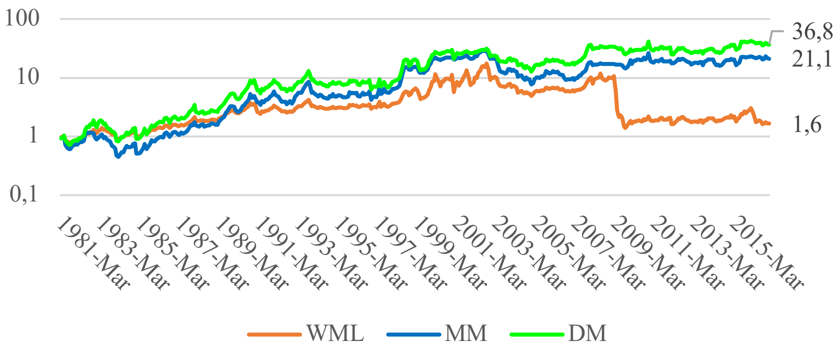
If we involve transaction costs, we expect returns to go down; however, the actual drop turns out to be so substantial that it almost erodes all the WML returns: with one dollar invested in the beginning of 1981, simple momentum portfolio has cumulative returns ranging, depending on transaction costs estimate, between 0.5 (which basically means a loss) and 1.6 dollars instead of 204 (no transaction costs considered; Figure 1) by the end of 2016. After involving transaction costs, momentum portfolio returns become surprisingly low. Moreover, not only do the transaction costs themselves show a discernible impact on the returns of the momentum portfolios, but also their magnitude. Thus, if the transaction costs consist of only bid-ask spread estimates, the returns on the zeros-cost WML portfolios decrease strikingly compared to those without transaction costs. Nevertheless, if we add commission fee to the transaction cost, the decline in the portfolio returns is even more pronounced.

The volatility timing strategies still improve returns on momentum portfolios considerably as before. Having said that, we should point out that the comparison between returns on portfolios formed from MM and DM strategies with their counterparts without transaction costs show a steep downfall. That is, WML returns drop from 35,159 to 36.8 (with transaction costs estimated by bid-ask spread only) when using the DM strategy; with the MM strategy WML returns are reduced from 22,157 to 21.1.

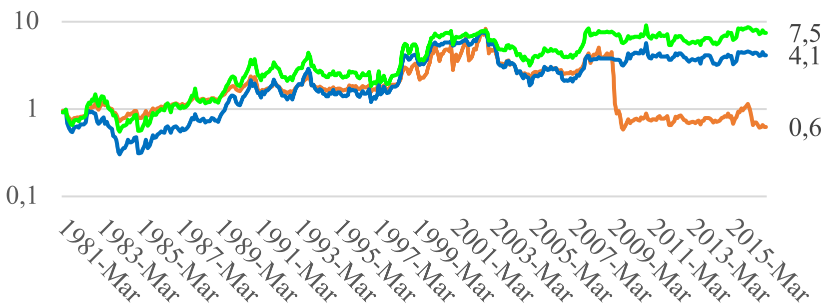
Figure 4. Cumulative returns on WML portfolios after transaction costs

The plots describe cumulative returns on WML portfolios after taking into account trading costs for the period 1981 to 2016. Panel A shows portfolio returns when trading costs include only bid-ask spread estimates from Abdi and Ranaldo (2017). Transaction costs in Panel B involve additionally commission fees shown in Table 1. Panel C depicts the cost-adjusted cumulative returns when trading costs consist of bid-ask spreads, commission fees and short-selling fees (1% per year).

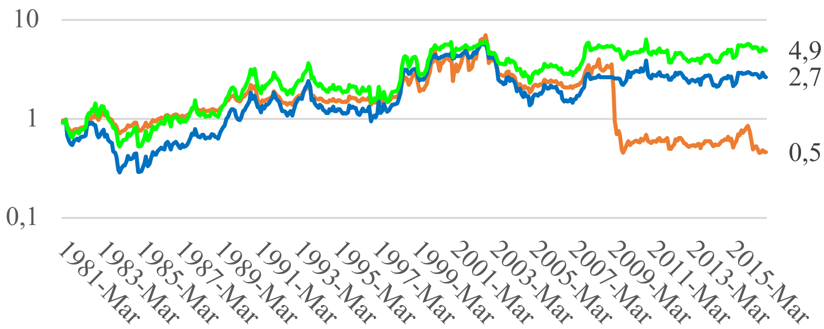
Panel A: bid-ask spreads only



Panel B: bid-ask spreads + commission fees



Panel C: bid-ask spreads + commission fees + short sale fees



If we add commission fees and a 1% fee for short positions, the cumulative return on the unscaled momentum portfolio does not exceed one, meaning that that one dollar invested in these strategies in the beginning of 1981 has not produced any profits by the end of 2016. This powerful result implies that if transaction costs are not adequately managed, momentum strategies will not be able to generate adequate returns. Meanwhile cumulative returns on the volatility timing portfolios stay positive even after imposing additional fees. Thus, the cumulative return by the end of 2016 for the DM and the MM strategy is equal to 7.5 and 4.1, respectively if commission fees are added and 4.9 and 2.7 if commission fees and short position fees are added.

Overall, it is better to involve the commission fees besides the bid-ask spread estimates in the transaction costs since the results tend to be more genuine. Moreover, it is more plausible to apply different commission fees to winner and loser portfolios (i.e. by adding an additional 1% annual fee for short positions) than to use the same average commission fee for all. From now on, all the analysis results involving transaction costs will take into consideration the different commission fees mentioned above (i.e., the results corresponding to Panel C in Figure 4).

4.3. Cost-mitigation strategies

As discussed in the methodology section, we apply several versions of the buy/hold cost-mitigation strategy. Let us first discuss the effect of these strategies on the portfolio's turnover. Figure 5 depicts the turnover, smoothed with a 12-month moving average, for each cost mitigation strategy while the summary of average turnovers for different strategies can be found in Table 3.

Table 3. Average turnovers during 1981–2016 for different momentum strategies

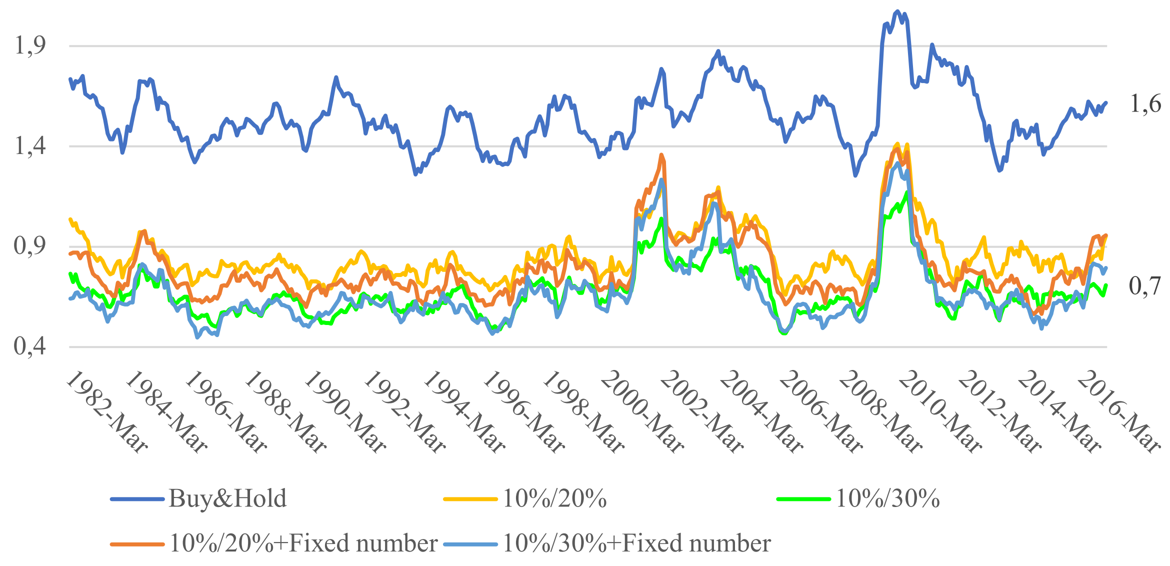
The table shows average turnovers over the period 1981-2016 for WML portfolios after implementing volatility timing strategies (MM, DM approach) and different versions of the buy/hold cost mitigation strategy.

	WML	**MM**	**DM**
Buy&Hold	1.557	2.339	2.259
10%/20%	0.863	1.483	1.369
10%/30%	0.678	1.245	1.132
10%/20% Fixed number	0.793	1.358	1.247
10%/30% Fixed number	0.673	1.225	1.108

It is not surprising that the average turnovers of the buy-and-hold portfolio are the highest and remarkably higher than the ones that use the cost-mitigation strategies. Both the 10%/20% rule and its fixed number version exhibit higher average turnover than the 10%/30% and 10%/30% fixed number cost strategies. However, the decrease in the turnover as a result of imposing additional fixed number condition is moderate for the 10%/20% strategy, and almost unnoticeable for the 10%/30%. This can be partly explained by the fact that the 10%/30% strategy has such a low turnover that additional rule (fixed number) does not produce any substantial gain (there is some gain though, as 10%/30% average turnover for 1981-2016 is 0.793 and 10%/30% fixed number turnover is 0.673). The illustration of the observations above can be found in Figure 5.

Figure 5. Moving average turnovers of WML portfolios with different cost-mitigation strategies[11]

The plot depicts turnovers smoothed with a simple moving average for 12-month period over the period 1981-2016 for the simple buy-and-hold momentum portfolio after imposing different rules of the buy/hold spread cost mitigation technique.



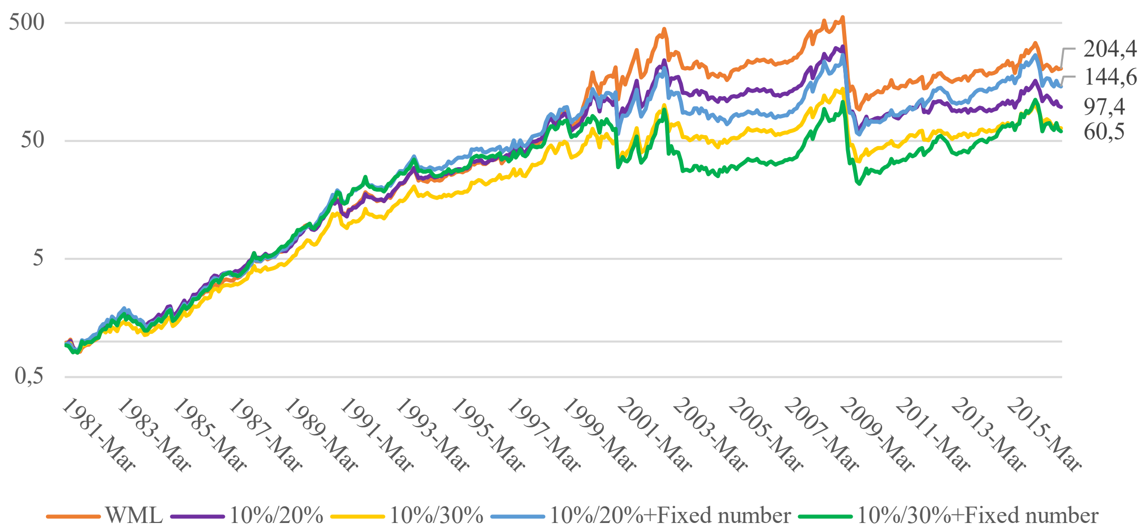
The turnovers of other strategies specifically MM and DM with cost mitigation techniques share similar patterns. The illustrations of these patterns together with observations on differences between turnovers of simple buy-and-hold and volatility timing strategies can be found in Appendix C.

[11] Simple moving average for 12-month period

In terms of the returns, there is apparently a trade-off between the turnovers and the returns on the momentum portfolios (formed with these cost strategies), as can be seen in Figure 6.

Figure 6. Cumulative returns on WML portfolios with different cost-mitigation strategies

The plot depicts cumulative returns for the for the simple buy-and-hold momentum portfolio after imposing different buy/hold spread cost mitigation techniques over the period 1981-2016 without involving transaction costs.



As expected, there is a fall in the returns on WML portfolios both with and without cost mitigation strategies during the financial crises 1999-2000 and 2008-2009. Despite having the highest average turnover over the whole period, the buy-and-hold WML portfolio has the highest cumulative returns compared with the ones formed using all four cost strategies introduced. By contrast, the WML portfolio formed from the 10%/30% fixed number strategy trades off the lowest turnover averages with the lowest cumulative returns. Noticeably, 10%/20% fixed number strategy produces the second highest cumulative returns while maintaining low average turnovers over time. In order to confirm the superiority of this cost strategy over the others, we check the performance of WML portfolios when the cost mitigation strategies are combined with volatility timing ones. The detailed results are given in the next part.

4.4. Combing volatility timing strategies with cost-mitigation strategies

In order to combine volatility timing and cost mitigation strategies, we first form the WML portfolio with different versions of buy/hold spread rule and then implement volatility timing via scaling portfolio's weights up and down. For the sake of brevity, we include the charts with cumulative returns for these strategies in Appendix D (with and without transaction costs, for the period from 1981 to 2016). The most important finding is that the 10%/20% fixed number strategy provides a good trade-off between the returns and transaction costs decrease. In addition, this cost strategy has the second lowest turnover, similar to its performance without volatility timing strategies. Thus, we decide on choosing this buy/hold rule for further analysis in the rest of the paper.

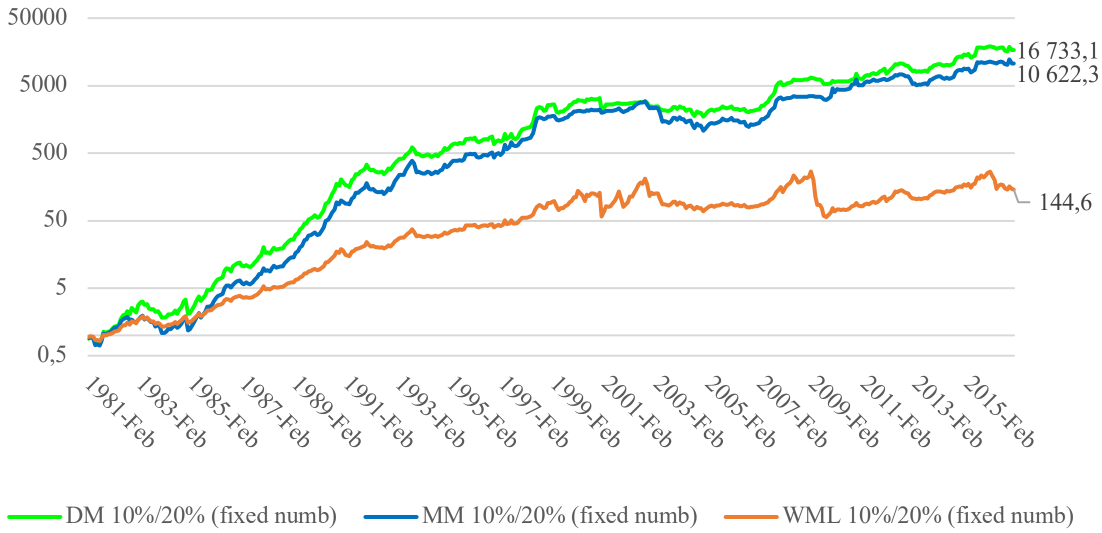
A closer look at the performance of the WML portfolios returns after implementing 10%/20% fixed number strategy can bring interesting findings (see Figure 7). Remarkably, when we compare Panel A of Figure 7 with Figure 1, it is evident that cumulative returns (without taking into account transaction costs) fall considerably after implementing the chosen cost mitigation strategy. For the simple buy-and-hold WML portfolio, returns in the end of 2016 would be 204 without the cost mitigation strategy and 145 with it. When it comes to the comparison for MM and DM strategies, the returns drop similarly with 35,159 and 22,157 being the cumulative returns at the end of 2016 (Figure 1) for DM and MM strategies, respectively, while with the implementation of the 10%/20% fixed number strategy the returns become 16,733 for DM and 10,622 for MM strategy (Panel A of Figure 7).

Accounting for transaction costs (which comprise bid-ask spreads, commission fees and short sale fees) leads to the opposite conclusion; returns after the transaction costs are improved considerably with the cost mitigation strategy implementation. A comparison of Panel B in Figure 7 and Panel C in Figure 4 supports this statement. The simple momentum strategy generates a loss, i.e. the cumulative return after the transaction costs in the end of 2016 is only 0.5 (see Panel C in Figure 4), however with the 10%/20% fixed number strategy the returns jump to 5.1 (Panel B of the Figure 7). The MM and DM strategy returns after the transaction costs in the end of 2016 go up from 2.7 and 4.9 without the cost mitigation strategy to 43 and 93, respectively, with the 10%/20% fixed number strategy.

Figure 7. Cumulative returns on WML portfolios formed with 10%/20% fixed number cost strategy

The plot depicts cumulative returns for simple momentum, MM and DM portfolios after imposing cost mitigation strategy 10%/20% and fixed number of stocks over the period 1981-2016. Panel A shows cumulative returns without transaction costs involved. Panel B shows cumulative return after the transaction costs which include bid-ask spreads estimated by Abdi and Ranaldo (2017), commission fees (different by years) and short-selling fees (1% per year).

Panel A: without transaction costs involved



Panel B: after transaction cost: bid-ask spreads + commission fees + short sale fees

Our findings are summarized in the Table 4, where we present cumulative returns, Sharpe ratios and Maximum drawdown after transaction costs for the simple buy-and-hold, MM and DM portfolios with and without different versions of buy/hold spread cost mitigation strategy. First, we can note that the Sharpe ratio for simple WML portfolios drops from 0.1486 to -0.0053 after transaction costs involved (compare the results for “Plain” in Table 4 to the results in Table 2). This is a striking result, which indicates that if investors utilize a simple momentum strategy without necessary attention to the transaction costs, the risk-adjusted returns on this portfolio will be lower than the risk-free rate (Sharpe ratio of simple momentum portfolio for monthly returns is -0.0053). This result also supports the idea proposed by Lesmond et al. (2004) that in reality, the momentum strategy does not generate excessive returns after considering transaction costs. If we impose volatility timing without explicitly addressing transaction costs, the Sharpe ratios are improved significantly versus the simple momentum (0.0324 and 0.0491 for MM and DM, respectively), but they are considerably lower than US Average Sharpe ratio (0.143) during the same period 1981-2016.

Involving cost mitigation strategies leads to a sharp rise in returns and Sharpe ratios. Thus, for the simple momentum portfolios Sharpe ratio goes up from -0.0053 to 0.0535 with 10%/20% fixed number strategy. For MM and DM strategies, Sharpe ratios go up from 0.0324 and 0.0491 to 0.1050 and 0.1257 respectively. However, even the highest Sharpe ratio of 0.1262 (DM strategy and 10%/30% fixed number cost mitigation strategy) falls behind the US average Sharpe ratio (0.143).

We should also mention Maximum drawdown and its changes after volatility timing strategies implementation. This indicator goes down considerably as a result of volatility timing strategy implementation, indicating that maximum loss of the portfolios decreased, and the strategy has become less risky. Maximum drawdown for the simple WML portfolio is 0.94, while for MM and DM is 0.77 and 0.63 accordingly, which means a decrease by 18% to 33%.

Appendix E contains the summary of the same indicators without the transaction costs and with the transaction costs including only bid-ask spreads.

Table 4. A summary of several indicators before and after applying volatility timing strategies and cost-mitigation strategies with transaction costs involved

The table presents the summary of returns, Sharpe rations and Maximum drawdown for WML portfolios after implementing volatility timing strategies (MM, DM approach) and imposing different buy/hold cost mitigation techniques over the period 1981-2016. The cumulative returns are after transaction costs which include bid-ask spreads estimated by Abdi and Ranaldo (2017), commission fees (different by years) and short-selling fees (1% per year).

		WML	MM	DM
Plain	*Cumulative returns*	0.5	2.7	4.9
	Sharpe ratio	-0.0053	0.0324	0.0491
	Max drawdown	0.9355	0.7699	0.6323
10%/20%	*Cumulative returns*	3.7	26.1	52.8
	Sharpe ratio	0.0402	0.0923	0.1125
	Max drawdown	0.8676	0.7328	0.5149
10%/30%	*Cumulative returns*	5.0	39.4	65.3
	Sharpe ratio	0.0456	0.1059	0.1213
	Max drawdown	0.7968	0.6568	0.5703
10%/20% fixed number	*Cumulative returns*	5.1	42.8	92.9
	Sharpe ratio	0.0535	0.1050	0.1257
	Max drawdown	0.8628	0.7532	0.6095
10%/30% fixed number	*Cumulative returns*	3.1	39.2	97.6
	Sharpe ratio	0.0429	0.1026	0.1262
	Max drawdown	0.8963	0.7212	0.6748

4.5. Robustness tests

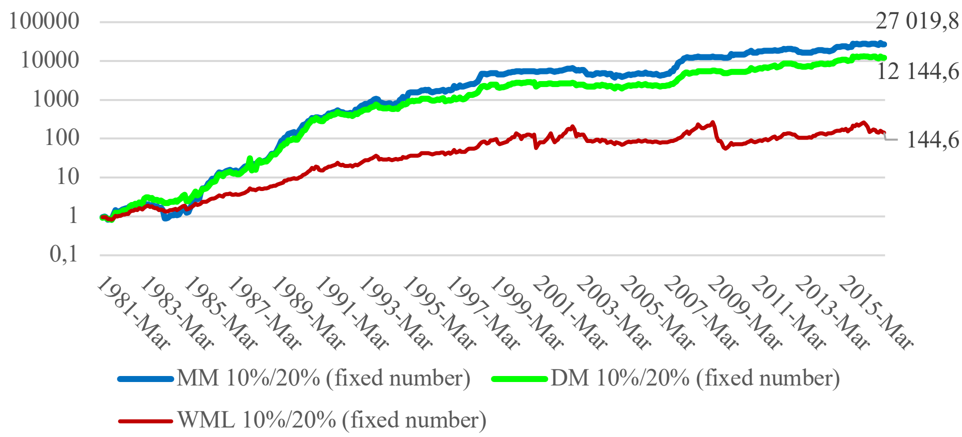
4.5.1. Restrictions on scaling weights

When putting any restrictions on the optimal portfolio weights while applying volatility timing strategies (i.e., on the value of w_t from equations (10) and (14)), we should be aware that even though the momentum crash problem is fixed, there is a lower gain in the portfolio returns during the stable periods since the portfolio weights cannot exceed the threshold. For instance, compared to Panel A in Figure 7 where the upper threshold value for scaling weights are set at 2, Panel A in Figure 8 shows that without upper bound for scaling weights, the cumulative returns by the end of 2016 increase significantly from 10,622 to 27,019 for MM portfolios. Interestingly, the cumulative returns on DM portfolios fall remarkably from 16,733 (with an upper bound of 2) to 12,144 (without upper bound).

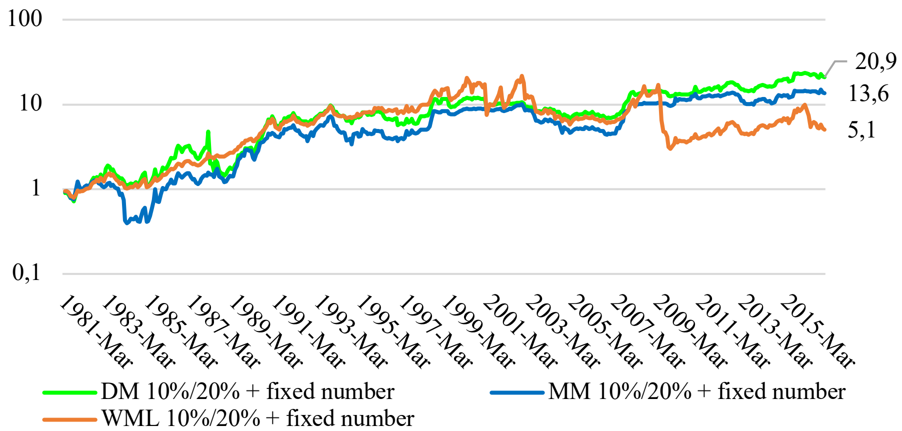
Figure 8. Cumulative returns on WML portfolios without upper bound for scaling weights

The figure shows the cumulative returns of the simple buy-and-hold, MM and DM portfolios without any restriction on the portfolio scaling weights over the period 1981-2016. Panel A demonstrates the cumulative returns without taking into account transaction costs while Panel B depicts the cost-adjusted cumulative returns when trading costs include bid-ask spreads, commission fees and short-selling fees (1% per year).

Panel A: without the transaction cost involved



Panel B: with the transaction costs involved: bid-ask spreads + commission fees + short sale fees



However, if there are no restrictions on the scaling weights in the volatility timing strategies, they can be abnormally large. Table 5 shows the maximum scaling weight over the sample period for different strategies. As it can be seen, the value can go as high as 10, implying that the investor should take a bet 10 times larger than what the simple momentum strategy takes. Such extreme values would lead to a striking rise in the transaction costs of the unrestricted MM and DM strategies over the period 1981-2016. As a result, the cumulative returns on these volatility-managed portfolios drop noticeably by approximately four times (compare Panel B in Figure 7 and in Figure 8). The aforementioned dramatic changes in the cumulative returns of the MM and DM portfolios, before and after the restriction is imposed on the scaling weights, question the feasibility of the unrestricted volatility timing strategies and justifies our decision to set an upper bound for the scaling weights. In order to keep the upper bound value not too low while preserving the feasibility of the strategies, we decided to choose the scaling bound value of 2 for our main analysis.

Table 5. The maximum scaling weights of the WML portfolios without the upper bound

The table presents the maximum values of the scaling weights (without any restriction on them) of both MM and DM portfolios before and after implementing 10%/20% fixed number cost strategy.

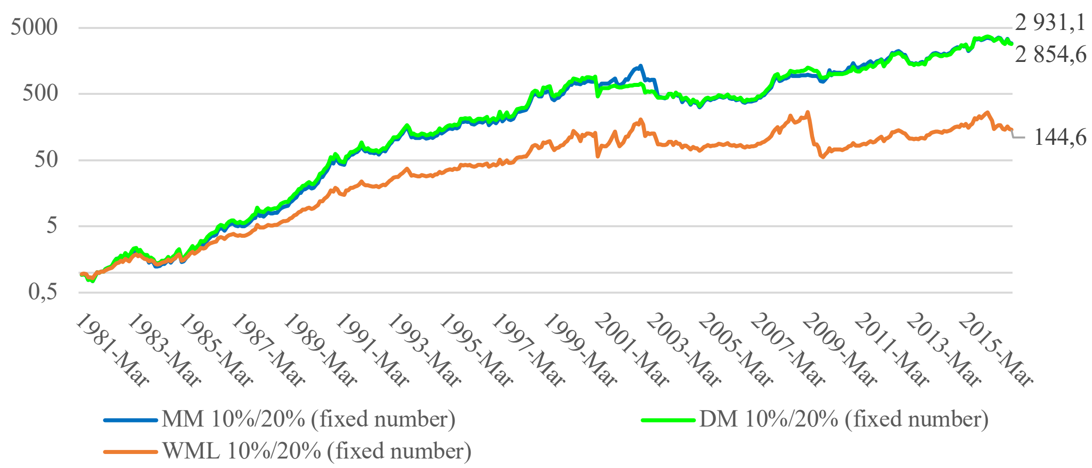
Strategy	Max scaling weight without restrictions
DM	9.95
DM 10%/20% Fixed number	10.04
MM	8.47
MM 10%/20% Fixed number	8.07

We also discuss the possibility of using a smaller upper bound. We expect that when we decrease the upper bound value, the additional gains in returns during good times decline. Hence, we decide to lower the upper bound of the scaling weights from 2 to 1.5 and take a look at the cumulative returns on the WML portfolios shown in Figure 9. It can be easily seen that the returns on WML portfolios for two momentum strategies show a big fall from above 10,000 (see Panel A Figure 7) to about 2,900 by the end of 2016 (see Panel A Figure 9). Notably, the drop in WML returns for DM strategy is larger than that for MM approach. In addition, the cumulative returns from the two volatility timing strategies are closer to each other. Thus, we predict that if the scaling bound gets smaller and approaches 1, the returns on WML portfolios of two volatility timing strategies not only become the same but also converge the returns on the simple buy-and-hold WML portfolio. We observe similar patterns after taking the transaction costs into consideration (see Panel B in Figure 7 and in Figure 9).

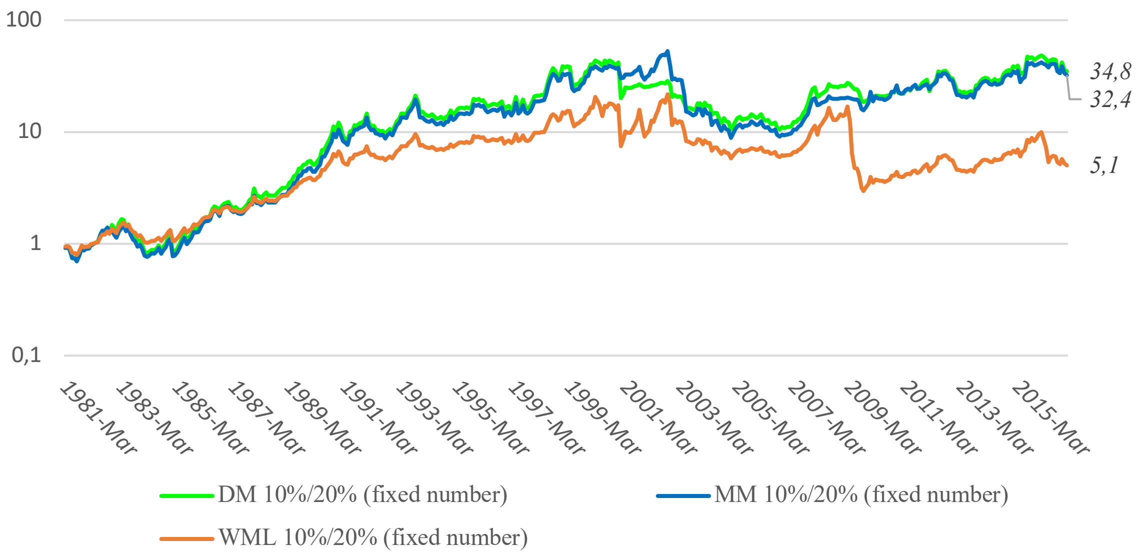
Figure 9. Cumulative returns on WML portfolios when combining both volatility timing strategies and cost strategy (10%/20% fixed number) with scaling bound of 1.5

The plots depict cumulative returns for simple momentum, MM and DM portfolios after imposing cost mitigation strategy 10%/20% and fixed number of stocks over the period 1981-2016. Panel A shows cumulative returns without transaction costs involved. Panel B shows cumulative return after the transaction costs which include bid-ask spreads estimated by Abdi and Ranaldo (2017), commission fees (different by years) and short-selling fees (1% per year). Noticeably, the upper threshold on the scaling factor is 1.5 for this figure compared to 2 in Figure 7.

Panel A: no transaction costs involved



Panel B: after transaction costs: bid-ask spreads + commission fees + short sale fees

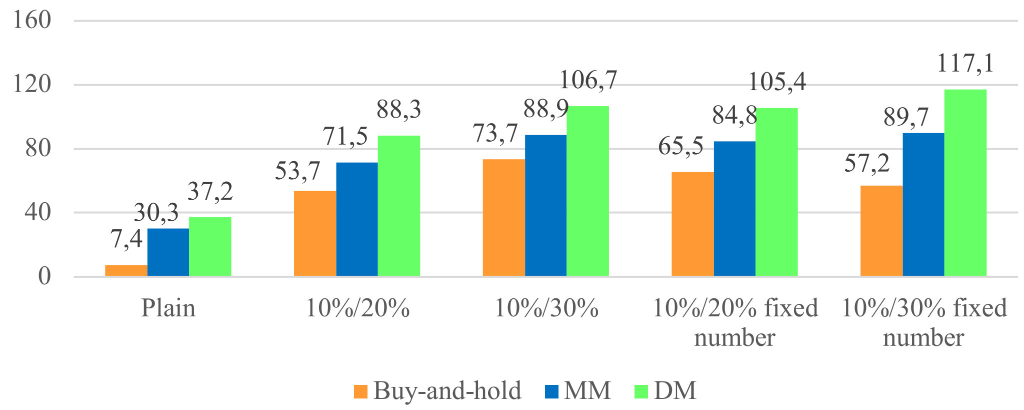


4.5.2. Break-even commission fees

As shown in previous sections, when transaction costs include both bid-ask spread estimates and commission fees, we observe a remarkable fall in the returns on the WML portfolios. However, with the implementation of the buy/hold cost-mitigation strategies, the returns on the volatility-managed WML portfolios still stay positive. Since the commission fees for the US markets take different values in various studies, we decide to do a further analysis on the level of commission fee at which the zero-cost WML portfolios break even when trading costs include the same bid-ask spread estimates by Abdi and Ranaldo (2017) and commission fees. The results are shown in the below table.

Figure 10. The break-even commission fees for the zero-cost WML portfolios before and after applying volatility timing and cost mitigation strategies, bps

The figure shows which commission fees should be added to bid-ask spread in order to break-even for different momentum portfolios (simple buy-and-hold momentum portfolio, MM, DM) over the period 1981-2016.



It is noteworthy that the results displayed in the table are the one-way commission fees per trade and are assumed to remain unchanged for the period 1981-2016. Compared to the commission fees shown in Table 1, the values shown in Table 6, on average, are much higher, except for the break-even commission fee for the simple buy-and-hold WML portfolio without cost-mitigation strategies. In addition, with the implementation of the cost mitigation strategies, the break-even levels increase obviously, once again supporting our finding that the buy/hold strategies help recover the returns that have been wiped away by the large trading costs of both the momentum and volatility timing strategies.

5. Conclusions

Our paper investigates two volatility timing strategies which are suggested by Moreira and Muir (2017) and Daniel and Moskowitz (2016) so as to alleviate the momentum crashes problem. The basic idea of the named strategies is to scale up or down momentum portfolio weights when the market is in calm and volatile states accordingly. The results of implementing two volatility timing strategies support the conclusions made earlier by the authors; the problem of momentum crashes is effectively solved, and the momentum returns subsequently go up compared to simple momentum strategy. The cumulative returns by the end of 2016 are 204, 22,157 and 33,159 for simple buy-and-hold momentum strategy, MM and DM volatility timing strategies respectively. Moreover, the Sharpe ratio goes up from 0.15 to 0.25 and 0.26 and the distribution of the returns become more normal with the implementation of volatility timing strategies.

In terms of transaction costs, we decided to investigate three versions of them: the first version includes only bid-ask spread estimates made by Abdi and Ranaldo (2017); the second version consists of both the first one and different commission fees by years (which nicely reflects the downward trend in total transaction costs over time); and the third version involves both the second one and an annual 1% commission fee only for short positions (which reflects that short positions suffer from higher transaction costs). Additional commissions do not change the picture much; after involving transaction costs, the momentum returns drop significantly by several orders of magnitude. Thus, if we include only bid-ask spreads, the cumulative return by the end of 2016 for the simple momentum strategy go down from 204 to 1.6. Having said that, we should note that though volatility timing strategies perform by large distance better than the simple momentum strategy, even after involving transaction costs (estimated with only bid-ask spreads) with the final cumulative returns of 21 and 37 for MM and DM strategies respectively, these figures do not seem extraordinarily high as before (without the transaction costs). Imposing additional commission fees and short-selling fees makes returns go down even steeper, to 2.7 and 4.9 for MM and DM momentum portfolios respectively and even not exceeding one for the simple momentum one. We should emphasize here that there has been no agreement about whether the momentum strategy pays off after the transaction costs. Some authors (Lesmond et al. (2004)) say that the strategy pays off anyway, others (Barroso et al. (2015), Moreira and Muir (2017)) are more inclined to think that the high turnover associated with the momentum strategy wipes away the returns. Our view is that though simple momentum strategy's returns after the transaction costs are questionable, additional techniques which not only cure momentum crashes but also mitigate high transaction costs are able to partly recover momentum returns.

Talking about the transaction costs mitigation strategies, we turned our attention to Novy-Marx and Velikov (2016)'s technique, which is based on a simple rule and helps decrease turnover of the portfolio significantly. We tested different modifications of the strategy (10%/20% and fixed number of stocks, 10%/30% and 10%/30% and fixed number of stocks) and chose the combination of 10%/20% and fixed number of stocks as it provides the best trade-off between the turnover reduction and decrease in portfolio returns. For example, for the simple momentum strategy, WML portfolio's final cumulative return (without the transaction costs) goes down from 204 to 145 after implementing cost mitigation technique, however at the same time the average turnover also decreases from 1.6 to 0.8. The consequences of imposing 10%/20% and fixed number cost mitigation strategy on volatility-managed portfolios are similar; the final cumulative returns on MM and DM momentum portfolios without transaction costs fall from 22,157 to 10,622, and from 35,159 to 16,733 respectively. Meanwhile, the average turnover declines from 2.3 to 1.4 for the former and from 2.3 to 1.2 for the latter.

When taking into account the transaction costs, the imposing of cost mitigation strategies leads to a boost in the final cumulative returns by 9-10 times. The final cumulative returns (after the highest transaction costs) for MM strategy increases from 2.7 to 42.8 after implementing the cost mitigation technique and for DM strategy, the comparison is 4.9 vs. 92.9. Similarly, the simple momentum strategy's cumulative return goes up from 0.5 to 5.1.

For further investigations, we believe that other volatility timing strategies can be explored, such as Barroso et al (2014)'s strategy or the adjusted approach of Moreira and Muir (2017) suggested by Chen et al (2019). Researchers could also consider looking at the transaction costs more attentively, testing other estimates of transaction costs and combining momentum strategy with other cost mitigation techniques (e.g. including the stocks based on their bid-ask spreads). Regarding international evidence, some authors (Fan et al (2018), Zaremba et al. (2018), master thesis by Ye and Österberg (2018)) checked results for consistency across the markets, however there is still room for additional analysis here.

References

Abdi, F. (2019, April 4). *Supplementary Data*. Retrieved from Farshid Abdi. Ph.D. Candidate in Finance: http://www.farshidabdi.net/data/index.html

Abdi, F., & Ranaldo, A. (2017). A Simple Estimation of Bid-Ask Spreads from Daily Close, High, and Low Prices. *Working Papers on Finance*.

Asness, C. S., Moskowitz, T. J., & Pedersen, L. H. (2013). Value and Momentum Everywhere. *Journal of Finance*, 929-985.

Barroso, P., & Santa-Clara, P. (2015). Momentum has its moments. *Journal of Financial Economics*, 111-120.

Brandt, M. W., Santa-Clara, P., & Valkanov, R. (2009). Parametric Portfolio Policies: Exploiting Characteristics in the Cross-Section of Equity Returns. *The Review of Financial Studies*, 3411-3447.

Chen, A.-S., Chang, H.-C., & Cheng, L.-Y. (2019). Time-varying Variance Scaling: Application of the Fractionally Integrated ARMA Model. *North American Journal of Economics and Finance*, 1-12.

Damodaran, A. (2019, April 2). *Trading Costs and Taxes - NYU Stern*. Retrieved from Stern School of Business at New York University: http://people.stern.nyu.edu/adamodar/pdfiles/invphiloh/tradingcosts.pdf

Daniel D., K., Jagannathan, R., & Kim, S. (2012). *Tail Risk in Momentum Strategy Returns.* New York: Working Paper, available at http://www.kentdaniel.net.

Daniel, K., & Moskowitz, T. J. (2016). Momentum crashes. *Journal of Financial Economics*, 221-247.

Desrosiers, S., L'Her, J.-F., & Plante, J.-F. (2007). Importance of style diversification for equity country selection. *Journal of Asset Management*, 188-199.

Do, B., & Faff, R. (2012). Are pairs trading profits robust to trading costs? *The Journal of Financial Research*, 261-287.

Fama, E. F., & French, K. R. (2012). Size, value, and momentum in international stock returns. *Journal of Financial Economics*, 457-472.

Fan, M., Li, Y., & Liu, J. (2018). Risk adjusted momentum strategies: A comparison between constant and dynamic volatility scaling approaches. *Research in International Business and Finance*, 131-140.

Fleming, J., Kirby, C., & Ostdiek, B. (2003). The Economic Value of Volatility Timing using "realized" volatility. *Journal of Financial Economics*, 473-509.

Foltice, B., & Langer, T. (2015). Profitable momentum trading strategies for individual investors. *Financial Markets and Portfolio Management*, 85-113.

Frazzini, A., Israel, R., & Moskowitz, T. J. (2014, March 28). *Trading Costs of Asset Pricing Anomalies*. Retrieved from SSRN´s eLibrary: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2294498

French, K. R. (2019, April 13). *Data Library*. Retrieved from Kenneth R. French: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, 1779-1801.

Grundy, B., & Martin, J. (2001). Understanding the nature of the risks and the source of the rewards to momentum investing. *Review of Financial Studies*, 29-78.

Investment Technology Group, i. (2019, April 2). *Global Cost Review, Q4/2018*. Retrieved from Investment Technology Group: https://www.virtu.com/uploads/2019/02/ITG-Global-Cost-Review-4Q18.pdf

Jegadeesh, N., & Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance*, 65-91.

Kociński, M. A. (2017). On Transaction Costs in Stock Trading. *Quantitative methods in economics*, 58-67.

Law, J. (2014). *A Dictionary of Finance and Banking.* Oxford University Press.

Lesmond, D., Schill, M., & Zhou, C. (2004). The illusory nature of momentum profits. *Journal of Financial Economics*, 349-380.

Li, X., Brooks, C., & Miffre, J. (2009). Low-cost momentum strategies.(Original Article). *Journal of Asset Management*, 366.

Li, X., Brooks, C., & Miffre, J. (2009). *Transaction Costs, Trading Volume and Momentum Strategies.* Whiteknights, UK: ICMA Centre Discussion Papers in Finance DP2009-04.

Mendes, B. V., & Lavrado, R. C. (2017). Implementing and testing the Maximum Drawdown at Risk. *Finance Research Letters*, 95-100.

Michael W., B., Pedro, S.-C., & Rossen, V. (2009). Parametric Portfolio Policies: Exploiting Characteristics in the Cross-Section of Equity Returns. *The Review of Financial Studies*.

Miffre, J., & Rallis, G. (2007). Momentum strategies in commodity futures markets. *Journal of Banking and Finance*, 1863-1886.

Moorman, T. (2014). An empirical investigation of methods to reduce transaction costs. *Journal of Empirical Finance*, 230-246.

Moreira, A., & Muir, T. (2017). Volatility-Managed Portfolios. *Journal of Finance*, 1611-1644.

Moskowitz, T. J. (1999). Do industries explain momentum? *The journal of finance: the journal of the American Finance Association*, 1249-1290.

Novy - Marx, R., & Velikov, M. (2016). A Taxonomy of Anomalies and Their Trading Costs. *The Review of Financial Studies*, 104-147.

Okunev, J., & White, D. (2003). Do Momentum-Based Strategies Still Work in Foreign Currency Markets? *Journal of Financial and Quantitative Analysis*, 425-447.

Rouwenhorst, K. G. (1998). International Momentum Strategies. *Journal of Finance*, 267-284.

Thomas, D. C., & Ellis, M. (2004). Momentum and the FTSE 350. *Journal of Asset Management*, 25-36.

Wharton, U. o. (2019, April 13). *WRDS Wharton Research Data Services*. Retrieved from Wharton Research Data Services: https://wrds-web.wharton.upenn.edu/wrds/

Zaremba, A., Umutlu, M., & Maydybura, A. (2018). Less pain, more gain: Volatility-adjusted residual momentum in international equity markets. *Investment Analysts*, 165-191.

Zhuang, C. (2018). Improving performance of exchange rate momentum strategy using volatility information. *Physica A*, 741-753.

Appendixes

A.

The first table presents the values of the constants λ and c shown in Equation (10) and (14) when we combine different versions of the buy/hold spread rule with Daniel and Moskowitz (2016)'s and Moreira and Muir (2017)'s volatility timing strategies respectively.

Table A1. Values of λ and c in Equation (10) and (14) before and after applying cost mitigation strategies

Buy/hold cost mitigation strategies	λ	c
Simple buy-and-hold	2.5133	0.0025
10%/20%	2.0689	0.0023
10%/30%	1.9712	0.0022
10%/20% + fixed number	2.2327	0.0021
10%/30% + fixed number	2.3115	0.0022

The second table shows the results of the regression model (13).

Table A2. Regression results of model (13)

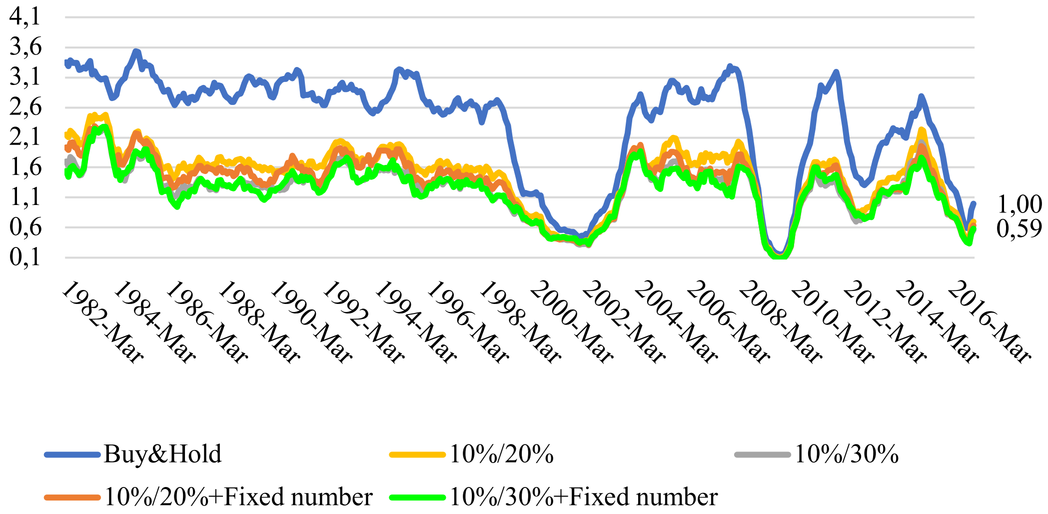
Estimate	WML	10%/20%	10%/30%	10%/20%+FN	10%/30%+FN
$\widehat{\gamma_0}$	0.0088	0.0068	0.0063	0.0063	0.0081
t-stat	*0.0048*	*0.0047*	*0.0043*	*0.0044*	*0.0051*
$\widehat{\gamma_B}$	-0.0064	-0.0095	-0.0088	-0.0082	-0.0135
t-stat	*1.0092*	*0.9861*	*0.9019*	*0.9309*	*1.0763*
$\widehat{\gamma_{\sigma_m^2}}$	63.1815	50.6076	45.2640	60.9224	46.8633
t-stat	*0.3881*	*0.3988*	*0.4399*	*0.4254*	*0.3585*
$\widehat{\gamma_{int}}$	-24.9061	-12.1499	-13.1695	-25.0207	-9.8968
t-stat	*0.0069*	*0.0059*	*0.0050*	*0.0071*	*0.0060*

B.

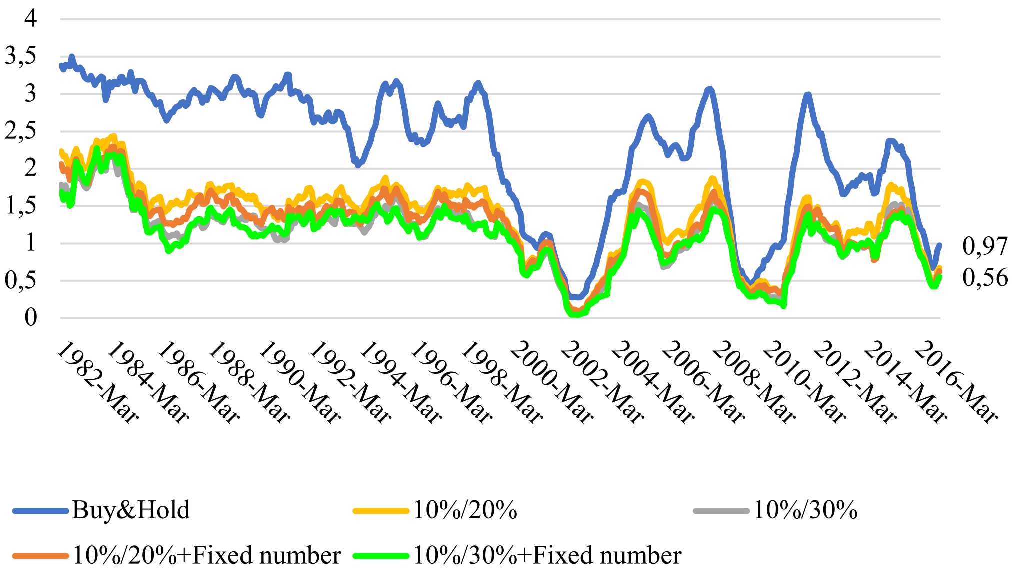
In Appendix B, we present the 12-month moving average turnovers on the zero-investment WML portfolios when we combine cost mitigation strategies with MM and DM volatility timing strategies respectively.

Figure B. Moving average turnovers for volatility timing strategies

Panel A: MM volatility timing strategy



Panel B: DM volatility timing strategy



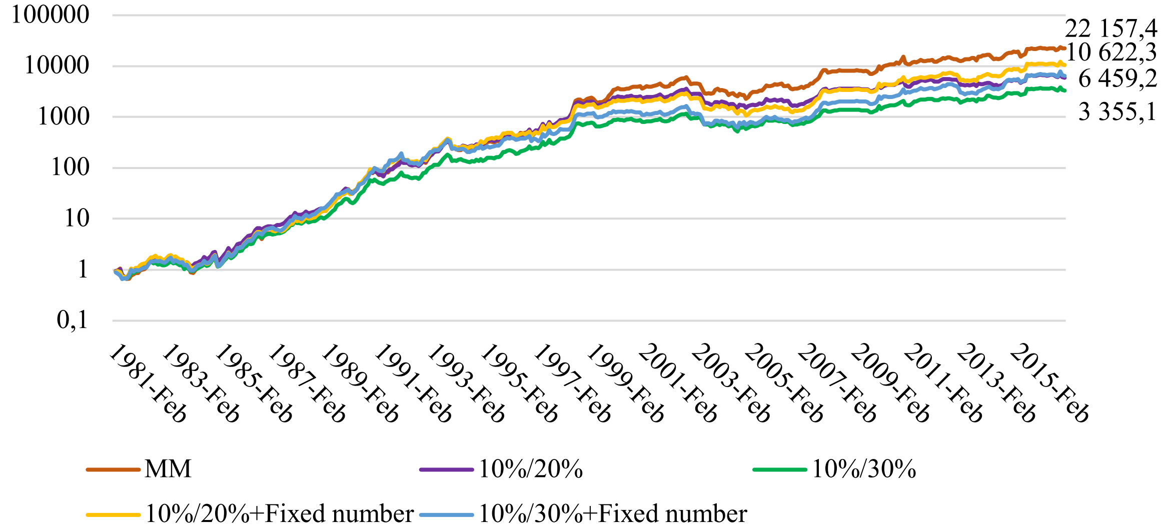
Strikingly there are longer periods of higher and lower turnovers for volatility timing strategies compared with the plain WML strategy turnover.

C.

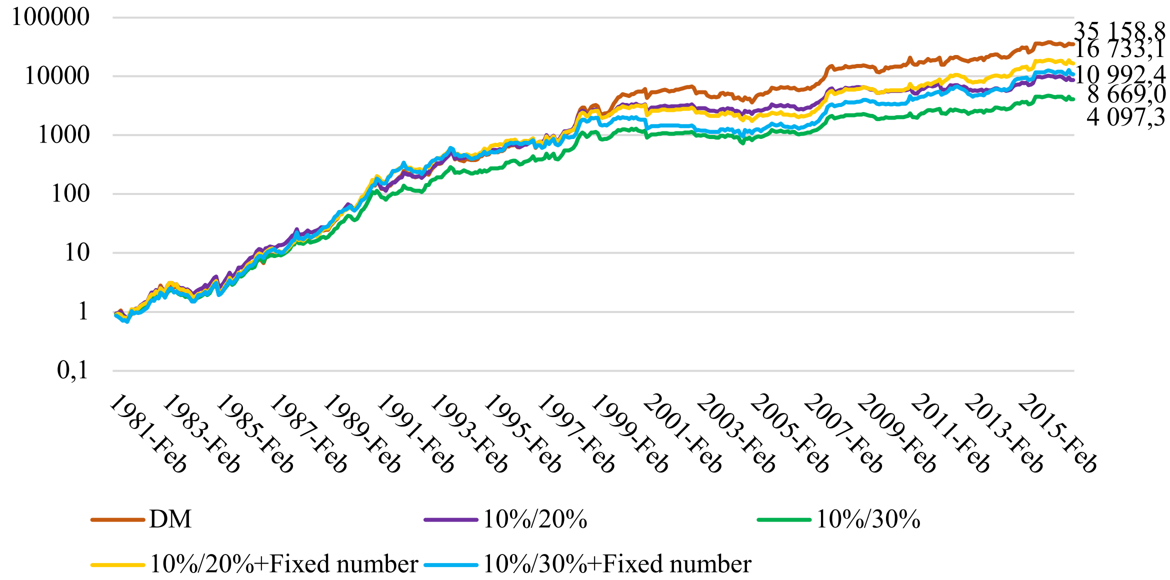
Appendix C shows the returns accumulated of the zero-cost WML portfolios before and after implementing both cost mitigation and volatility timing strategies without taking into account transaction costs over the sample period.

Figure C. Cumulative returns on WML portfolios without transaction costs

Panel A: MM volatility timing strategy



Panel B: DM volatility timing strategy

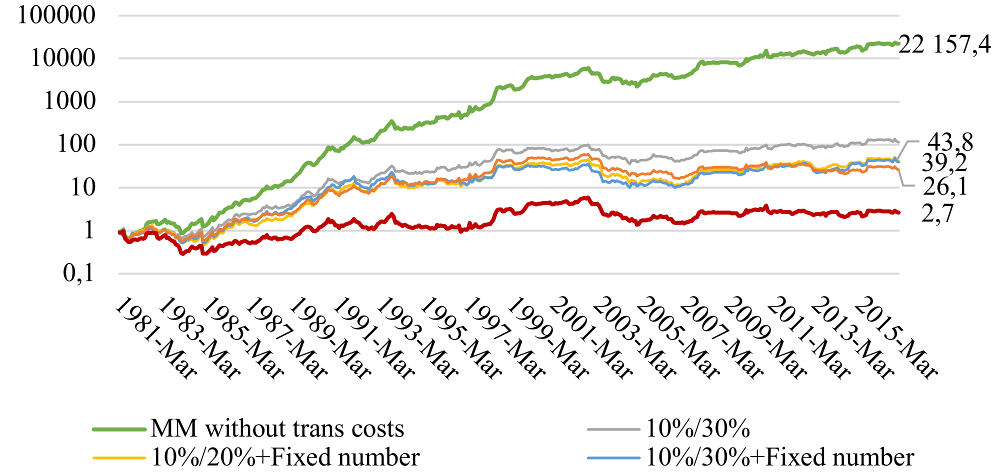


D.

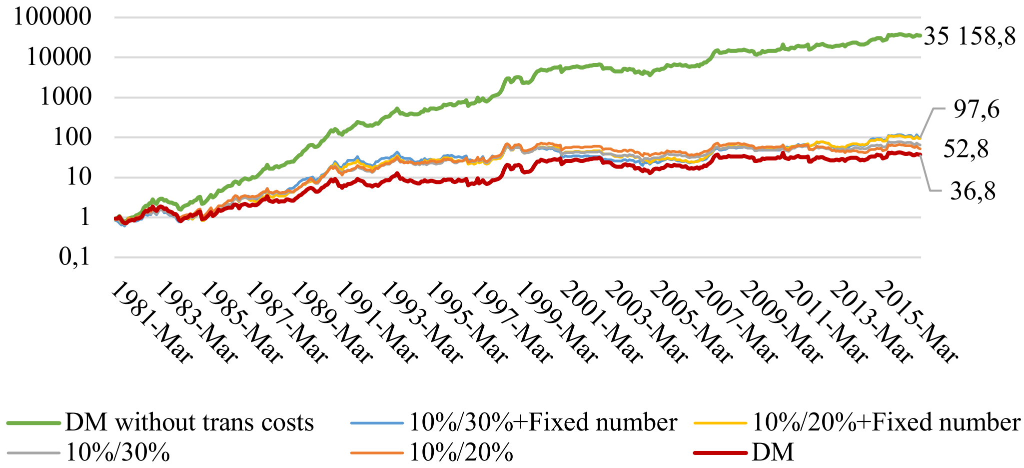
Similar to Appendix C, Appendix D illustrates the cumulative returns on the zero-investment WML portfolios before and after applying cost mitigation and volatility timing strategies from Jan 1981 to Dec 2016. However, figures in Appendix D show returns after involving transaction costs, which include bid-ask spreads and commission fees.

Figure D. Cumulative returns on WML portfolios after involving transaction costs

Panel A: MM volatility timing strategy



Panel B: DM volatility timing strategy



As can be easily observed, for both DM and MM strategies, taking into account transaction costs reduces the returns on the WML portfolios remarkably. However, with the implementation of cost mitigation strategies, the returns are noticeably recovered but still significantly lower than those without transaction costs.

E.

In Appendix E, we summarize the cumulative returns as well as two key performance measures (Sharpe Ratio and Maximum Drawdown) of the WML portfolios before and after applying cost mitigation and volatility timing strategies without transaction costs (Table E1) and with transaction costs including only bid-ask spread estimates introduced by Abdi and Ranaldo (2017) (Table E2).

Table E1. A summary of several indicators before and after applying volatility timing strategies and cost-mitigation strategies without transaction costs

		WML	MM	DM
Plain	*Cumulative returns*	204.4	22.157.4	35.158.8
	Sharpe ratio	0.1486	0.2663	0.2767
	Max drawdown	0.9355	0.7699	0.6323
10%/20%	*Cumulative returns*	97.4	5.942.1	8.669.0
	Sharpe ratio	0.1330	0.2482	0.2580
	Max drawdown	0.8052	0.5779	0.3540
10%/30%	*Cumulative returns*	63.8	3.355.1	4.097.3
	Sharpe ratio	0.1231	0.2427	0.2482
	Max drawdown	0.7567	0.5603	0.4315
10%/20% fixed number	*Cumulative returns*	144.6	10.622.3	16.733.1
	Sharpe ratio	0.1417	0.2523	0.2636
	Max drawdown	0.7895	0.6341	0.4618
10%/30% fixed number	*Cumulative returns*	60.5	6.459.2	10.992.4
	Sharpe ratio	0.1189	0.2363	0.2495
	Max drawdown	0.7980	0.6308	0.5258

If the returns and performance measure are calculated without involving transaction costs, then after implementing cost mitigation strategies they go down considerably.

Table E2. A summary of several indicators before and after applying volatility timing strategies and cost-mitigation strategies with transaction costs (only bid-ask spreads)

		WML	MM	DM
Plain	*Cumulative returns*	1.6	21.1	36.8
	Sharpe ratio	0.0268	0.0864	0.1011
	Max drawdown	0.9195	0.7443	0.5882
10%/20%	*Cumulative returns*	7.5	95.3	176.8
	Sharpe ratio	0.0601	0.1298	0.1473
	Max drawdown	0.8510	0.6956	0.4816
10%/30%	*Cumulative returns*	8.8	116.1	178.0
	Sharpe ratio	0.0624	0.1395	0.1523
	Max drawdown	0.7801	0.6376	0.5368
10%/20% fixed number	*Cumulative returns*	9.6	140.8	282.6
	Sharpe ratio	0.0705	0.1372	0.1555
	Max drawdown	0.8463	0.7228	0.5836
10%/30% fixed number	*Cumulative returns*	5.4	113.2	262.8
	Sharpe ratio	0.0569	0.1306	0.1523
	Max drawdown	0.8805	0.7051	0.6525

From the table, we can see that the patterns are similar to what we found in Table 4.