



UNIVERSITY OF GOTHENBURG
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Master of Science in Finance

Modeling the evolution of market uncertainty

**Hedge Fund returns and Volatility of Aggregate Volatility within a
dynamic perspective**

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Abstract

This paper investigates, in a dynamic perspective, whether uncertainty about equity market returns can have implications on hedge fund portfolio decisions over time. Therefore, the thesis wants to ascertain if the risk originated by that uncertainty is an explanatory factor for cross-sectional differences in returns over time. I develop this research employing an expanded version of the seven-factor Fung and Hsieh model (2004). To model exposures' time-variation, I use three different Generalized Autoregressive Score models where: (i) all loadings are time-varying; (ii) only volatility-of-aggregate-volatility loading is time-varying; (iii) selected loadings are time-varying. I analyze a 9,381 hedge funds sample in the period between January 1994 and December 2013 and I find a negative and significant relation between time-varying volatility-of-aggregate-volatility exposures and hedge fund returns. Results show that exposure to uncertainty about volatility is a priced factor in the cross-section of hedge fund returns at a 0.01 significance level. The use of the 'All time-varying parameters' GAS model improves hedge fund performance evaluation, highlighting a clear time-variation in the data. Results are robust to other volatility-of-aggregate-volatility proxies.

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Chapter 1: Introduction

Uncertainty about aggregate volatility in the equity market is likely to affect current economy, as it is heavily characterized by changing investment opportunities over time. Volatility-of-aggregate-volatility can be a source of risk for hedge fund returns, since investors aim to profit from state-contingent bets on dynamic trading conditions. Hedge funds promise to earn significant excess returns employing multiple sets of strategies. On the one hand, these are allegedly results of complex portfolio-construction and risk management methods; on the other hand, they are vulnerable to the effects of unexpected economic shocks.

This research aims to answer, within a dynamic perspective, the question whether better (worse) performance of hedge funds can be attributed to lower (higher) exposure to volatility-of-aggregate-volatility both in the cross-section and over time. Answering this question, I am also able to ascertain whether the estimation of time-variation in risk exposures positively contributes to improving hedge fund returns modeling.

This inquiry further develops the research of Agarwal, Arisoy and Naik (2017) who also investigate the relation of the change in aggregate volatility and hedge fund returns. Their analysis uses a general split sample and unrealistically assumes constant risk exposures within time windows. This research instead avoids making those assumptions and exploits a more advanced machinery to analyze this relation.

Following the framework delineated by Fung and Hsieh (2004), this thesis is composed of a time-series analysis of hedge fund portfolios risk exposures and a cross-section of individual returns. The former evaluates to what loadings hedge funds are responsive over time. The latter examines whether uncertainty in the market is a determinant of cross-sectional differences in hedge fund returns. This is estimated controlling for multiple individual fund characteristics such as the minimum investment period, expressly built for this study.

This thesis contributes to the extant literature by providing a new measure of volatility-of-aggregate-volatility, the *VOV* risk factor, built as the conditional volatility obtained by

fitting a t-GARCH(1,1) model to VIX index returns. The main innovation of this thesis is the use of the Generalized Autoregressive Score (GAS) model by Creal, Koopman and Lucas (2013) for time-varying parameters. I develop three different GAS models to estimate the loadings from an expanded version of the Fung and Hsieh (2004) seven-factor model: (i) all loadings are time-varying; (ii) only *VOV* loading is time-varying; (iii) selected loadings are time-varying.

In the first stage of analysis, I develop the aforementioned expanded time-varying version of the Fung and Hsieh (2004) seven-factor model. Overall, there is a clear time-variation in the data. I analyze eleven equally weighted strategy portfolios built upon 9,381 hedge funds from January 1994 to December 2013 and I find that all the three different time-series models regressions exhibit a significant and negative exposure to the *VOV* risk factor over time.

I also determine that using time-varying risk exposures improves hedge fund returns' estimation. The quality of estimated model is tested through the Akaike information criterion. This provides evidence supporting 'All time-varying parameters' GAS as the best model in terms of data fit and parameters parsimony in six out of the eleven strategy portfolios under inquiry. This constitutes an interesting addition to this stream of literature and mainly to the findings of Bollen and Whaley (2009).

In the second stage of the analysis, results show that funds' *VOV* betas have a negative exposure to hedge fund excess returns at a 0.01 significance level for all the three estimated models over the 20 years long sample. This finding happens to confirm what concluded by Agarwal, Arisoy and Naik (2017).

Individual fund characteristics determine cross-sectional differences in hedge fund returns as well. Among them, the more significant are: age, incentive fee structure, minimum investment requirement and presence of high water mark clause.

The obtained results are robust to the use of two alternative statistical measures of *VOV* risk factor, namely *RVIX* and *SDVIX*.

In conclusion, I find evidence that factor exposures vary over time. Despite differences in the estimation model and assumptions, this new research ascertains that findings of Agarwal, Arisoy and Naik (2017) are resistant to time-variation.

The remainder of this paper is organized as follows: Chapter 2 provides a primer on hedge funds and illustrates the development of literature on hedge fund return-generating process. Chapter 3 explains how uncertainty on market volatility is inserted in time-series and cross-sectional models. Chapter 4 presents data and details the construction of *VOV* risk factor and GAS models for time-varying parameters. Chapter 5 and 6, respectively, show the analysis of results and robustness checks. Chapter 7 concludes.

Chapter 2: Hedge fund performance

2.1 A primer on hedge funds

Before getting in the hearth of the research, this first section introduces hedge funds as an alternative investment vehicle.

Hedge funds are typically pooled funds, as they raise capital from multiple shareholders. This allows them to spread the high risk, arising from the various asset classes in which they invest, among several different investors. These funds promise to earn significant excess returns, employing numerous kinds of strategies with different levels of riskiness. Each fund is allegedly the result of complex portfolio-construction and risk management methods, built in such a way to exploit particular opportunities arising in the market at certain points in time, such as arbitrage and market mispricing opportunities. Due to this high strategic specialization, they are often classified according to their investment 'style'. Historically, this type of financial intermediary was named 'hedge fund' after the 'market neutrality' it sought in its early phase. Nowadays, even though this situation has been changing, the same name is still used (Connor and Woo, 2004).

Hedge funds distinguish themselves from other pooled funds, such as mutual funds, as they can include a wider range of assets in their investment portfolios. Given that the only formal restriction to hedge funds consists in what is stated in the mandate stipulated between investors and managers, managers can invest in any asset class potentially enlarging profits with respect to comparable funds (Agarwal, Mullally and Naik, 2015). Other distinguishing features are the use of leverage, short selling, and derivatives to amplify returns.

Hedge funds are generally regarded as private ventures, regulated in almost all the jurisdictions as limited partnerships. Investors assume the role of the limited partners, while fund managers are referred to as the general partners. Because of this structure, managers are not only highly involved in the fund supervision, but also in its performance. In some cases, managers are called to contribute a considerable amount of their personal wealth to initiate the venture, in order to better align their incentives.

Even if hedge funds are not exclusively open to a restricted number of qualified or institutional investors, the often high minimum investment requirement prevents the majority of unqualified investors from entering the venture (Fung and Hsieh, 1999). As a consequence, investors in hedge funds are made aware of the risks taken and may be able to actively monitor managers' decisions.

Furthermore, at the moment of investment shareholders commit to pay two fees to fund managers: a management fee and an incentive fee. This fee structure is also commonly referred to as the 'Two Twenty structure'. The 2% of managed funds is the yearly management fee that investors are entailed to pay. Managers can usually retain 20% of profits exceeding a hurdle rate, after having returned the entire investors' capital.

Since an hedge fund is a particular type of private equity funds, most of the information regarding its composition, its investors' identities and its returns is not disclosed to the public, mainly for privacy and anti-competition reasons. Nonetheless, the Security Exchange Commission (SEC) mandates hedge funds to register for: a specified range of securities; managers' names, especially when fund dimensions are considerable; and other few cases.

According to Hedge Fund Research (HFR, 2018), hedge funds currently cover a substantial part of the investment market: they recorded \$3.11 trillion of Assets Under Management (AUM) all over the world in the 2018 fourth quarter, highlighting a steady growth from the \$39 billion in 1990 (Fung and Hsieh, 1999). As stated by the Chartered Financial Analyst Institute (CFA Institute, 2018), in this same period the number of active hedge funds raised from 610 to 14,800: all those pieces of information together suggest that the interest regarding this pooled investment form spreads all over the professional environment as the size of the industry grows, pushing at the same time scholars' curiosity (Agarwal, Mullally and Naik, 2015).

Even if only the last decades recorded an important increase in the diffusion of these investment vehicles, hedge fund history actually started almost 70 years ago. In 1949, Albert Wislow Jones created the first pooled investment fund to be conceived in this particular way, where he actually put in practice the strategy which is now commonly referred to as *Long/Short Equity Hedge* and built the fee structure still used nowadays. This

‘style’ of management is based on being short/long on stocks expected to lose/acquire value to minimize market risk and maximize gains. The innovation passed nearly unnoticed until the last years of the 1960’s when Loomis (1966) wrote an article in *Forbes* magazine: he illustrated hedge funds functioning and their great potential in performing better than other asset classes, such as mutual funds. This sparked enthusiasm in the investors and just by 1968, according to Caldwell (1995), SEC recorded that 140 funds had been formed. Despite the initial burst, hedge funds experienced an abrupt spread slowdown in the years that followed, only to come back in the late 1980s with a mechanism similar to the one before. From the moment in which Rohrer (1986) reported in the *Institutional Investor* that the Julian Robertson’s Tiger Fund was able to gain an abnormal return of the 43% in 1986, the rise and the diffusion of this type of investment was reinvigorated.

2.2 Literature review

This paper aims to investigate the relation between hedge fund returns and volatility-of-aggregate-volatility within a dynamic perspective. This section illustrates the development of the hedge fund return-generating process literature and, then, recent additions in terms of new different risk factors in this stream of research. Among them, I focus in particular on the volatility-of-aggregate-volatility risk factor (Agarwal, Arisoy and Naik, 2017), which constitutes the basis of this research.

2.2.1 Hedge fund return-generating processes

A vast literature exists on how hedge fund return-generating processes and the performance evaluations function. The majority of studies on this topic use a linear multifactor model to analyze excess returns and to decompose them into different factors.

Generally, identified components in excess returns are classified among alpha and betas constituents: the former is the idiosyncratic asset characteristic, often attributed to managers’ ability; the latter refer to the return parts easily reproducible by an investor through portfolio replication.

Originally, most of hedge funds intended to reach market neutrality, achievable by including in portfolios those assets which are less correlated to the market and, consequently, less exposed to systematic risk factors resulting into a better performance. Fung and Hsieh (1997) and Liang (1999) argue that this is a profitable and safe strategy. However, the studies of Asness, Krail and Liew (2001), before, and of Bali, Brown and Caglayan (2012), later, refute these findings and determine that hedge fund performance is highly influenced by exposure to systematic risk.

Given this, some strands of literature have developed numerous methods to ascribe fund performance to different risk factors. They are mainly grouped into two veins: the ‘top-down’ and the ‘bottom-up’ approaches. The first one recognizes what factors, among the ones defined in previous studies, are able to explain hedge fund excess returns; the second one consists of replicating portfolios, trading in the hedge fund underlying assets. This last method is also referred to as the ‘Asset-Based Style’ (ABS) factors approach: it is originally developed by Fung and Hsieh (2002a) with the intent of predicting future returns, following a similar empirical implementation to the Capital Asset Pricing (CAPM) by Sharpe (1964) and Lintner (1965) and to the Arbitrage Pricing Theory (APT) by Ross (1976) models.

Fung and Hsieh apply this approach to identify a number of risk factors present in the different hedge fund strategies. They model ‘trend-following’ hedge fund returns using look-back straddles (Fung and Hsieh, 2001). They employ principal component analysis to fixed-income hedge funds to identify common roots of risks and returns (Fung and Hsieh, 2002b). They find that the spread between small versus large cap stocks is a factor of exposure for *Long/Short Equity Hedge* funds (Fung and Hsieh, 2011).

Bollen and Whaley (2009) develop an alternative methodology to quantify hedge fund alpha and betas components: this approach takes into account time-variation in the risk exposures. They compare different techniques to achieve this purpose: a rolling window regression, a stochastic autoregressive beta model, and an optimal changepoint regression model. They find significant evidence supporting that the last regression method is the most effective and that using time-varying risk exposures clearly contributes to funds’ alphas estimation, improving hedge fund returns predictions.

After this first wave of research, some authors stood out for having tried to capture broader influences on hedge fund returns. Bali, Brown and Caglayan (2011) propose to use macroeconomic risks factors, based, for instance, upon default premium and inflation rate, to better understand in a cross-sectional analysis the loadings of this investment vehicle returns. Bali, Brown and Caglayan (2014) build up on their previously stated model adding measures of macroeconomic uncertainty, just as the time-varying change in short-term interest rate, the default spread conditional volatility, and so on.

Avramov, Barras and Kosowski (2013) deepen this kind of analysis inspecting whether macroeconomic variables are able to predict future hedge fund individual returns. The study is able to confirm this intuition, together with ascertaining the presence of a significant causal relation between individual excess returns and the change in aggregate volatility.

2.2.2 Hedge funds and volatility-of-aggregate-volatility

Other scholars further develop the last finding. Anderson, Bianchi and Goldberg (2015) apply it to a broader category of assets, grouped into portfolios, by using an ex-post factor, which they refer to as FVIX. The factor is built as a time-varying portfolio of equities aiming to reproduce the daily movements in the Chicago Board Options Exchange Market Volatility Index (VIX).

Baltussen, van Bakkum and van der Grient (2018) create a volatility-of-aggregate-volatility factor using Implied Volatilities (IVs) from option prices. They analyze in this manner the mechanism of stock pricing, in view of the fact that IVs constitute a reliable measure of forward-looking stock returns' volatility.

Agarwal, Arisoy and Naik (2017) empirically analyze, both from a cross section and a time series perspectives, whether uncertainty about change in aggregate volatility in the equity market can be considered an explanatory factor for the hedge fund excess returns. This builds up on the fact that hedge fund positions can be juxtaposed with gamblers' bets: they are placed with the purpose of following rapid market changes through a dynamic model, exposing investors to a considerable amount of unpredictability and, consequently,

volatility in the results. Given that hedge funds generally try to earn excess returns exploiting particular market opportunities for short periods of time, Agarwal, Arisoy and Naik (2017) extend the classical Fung and Hsieh (2004) seven-factor model to incorporate their change of aggregate volatility risk factor.

In order to capture the effect of market uncertainty over hedge fund excess returns, they develop a volatility-of-aggregate-volatility risk factor, *VOV*. Implementing the methodology of Fung and Hsieh (2001), they estimate fund exposures to *VOV* by creating a look-back straddle option written on the VIX index, referred to as *LBVIX*.

Using this *VOV* factor in a static Ordinary Least Squares regression (OLS) model for a sample of 13,283 hedge funds between April 2006 and December 2012, they show that hedge fund returns are significantly and negatively exposed to the aggregate uncertainty of the market, especially during the 2008-2009 Great Financial Crisis period, and that this risk factor is also priced in the cross-section of returns.

Furthermore, the study from Agarwal, Arisoy and Naik (2017) constitutes one of the first investigation relating uncertainty about market volatility to hedge fund returns. Previous research treats unpredictability in other contexts, such as: Zhang (2006) which studies it with respect to quality of information in the market; Cremers and Yan (2016) and Pástor and Veronesi (2003) in the future profitability of firms; Bansal and Shaliastovich (2013) focusing on expected growth and inflation in the bond market.

In conclusion, it is possible to affirm that the literature about the hedge fund performance evaluation is extensive in its specification, ranging from the different methods used to identify factors to the diverse risk components, through the different estimation approaches.

Nevertheless, the main contribution of my research would be broadening this strand of knowledge through new perspectives and techniques. Avoiding to make unreasonable assumptions, such as static loadings or split sample, this thesis tries to recognize the effect of the change in aggregate volatility on these investment vehicle excess returns.

Chapter 3: Theoretical models

This research is composed by two main elements: a time-series analysis of hedge fund excess returns and a cross-sectional analysis. The former evaluates factor exposures, while the latter investigates the time variations of hedge fund exposures to market uncertainty and the deterministic differences in fund returns. The following sections introduce the methods used in each analysis.

3.1 Time-series analysis model

According to the framework delineated by Agarwal, Arisoy and Naik(2017), the time-series analysis I develop is an expanded version of the Fung and Hsieh (2004) seven-factor model. I investigate the relation between the volatility-of-aggregate-volatility and hedge fund excess returns.

Through their model, Fung and Hsieh (2004) decompose fund excess returns into several constituents: the risk-adjusted performance factor, α_i , and the single factor exposures, β_i^k . This method aims to explain a large portion of a well-diversified portfolio performance through the aforementioned risk factors. From a more operational point of view, the regression is a linear multi-factor model describing hedge fund returns. This regression heavily resembles the Fama and French (1992) three-factor model conceived to explain individual stock returns.

This model can be estimated with an OLS. The β_i^k s give information about magnitude and the direction of the exposures of excess returns:

$$\begin{aligned} r_{i,t}^e = & \alpha_i + \beta_i^{(1)} PTF_{SB_t} + \beta_i^{(2)} PTF_{SFX_t} + \beta_i^{(3)} PTF_{SCOM_t} + \beta_i^{(4)} BD10RET_t \\ & + \beta_i^{(5)} BAAMTSY_t + \beta_i^{(6)} SNPMRF_t + \beta_i^{(7)} SCMLC_t + \varepsilon_{i,t}, \end{aligned} \tag{1}$$

where: i represents the order of a particular hedge fund in the dataset; t is the month time index in the time series of observations; and $r_{i,t}^e$ is the monthly hedge fund return i in excess of the 1-month T-bill return, assumed to be the risk-free return. The regression in

Equation (1) is applied to all the different hedge funds for all the time periods present in the database.

The seven factors inserted in Equation (1) respectively correspond to: the bond trend following factor, the currency trend following factor, the commodity trend following factor, the equity market factor, the equity size spread factor, the bond market factor, and the bond size spread factor.

I follow the outline of Agarwal, Arisoy and Naik (2017) and extend the model in Equation (1) to include a new risk component describing the uncertainty of the equity market through the *VOV* factor. To let the research analyze the dynamic development of the single risk influences on excess returns, I update in the following way the model to accommodate possible time variations in exposures:

$$r_{i,t}^e = \alpha_{i,t} + \beta_{i,t}^{(1)} PTFSB_t + \beta_{i,t}^{(2)} PTFSTX_t + \beta_{i,t}^{(3)} PTFSCOM_t + \beta_{i,t}^{(4)} SNPMRF_t + \beta_{i,t}^{(5)} SCMLC_t + \beta_{i,t}^{(6)} BD10RET_t + \beta_{i,t}^{(7)} BAAMTSY_t + \beta_{i,t}^{(8)} VOV_t + \varepsilon_{i,t}, \quad (2)$$

where: *VOV* represents the volatility-of-aggregate volatility risk factor; *t* is the time index in months time series of observations, now, also applied to each parameter.

These constitute my contribution to the literature.

3.2 Cross-sectional analysis model

In this section, a multivariate cross-sectional approach for asset pricing tests whether the volatility-of-aggregate-volatility determines cross-sectional differences in hedge fund performance.

This model differs from the multivariate cross-sectional regression of Agarwal, Arisoy and Naik (2017): its innovation lies in the use of time-varying betas instead of the static parameters used in previous literature.

In my research the model follows a multivariate panel regression: the different estimated time-series of volatility-of-aggregate-volatility loadings are taken into account to estimate

their relation with hedge fund excess returns. Operating in this framework, I can ascertain the presence of dynamic trends in hedge fund return differences.

This regression not only allows for time-variation in the parameters produced for the different hedge fund categories in the time-series analysis, but it also controls for other effects, mainly idiosyncratic hedge fund characteristics.

I devote special attention to individual traits such as minimum investment period and leverage. The former is developed expressly in this analysis. This measures how much investors in hedge funds are required to wait for redeeming their shares. It is worth analyzing how this characteristic relates to the differences in individual fund returns since it is a signal of investment illiquidity and strategy riskiness. The latter is commonly used by hedge fund managers to put in practice more profitable strategies.

Equation (5) illustrates the general set of the multivariate panel regression:

$$\begin{aligned}
 r_{i,t}^e = & \lambda_{0,t} + \lambda_{VOV,t} \beta^{VOV}_{i,t} + \lambda_{MinInvPeriod,t} MinInvPeriod_{i,t} + \lambda_{Size,t} Size_{i,t} + \\
 & \lambda_{Age,t} Age_{i,t} + \lambda_{IncFee,t} IncFee_{i,t} + \lambda_{MgmtFee,t} MgmtFee_{i,t} + \\
 & \lambda_{MiniInvestment,t} MiniInvestment_{i,t} + \lambda_{Futures,t} Futures_{i,t} + \\
 & \lambda_{Derivatives,t} Derivatives_{i,t} + \lambda_{MaxLeverage,t} MaxLeverage_{i,t} + \\
 & \lambda_{Leverage,t} Leverage_{i,t} + \lambda_{AvgLeverage,t} AvgLeverage_{i,t} + \\
 & \lambda_{HighWaterMark,t} HighWaterMark_{i,t} + \epsilon_{i,t+1}
 \end{aligned}
 \tag{5}$$

The left-hand side of the Equation (5) represents the excess hedge fund return, meaning that yield investors claim to gain in order to bear an additional risk; the right-hand side includes risk measures and factor risk prices, namely the λ s.

The different risk measures and the panel regression functioning will be illustrated in the next chapter

Chapter 4: Data and methodology

implementation

In the first part of this chapter I describe hedge fund returns, their characteristics and data sources. In the second, I show how risk factors building with a special attention to *VOV*. Lastly, the methods I use to autonomously develop different regression models for dynamic time-series and cross-section of returns are explained.

4.1 Hedge fund database

I use Thomson Reuters Lipper TASS hedge fund data, which includes hedge fund returns and characteristics. The full database contains information for more than 20,000 hedge funds collected over the 240 months from January 1994 to April 2014. When imposing some restrictions, such as treating outliers in the first and last twentieth percentiles, the sample results to contain 534,116 observations for 9,381 hedge funds. This set is framed in the period going from January 1994 to December 2013 to have a more complete sample of data.

For the time-series regression, hedge fund returns are pooled into eleven equally weighted portfolios each corresponding to one of the eleven ‘styles’. By doing so, I am able to compute a return observation for every month in the sample and corresponding to the particular strategy followed by the funds. Below, I describe the eleven different ‘styles’ and I also report the number of funds using the mentioned strategy together with their average survival period:

1. *Convertible Arbitrage* (247 funds, 4 years and 3 months): a long position in a convertible security and a short position in the underlying asset are assumed to take advantage of pricing inefficiencies;
2. *Dedicated Short Bias* (43 funds, 4 years and 4 months): it is a directional trading strategy in which the investor exposes herself to the market to gain profits during bearing market periods by being short on securities characterized by a selling side;

3. *Emerging Markets* (853 funds, 4 years and 1 months): the investment majority is reserved to securities coming from countries in their emerging growth development moment, to have a high risk exposure, as well as high profitability;
4. *Equity Market Neutral* (579 funds, 4 years and 6 months): investments are made for matching long and short positions in order to profit both from bulling and bearing times and to avoid the specific market risk;
5. *Event Driven* (658 funds, 4 years and 1 months): by exploiting their superior market knowledge, managers try to profit from short-lived shares mispricing moments in which specific corporate events take place, i.e. Mergers & Acquisitions, restructuring, bankruptcy, etc.
6. *Fixed-Income Arbitrage* (394 funds, 4 years and 1 months): the main focus consists in realizing gains from pricing discrepancies between the different interest rate securities, regardless of the state of the market;
7. *Global Macro* (718 funds, 4 years and 3 months): hedge funds invest according to predictions market reactions to significant macro-economic events over national, continental and global scenarios: i.e. being short (long) on markets expected to bear (bull) as a specific event consequence;
8. *Long/Short Equity Hedge* (3,189 funds, 4 years and 6 months): to minimize market risk exposures and maximize overall gains, hedge fund managers include in their portfolios those stocks expected to appreciate and they liquidate those securities whose price is predicted to decline;
9. *Managed Futures* (858 funds, 4 years and 4 months): following Modern Portfolio Theory (Markowitz,1952), managers aim to achieve both portfolio and market diversification by including in their investment baskets derivative instruments, such as futures, which, generally, record inverted tendencies in performance compared to stocks and bonds;
10. *Multi-Strategy* (1,791 funds, 3 years and 6 months): as it evidently appears from its name, this kind of investment vehicles includes in its portfolio stakes of different strategies that hedge funds use to obtain diversification and flexibility;
11. *Options Strategy* (51 funds, 3years and 11 months): managers enter the option market assuming multiple positions to achieve coverage and stability of their investments, meet specific performance goals, and, eventually, gain leverage.

Observing data about numbers of funds belonging to each strategy, *Long/Short Equity Hedge* ‘style’ emerges as the most popular with 34% of the funds in the sample. The second most widespread fund ‘style’ is *Multi-Strategy* with 19% share, while the third is *Managed Futures* with only 9% share.

As previously illustrated, I use different control variables in the cross-sectional analysis, mainly different hedge funds individual characteristics. In the following paragraph more information is provided.

Variables used in the analysis as controls are: the new measure of minimum investment period I develop following the rules presented below; lagged values of AUM for size to avoid bias and autocorrelation effects; age measured in years from inception date; incentive fee as a percentage of annual gains; management fee as a fixed percentage of AUM; minimum investment required to enter the fund; a dummy for funds investing in futures; a dummy for the use of derivatives; the amount of maximum leverage; a dummy for leverage; the amount of average leverage; and a dummy for presence of a high water mark¹.

The minimum investment period combines different information contained in the dataset. I calculate it as the sum of the following three components: the lockup, the payout and the redemption frequency periods. These variables express the time constraints investors face from the moment of investment to the complete capital redemption. In general, this minimum investment period is defined as the shortest possible amount of time the investor has to wait to withdraw the entire investment without being subject to any early redemption fee.

The lockup period is defined as the time length in which investors are restricted from withdrawing their initial fund investment. In this paper I measure this interval of time in months and I consider it elapsed only when no expense is charged for early withdrawal. When computing the lockup period I make the following adjustments to the raw data: if a percentage is expressed without a time period, it is assumed that the percentage refers to the first 12 months from entrance; if ‘see notes’ is specified as a comment the period is automatically assumed to be zero months because precise data differently stating is absent;

¹ Namely, a clause requiring hedge funds to recover any losses before an incentive fee is provided for outperformance

if 'no lockup' is specified as a comment the period is considered to be of zero months, regardless of the number provided in the original database.

The redemption frequency period measures how often investors are allowed to redeem their shares. To calculate this, the investor is assumed to have her investment redeemed as soon as possible.

The payout period corresponds to the time investor needs to wait from the official redemption moment to the entire investment reception. I make only two adjustments to the raw data to ensure that no performance fee is due: if a percentage of investment withdrawal is expressed for a certain time period, the payout is assumed to happen just after the time in which obligation is no more binding; and, if a percentage is expressed without a time period, I assume that the stated percentage has to be paid in the first month, as general evidence shows this amount of time to be the most often indicated by the funds.

Table 1 reports summary statistics for the elements of cross-sectional analysis.

Table 1

Summary Statistics for Hedge Funds Thomson Reuters Lipper TASS Database

Summary statistics for the period January 1994–December 2013 (full sample) for a total of 9,381 funds. *Ex.returns* are the monthly hedge fund percent returns in excess of the risk-free rate, estimated as the 1-year T-bill. *MinInvPeriod* is the shortest time the investor has to wait to be able to withdraw the entire investment without the obligation of pay any early redemption fee (in months). *LagAUM* are the 1-year lagged values of Assets Under Management (AUM) are in millions of dollars. All AUMs denominated in currencies other than USD are converted using month-end exchange rates provided by Datastream. *Age* is the number of years from fund inception. *IncFee* is a fixed percentage fee of fund’s net asset value. *MgmtFee* is a fixed percentage fee of AUM. *MinInvestment* is the minimum requirement to enter a fund in millions of dollars. *Futures* is a dummy for funds investing in futures. *Derivatives* is a dummy for funds investing in derivatives. *MaxLeverage* is the amount of maximum leverage expressed as a percentage of equity. *Leverage* is a dummy for the funds using leverage. *AvgLeverage* is the average amount of leverage as a percentage of equity. *HighWaterM* is a dummy for funds with high water mark clause. Values rounded to the 4th decimal place.

Fund Characteristic	Mean	Standard Deviation	Min	Median	Max	N. of obs.
<i>Ex.returns</i> (monthly %)	0.5212	4.2885	-18.6998	0.5341	20.2344	534,116
<i>MinInvPeriod</i> (months)	7.8914	9.1359	0.0	3.0	96.0000	269,268
<i>LagAUM</i> (\$100m)	1.2301	2.6737	0.0001	0.2821	16.9343	524,804
<i>Age</i> (years)	4.0064	3.4561	0.0	3.01	19.11	534,116
<i>IncFee</i> (%)	17.4152	6.9238	0.0	20.0	50.0	503,666
<i>MgmtFee</i> (%)	1.4463	0.7106	0.0	1.5	22.0	531,410
<i>MinInvestment</i> (\$ 100 millions)	0.0074	0.0418	0.0	0.0025	10.0	531,636
<i>Futures</i> (dum. 0-1)	0.1529	0.3598	0.0	0.0	1.0	534,116
<i>Derivatives</i> (dum. 0-1)	0.1363	0.3432	0.0	0.0	1.0	534,116
<i>MaxLeverage</i> (%)	111.053	257.9046	0.0	0.0	8,000.0	374,145
<i>Leverage</i> (dummy 0-1)	0.6160	0.4863	0.0	1.0	1.0	534,116
<i>AvgLeverage</i> (%)	54.2855	169.0434	-40.0	0.0	6,000.0	374,145
<i>HighWaterM</i> (dum. 0-1)	0.6226	0.4847	0.0	1.0	1.0	534,116

As shown in Table 1, the full-sample presents an average return slightly lower than the median suggesting that results are negatively skewed. The same cannot be asserted for the minimum investment period and lagged AUM which are portrayed by highly skewed to the left distributions: this is obviously due to the non-negativity restriction characterizing them. Management fees show a smaller variation over the entire sample in comparison to the incentive fee. The majority of funds uses leverage, while only a small fraction of hedge funds includes derivatives in their portfolio.

4.2 Hedge fund risk factors

This section illustrates how I obtain the risk factors used in time-series regression of Equation (2).

The three trend-following risk factors are bond trend following factor, currency trend following factor, and commodity trend following factor, referred to from now on, respectively, as *PTFSB*, *PTFSFX* and *PTFSCOM*. Those are built as the returns from portfolios containing look-back straddle options, respectively written upon bonds, currencies, and commodities. Subsequently to Fung and Hsieh (2001) development of those factors, David Hsieh created a constantly updating online database, which I use to obtain the data.

The two equity-oriented risk factors, the equity market factor and the equity size spread factor, are downloaded from the Kenneth R. French Data Library. The former, *SNPMRF*, is the excess return on the market portfolio (Mkt-Rf); while the latter, *SCMLC*, is the size factor (Small Minus Big, SMB).

To construct bond-oriented risk factors, the bond market factor and the bond size spread factor, I download data from FRED website and Moody's Baa website. The first one, referred to as *BD10RET*, is computed using the bond yields for the ten-years T-bill constant maturity; while the second, referred to as *BAAMTSY*, is the monthly change in yield difference between Moody's Baa bonds and the Treasury rates.

The data necessary to develop the last factor *VOV* are retrieved, as well as the T-bill yields, from FRED.

Given that the time-series model chosen is linear, it is important to investigate the linear relation between risk factors. Table 2 reports Pearson correlations among factors.

Table 2

Pearson correlation among factors

This table reports the Pearson linear correlation coefficients for the risk components of the modified version of the Fung and Hsieh seven factor plus *VOV* model. *PTFSB*, *PTFSFX*, *PTFSCOM* are the bond, currency and commodity trend following factors as defined in Fung and Hsieh (2004). *SNPMRF* is the excess return on the market portfolio. *SCMLC* is the size factor. *BD10RET* is the monthly change in the 10-years T-bill constant maturity bond yields. *BAAMTSY* is the monthly change in difference in yield between Moody's Baa bonds and the Treasury rates. *VOV* is the conditional volatility obtained from fitting a t-GARCH(1,1) model to VIX index demeaned log-returns. *, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels, respectively, for the p-values. Values rounded to the 3rd decimal place.

Factors	<i>PTFSB</i>	<i>PTFSFX</i>	<i>PTFSCOM</i>	<i>SNPMRF</i>	<i>SCMLC</i>	<i>BD10RET</i>	<i>BAAMTSY</i>	<i>VOV</i>
<i>PTFSB</i>	1							
<i>PTFSFX</i>	0.319**	1						
<i>PTFSCOM</i>	0.252***	0.39 ***	1					
<i>SNPMRF</i>	-0.301**	-0.256*	-0.203*	1				
<i>SCMLC</i>	-0.101*	-0.019*	-0.072*	0.282***	1			
<i>BD10RET</i>	-0.331**	-0.171**	-0.132**	0.306*	0.209**	1		
<i>BAAMTSY</i>	0.264***	0.363***	0.218**	-0.527**	-0.265**	-0.511**	1	
<i>VOV</i>	0.192***	0.078***	0.042***	-0.233**	-0.119**	-0.236**	0.351***	1

The Pearson coefficient measures strength and direction of the pairwise correlation between the presented factors. *VOV* factor is especially correlated to bond market and size-spread elements with, respectively, negative and positive signs. There is no considerable multicollinearity problem due to the not extremely high correlations. As visible from Table 2, all components exhibit a significant linear correlation.

4.3 Construction of *VOV* risk factor

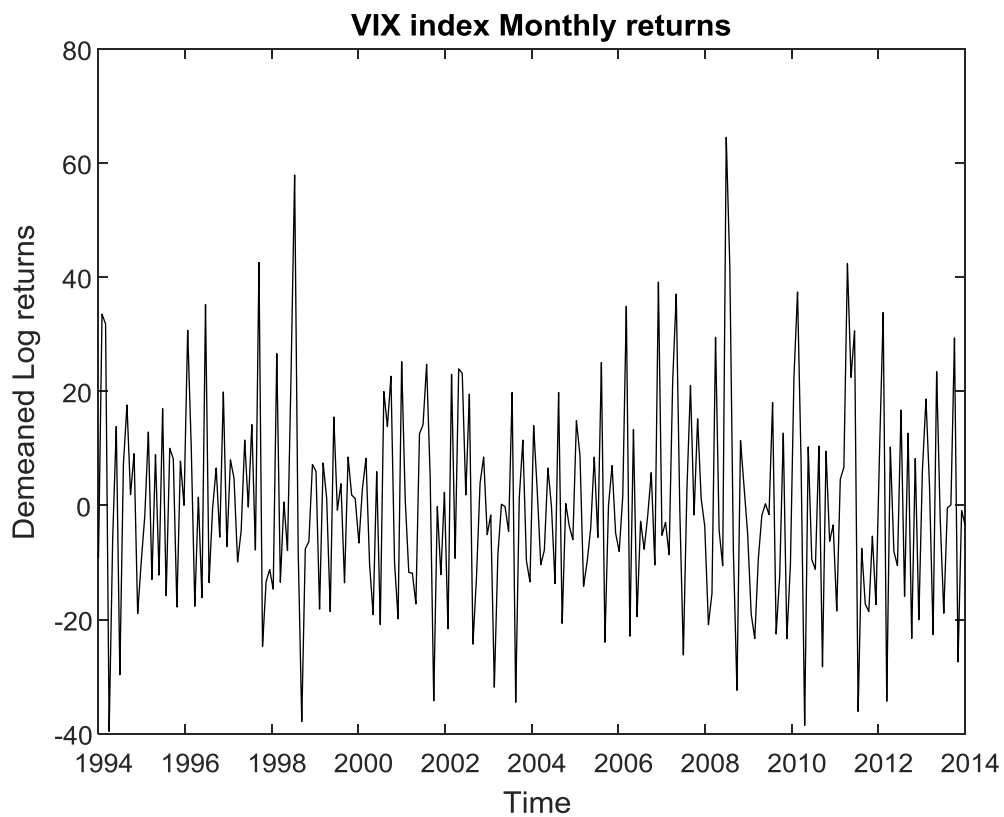
In this analysis, the volatility-of-aggregate-volatility is measured by the *VOV* risk factor. I build it starting from VIX index demeaned monthly log-returns, as they provide a

reasonable measure of aggregate volatility in a near-term perspective. Figure 1 depicts data on which estimation is conducted:

Figure 1

Demeaned VIX index returns (January 1994-April 2014)

This image plots the time-series of VIX index monthly returns for the period between January 1994 to April 2014. Data are obtained from FRED. Returns are demeaned and logged in order to have a stable series of observations over time.



Since VIX measures only volatility, I calculate the volatility-of-aggregate-volatility risk factor by modeling these returns as the conditional variance of a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model by Bollerslev (1986) and Engle (2001). By doing so, I obtain a measure of volatility changes in the equity market which strongly differs from simple volatility, as the following lines simply point out. There can exist situations characterized at the same time by both a considerable amount of risk,

i.e. VIX is high, and a low *VOV*. This happens when VIX is persistent and does not change too much from period to period. There can also be situations where VIX exhibit large changes and *VOV* is elevated, as the market transitions from low risk to high risk regimes and back.

GARCH model is used in econometrics when it is possible to represent innovation in the time series of data by a function of its past observations. I select this model since it efficiently describes financial time series characterized by a changing volatility over time.

In practice, a GARCH model is used to fit VIX returns. By doing so, I obtain the necessary parameters to infer conditional volatility, the *VOV* factor. In general, the innovation is assumed to be normally distributed with zero mean and a variance equal to σ_t^2 .

Hereafter, to make sure that the factors are both positive and negative, I use calculate the log-returns. To make sure that the autocorrelation between time series data is eliminated, I demean the log-returns. I make different trials to determine which model specification better fits the data. First, the GARCH(1,1) model with Normally distributed innovations is used. Then, a GARCH(1,1) model with Student's *t* distribution is estimated. Afterwards, different GARCH model specifications with varying number of lags are considered.

With the purpose of assessing the best model specification between the different GARCH models, I calculate an information criterion. Information criteria include two components: an inverse function of model fit measure and a function proportionally increasing with the number of parameters. The most valid candidate is identified in the model with the lowest value of the chosen class of information criteria: this represents the best compromise between fit and parsimony. According to the Akaike Information Criterion (AIC) (Akaike, 1974), the best model is the simple GARCH(1,1) model with Student's *t* distribution.

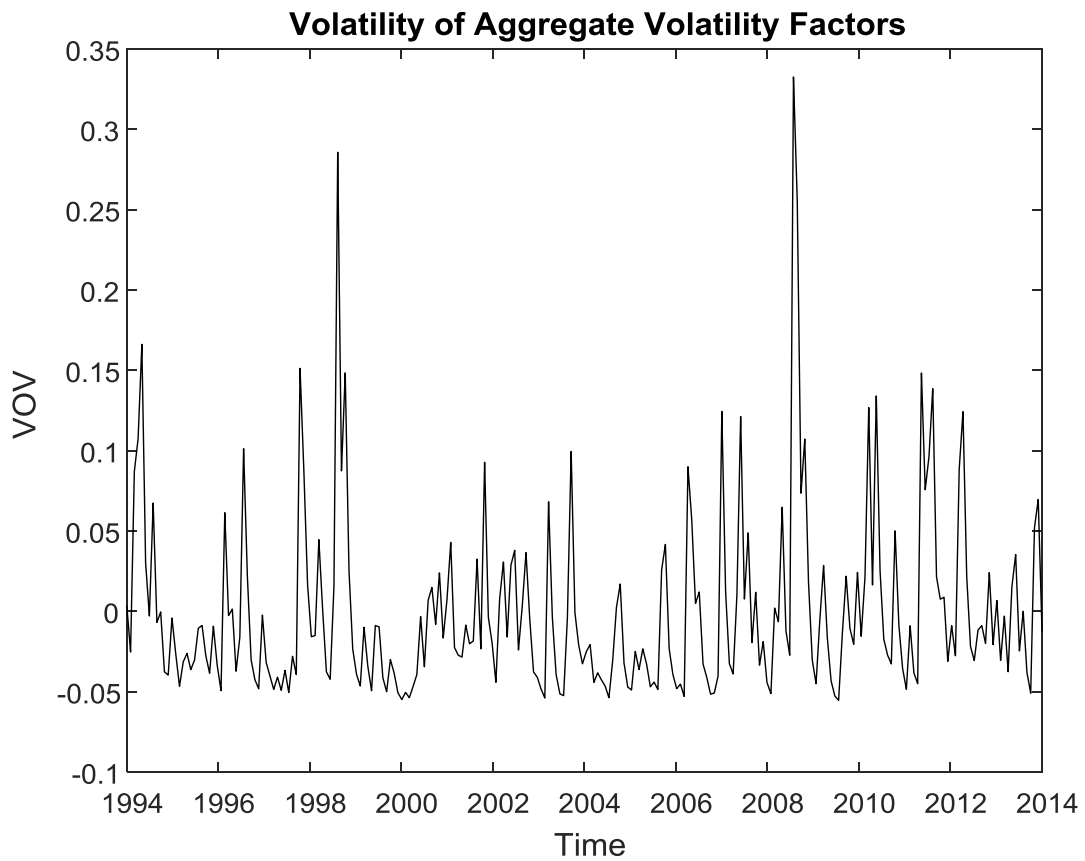
To validate the obtained risk factors, I perform a set of nested tests, such as the Ljung-Box test for residual autocorrelation and the Jarque-Bera test for Normality of standardized residuals. Those tests provide significant evidence supporting the validity of GARCH(1,1) model with Student's *t* distribution model as the best fitting model. Appendix 1 reports details for the test-statistics and their results.

Below, Figure 3 plots the time-series of the *VOV* risk factors. As shown, the change in volatility is on average positive and it peaks during the late 90's and the 2008-2009 Great Financial crisis.

Figure 2

Volatility of Aggregate Volatility Factors

This image plots the time-series of *VOV* factors for the period between January 1994 to April 2014. *t*-GARCH(1,1) model estimation is based on VIX index returns data obtained from FRED.



Compared to the ABS approach used by Agarwal, Arisoy and Naik (2017) to measure volatility-of-aggregate-volatility, the method just described is arguably more intuitive and it requires a lower computational effort and avoids to make many predictions and assumptions.

4.4 Generalized Autoregressive Score model

The main innovation in this research lies in the regression method. I use the Generalized Autoregressive Score (GAS) by Creal, Koopman and Lucas (2013) which allows for intuitive modeling time-varying parameters.

The main advantage of this method is that it provides an integrated structure to obtain dynamic parameters, as part of the non-linear model class. In the classification of Cox (1981), GAS is an ‘observation-driven method’, unlike state space models which are ‘parameter driven’. In observation-driven models, like GAS or GARCH, parameters’ time variation is modeled through functions of lagged response variables together with predetermined variables. In this way, it is possible to perfectly predict the one-step ahead parameters’ values analyzing only the set of available information. This results in a more computationally friendly estimation of maximum likelihood. In the parameter-driven framework, parameters are modeled as autonomous stochastic processes. In those models estimations and predictions often require computationally expensive methods of simulation, such as in the case of Stochastic Volatility model (Heston, 1993) in which the likelihood function is not available in closed form.

From a more practical point of view, the dynamic nature of parameters is given by an updating mechanism of the scaled score likelihood function, also referred to as the predictive model density at time t . In this model, the likelihood evaluation is straightforward and it takes into account all the information in the complete density structure for parameters’ estimation, unlike those models that include distribution moments in their method of regression.

In the next two sections, I illustrate how GAS model mechanism works and how it is application in this research.

4.4.1 *General illustration*

Let f_t , θ , and \mathcal{F}_t denote, respectively, the time-varying parameters, a vector of static parameters and the information set, containing past observations for dependent and

independent variables and parameters. For a time-series of observations, y_t , built as a $N \times 1$ vector, is distributed as:

$$y_t \sim p(y_t | f_t, \mathcal{F}_t; \theta) \quad (6)$$

Creal, Koopman, and Lucas (2013) propose to use the following updating mechanism for time-varying parameters:

$$f_{t+1} = \omega + \sum_{i=1}^p A_i s_{t-i+1} + \sum_{i=1}^p B_i f_{t-i+1}, \quad (7)$$

where: ω is a vector of constants, A_i and B_j are coefficient matrices with customizable dimensions, and s_t is the scaled score of the likelihood function;

The three static parameters, ω , A_i and B_j , are contained in θ which needs to be estimated with a specific optimization function to obtain the updating mechanism of the main time-varying parameters.

The innovation or driving mechanism in the factor recursion is given by s_t :

$$s_t = S_t \cdot \nabla_{t,f_t}, \quad (8)$$

where:

$$\nabla_{t,f_t} = \frac{\partial \ln p(y_t | f_t, \mathcal{F}_t; \theta)}{\partial f_t}, \quad (9)$$

and

$$S_t = S(t, f_t, \mathcal{F}_t; \theta) = J_{t,1,f_t}^{-1} = E_{t-1} [\nabla_{t,f_t}^2] = -E_{t-1} \left[\frac{\partial^2 \ln p^2(y_t | f_t, \mathcal{F}_t; \theta)}{\partial f_t^2} \right]. \quad (10)$$

4.4.2 Time-series application

In this section, I show how expressly for this paper I build the basic general GAS model to estimate time-varying parameters for the time-series regression shown in Equation (2) and how I realize the different specifications.

For notation convenience, I group the asset-based factors into the X matrix :

$$y_t = X_t' \beta_t + \varepsilon_t \quad (11)$$

All the error terms are assumed to be independent and identically distributed according to a Normal distribution:

$$\varepsilon_t \sim N(0, \sigma^2)$$

The conditional likelihood function is computed as:

$$\mathcal{L}_t(\beta_t, \sigma^2; y_t, x_t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2} \frac{(y_t - x_t' \beta_t)^2}{\sigma^2}} \quad (12)$$

The log-likelihood function is:

$$l_t(\beta_t, \sigma^2; y_t, x_t) = -\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln \sigma^2 - \frac{1}{2} \frac{(y_t - x_t' \beta_t)^2}{\sigma^2} \quad (13)$$

Having derived the necessary log-likelihood function, I can start calculating the innovation mechanism used in GAS, namely s_t . The first derivative of Equation (13) is:

$$\nabla_{t,f_t} = \frac{\partial l_t(\beta_t, \sigma^2; y_t, x_t)}{\partial \beta_t'} = \frac{1}{\sigma^2} (y_t - x_t' \beta_t) x_t \quad (14)$$

The inverse Fisher information matrix is obtained by:

$$\begin{aligned} S_t &= S(t, f_t, \mathcal{F}_t; \theta) = \mathcal{J}_{t,1,f_t}^{-1} = E_{t-1} \left[\nabla_{t,f_t} \nabla_{t,f_t}' \right]^{-1} \\ &= \left(E_{t-1} \left[\frac{1}{\sigma^2} x_t (y_t - x_t' \beta_t) (y_t - x_t' \beta_t)' x_t \frac{1}{\sigma^2} \right] \right)^{-1} \\ &= \left(\frac{1}{(\sigma^2)^2} E_{t-1}(x_t \varepsilon_t \varepsilon_t' x_t') \right)^{-1} \end{aligned}$$

$$\begin{aligned}
&= \left(\frac{1}{(\sigma^2)^2} \sigma^2 E_{t-1}(x_t x_t') \right)^{-1} \\
&= \sigma^2 (E_{t-1}(x_t x_t'))^{-1}
\end{aligned} \tag{15}$$

In general, homoskedasticity cannot be assumed for the $(x_t x_t')^{-1}$ matrix since market risk factors do not move together. In addition, VOV factor is modeled as an autoregressive model for conditional heteroskedasticity due to its persistent variance. Therefore, I model the inverse Fisher information matrix as an Exponentially Weighted Moving Average (EWMA) with a fixed parameter λ , assumed to be 0.98:

$$\begin{aligned}
S_t &= \lambda S_{t-1} + (1 - \lambda)(x_t x_t')^{-1} \sigma^2 \\
\frac{S_t}{\sigma^2} &= \lambda \frac{S_{t-1}}{\sigma^2} + (1 - \lambda)(x_t x_t')^{-1}
\end{aligned} \tag{16}$$

The driving mechanism is then defined as:

$$\begin{aligned}
s_t &= S_t \cdot \nabla_{t,f_t} \\
&= S_t \cdot \left[\frac{1}{\sigma^2} (y_t - x_t' \beta_t) x_t \right]
\end{aligned} \tag{17}$$

The derived updating score function has the following form:

$$\beta_{t+1} = \omega + B (\beta_t - \omega) + A \left\{ S_t \cdot \left[\frac{1}{\sigma^2} (y_t - x_t' \beta_t) x_t \right] \right\} \tag{18}$$

4.4.3 GAS parameters estimation

Static parameters in θ are estimated with maximum likelihood. To initialize this process, consistent starting values for θ and initial $\beta_{t=0}$ must be selected. In the following paragraphs, I explain how I pick these initial parameters.

The first component in θ is ω , which corresponds to a vector of initial values for the time-varying parameters with a length equal to the number of risk factors used in the time-series regression. Those are estimated with OLS for a static version of the model.

The second and third elements are A and B which I assume to be diagonal square matrices. On their main diagonal, they, respectively, have low and close to 1 values which are always selected to sum to 1 (integrated GAS model). Modifying the order and the position of values on those two matrices diagonals, it is possible to model diverse GAS specifications each one differently describing how risk factors affect hedge fund excess returns. In particular, for $A_i = 0$ and $B_i = 0$, I obtain a model with parameter i assumed to be static.

The last parameter to be estimated as part of θ is the variance of the error term, σ^2 .

The different GAS models I develop are: (i) all loadings are time-varying; (ii) only *VOV* loading is time-varying; (iii) selected loadings are time-varying.

The first GAS model estimates all the risk factors loadings as time-varying with the purpose of estimating all the movements in their relations with excess returns. To achieve this, the initial values for A and B are set different from zero.

According to the second model specification, the loading of the *VOV* factor is the only to vary over time, among all the estimated by Maximum Likelihood ones. In this case, I impose all the parameters in A and B to be zeros, to model static parameters, with the exception for A_i and B_i corresponding to the *VOV* factor.

The third and last estimation builds upon the results obtained from the first GAS with all time-varying loadings. Here, I select factors whose relation with hedge fund excess returns clearly appears to be time-varying.

Below, I provide an example: let A and B be two diagonal matrices for a GAS model. Provided that this specification is modeled to estimate only two loadings out of three as time-varying, and having in mind that the order of the elements on the main diagonal correspond to the order of the factors, I would have the following matrices:

$$A = \begin{matrix} a_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_j \end{matrix}; \quad B = \begin{matrix} b_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_j \end{matrix}.$$

The zeros on the main diagonal correspond to those parameters assumed to be static. Other parameters are allowed to be time-varying.

4.5 Panel data regression

In the cross-sectional regression of excess returns, the panel data is made up of observations for each of the individual hedge funds. For each investment vehicle, I report time-series for excess returns, the obtained exposures to the *VOV* factor and individual hedge fund characteristics. A panel data regression is used to control for the individual heterogeneity over repeated observations.

This analysis aims to investigate the relation underlying hedge funds excess returns and the time-varying volatility-of-aggregate-volatility.

Having estimated the three GAS models time-series regression, I extract the three time-series of the time-varying loadings for *VOV* factor. In the time-series analysis, I obtain eleven time-series of *VOV* loadings, one for each of the equally weighted strategy portfolios created. I merge each of them to the original dataset containing hedge fund returns and individual characteristics by linking each investment vehicle to its exposure according to its style and the time of observation². By doing so, I obtain three unbalanced panel datasets.

For each of the panel datasets, I estimate a linear regression of the following form:

$$y_{i,t} = \alpha_i + \sum_{j=1}^N x_{i,t,j} \beta_j + \epsilon_{i,t} + \eta_k + \xi_t$$

(19)

² This approach was chosen for time constraint reasons. Otherwise, it would have been better to estimate a time series analysis for all of the 9,381 hedge funds present in the sample to be able to obtain more precise panel regression results.

Following Siegmann, Stefanova and Zamojski (2013), I select as the method of estimation the Generalized Least Square (GLS) regression with time-fixed, ξ_t , and style-fixed effects, η_k . Standard errors are clustered by individual hedge funds.

In this way, I apply the estimation method, just stated, to every time series of parameters obtained by the three GAS model specifications for each of the eleven investment strategies.

Chapter 5: Results

In this chapter, I present the results obtained from the previously illustrated regression models: I show the necessary evidence to understand either the relation underlying hedge fund excess returns and the change in aggregate volatility, or the cross-section of returns of these investment vehicles.

5.1 Hedge funds performance time-series analysis

In this first stage of analysis, I regress time-series data to examine the exposures of the eleven strategy equally weighted portfolios of returns to the volatility-of-aggregate-volatility risk factor.

For brevity sake, I first present the results regarding the portfolio built on the most popular strategy, namely that *Long/ Short Equity Hedge* management ‘style’ followed by 3,189 funds. Then, I delineate a broader picture of factor exposures over time by providing a brief general overview for each of the eleven strategies under inquiry.

5.1.1 *All time-varying parameters estimation*

First, general insights gathered from the obtained results resemble the findings of Agarwal, Arisoy and Naik (2017). This suggests that the method I employ in this study to measure *VOV* is a valid alternative for the one used in previous research. At this stage, I elaborate on how loadings changed over time for the three different GAS estimation models.

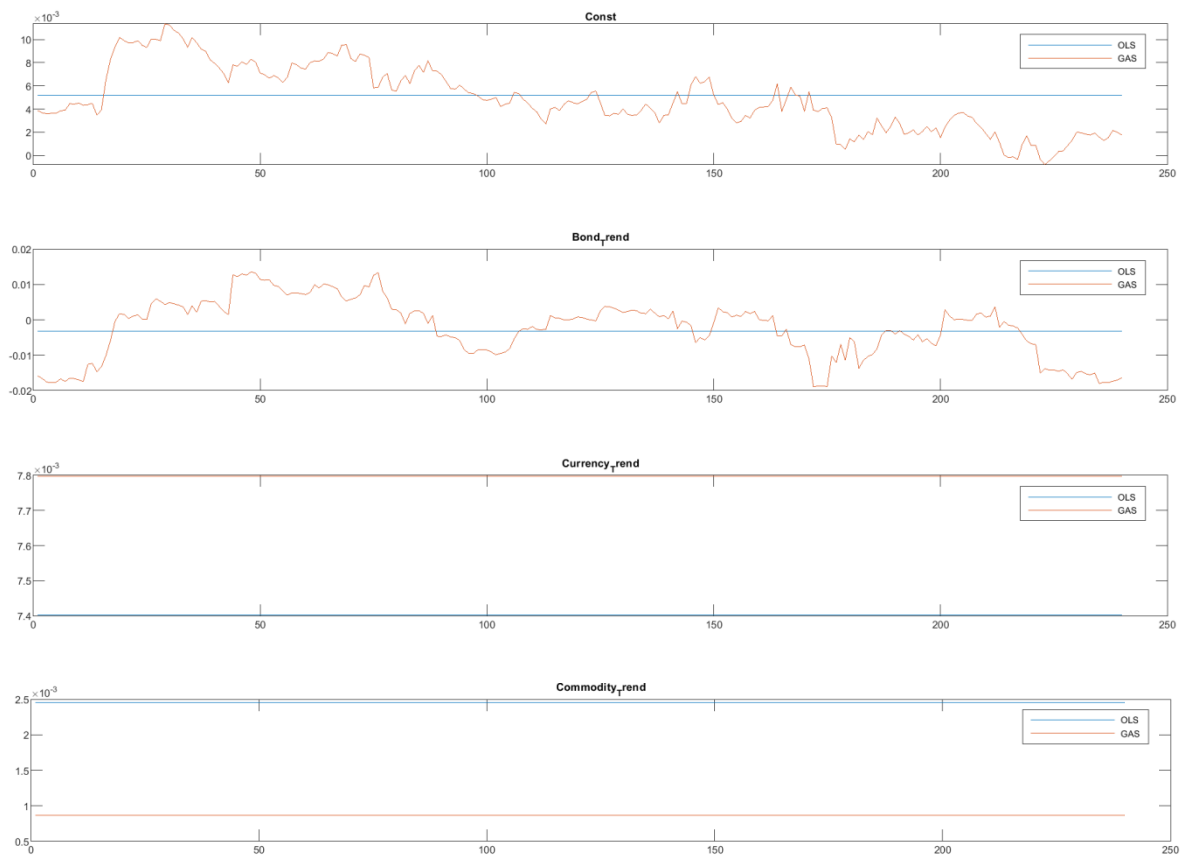
I fit code at the *Long/Short Equity Hedge* strategy. When all parameters are allowed to change over time, not all risk factors loadings actually exhibit dynamic trends. In general, it is clear that the aggregate exposure of hedge fund excess returns to risk factors has been changing during the two analyzed decades.

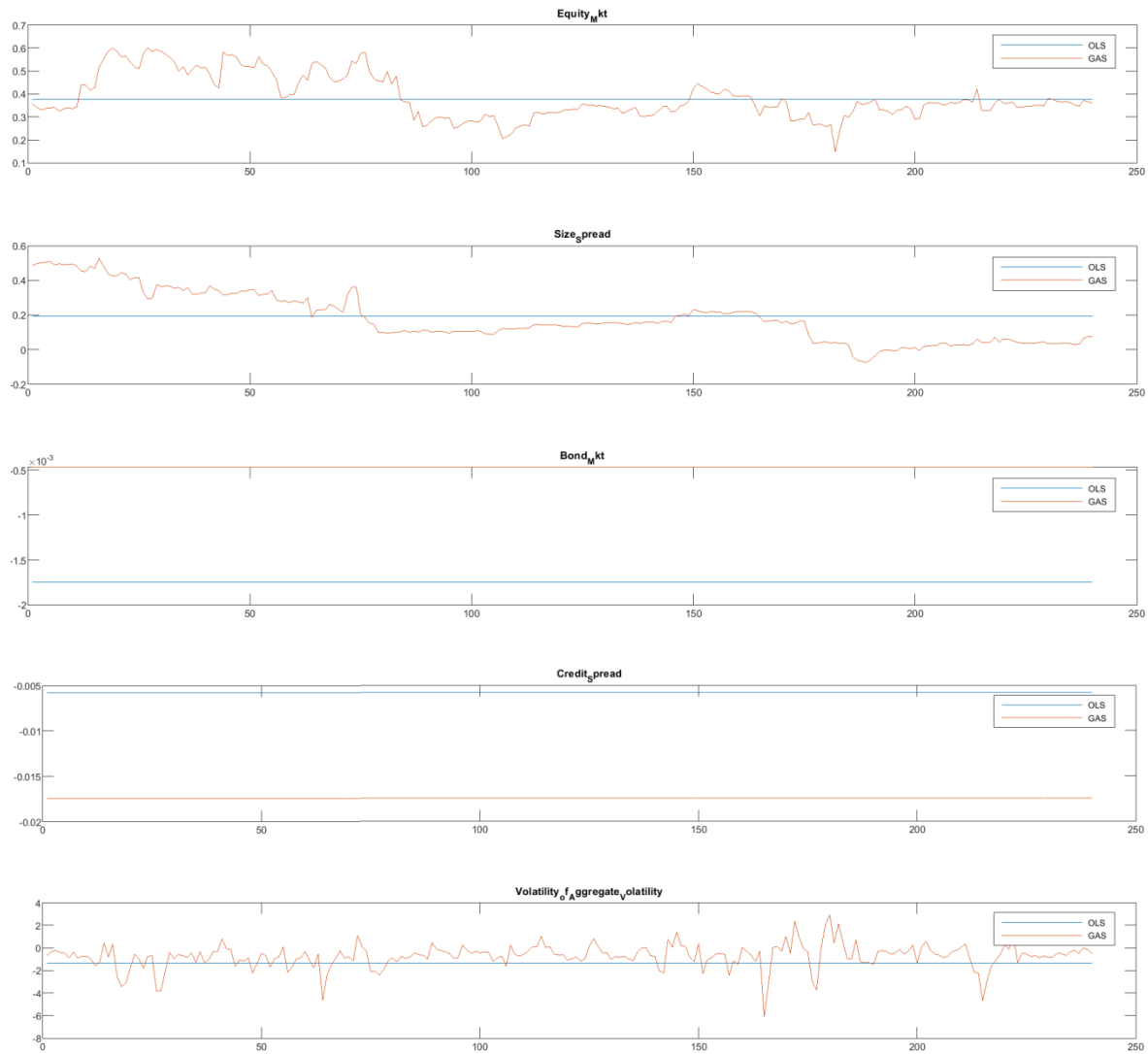
Figure 3

GAS All time-varying parameters vs. OLS for Long/ Short Equity Hedge Strategy

I show how the loadings for each risk factor have been changing over time. The red line represents the GAS estimation, while the blue one represents the OLS equivalent. The data refer to the equally weighted portfolio for 3,189 the Long/Short Equity Hedge strategy funds between 1994 and 2013. The risk factors represented are in order: constant, bond trend following, currency trend following, commodity trend following, equity market, size spread, bond market, credit spread, volatility-of-aggregate-volatility.

Based on author's calculation.





The intercept parameter displays an interesting downward moving trend from the early years of the 1990s: this fact underlines that *Long/Short Equity Hedge* funds saw their idiosyncratic portion of profits decrease over this period. Given that *Long/Short Equity Hedge* fund was the first category to appear on the market, it is possible that the number of funds using this strategy has exhausted the existing opportunities of profit.

The bond trend following factor loading exhibits a clear time variation. Over the years, managers have assumed different positions to increase profits. In the late 1990s and early 2000s a tendency to be long on bonds was spread. On the contrary, managers short sold in bulk during the 2008-2009 Great Financial Crisis. Interestingly, even if after the crisis a buying period followed, short selling was renovated during 2014, the last sample year.

The two equity-oriented risk factor loadings have in common the same decreasing tendency: hedge fund excess returns were higher in the early stages compared to more recent sample years.

I find evidence for structural breaks in the exposure of returns to the size spread factor. This may be due to shifts in where profit opportunities arise or shifts in investor preferences when it comes to picking smaller or bigger funds.

This portfolio displays significant negative loadings to the change in aggregate volatility throughout the analyzed sample period. There is no structural break in the path delineated over time by the volatility-of-aggregate-volatility suggesting that the analysis in Agarwal, Arisoy and Naik (2017) is flawed. A stochastic process can be identified instead.

The dynamic trend of this risk factor exposure appears clear throughout all the sample period: in fact, loadings tend to oscillate around small and negative values. This behavior can be explained by the intrinsic features of these funds and their form of hedging. Hedge funds in this category try to minimize risk exposures by being long on assets expected to appreciate and short on those forecasted to depreciate. By doing so, managers can counterbalance the change in aggregate volatility effect and open new sources of returns. According to this mechanism, the risk connected to volatility-of-aggregate-volatility should be constantly hedged.

Nevertheless, an exception to this consideration can be identified in the crisis period when the negative effect of the volatility-of-aggregate-volatility risk was stronger and more evident. By observing the results shown in Figure 3, one may advance the hypothesis that funds appeared to be exposed to the change of volatility risk due to the managers' short-selling inability, particularly evident during the 2008-2009 Great Financial Crisis and transition times.

In a broader perspective, there is a clear time-variation for data in all the strategies. Nonetheless, some risk factors are time-changing, while others appear to remain static over the whole sample period.

Overall, eight of the eleven equally weighted portfolios for the different strategies exhibit a significant and negative exposure to change in aggregate volatility. In eight out of eleven

strategies, the findings are similar both to those identified in the analysis for *Long/Short Equity Hedge* funds and to the previous research results: for all the different strategies, the uncertainty component, brought to the market from volatility, has had strong downgrading effects for hedge fund performance, especially for the surrounding period of 2008-2009 Great Financial Crisis. Appendix 2 reports the results produced for each of the eleven strategies.

Furthermore, AIC suggests that in six out of eleven strategies the ‘All time-varying parameters’ GAS is the best model for estimating regression over this time-series of data compared to the static OLS used in the previous study. See Table 5.

5.1.2 VOV only time-varying parameters estimation

In this second model specification, only the loading for the *VOV* risk factor is allowed to move over time: therefore, the diagonal matrices, A and B , are composed by mostly zeros with the exception of the coefficient corresponding to *VOV*. Table 3 shows that all the optimal estimated GAS parameters are small and positive: A is always close to zero, while B is constantly close to one.

Table 3

VOV-only time-varying GAS parameters

This table displays the estimated ‘VOV-only time-varying’ GAS parameter values for each of the 11 equally weighted portfolio strategies. Their robust standard errors, which are reported in squared brackets. The strategies’ names are abbreviated as follows: CA stands for *Convertible Arbitrage*; DS for *Dedicated Short Bias*; EM for *Emerging Markets*; EN for *Equity Neutral*; FA for *Fixed Income Arbitrage*; GM for *Global Macro*; LS for *Long/Short Equity Hedge*; MF for *Managed Futures*; MS for *Multi-Strategy*; OS for *Options Strategy*. *, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels for the A t-test statistic, respectively. Robust standard errors are reported in squared brackets. Values rounded to the 3rd significant digit.

	CA	DS	EM	EN	ED	FA	GM	LS	MF	MS	OS
			*			***					**
A	0.010 [0.05]	0.010 [0.03]	0.010 [0.01]	0.050 [0.13]	0.010 [0.03]	0.030 [0.02]	0.010 [0.04]	0.010 [0.07]	0.010 [0.02]	0.050 [0.06]	0.010 [0.00]
B	0.990 [0.08]	0.990 [0.00]	0.990 [0.02]	0.095 [0.09]	0.990 [0.05]	0.970 [0.04]	0.990 [0.02]	0.990 [0.04]	0.990 [0.02]	0.950 [0.04]	0.990 [0.02]
α_{OLS}	0.003 [0.00]	0.004 [0.00]	0.005 [0.00]	0.005 [0.00]	0.005 [0.00]	0.005 [0.00]	0.003 [0.00]	0.005 [0.00]	0.005 [0.00]	0.006 [0.00]	0.004 [0.00]
β_{OLS}^{PTFSB}	-0.003 [0.01]	-0.014 [0.02]	-0.023 [0.02]	0.001 [0.00]	-0.015 [0.01]	-0.006 [0.00]	-0.011 [0.01]	-0.003 [0.01]	0.025 [0.01]	0.002 [0.01]	-0.007 [0.00]
β_{OLS}^{PTFSFX}	-0.005 [0.01]	-0.001 [0.02]	0.014 [0.01]	0.001 [0.00]	0.003 [0.01]	-0.005 [0.00]	0.020 [0.01]	0.007 [0.01]	0.031 [0.01]	-0.001 [0.00]	-0.003 [0.00]
$\beta_{OLS}^{PTFSCOM}$	-0.009 [0.01]	-0.015 [0.02]	0.002 [0.02]	0.001 [0.00]	-0.005 [0.01]	0.001 [0.00]	0.010 [0.01]	0.002 [0.01]	0.044 [0.01]	0.003 [0.00]	0.002 [0.00]
$\beta_{OLS}^{BD10RET}$	0.099 [0.02]	-0.594 [0.06]	0.402 [0.06]	0.072 [0.02]	0.183 [0.02]	0.023 [0.01]	0.123 [0.02]	0.378 [0.04]	0.024 [0.04]	0.123 [0.02]	0.072 [0.02]
$\beta_{OLS}^{BAAMTSY}$	0.022 [0.03]	-0.370 [0.08]	0.138 [0.07]	0.002 [0.02]	0.080 [0.03]	0.006 [0.02]	0.043 [0.03]	0.195 [0.05]	0.020 [0.05]	0.052 [0.02]	0.020 [0.02]
β_{OLS}^{SNPMRF}	-0.018 [0.00]	-0.006 [0.01]	-0.008 [0.01]	-0.004 [0.00]	-0.003 [0.00]	-0.012 [0.00]	-0.016 [0.00]	-0.002 [0.01]	-0.018 [0.01]	-0.005 [0.00]	-0.003 [0.00]
β_{OLS}^{SCMLC}	-0.050 [0.01]	-0.012 [0.02]	-0.031 [0.01]	-0.006 [0.00]	-0.021 [0.01]	-0.023 [0.00]	-0.012 [0.00]	-0.006 [0.01]	-0.006 [0.01]	-0.010 [0.00]	0.003 [0.00]
β_{OLS}^{VOV}	-3.029 [3.28]	-5.782 [8.70]	-6.609 [10.5]	-1.211 [1.74]	-2.401 [3.00]	-1.556 [1.64]	2.129 [2.08]	-1.367 [4.26]	2.376 [4.51]	-0.530 [2.14]	-1.839 [2.06]
ω	2.95 [0.60]	16.59 [3.38]	12.16 [4.51]	0.69 [0.08]	2.37 [0.32]	0.77 [0.19]	1.848 [0.29]	5.322 [0.67]	5.85 [0.82]	1.32 [0.25]	1.048 [0.12]

What emerges from this analysis is that nearly all the equally weighted portfolios built upon the different strategies bring evidence of the negative exposure of hedge fund performance to the *VOV* risk factor.

The only strategy which has to be excluded from this consideration is *Global Macro*. However, this result does not come unexpected: in fact, in the analysis of Agarwal, Arisoy and Naik (2017), this was the only ‘style’ to have a positive loading on change in volatility. Managers of funds following *Global Macro* investment ‘style’ are more concerned than others with tracking global events and making predictions about their consequences. While managers compose their portfolios, they pay attention to macro-economic trends and events so that they have in mind a broader picture of how and how fast the markets all over the world move. This may constitute a successful practice in effectively hedging the risk connected to change in volatility and gaining excess returns.

From a general point of view, the results produced by this analysis heavily resemble the ones reported for the ‘All time-varying’ GAS model specification.

The robust standard errors reported in Table 3 have negligible dimensions, signaling a high precision of model estimation for the sample data. The initial parameter for the volatility-of-aggregate-volatility risk factor estimated by OLS shows a slightly lower fitting quality.

Nonetheless, this model does not produce significantly accurate results. As a rule of thumb, I consider t -values as the most meaningful statistical coefficients for assessing a GAS model quality of fit. By examining them, I find that only three out of eleven strategies are statistically accurate. This may be caused by a lack of reliability of the assumption underlying this GAS specification: in fact, only the *VOV* loading is assumed to be dynamic.

5.1.3 Selected factors parameters estimation

This last GAS model estimates, as time-varying, only those loadings actually exhibiting dynamic trends when all parameters are allowed to change over time, such as in the ‘All-time varying parameters’ GAS.

Table 4

Selected factors time-varying GAS parameters

This table displays the estimated ‘Selected factors time-varying’ GAS parameter values for each of the 7 equally weighted portfolio strategies. Their robust standard errors, which are reported in squared brackets. The strategies’ names are abbreviated as follows: DS stands for *Dedicated Short Bias*; EM for *Emerging Markets*; FA for *Fixed Income Arbitrage*; LS for *Long/Short Equity Hedge*; MF for *Managed Futures*; MS for *Multi-Strategy*; OS for *Options Strategy*. *, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels for the A t-test statistic, respectively. Robust standard errors are reported in squared brackets. Values rounded to the 3rd significant digit.

	DS	EM	FA	LS	MF	MS	OS
	*	*	**	***	*	*	**
A	0.039 [0.00]	0.020 [0.01]	0.038 [0.01]	0.085 [0.02]	0.075 [0.01]	0.145 [0.03]	0.079 [0.02]
B	1.000 [0.00]	0.964 [0.01]	0.946 [0.02]	0.967 [0.01]	0.633 [0.01]	0.420 [0.17]	0.293 [0.28]
α_{OLS}	0.016 [0.01]	0.006 [0.00]	0.005 [0.00]	0.004 [0.00]	0.003 [0.00]	0.006 [0.00]	0.004 [0.00]
β_{OLS}^{PTFSB}	-0.031 [0.01]	-0.014 [0.01]	-0.008 [0.00]	-0.006 [0.01]	0.040 [0.01]	0.000 [0.00]	-0.007 [0.00]
β_{OLS}^{PTFSFX}	-0.004 [0.01]	0.004 [0.01]	-0.004 [0.00]	0.006 [0.00]	0.024 [0.01]	0.000 [0.00]	-0.003 [0.00]
$\beta_{OLS}^{PTFSKOM}$	-0.051 [0.03]	0.010 [0.02]	0.001 [0.00]	0.002 [0.00]	0.046 [0.01]	0.007 [0.00]	0.002 [0.00]
$\beta_{OLS}^{BD10RET}$	-0.929 [0.30]	0.365 [0.06]	0.019 [0.01]	0.351 [0.05]	0.052 [0.04]	0.127 [0.02]	0.105 [0.02]
$\beta_{OLS}^{BAAMTSY}$	-0.429 [0.60]	0.118 [0.04]	0.009 [0.01]	0.201 [0.06]	0.022 [0.04]	0.064 [0.02]	0.020 [0.02]
β_{OLS}^{SNPMRF}	-0.019 [0.05]	-0.006 [0.01]	-0.009 [0.00]	-0.000 [0.00]	-0.021 [0.00]	-0.003 [0.00]	-0.008 [0.00]
β_{OLS}^{SCMLC}	0.087 [0.18]	-0.028 [0.01]	-0.020 [0.00]	-0.015 [0.00]	-0.015 [0.01]	-0.009 [0.00]	0.000 [0.00]
β_{OLS}^{VOV}	1.645 [2.67]	-6.500 [2.83]	-0.018 [0.99]	-1.348 [2.43]	1.693 [2.30]	0.127 [0.98]	-1.768 [1.32]
ω	5.26 [0.50]	5.16 [0.49]	0.45 [0.04]	0.95 [0.08]	3.88 [0.36]	0.75 [0.07]	0.85 [0.08]

The results in Table 4 are available only for seven out of the eleven strategies present in the entire sample. This is due to the fact that the other strategies show time-variation for all the loadings, making the estimation of a new model only for dynamic loadings redundant.

Even though results for the volatility-of-aggregate-volatility loading resemble previous findings from the ‘VOV only time-varying’ GAS model, in this case, the estimated coefficients are more significant. The robust standard errors are smaller for all the parameters. All the strategies investigated exhibit negative and significant VOV time-varying loadings at different confidence levels ranging from the 0.1 to the 0.01 confidence level, as shown in Table 4.

5.1.4 Comparing results for the three GAS models

In this last paragraph of time-series regression, I provide a global analysis of the quality of the different GAS model specifications used.

I compute the AIC for all the four nested model estimated in this thesis to assess which model best fits the data. I also report AIC for OLS model as a benchmark of previous studies. As explained before, AIC highlights the best model by assigning it the lowest value in absolute terms.

Results in Table 5 suggests that the ‘All time-varying parameters’ GAS is the best fitting model for six out of eleven strategies including *Long/Short Equity Hedge*, which is the most popular management ‘style’. Interestingly, even if *Multi-Strategy* hedge fund returns are better estimated by OLS, the AIC results present a small and almost negligible difference between the static and the dynamic model.

Table 5**Akaike Information Criterion for best fitting model selection**

This table displays the results of the Akaike Information Criterion which compares the quality of four different model specifications. The AIC is computed for all the 11 equally weighted portfolio strategies. ‘-’ is displayed for those strategies not estimated by ‘Selected factors time-varying’ GAS model. ‘...’ is displayed in those cases in which the criterion was not able to converge. Values rounded to the 2nd decimal place. Values in bold are best data fitting models.

	Ordinary Least Squares	GAS all time- varying	GAS VOV- only	GAS Selected factors
Convertible Arbitrage	834.49	693.88	815.12	-
Dedicated Short Biased	1,197.70	941.23	1,114.40	2,720.30
Emerging Market	971.06	1,001.00
Equity Market Neutral	627.70	503.56	559.60	-
Event Driven	959.21	582.98	739.11	-
Fixed Income Arbitrage	754.64	473.65	530.08	1,068.50
Global Macro	641.51	763.86	772.12	-
Long/Short Equity Hedge	1,119.20	706.12	923.46	1,373.20
Managed Futures	832.57	1,033.23	1,060.20	1,554.8
Multi-Strategy	638.64	648.07	680.30	1,136.30
Options Strategy	576.27	595.42	585.41	1,106.40

5.1.5 Time-series analysis conclusions

After having computed the different risk factors exposures to explain the relation between hedge fund excess returns and the volatility-of-aggregate-volatility, it is possible to answer part of the research question. Overall, in the sample period ranging from January 1994 to December 2013, all the three different time-series regressions point over time towards a negative exposure to change in volatility, also substantiating the findings from Agarwal, Arisoy and Naik (2017). In more practical terms, whenever the unpredictability of the market increases, hedge fund excess returns suffer a decline, whatever strategy is employed and regardless of the idiosyncratic characteristics of the individual investment vehicle.

Moreover, the tests performed upon those models confirm the statistical validity of all the three different GAS models and, in particular, for the ‘All time-varying parameters’ GAS.

In addition, based on AIC, the use of the time-varying risk exposures clearly contributes to funds' alphas and betas estimation of hedge fund returns.

5.2 Hedge fund performance cross-section analysis

In this second regression stage, I study whether the change of the volatility in the equity market is a determinant of cross-sectional differences in hedge fund returns.

The multivariate cross-sectional regression is estimated for all the three different time-varying change in aggregate volatility models controlling for: selected fund characteristics, time and strategy fixed effects, as illustrated in Equation (5).

Table 6 reports the coefficients resulting from the monthly cross-sectional panel regression over the period between January 1994 and December 2013. All the estimations are carried out over the complete sample size and report a fit precision measure, R^2 , around 12%.

All the three specifications show that the relation between fund strategies' *VOV* betas and hedge fund returns in excess of risk free rate is negative and robust at a significance level of 0.01. This may be a sign of the inability of most of managers to adjust their strategies to hedge this risk component.

Table 6

Panel Regression results

Panel regression estimated with Equation (5). The dependent variable is the monthly individual fund excess return. The main explanatory variable is β_{VOV} estimated in the time-series analysis as 11 time-series of coefficients. Each vector corresponds to one of the 11 equally weighted strategy portfolio built for the first analysis. Panel regression is computed on grouped sets of funds β_{VOV} and individual hedge fund returns controlling for time and style-fixed effects. *MinInvPeriod* is the shortest time the investor has to wait to be able to withdraw the entire investment without the obligation of pay any early redemption fee (in months). *LagAUM* are the 1-year lagged values of Assets Under Management (AUM) are in millions of dollars. All AUMs denominated in currencies other than USD are converted using month-end exchange rates provided by Datastream. *Age* is the number of years from fund inception. *IncFee* is a fixed percentage fee of fund's net asset value. *MgmtFee* is a fixed percentage fee of AUM. *MinInvestment* is the minimum requirement to enter a fund in millions of dollars. *Futures* is a dummy for funds investing in futures. *Derivatives* is a dummy for funds investing in derivatives. *MaxLeverage* is the amount of maximum leverage expressed as a percentage of equity. *Leverage* is a dummy for the funds using leverage. *AvgLeverage* is the average amount of leverage as a percentage of equity. *HighWaterM* is a dummy for funds with high water mark clause. *, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels, respectively for p-values. Values rounded to the 4th significant digit.

Fund Characteristic	All time-varying	VOV-only	Selected Factors
Intercept	-0.447	-0.655	-0.451
β_{VOV}	*** -0.0152	*** -0.0669	*** -0.014
Min Inv Period	-0.0002	-0.0001	-0.0017
Lagged AUM	0.006	0.0052	0.0059
Age	*** -0.022	*** -0.0221	*** -0.0219
Incentive Fee	* 0.0067	* 0.0061	* 0.0067
Management Fee	0.0246	0.0347	0.0234
Minimum Investment	*** 5.995	*** 5.874	*** 6.003
Futures	-0.0117	-0.0005	-0.0115
Derivatives	-0.0642	-0.0621	-0.0642
Maximum Leverage	0.0001	0.0001	0.0001
Leverage	-0.011	-0.0141	-0.0133
Average Leverage	-0.0001	-0.0001	-0.0001
High Water Mark	*** 0.382	*** 0.372	*** 0.381
Time-fixed effect	True	True	True
Strategy-fixed effect	True	True	True
N	234,310	234,310	234,310
R ²	11.83%	11.86%	11.82%

As already shown by previous papers on cross-section of hedge fund returns, the age component negatively influences the probability of an investment outperformance. When funds stay on the market for longer periods their strategies are vulnerable to imitation and, consequently first movers tend to lose their competitive advantages over time (Seigmann, Stefanova, and Zamojski, 2013).

When funds set higher incentive fees, managers are signaling that they are more certain about their ability to deliver positive returns: this generates a significant positive correlation between incentive fees and hedge fund excess returns.

There is also a positive and significant relation between minimum investment and performance. Investors in hedge funds are required to contribute enormous amounts of money to get access to the venture: when managers set high minimum investment requirements, they expect to be able to get higher returns, sending a power signal to investors.

High water mark is a significantly positive determinant in the cross-section of returns. Managers autonomously deciding to be subject to this clause want to make sure that their investors know capability of generating positive returns and getting rewarded.

When managers prolong minimum investment period, they are signaling to investors their intention to invest in illiquid assets. Given this, I initially expected minimum investment period to present a negative relation to excess returns. Even if this economic intuition is proven to be right by panel regression, results appear insignificant.

To summarize, from this multivariate panel regression analysis I find that all the three different estimated GAS *VOV* betas are negative and significant cross-sectional determinants of hedge fund excess returns differences over the entire 20 years sample period.

In general, when volatility-of-aggregate-volatility has a negative relation to hedge fund returns, it may be caused by managers' inability to hedge effectively. Therefore, during periods of high changes in aggregate volatility, meaning both in highly bearing and highly bulling markets, hedge fund returns are vulnerable to severe losses.

Chapter 6: Robustness

To substantiate the findings presented in the main analysis, I perform different robustness checks. In this chapter, I built two alternative versions of *VOV* factor and I estimate the same time-series and cross-section regressions above shown.

6.1 Statistical proxies of *VOV*

To corroborate obtained results, I deliver the same analysis based upon two alternative statistical proxies for *VOV* factor. I follow Agarwal, Arisoy and Naik (2017) and construct these new factors using their same process to be able of comparing my study with previous literature.

The first *VOV* proxy is *RVIX*, a range-based volatility measure built as the monthly VIX index range just as in Bali and Weinbaum (2005). I define *RVIX* as:

$$RVIX_t = \ln[\text{Max}\{VIX_\tau\}] - \ln[\text{Min}\{VIX_\tau\}] \quad (20)$$

VIX is observed daily (denoted by subscript τ), while the obtained proxy is a monthly time-series.

The second statistical proxy is the monthly standard deviation of VIX index within a month, referred to as *SDVIX*, based upon daily data and calculated with the following Equation:

$$SDVIX_t = \sqrt{\frac{1}{T} \sum_{\tau=1}^T (VIX_\tau - \overline{VIX_t})^2} \quad (21)$$

Supplementary Appendix reports the obtained proxies time-series plots and their pairwise correlation matrices.

6.2 Time-series analysis robust results

This analysis supports the results stated before: I am able to show that the negative relation between *VOV* exposures and hedge fund returns is robust between January 1994 and December 2013. For the sake of brevity, I provide detailed results in the Supplementary Appendix.

In the following paragraphs, I assess the quality of the three different GAS model specifications developed to substantiate previous findings.

I compute AIC to assess which model best fits the analyzed data both for *RVIX* and *SDVIX* proxies. I report them, respectively, in Table 7 and in Table 8.

From Table 7, I can confirm previous findings: the ‘All time-varying parameters’ GAS model is the best fitting model for six out of the eleven investigated strategies.

Table 7

Akaike Information Criterion for the Range Volatility proxy

This table displays the results of the Akaike Information Criterion which compares the quality of four different model specifications. The AIC is computed for all the 11 equally weighted portfolio strategies. ‘-’ is displayed for those strategies not estimated by ‘Selected factors time-varying’ GAS model. ‘...’ is displayed in those cases in which the criterion was not able to converge. Values rounded to the 2nd decimal place. Values in bold are best data fitting models.

	Ordinary Least Squares	GAS all time- varying	GAS RVIX- only	GAS Selected factors
Convertible Arbitrage	848.77	689.11	815.21	1,454.10
Dedicated Short Biased	1,199.20	918.50	1,121.10	...
Emerging Market	977.44	992.34	1,055.30	6,850.60
Equity Market Neutral	640.10	515.94	554.24	1,433.10
Event Driven	1,006.80	554.37	734.81	1,192.10
Fixed Income Arbitrage	765.97	474.18	532.37	1,088.00
Global Macro	635.22	760.02	771.72	1,343.70
Long/Short Equity Hedge	1,146.50	700.45	920.70	1,375.10
Managed Futures	834.30	1,037.30	1,054.20	-
Multi – Strategy	642.03	645.58	678.26	1,206.30
Options Strategy	576.71	602.72	583.23	1,077.70

Table 8 shows that time-varying regression is the best fitting model for the majority of strategies. Unlike previous AIC, the *SDVIX* criterion suggests the ‘*SDVIX*-only time-varying’ GAS specification as the most suitable for two out of the eleven categories.

Table 8

Akaike Information Criterion for the Standard deviation Volatility proxy

This table displays the results of the Akaike Information Criterion which compares the quality of four different model specifications. The AIC is computed for all the 11 equally weighted portfolio strategies. ‘-’ is displayed for those strategies not estimated by ‘Selected factors time-varying’ GAS model. ‘...’ is displayed in those cases in which the criterion was not able to converge. Values rounded to the 2nd decimal place. Values in bold are best data fitting models.

	Ordinary Least Squares	GAS all time- varying	GAS SDVIX- only	GAS Selected factors
Convertible Arbitrage	842.53	712.96	817.78	-
Dedicated Short Biased	1,205.70	...	1,121.80	-
Emerging Market	976.50	...	1,056.90	-
Equity Market Neutral	660.38	570.62	548.02	-
Event Driven	1,043.20	594.15	734.31	-
Fixed Income Arbitrage	789.54	491.03	525.90	-
Global Macro	635.55	782.14	775.39	-
Long/Short Equity Hedge	1,137.10	830.06	922.60	-
Managed Futures	833.30	1,117.60	1,058.80	-
Multi – Strategy	640.43	657.85	680.72	-
Options Strategy	577.23	606.55	582.45	979.49

6.3 Panel regression robust results

In this second stage of robustness checks, I study whether these alternative proxies for change in aggregate volatility are priced factors in the cross-section of hedge fund returns.

To obtain comparable results, also this time the multivariate cross-sectional regression stated in Equation (5) is estimated. Table 9 reports the obtained results.

The change in uncertainty measure does not affect estimation precision described by R^2 , which remains stable and around the 12%.

The three panel regression estimations for *RVIX* provide clear evidence supporting the existence of a negative relation between funds' *VOV* betas and hedge fund returns. Unlike previous findings, the levels of significance are not equal for all the GAS specifications.

For *RVIX*, all the estimated models confirm that the volatility-of-aggregate-volatility is an explanatory factor of the cross-sectional differences in hedge fund returns, but only the 'All time-varying parameters' shows significant results.

All *SDVIX* results confirm findings stated above, with the exception being the 'SDVIX-only' GAS model.

The individual fund characteristics influence hedge fund performance in a consistent way with previous results when statistical alternatives of *VOV* are employed.

Table 9

Panel Regression results

Panel regression estimated with Equation (5). The dependent variable is the monthly individual fund excess return. The main explanatory variable is $\beta_{RVIX/SDVIX}$ estimated in the time-series analysis as 11 time-series of coefficients. Each vector corresponds to one of the 11 equally weighted strategy portfolio built for the first analysis. Panel regression is computed on grouped sets of funds $\beta_{RVIX/SDVIX}$ and individual hedge fund returns controlling for time and style-fixed effects. *MinInvPeriod* is the shortest time the investor has to wait to be able to withdraw the entire investment without the obligation of pay any early redemption fee (in months). *LagAUM* are the 1-year lagged values of Assets Under Management (AUM) are in millions of dollars. All AUMs denominated in currencies other than USD are converted using month-end exchange rates provided by Datastream. *Age* is the number of years from fund inception. *IncFee* is a fixed percentage fee of fund's net asset value. *MgmtFee* is a fixed percentage fee of AUM. *MinInvestment* is the minimum requirement to enter a fund in millions of dollars. *Futures* is a dummy for funds investing in futures. *Derivatives* is a dummy for funds investing in derivatives. *MaxLeverage* is the amount of maximum leverage expressed as a percentage of equity. *Leverage* is a dummy for the funds using leverage. *AvgLeverage* is the average amount of leverage as a percentage of equity. *HighWaterM* is a dummy for funds with high water mark clause. *, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels, respectively for p-values. Values rounded to the 4th significant digit.

Fund Characteristic	All RVIX	RVIXonly	S. F. RVIX	All SDVIX	SDVIXonly	S.F. SDVIX
Intercept	-0.518	-0.475	-0.516	-0.457	-0.287	-0.457
$\beta_{RVIX/SDVIX}$	***-0.0442	-0.0073	-0.0294	**0.0001	***0.732	**0.0001
Min Inv Period	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
Lagged AUM	0.0061	0.0061	0.0061	0.006	0.0053	0.006
Age	***-0.022	***-0.0221	***-0.0221	***-0.0221	***-0.0219	***-0.0221
Incentive Fee	*0.0068	*0.0068	*0.0068	*0.0068	*0.0062	**0.0068
Management Fee	0.0252	0.0252	0.0249	0.0249	0.0235	0.0249
Minimum Investment	***5.97	***5.978	***5.984	***5.981	***5.975	***5.981
Futures	-0.0119	-0.0112	-0.0117	-0.0116	-0.0157	-0.0116
Derivatives	-0.0636	-0.0641	-0.0639	-0.0641	-0.0615	-0.0641
Maximum Leverage	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Leverage	-0.0107	-0.011	-0.0111	-0.0109	-0.0129	-0.0109
Average Leverage	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
High Water Mark	***0.382	***0.382	***0.382	***0.382	***0.378	***0.382
Time-fixed effect	True	True	True	True	True	True
Strategy-fixed effect	True	True	True	True	True	True
N	234,310	234,310	234,310	234,310	234,310	234,310
R ²	11.82%	11.82%	11.82%	11.82%	11.86%	11.82%

Chapter 7: Conclusions

This paper investigates, in a dynamic perspective, whether exposure to uncertainty about aggregate volatility in the equity market is an explanatory factor of the cross-sectional differences in hedge fund returns. Furthermore, I analyze whether estimating risk exposures as time-varying improves hedge fund returns' modeling.

Overall, in the sample period ranging from January 1994 to December 2013, all the three different GAS time-series regression models exhibit a significant and negative exposure to the time-varying *VOV* risk factor. This effect negatively impacts returns and investors underlining a managers' inability to effectively hedge against this risk.

A clear time-variation is identified in the data and the performed tests suggest that the 'All time-varying parameters' GAS is the best model in terms of data fit and parameter parsimony.

Multivariate cross-sectional panel regressions show, for the entire 20 years sample, that all the three different GAS estimated risk exposures have significant explanatory power over hedge fund returns. Nonetheless, the minimum investment period, which was expressly built for this analysis, ends up showing a negative but insignificant relation to hedge fund excess returns.

Robustness checks, performed on the two alternative versions of *VOV* risk factor, *RVIX* and *SDVIX*, support these results.

This thesis contributes to the existing literature on aggregate volatility and hedge fund returns using a more advanced estimation method. Furthermore, this research avoids making some unrealistic assumptions regarding the static nature of returns loadings and split sample.

Building upon that, I am able to find evidence that the factor exposures actually vary over time, unlike what assumed by Agarwal, Arisoy and Naik (2017). Nonetheless, my research ascertains the robustness of their findings to this new method of estimation. I can also

determine a positive contribution of the proposed GAS models for time-variation in the hedge fund returns analysis.

Corroborating and expanding previous studies, this research ascertains that the relation between uncertainty of the equity market and hedge fund returns stays significant and negative when: (i) a larger sample period is considered, (ii) a different measure of change in aggregate volatility is developed, or (iii) a dynamic regression model is estimated.

Useful insights for building hedge fund investment portfolios can be gathered from the persistence of these results. Based on the strategy followed, hedge fund managers should take into account volatility-of-aggregate-volatility in their investment decision-making process, since this factor influences differently excess returns over time for each management 'style'. As a result, if managers will put in practice this recommendation, they would build strategies with butterfly-option like payoffs able to gain higher returns during moments of high change in volatility in the market.

Building on the robustness of presented results, new quests can be started. The 'All time-varying parameters' GAS, one of the estimation models proposed in this thesis, can be used in further research to analyze other shades of the hedge fund return-generating process in a dynamic perspective.

Appendices

The Appendix part of the text is organized as follows: Appendix 1 reports the test performed upon the *VOV* risk factor to state its statistical validity; Appendix 2 reports figures plots for the time-varying parameters estimated by the ‘All-time varying parameters’ GAS for *VOV*. This last section complements section 5.1.1 in the main text providing strategies in alphabetical order figures for the other 10 equally weighted portfolios strategies.

A Supplementary Appendix is provided in a separate document. Supplementary Appendix 1 illustrates all the details of the performed robustness checks for *RVIX*. Supplementary Appendix 2 does the same for *SDVIX*.

Appendix 1

For the purpose of validating the results obtained for the Normal GARCH(1,1), I perform a set of statistical tests. The first one is diagnostic checking for Fitted Volatility, which examines the results produced to verify the absence of autocorrelation between the standardized squared residuals values. This is performed through the Ljung-Box test for residual autocorrelation.

Under the Null Hypothesis: $H_0: \varepsilon_t \sim \text{White Noise} (0, \sigma^2)$

The test statistic converging to a $X^2 (m - p - q)$:

$$Q(m) = T(T+2) \sum_{k=1}^m (T-k)^{-1} \hat{\rho}_e^2(k)$$

I cannot reject the no residual autocorrelation null hypothesis, meaning that there is not enough evidence to say that the autocorrelation is different from zero, which suggests the model is well specified.

Additionally, also the Jarque-Bera test for Normality of standardized residuals, reported subsequently, highlights that the returns and the standardized residuals are not Gaussian.

Under the Null Hypothesis: $H_0: \varepsilon_t \sim i.i.d. N(0, \sigma^2)$

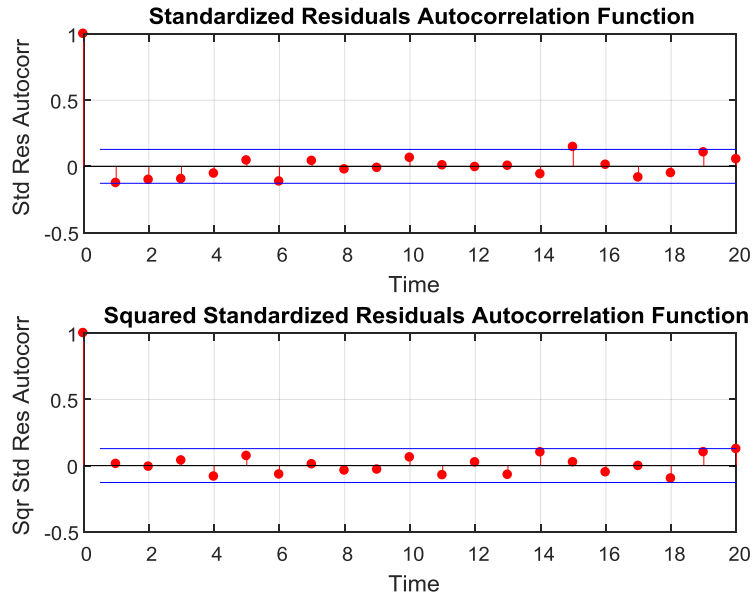
The test statistic converging to a $X^2(2)$:

$$e_t = \frac{\hat{\theta}(L)}{\hat{\phi}(L)} y_t$$

Therefore, the distribution assumed for the GARCH(1,1) model needs to be modified from a Normal distribution toward a more reasonable Student's t distribution. Applying all the tests stated above to this new GARCH(1,1) specification, it is confirmed that this model is better specified, since the autocorrelation of squared standardized residuals is minimized.

Appendix figure 1.1: Squared standardized residuals Autocorrelation function for GARCH(1,1) model with Student's t distribution

Based on author's calculation from FRED data for VIX index returns.



The equation for calculation of the Akaike Criterion is reported below:

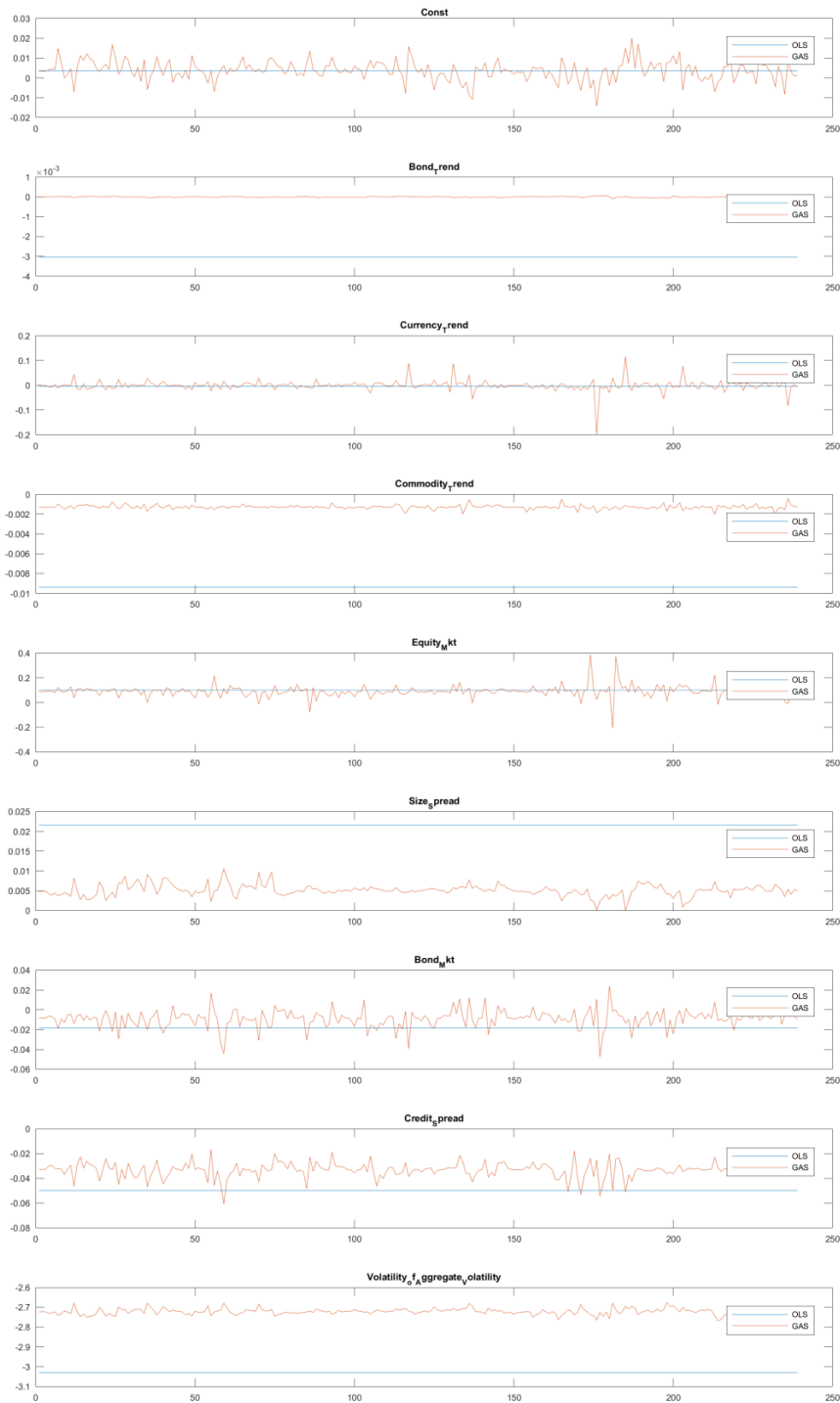
$$(p^*, q^*) = \arg \min \left\{ AIC(p, q) = \ln \sigma_t^2 + 2 \frac{p + q}{T} \right\}$$

where : ' T ' is the number of parameters in the estimation.

Appendix 2

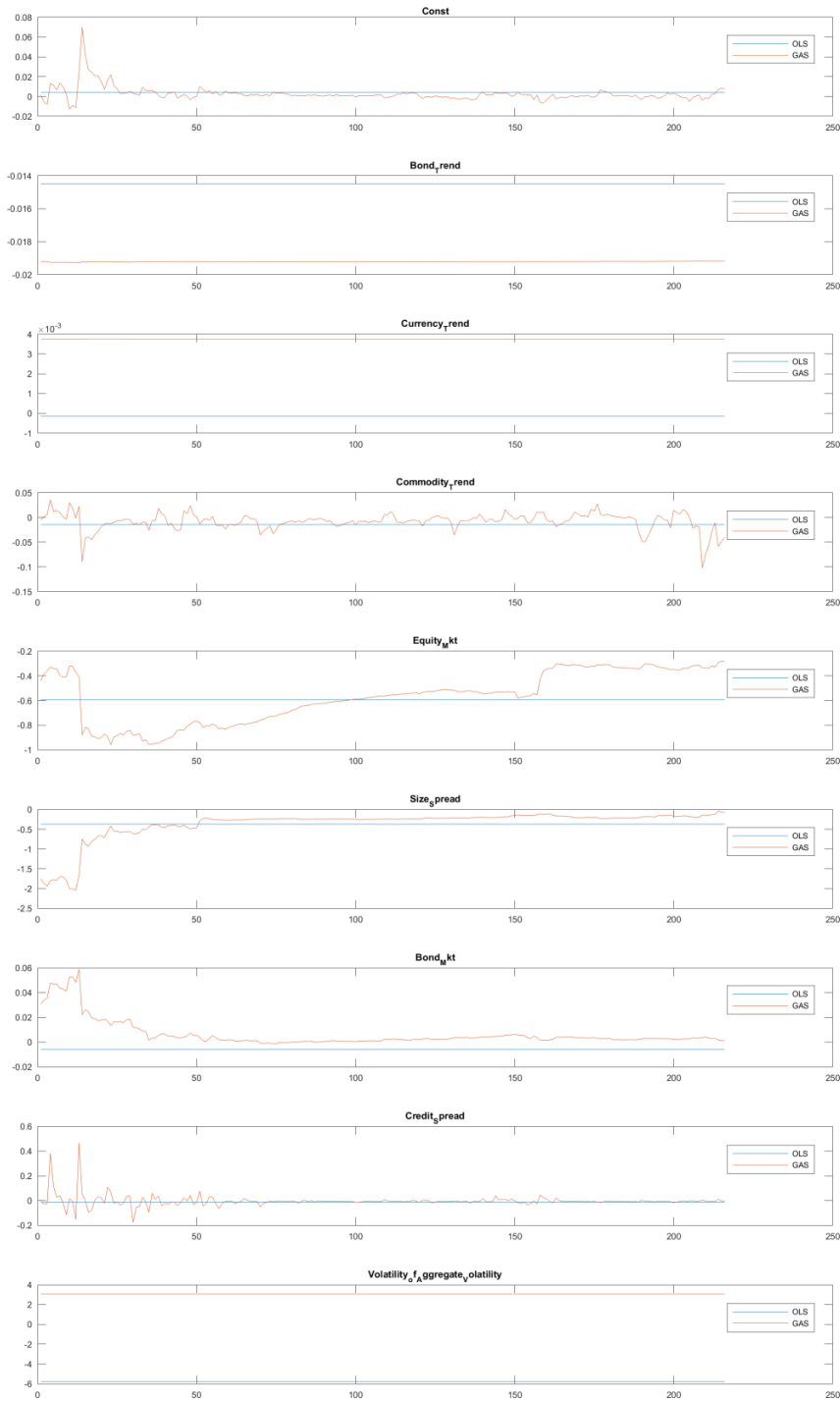
Appendix figure 2.1: GAS All time-varying parameters vs. OLS for Convertible Arbitrage Strategy

I show how loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 247 Convertible Arbitrage strategy funds between 1994 and 2013. Based on author's calculation.



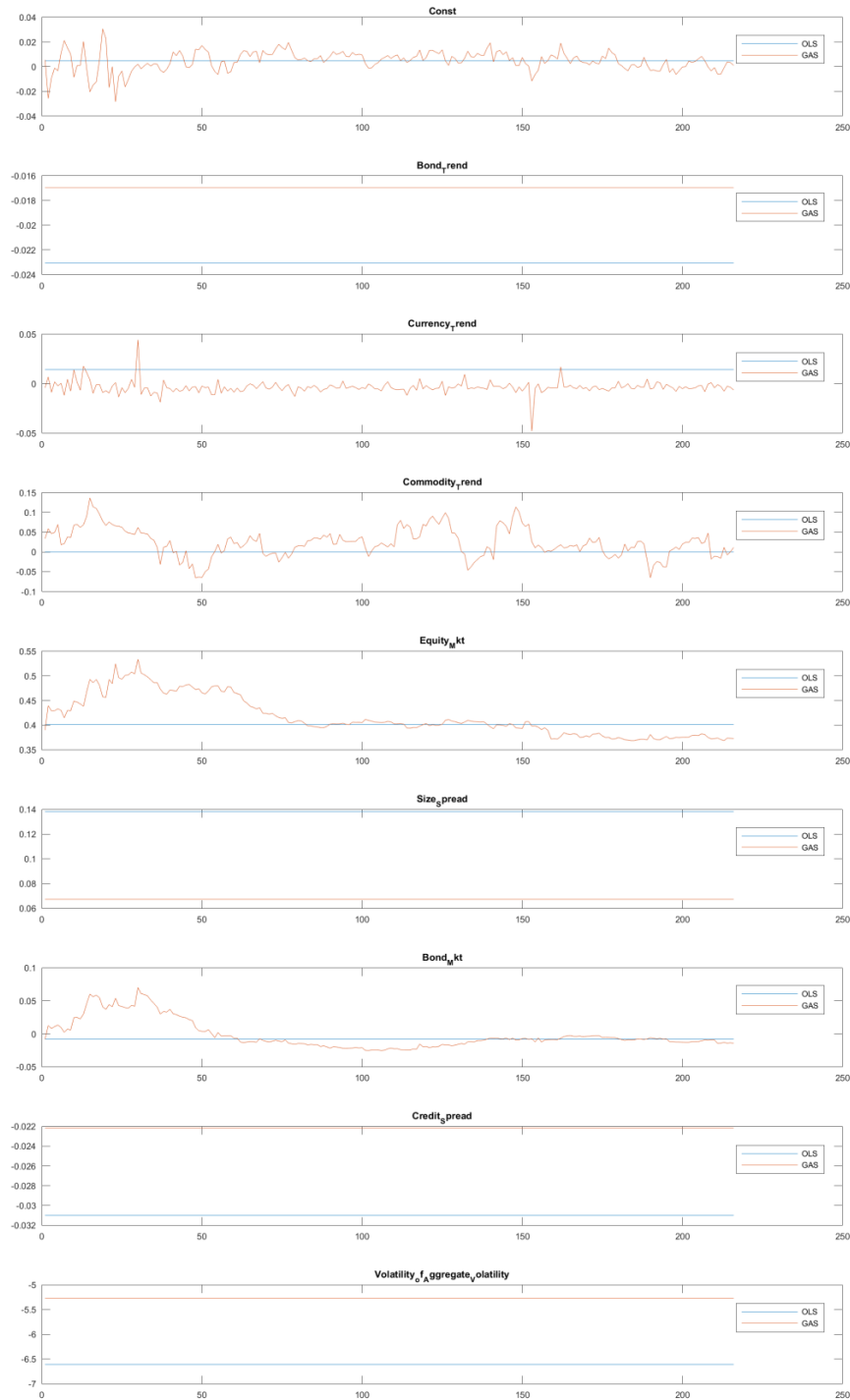
Appendix figure 2.2: GAS All time-varying parameters vs. OLS for Dedicated Short Bias Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 43 Dedicated Short Bias strategy funds between 1996 and 2013. Based on author's calculation.



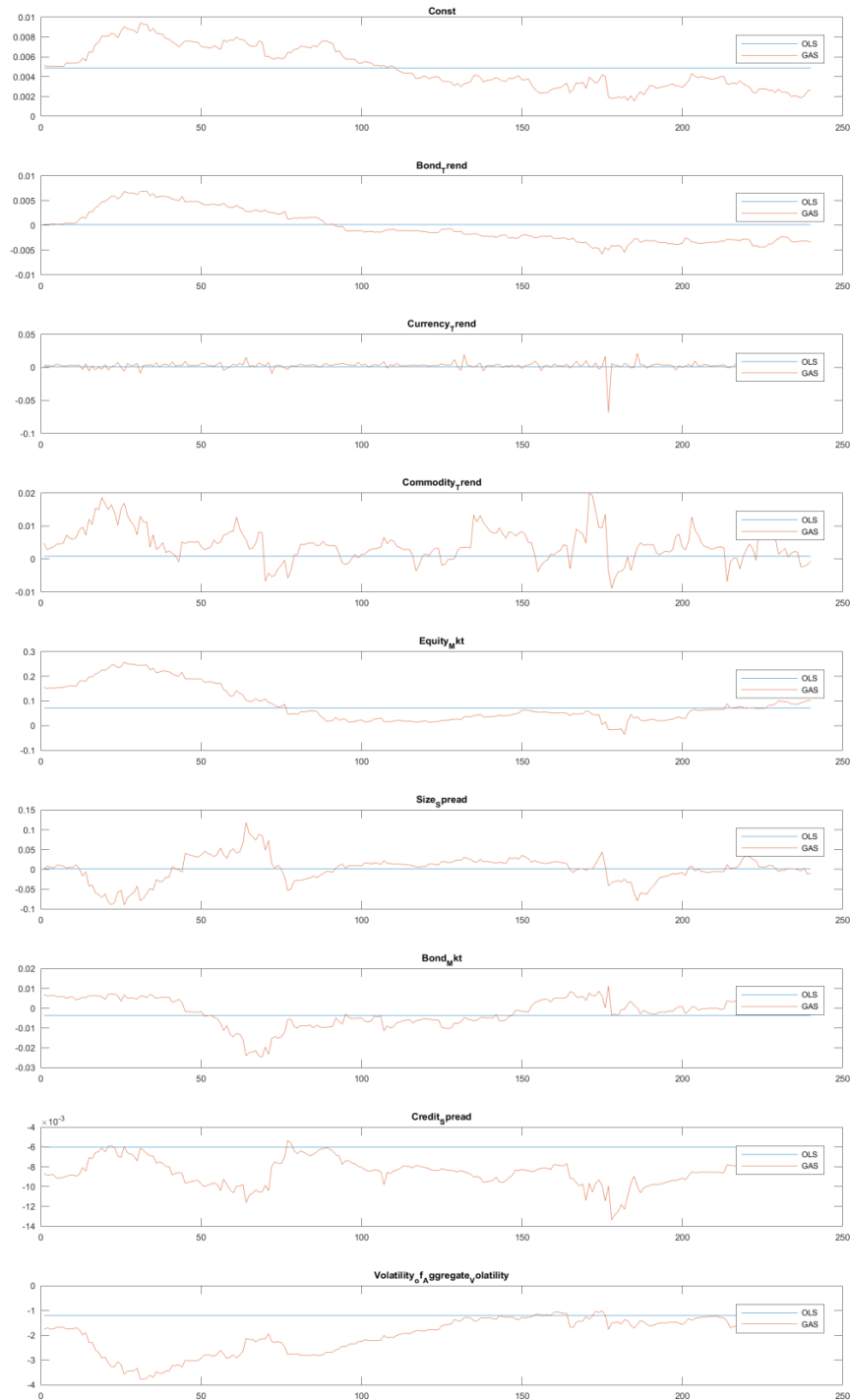
Appendix figure 2.3: GAS All time-varying parameters vs. OLS for Emerging Market Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 853 Emerging Market strategy funds between 1996 and 2013. Based on author's calculation.



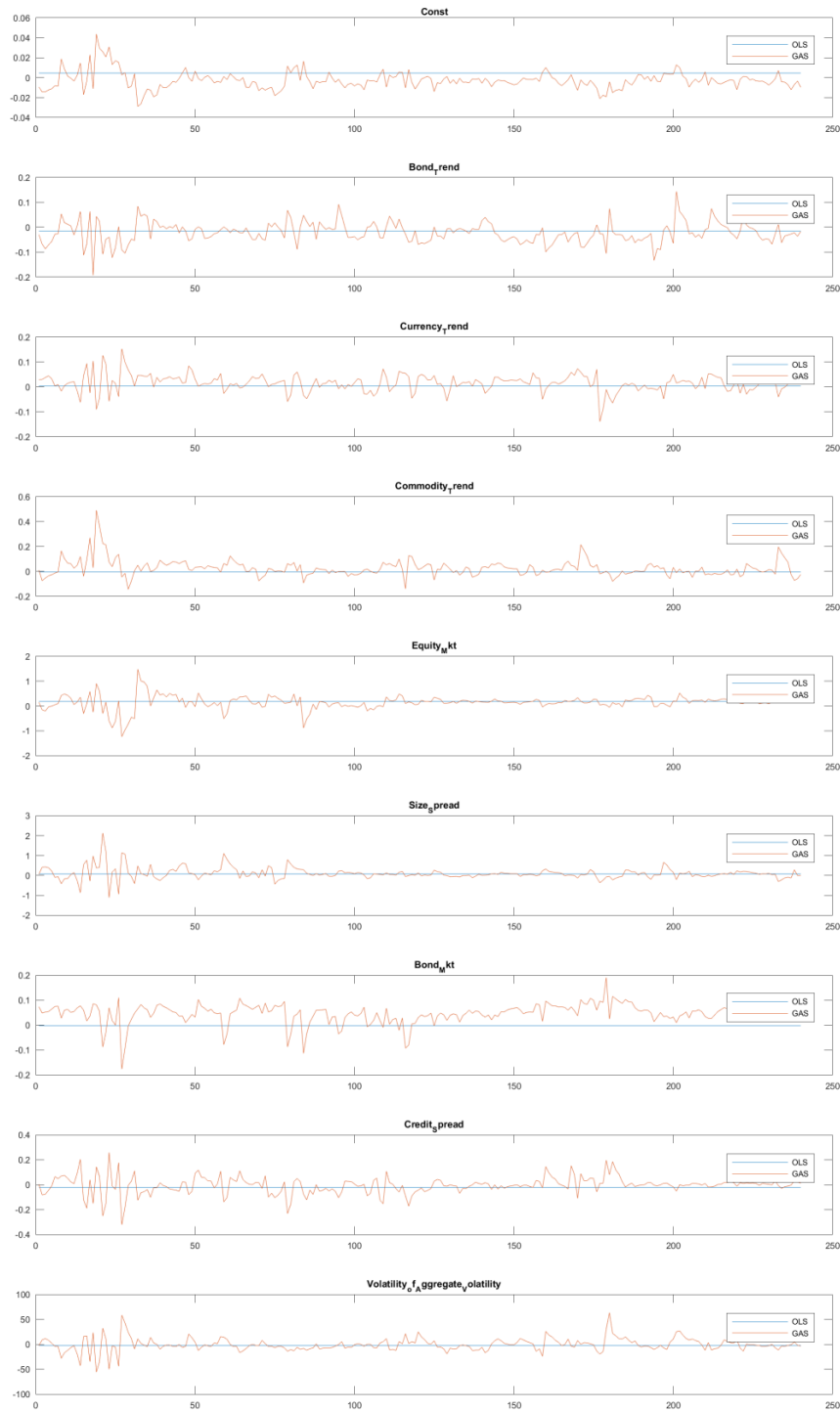
Appendix figure 2.4: GAS All time-varying parameters vs. OLS for Equity Market Neutral Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 579 Equity Market Neutral strategy funds between 1994 and 2013. Based on author's calculation.



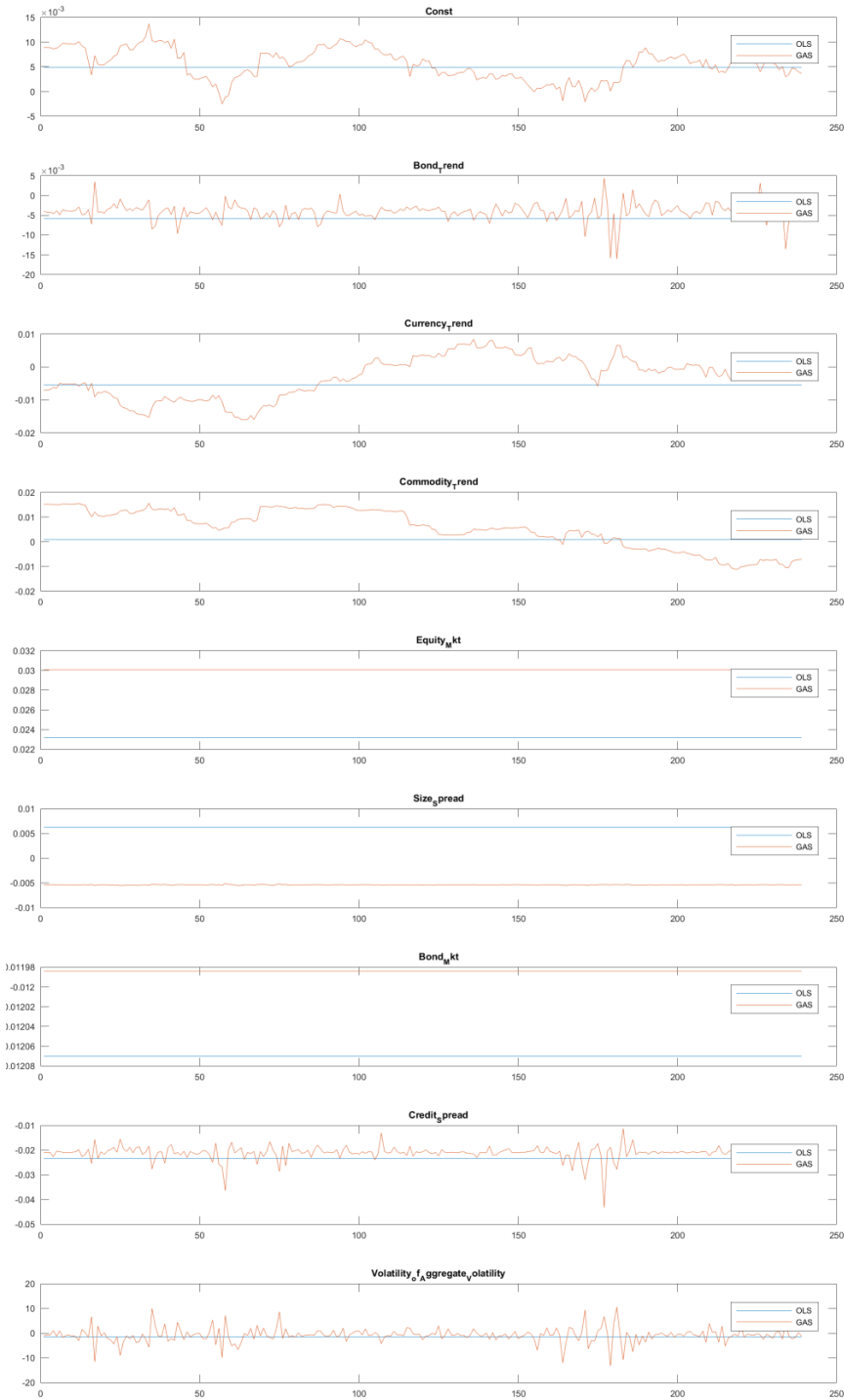
Appendix figure 2.5: GAS All time-varying parameters vs. OLS for Event Driven Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 658 Event Driven strategy funds between 1994 and 2013. Based on author's calculation.



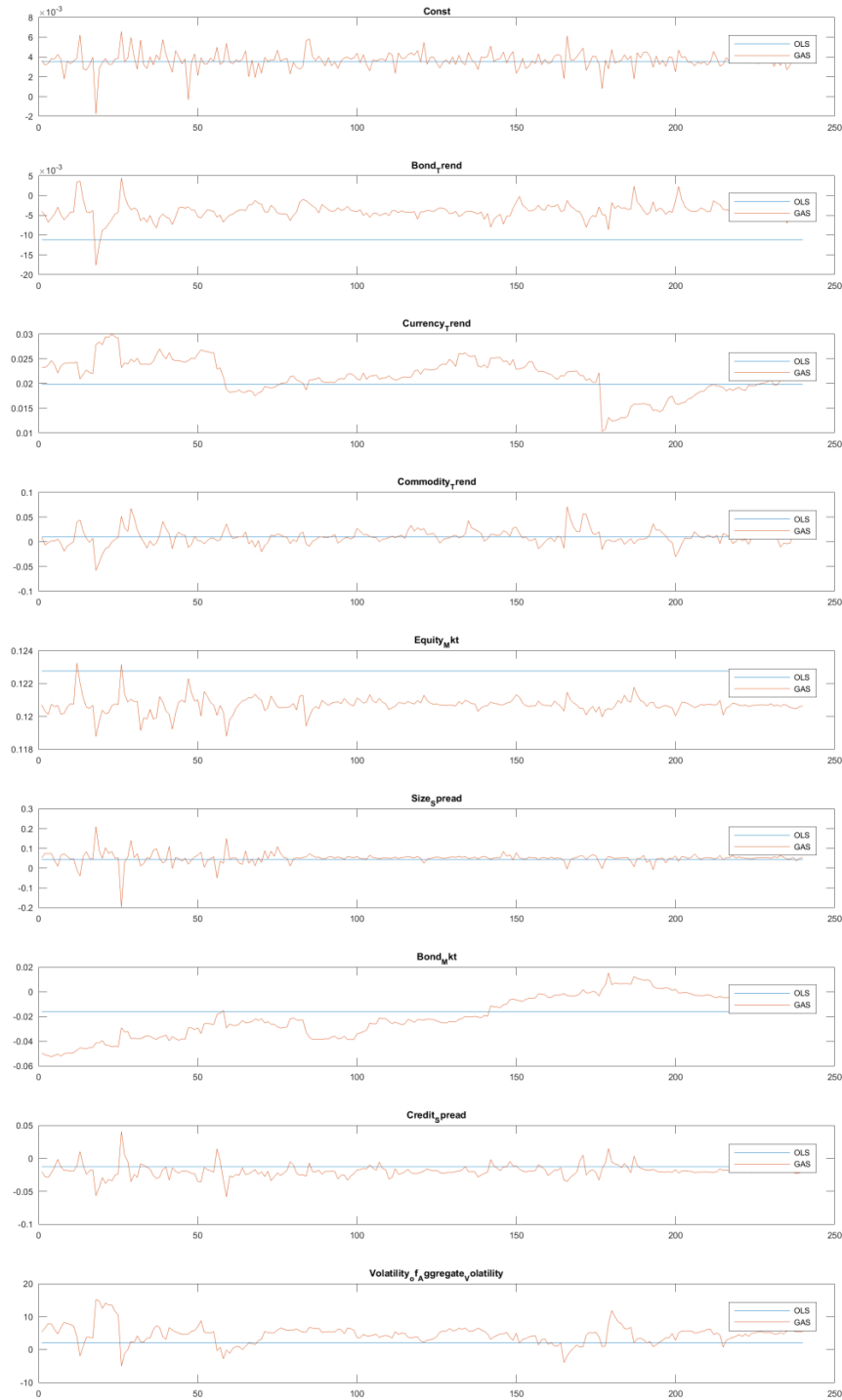
Appendix figure 2.6: GAS All time-varying parameters vs. OLS for Fixed-Income Arbitrage Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 394 Fixed-Income Arbitrage strategy funds between 1994 and 2013. Based on author's calculation.



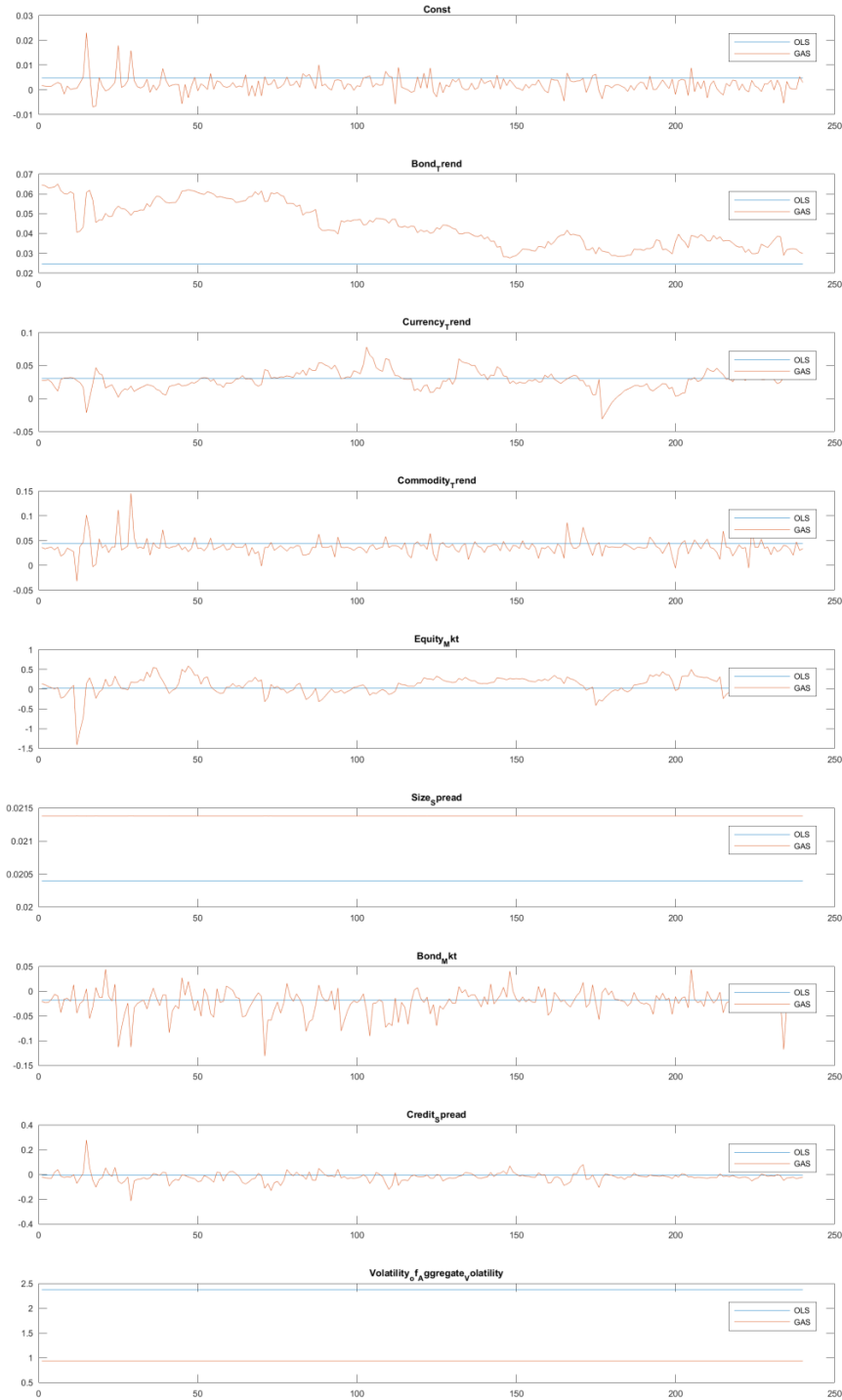
Appendix figure 2.7: GAS All time-varying parameters vs. OLS for Global Macro Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 718 Global Macro strategy funds between 1994 and 2013. Based on author's calculation.



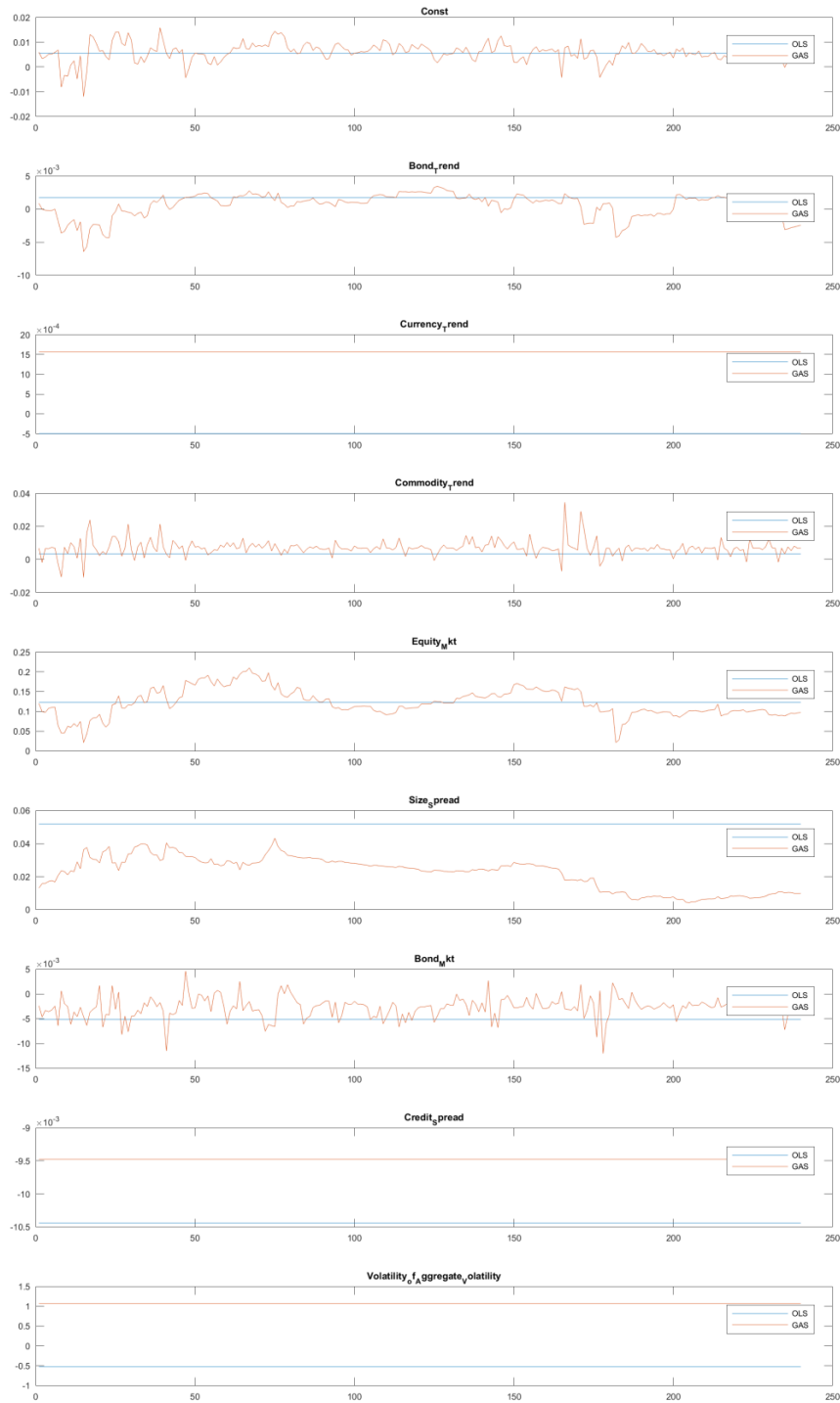
Appendix figure 2.8: GAS All time-varying parameters vs. OLS for Managed Futures Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 858 Managed Futures strategy funds between 1994 and 2013. Based on author's calculation.



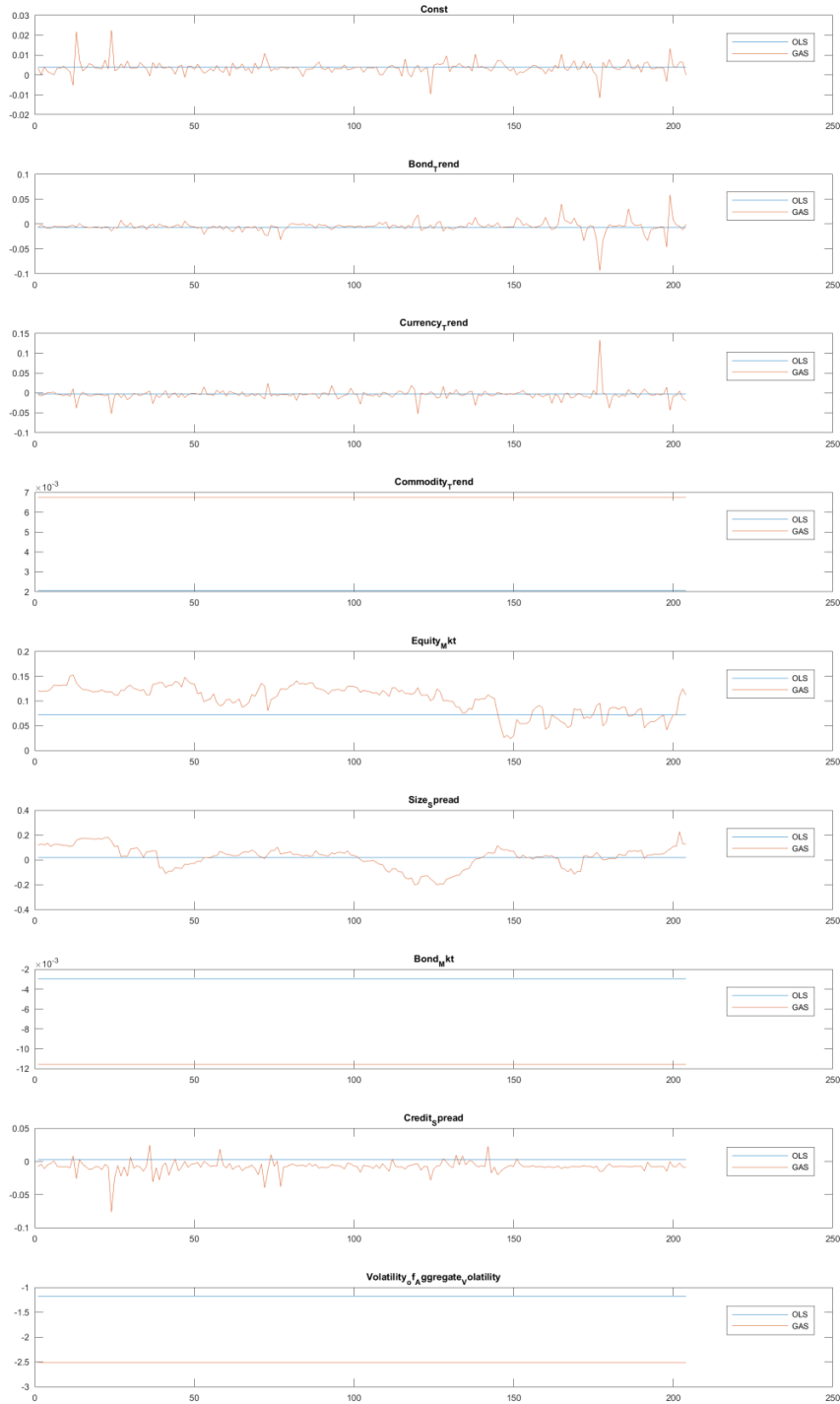
Appendix figure 2.9: GAS All time-varying parameters vs. OLS for Multi-Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 1,719 Multi-Strategy funds between 1994 and 2013. Based on author's calculation.



Appendix figure 2.10: GAS All time-varying parameters vs. OLS for Options Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 51 Options Strategy funds between 1994 and 2013. Based on author's calculation.



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Kenneth R. French - Data Library:
https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html .

Moody's Baa yield: <https://fred.stlouisfed.org/series/DBAA> .

Supplementary Appendix

For Master Thesis:

Modeling the evolution of market uncertainty

**Hedge Fund returns and Volatility of Aggregate Volatility within a
dynamic perspective**

Candidate:

Annalisa Carosi,

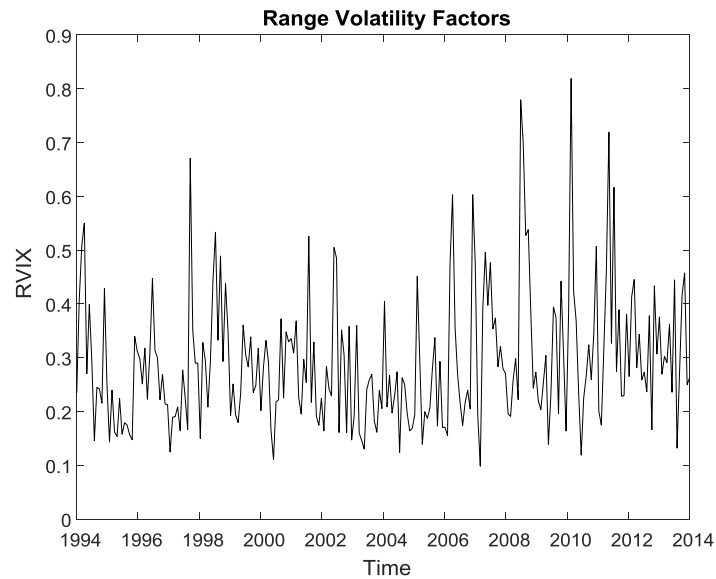
950912-T464

Academic Year: 2018/2019

Supplementary Appendix 1: *RVIX* results

Supplementary Appendix figure 1.1: *RVIX* Factors time series

This image plots the time-series of *RVIX* factors for the period between January 1994 to April 2014 from FRED data from daily *VIX* index returns.



Supplementary Appendix table1.2

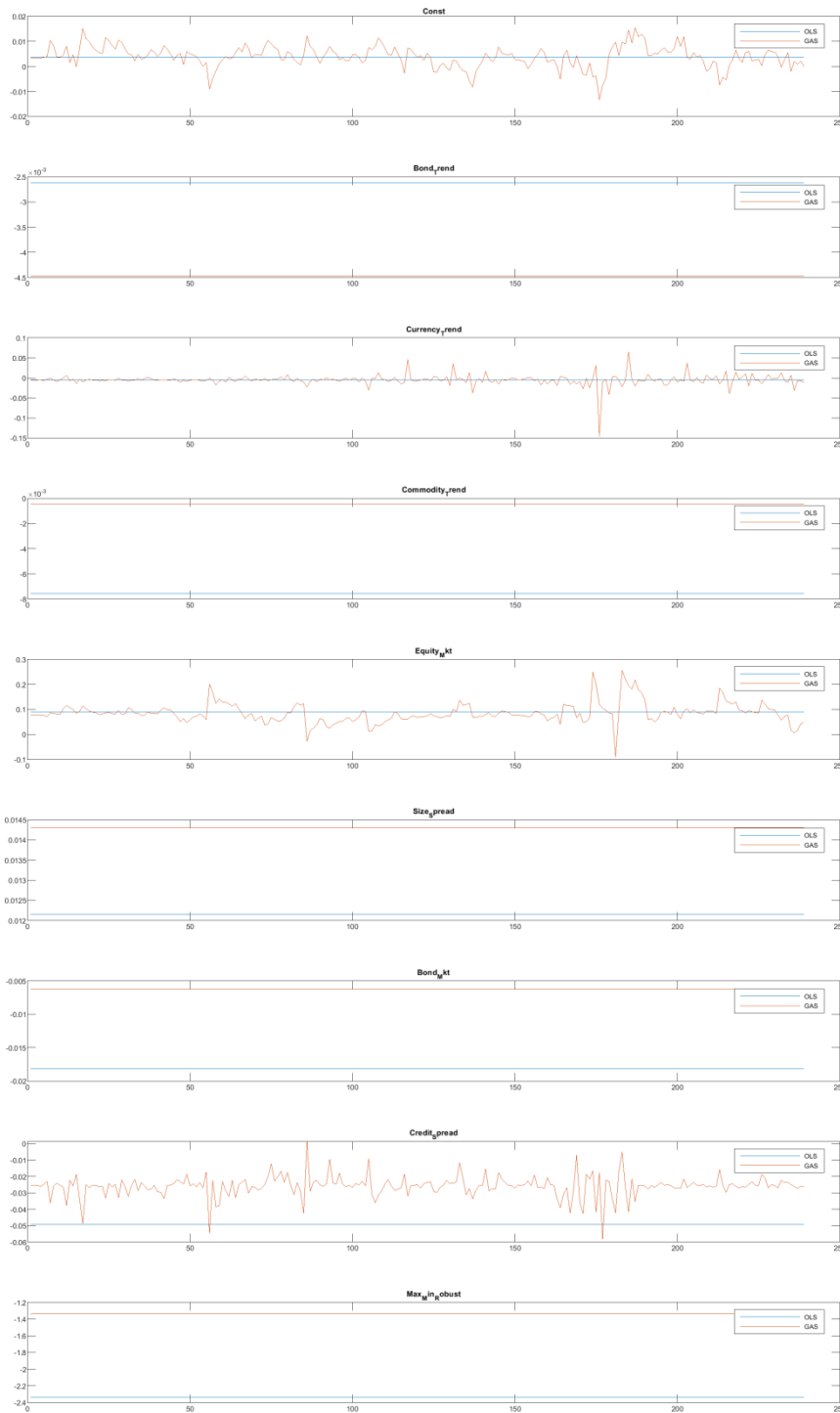
RVIX Pearson correlation among factors

*, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels, respectively, for p-values. Values rounded to the 3rd decimal place.

Factors	<i>PTFSB</i>	<i>PTFSFX</i>	<i>PTFSCOM</i>	<i>SNPMRF</i>	<i>SCMLC</i>	<i>BD10RET</i>	<i>BAAMTSY</i>	<i>RVIX</i>
<i>PTFSB</i>	1							
<i>PTFSFX</i>	0.273***	1						
<i>PTFSCOM</i>	0.211**	0.345***	1					
<i>SNPMRF</i>	-0.257**	-0.197**	-0.167**	1				
<i>SCMLC</i>	-0.074	-0.007	-0.061	0.215***	1			
<i>BD10RET</i>	-0.227**	-0.144*	-0.112*	0.199***	0.199***	1		
<i>BAAMTSY</i>	0.232***	0.296***	0.189***	-0.442**	-0.245**	-0.534**	1	
<i>RVIX</i>	0.203***	0.119*	0.157***	-0.311**	-0.211**	-0.166**	-0.334**	1

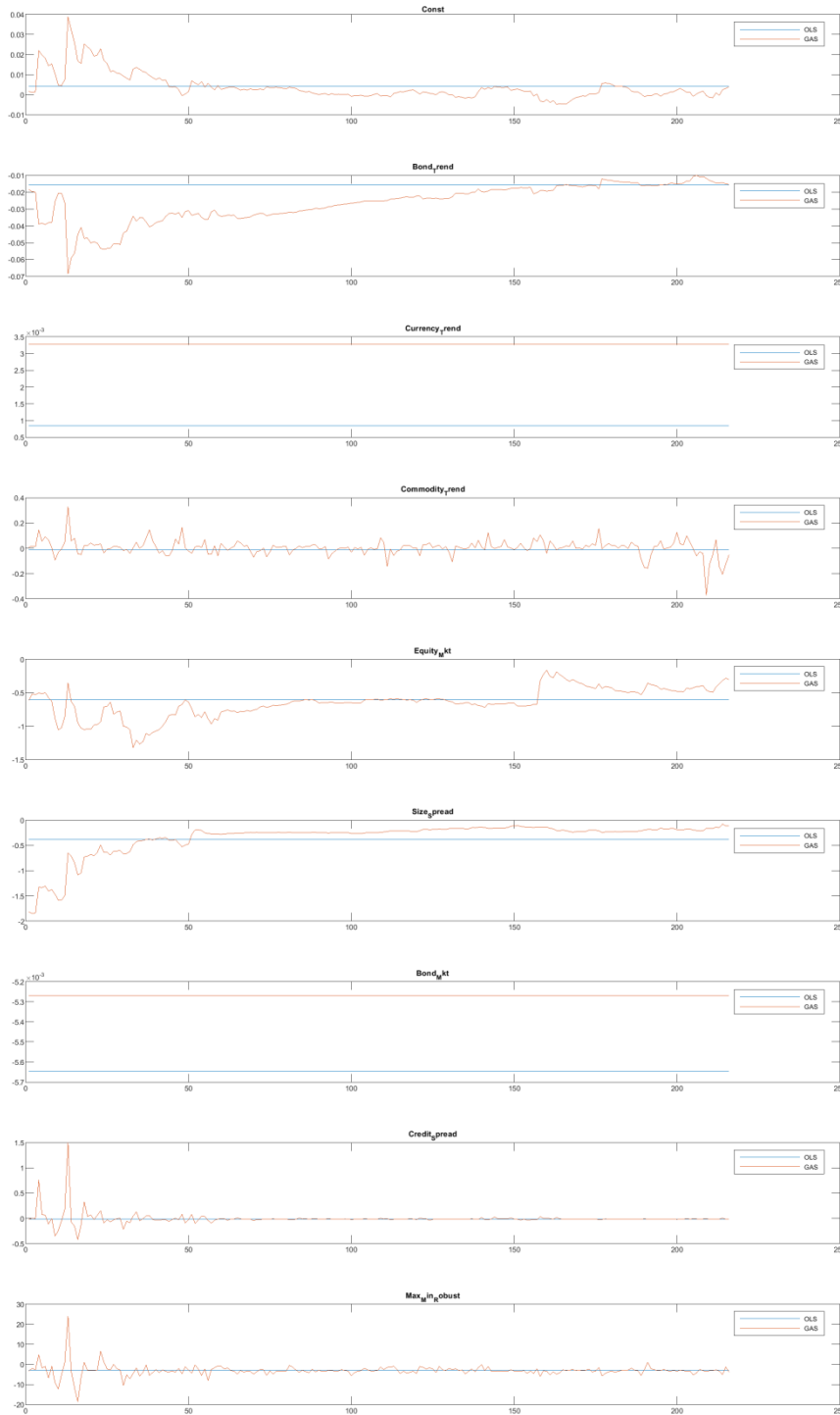
Supplementary Appendix figure 1.2: GAS All time-varying parameters vs. OLS for *RVIX* Convertible Arbitrage Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 247 Convertible Arbitrage strategy funds between 1994 and 2013. Based on author's calculation.



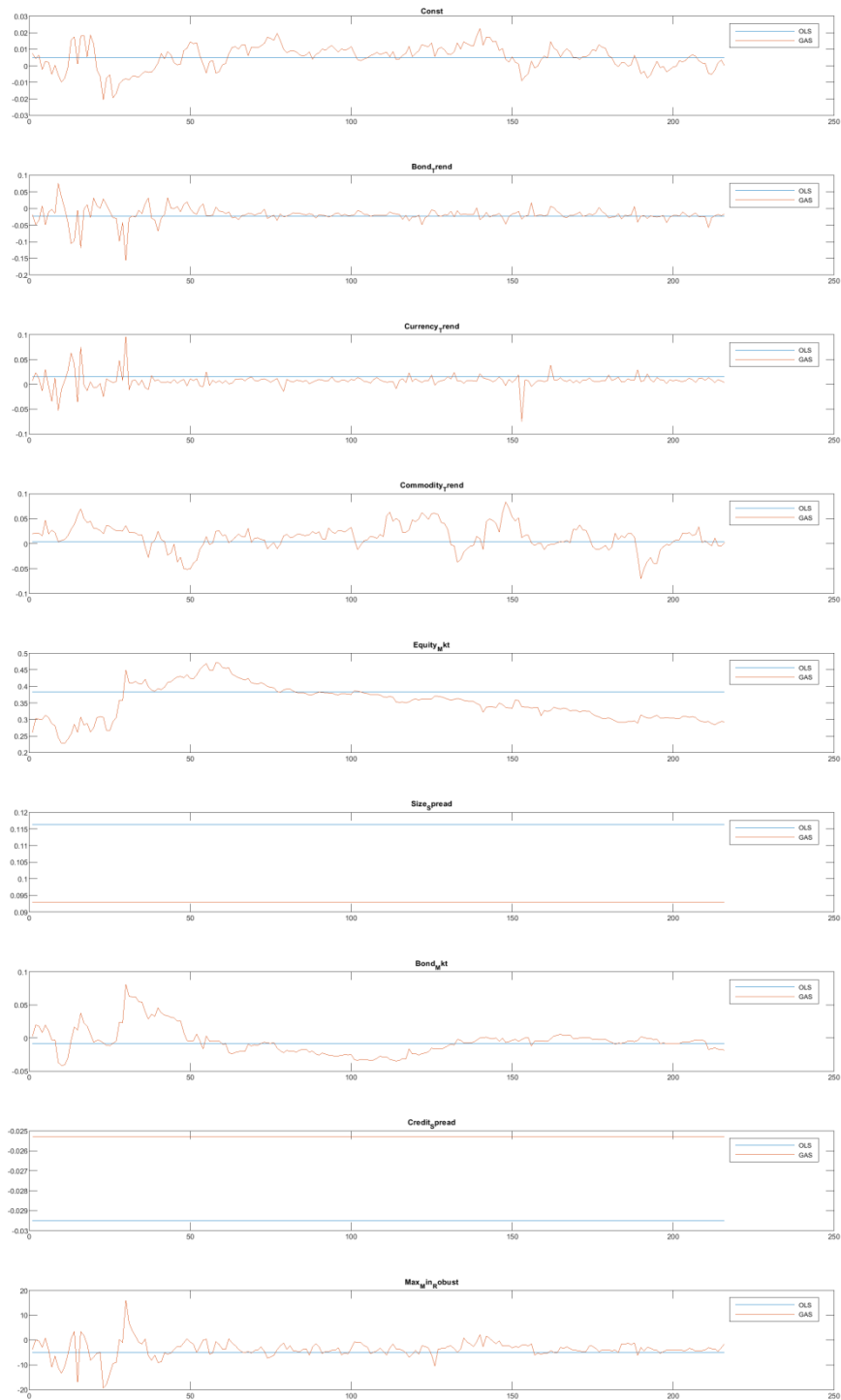
Supplementary Appendix figure 1.3: GAS All time-varying parameters vs. OLS for *RVIX* Dedicated Short Bias Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 43 Dedicated Short Bias strategy funds between 1996 and 2013. Based on author's calculation.



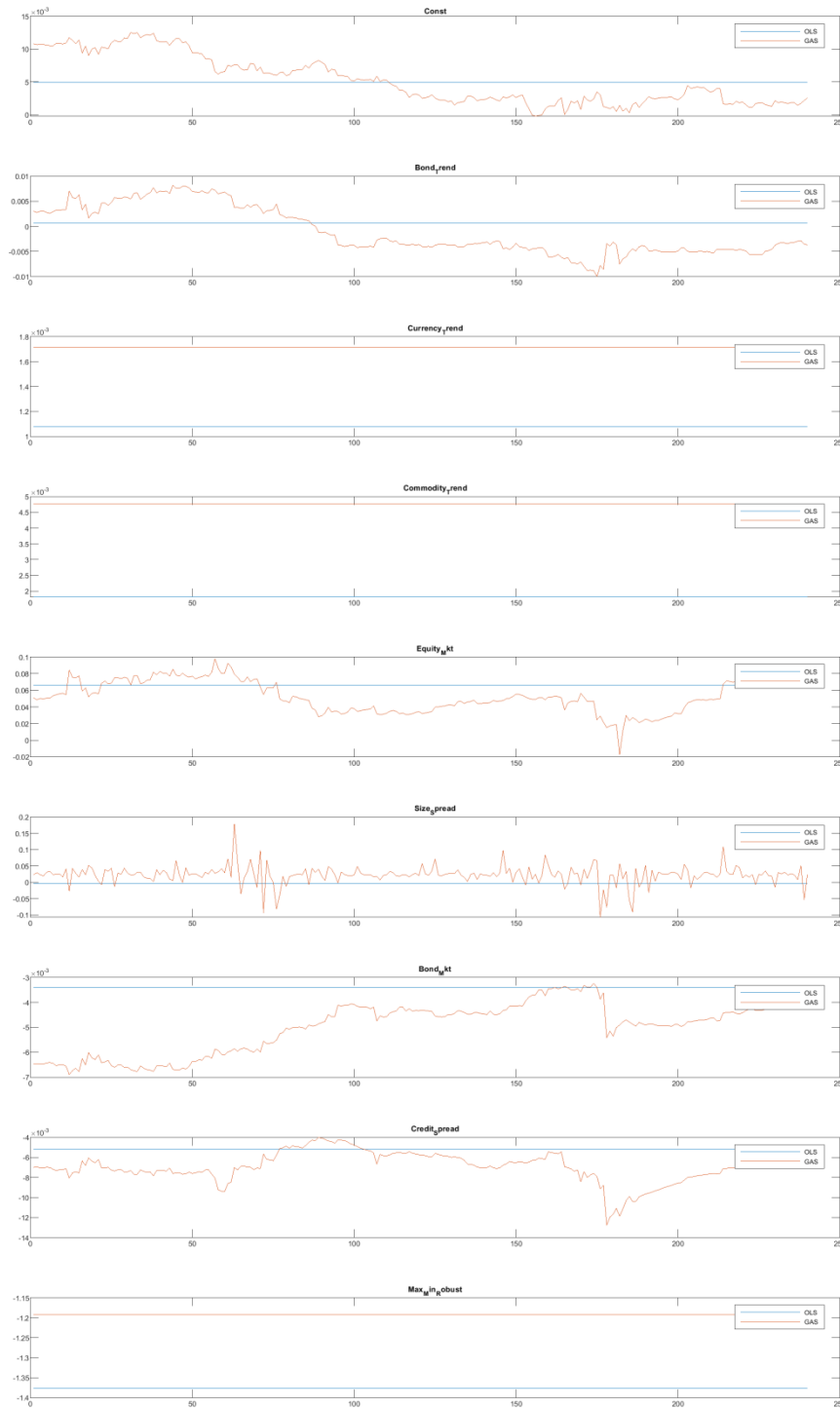
Supplementary Appendix figure 1.4: GAS All time-varying parameters vs. OLS for *RVIX* Emerging Market Strategy

I show how the loadings for each risk factor have been changing over time .Data refer to the equally weighted portfolio for the 853 Emerging Market strategy funds between 1996 and 2013. Based on author's calculation.



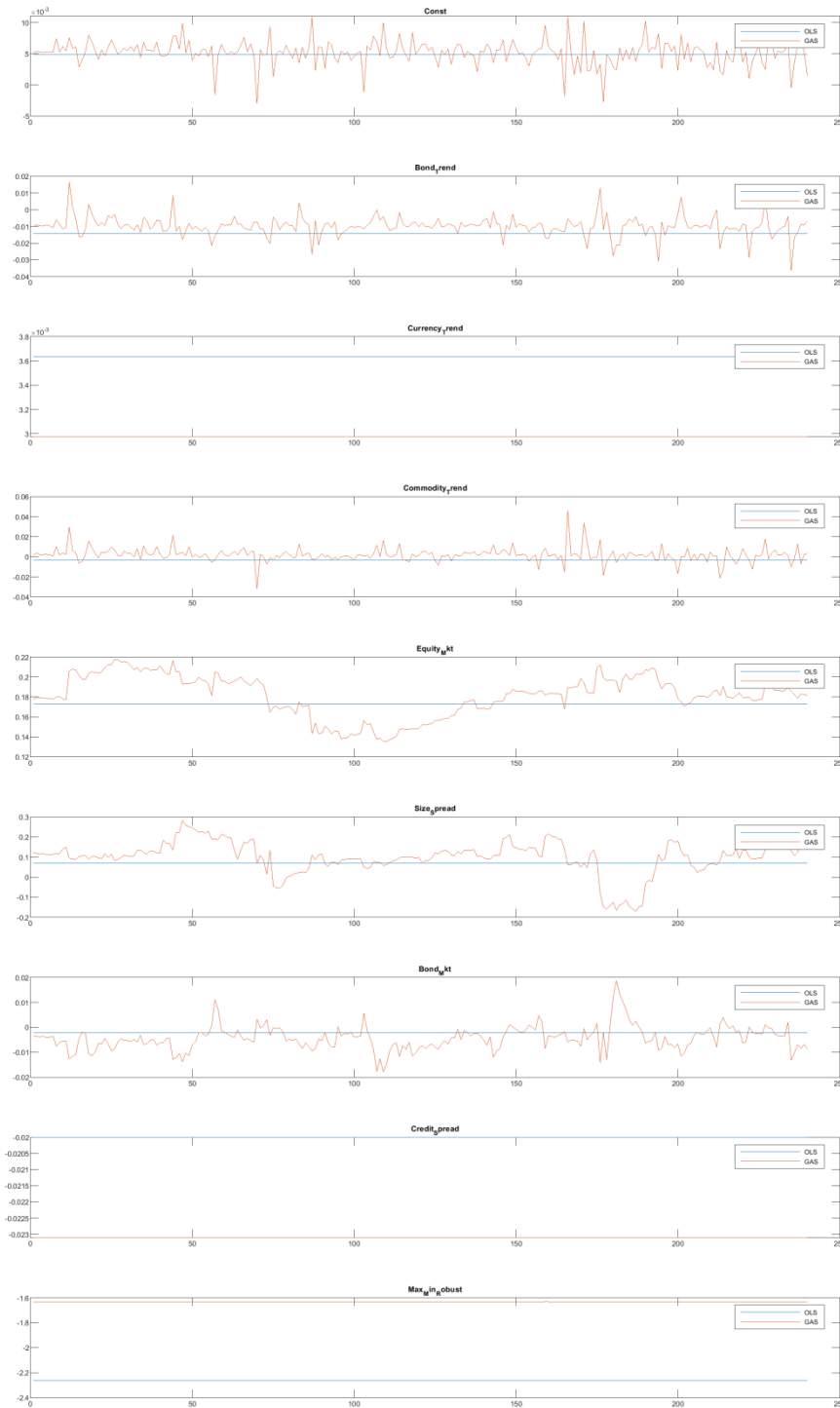
Supplementary Appendix figure 1.5: GAS All time-varying parameters vs. OLS for *RVIX* Equity Market Neutral Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 579 Equity Market Neutral strategy funds between 1994 and 2013. Based on author's calculation.



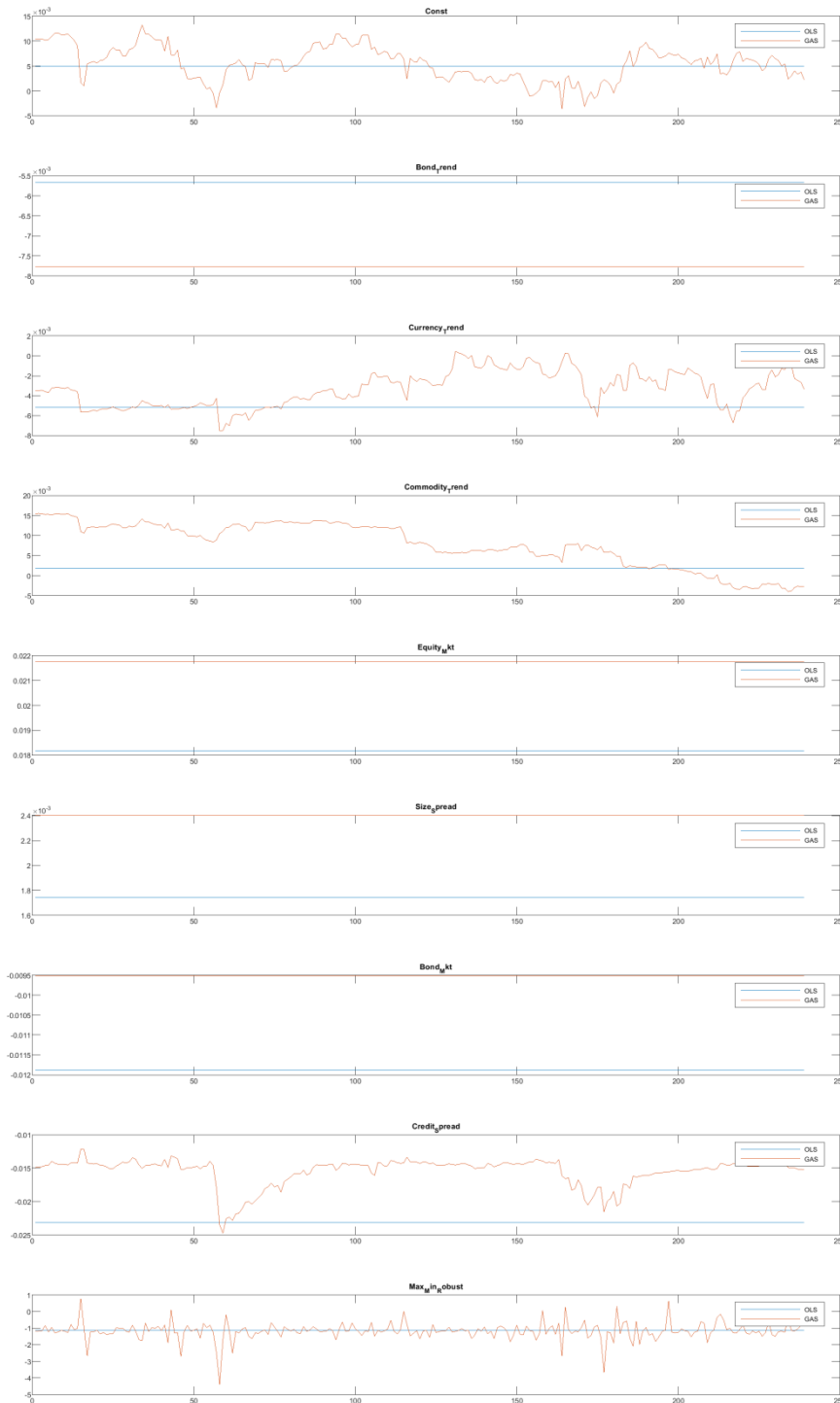
Supplementary Appendix figure 1.6: GAS All time-varying parameters vs. OLS for *RVIX* Event Driven Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 658 Event Driven strategy funds between 1994 and 2013. Based on author's calculation.



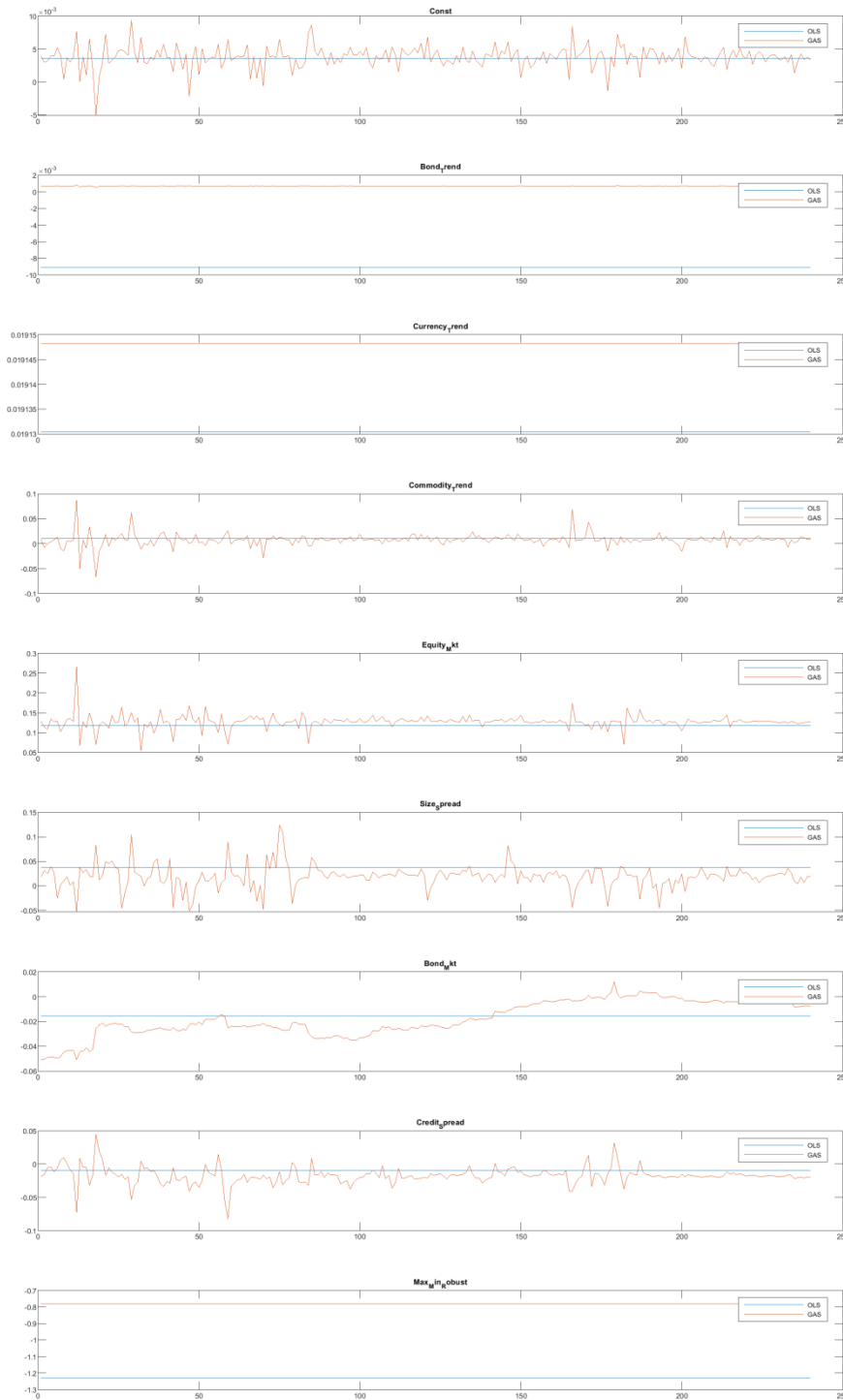
Supplementary Appendix figure 1.7: GAS All time-varying parameters vs. OLS for *RVIX* Fixed-Income Arbitrage Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 394 Fixed-Income Arbitrage strategy funds between 1994 and 2013. Based on author's calculation.



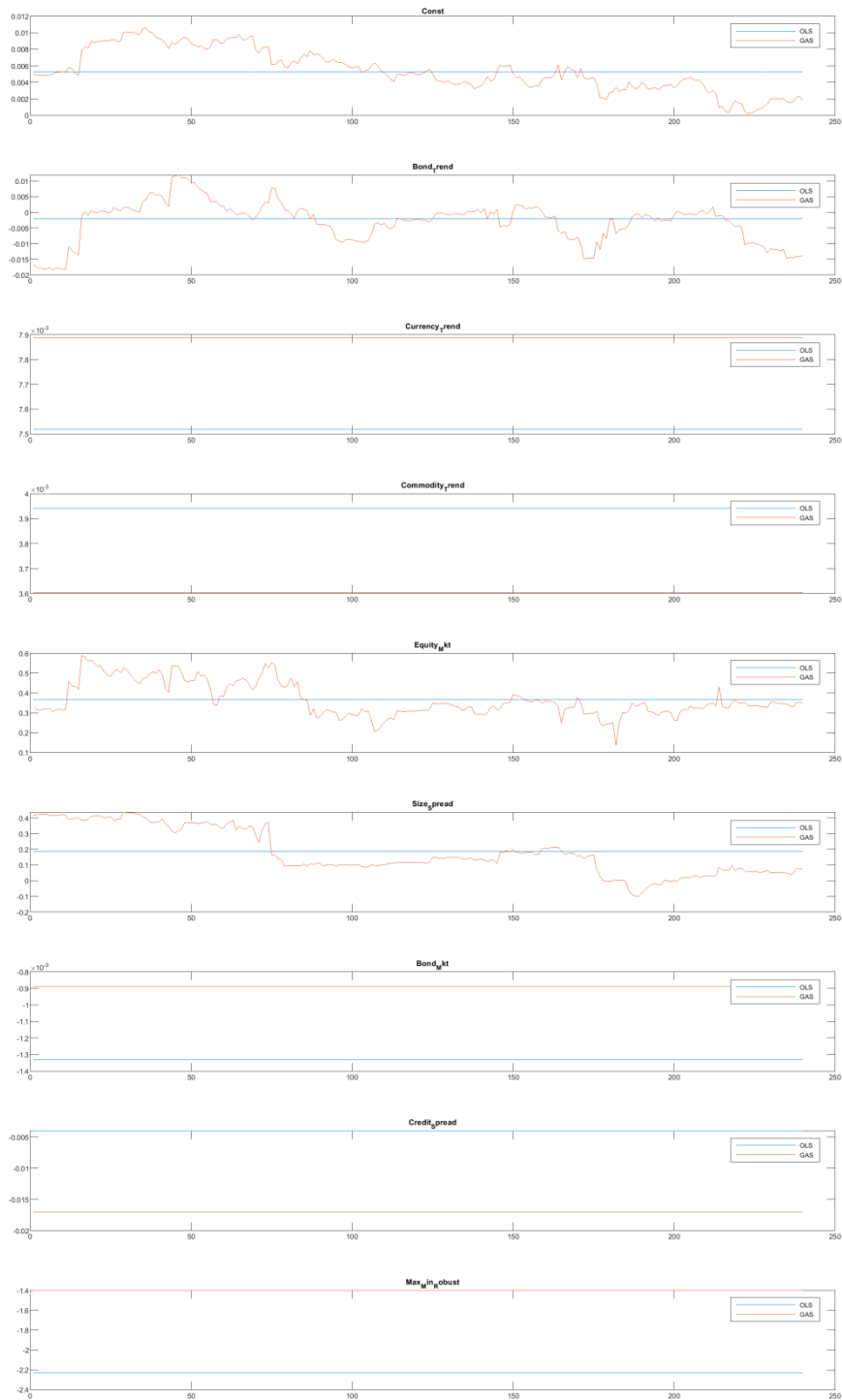
Supplementary Appendix figure 1.8: GAS All time-varying parameters vs. OLS for *RVIX* Global Macro Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 718 Global Macro strategy funds between 1994 and 2013. Based on author's calculation.



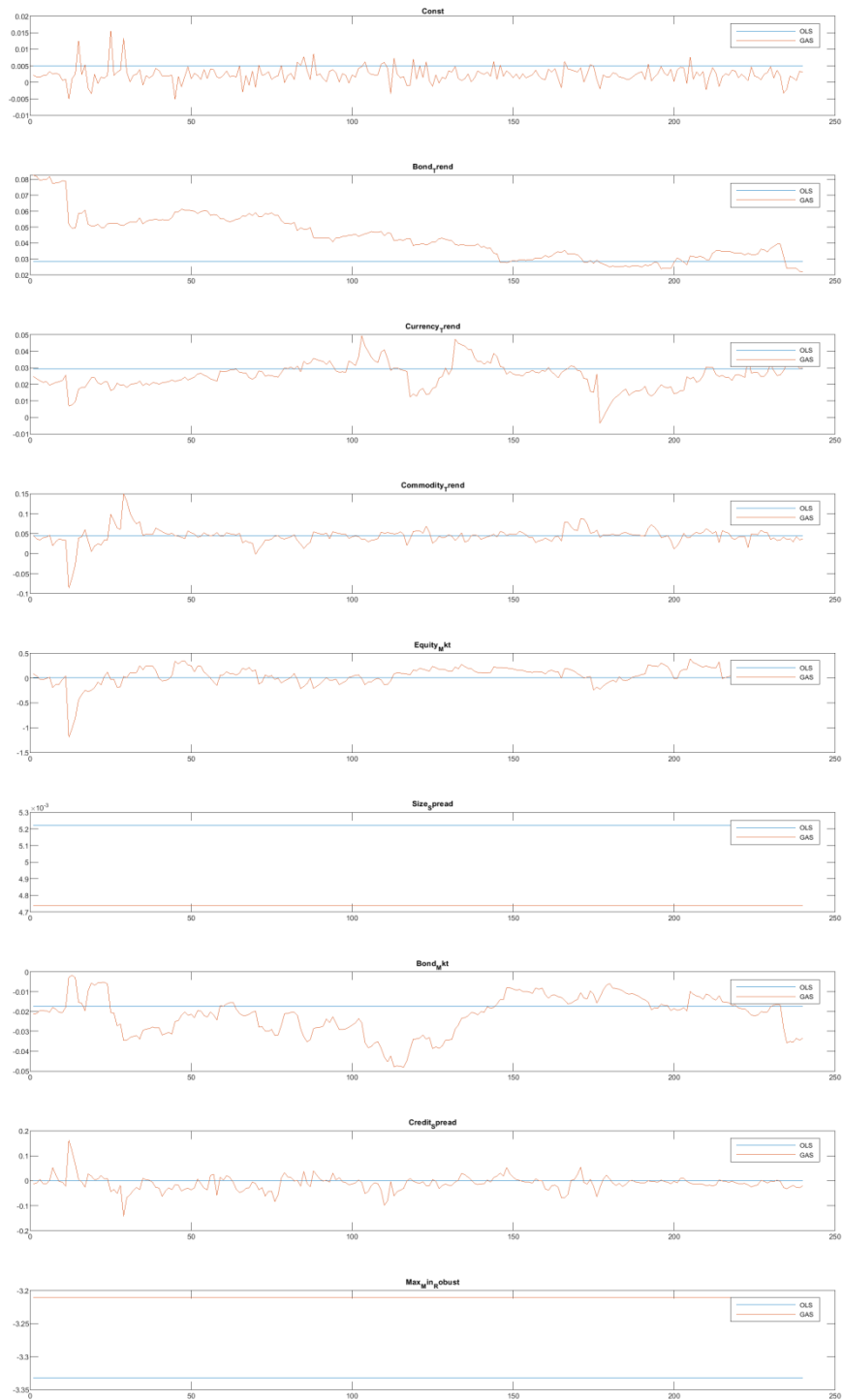
Supplementary Appendix figure 1.9: GAS All time-varying parameters vs. OLS for *RVIX* Long/ Short Equity Hedge Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for 3,189 the Long/Short Equity Hedge strategy funds between 1994 and 2013. Based on author's calculation.



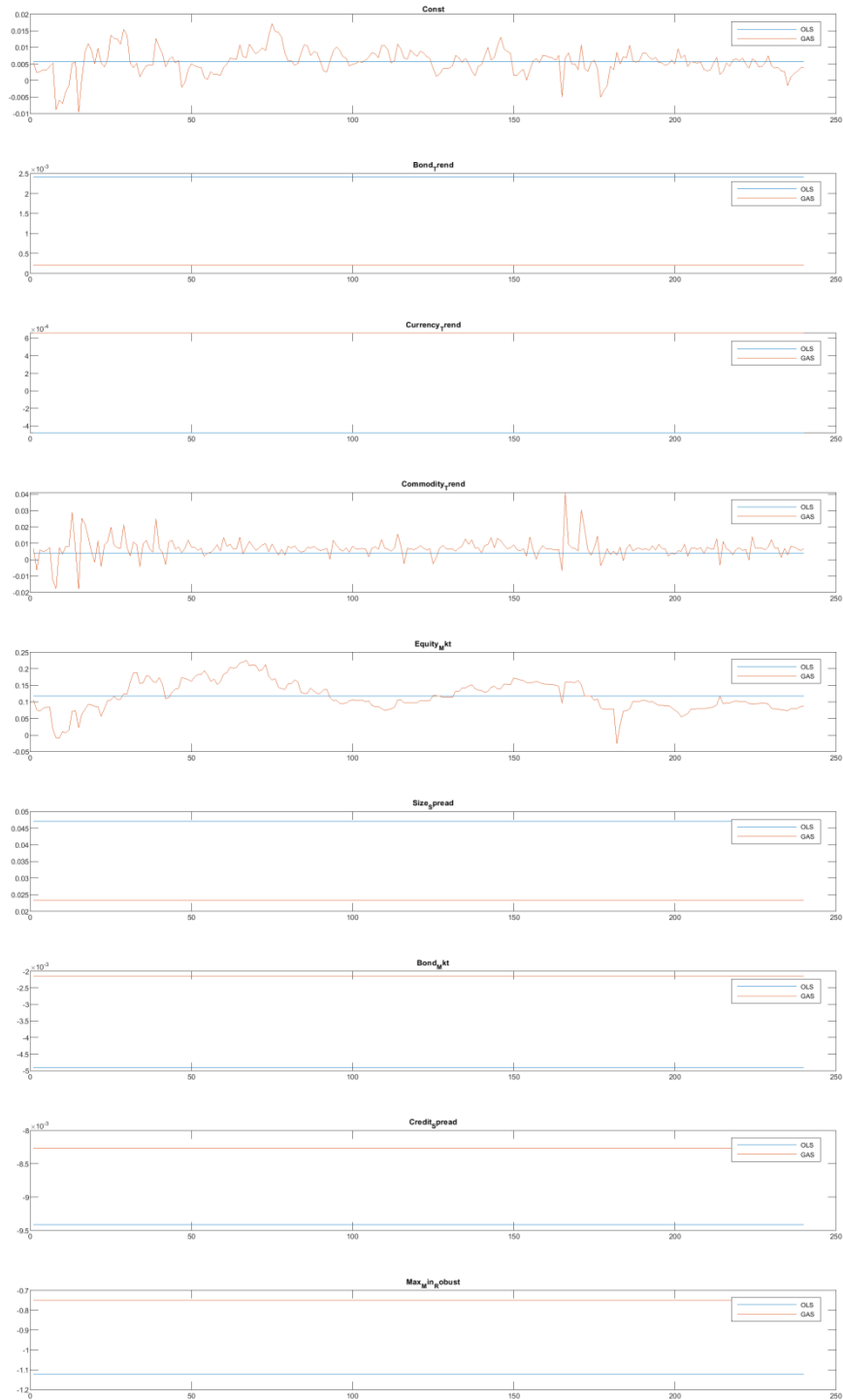
Supplementary Appendix figure 1.10: GAS All time-varying parameters vs. OLS for *RVIX* Managed Futures Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 718 Global Macro strategy funds between 1994 and 2013. Based on author's calculation.



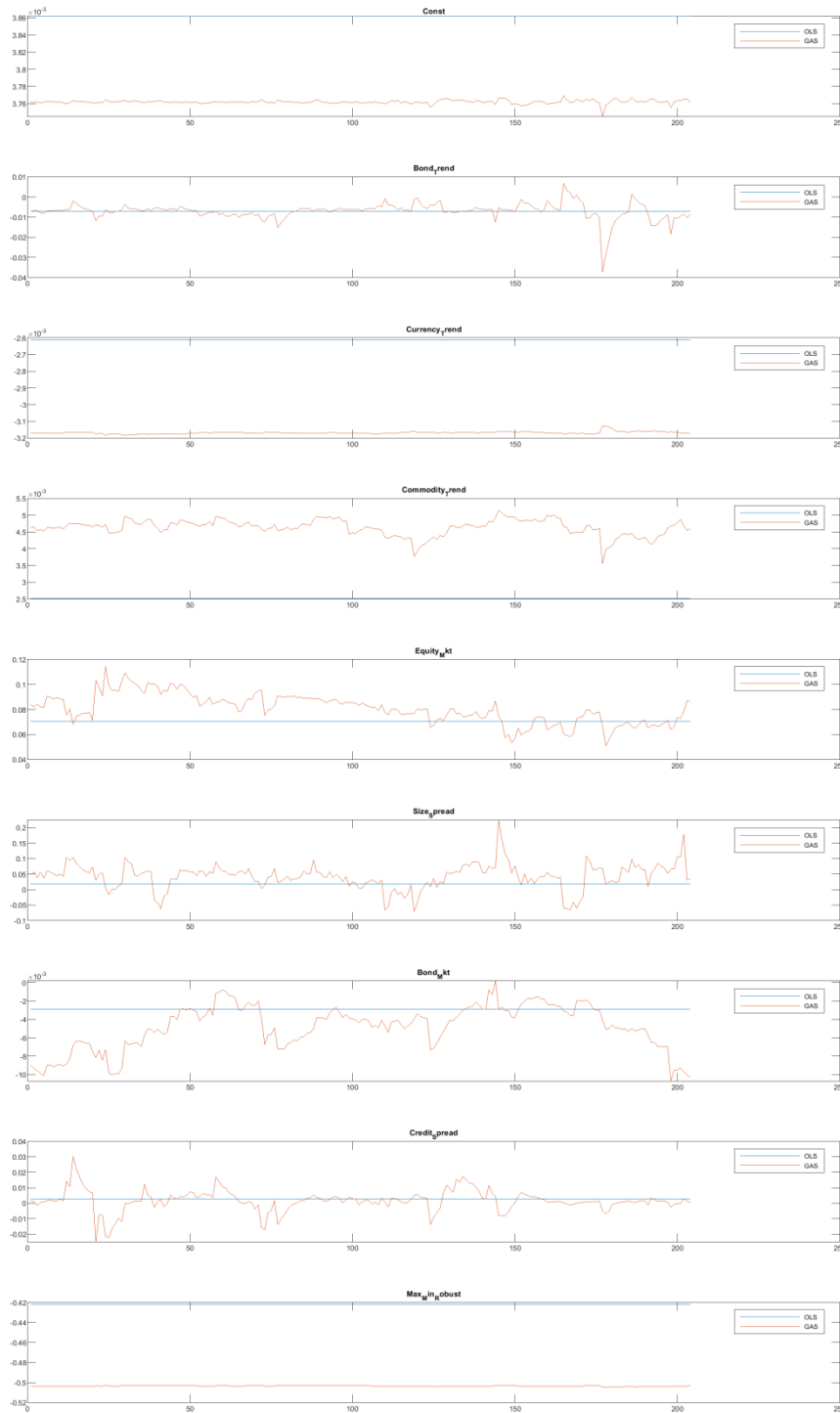
Supplementary Appendix figure 1.11: GAS All time-varying parameters vs. OLS for *RVIX* Multi-Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 1,719 Multi-Strategy funds between 1994 and 2013. Based on author's calculation.



Supplementary Appendix figure 1.12: GAS All time-varying parameters vs. OLS for *RVIX* Options Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 51 Options Strategy funds between 1994 and 2013. Based on author's calculation.



Supplementary Appendix table 1.2

RVIX-only time-varying GAS parameters

This table displays the estimated ‘RVIX-only time-varying’ GAS parameter values for each of the 11 equally weighted portfolio strategies. Their robust standard errors, which are reported in squared brackets. The strategies’ names are abbreviated as follows: CA stands for *Convertible Arbitrage*; DS for *Dedicated Short Bias*; EM for *Emerging Markets*; EN for *Equity Neutral*; FA for *Fixed Income Arbitrage*; GM for *Global Macro*; LS for *Long/Short Equity Hedge*; MF for *Managed Futures*; MS for *Multi-Strategy*; OS for *Options Strategy*. *, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels for the A t-test statistic, respectively. Robust standard errors are reported in squared brackets. Values rounded to the 3rd significant digit.

	CA	DS	EM	EN	ED	FA	GM	LS	MF	MS	OS
		*				**					
A	0.03 [0.07]	0.01 [0.01]	0.01 [0.01]	0.01 [0.01]	0.01 [0.03]	0.011 [0.11]	0.01 [0.03]	0.01 [0.03]	0.01 [0.01]	0.01 [0.02]	0.01 [0.01]
B	0.97 [0.05]	0.99 [0.01]	0.99 [0.02]	0.99 [0.01]	0.99 [0.04]	0.89 [0.17]	0.99 [0.06]	0.99 [0.02]	0.99 [0.02]	0.99 [0.01]	0.99 [0.02]
α_{OLS}	0.004 [0.00]	0.004 [0.00]	0.005 [0.00]	0.005 [0.00]	0.005 [0.00]	0.005 [0.00]	0.004 [0.00]	0.005 [0.00]	0.005 [0.00]	0.006 [0.00]	0.004 [0.00]
β_{OLS}^{PTFSB}	-0.003 [0.01]	-0.015 [0.02]	-0.024 [0.02]	0.007 [0.00]	-0.015 [0.01]	-0.006 [0.00]	-0.009 [0.01]	-0.002 [0.01]	0.029 [0.01]	0.002 [0.00]	-0.007 [0.00]
$\beta_{OLS}^{PTFSEFX}$	-0.004 [0.00]	-0.001 [0.02]	0.015 [0.01]	0.001 [0.02]	0.004 [0.00]	-0.005 [0.00]	0.019 [0.00]	0.007 [0.00]	0.030 [0.00]	-0.005 [0.00]	-0.003 [0.00]
$\beta_{OLS}^{PTFSCOM}$	-0.008 [0.00]	-0.012 [0.02]	0.004 [0.02]	0.002 [0.00]	-0.003 [0.01]	0.002 [0.00]	0.010 [0.00]	0.004 [0.01]	0.045 [0.01]	0.004 [0.00]	0.002 [0.00]
$\beta_{OLS}^{BD10RET}$	0.089 [0.03]	-0.605 [0.07]	0.384 [0.06]	0.066 [0.01]	0.173 [0.02]	0.018 [0.01]	0.117 [0.02]	0.367 [0.04]	0.009 [0.04]	0.118 [0.02]	0.071 [0.02]
$\beta_{OLS}^{BAAMTSY}$	0.012 [0.03]	-0.383 [0.08]	0.116 [0.07]	-0.004 [0.02]	0.071 [0.03]	0.002 [0.02]	0.037 [0.03]	0.185 [0.05]	0.005 [0.05]	0.047 [0.02]	0.018 [0.02]
β_{OLS}^{SNPMRF}	-0.018 [0.00]	-0.006 [0.01]	-0.008 [0.01]	-0.003 [0.00]	-0.002 [0.00]	-0.012 [0.00]	-0.016 [0.00]	-0.001 [0.00]	-0.017 [0.00]	-0.005 [0.00]	-0.003 [0.00]
β_{OLS}^{SCMLC}	-0.050 [0.01]	-0.013 [0.02]	-0.030 [0.01]	-0.005 [0.00]	-0.020 [0.00]	-0.023 [0.00]	-0.009 [0.00]	-0.004 [0.00]	-0.006 [0.00]	-0.010 [0.00]	0.003 [0.00]
β_{OLS}^{RVIX}	-2.335 [1.70]	-2.851 [4.72]	-4.876 [4.08]	-1.377 [0.71]	-2.26 [1.54]	-1.133 [0.79]	-1.228 [1.18]	-2.233 [2.48]	-3.332 [2.44]	-1.121 [1.05]	-0.422 [0.51]
ω	2.95 [0.56]	16.59 [4.17]	12.16 [3.36]	0.69 [0.08]	2.37 [0.31]	0.77 [0.22]	1.848 [0.29]	5.322 [0.65]	5.85 [0.87]	1.32 [0.26]	1.048 [0.12]

Supplementary Appendix table 1.3

Selected factors time-varying GAS parameters

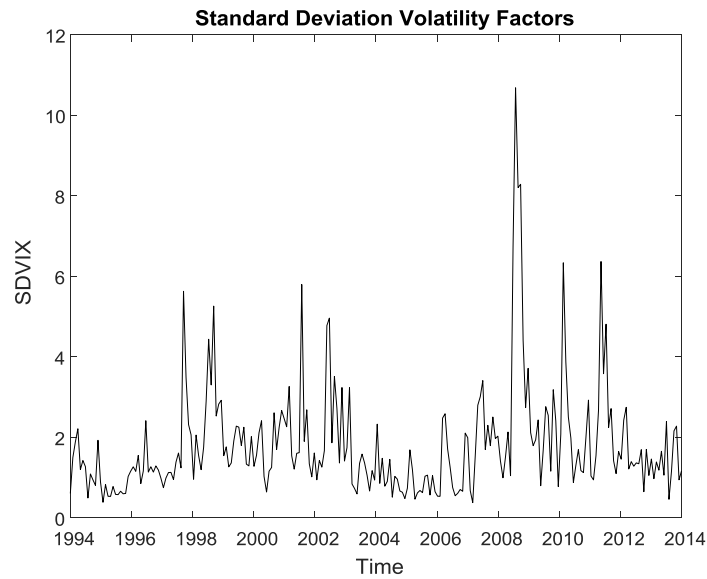
This table displays the estimated 'RVIX selected factors time-varying' GAS parameter values for each of the 11 equally weighted portfolio strategies. Their robust standard errors, which are reported in squared brackets. The strategies' names are abbreviated as follows: CA stands for *Convertible Arbitrage*; DS for *Dedicated Short Bias*; EM for *Emerging Markets*; EN for *Equity Neutral*; FA for *Fixed Income Arbitrage*; GM for *Global Macro*; LS for *Long/Short Equity Hedge*; MS for *Multi-Strategy*; OS for *Options Strategy*. *, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels for the A t-test statistic, respectively. Robust standard errors are reported in squared brackets. Values rounded to the 3rd significant digit.

	CA	DS	EM	EN	ED	FA	GM	LS	MS	OS
	***	***	**	***	***	***		***	*	*
A	0.019 [0.00]	0.032 [0.00]	0.031 [0.01]	0.02 [0.00]	0.015 [0.03]	0.034 [0.01]	0.018 [0.02]	0.083 [0.01]	0.017 [0.01]	0.051 [0.04]
B	0.934 [0.02]	1.0 [0.00]	0.859 [0.09]	0.97 [0.01]	0.673 [0.08]	0.946 [0.02]	0.97 [0.33]	0.968 [0.02]	0.89 [0.12]	0.011 [0.00]
α_{OLS}	0.004 [0.00]	0.023 [0.01]	0.005 [0.00]	0.005 [0.00]	0.004 [0.00]	0.005 [0.00]	0.005 [0.00]	0.005 [0.00]	0.006 [0.00]	0.004 [0.00]
β_{OLS}^{PTFSB}	-0.002 [0.00]	-0.019 [0.01]	-0.017 [0.01]	-0.001 [0.00]	-0.012 [0.00]	-0.007 [0.00]	-0.011 [0.00]	-0.012 [0.01]	0.002 [0.00]	-0.006 [0.00]
β_{OLS}^{PTFSFX}	-0.004 [0.00]	0.09 [0.04]	0.013 [0.01]	0.001 [0.02]	0.0 [0.00]	-0.005 [0.00]	0.01 [0.00]	0.009 [0.00]	-0.001 [0.00]	-0.003 [0.00]
$\beta_{OLS}^{PTFSCOM}$	-0.010 [0.00]	0.023 [0.03]	0.008 [0.01]	-0.001 [0.00]	-0.002 [0.00]	0.002 [0.00]	0.003 [0.00]	0.003 [0.00]	0.003 [0.00]	0.003 [0.01]
$\beta_{OLS}^{BD10RET}$	0.095 [0.02]	-0.889 [0.03]	0.359 [0.04]	0.074 [0.00]	0.179 [0.01]	0.017 [0.01]	0.322 [0.02]	0.322 [0.05]	0.12 [0.02]	0.071 [0.02]
$\beta_{OLS}^{BAAMTSY}$	0.021 [0.02]	-0.416 [0.05]	0.126 [0.04]	-0.003 [0.01]	0.107 [0.00]	0.004 [0.00]	0.205 [0.00]	0.205 [0.07]	0.047 [0.00]	0.029 [0.00]
β_{OLS}^{SNPMRF}	-0.013 [0.00]	-0.007 [0.00]	-0.003 [0.00]	0.001 [0.00]	-0.004 [0.00]	-0.01 [0.00]	-0.014 [0.00]	0.0 [0.00]	-0.003 [0.00]	-0.004 [0.00]
β_{OLS}^{SCMLC}	-0.035 [0.00]	0.307 [0.22]	-0.024 [0.00]	-0.002 [0.00]	-0.02 [0.00]	-0.016 [0.00]	-0.009 [0.00]	-0.014 [0.00]	-0.005 [0.00]	-0.001 [0.00]
β_{OLS}^{RVIX}	-1.953 [0.60]	-2.251 [7.72]	-4.366 [1.68]	-1.046 [0.41]	-1.995 [0.41]	-0.736 [0.48]	-1.351 [0.68]	-1.301 [0.57]	-1.029 [0.54]	-0.318 [0.59]
ω	1.06 [0.09]	5.21 [0.50]	5.15 [0.49]	0.47 [0.04]	0.53 [0.05]	0.41 [0.04]	1.218 [0.11]	0.916 [0.08]	0.79 [0.07]	0.88 [0.08]

Supplementary Appendix 2: *SDVIX* results

Supplementary Appendix figure 2.1: *SDVIX* Factors time series

This image plots the time-series of *SDVIX* factors for the period between January 1994 to April 2014 from FRED data from daily VIX index returns.



Supplementary Appendix table 2.1

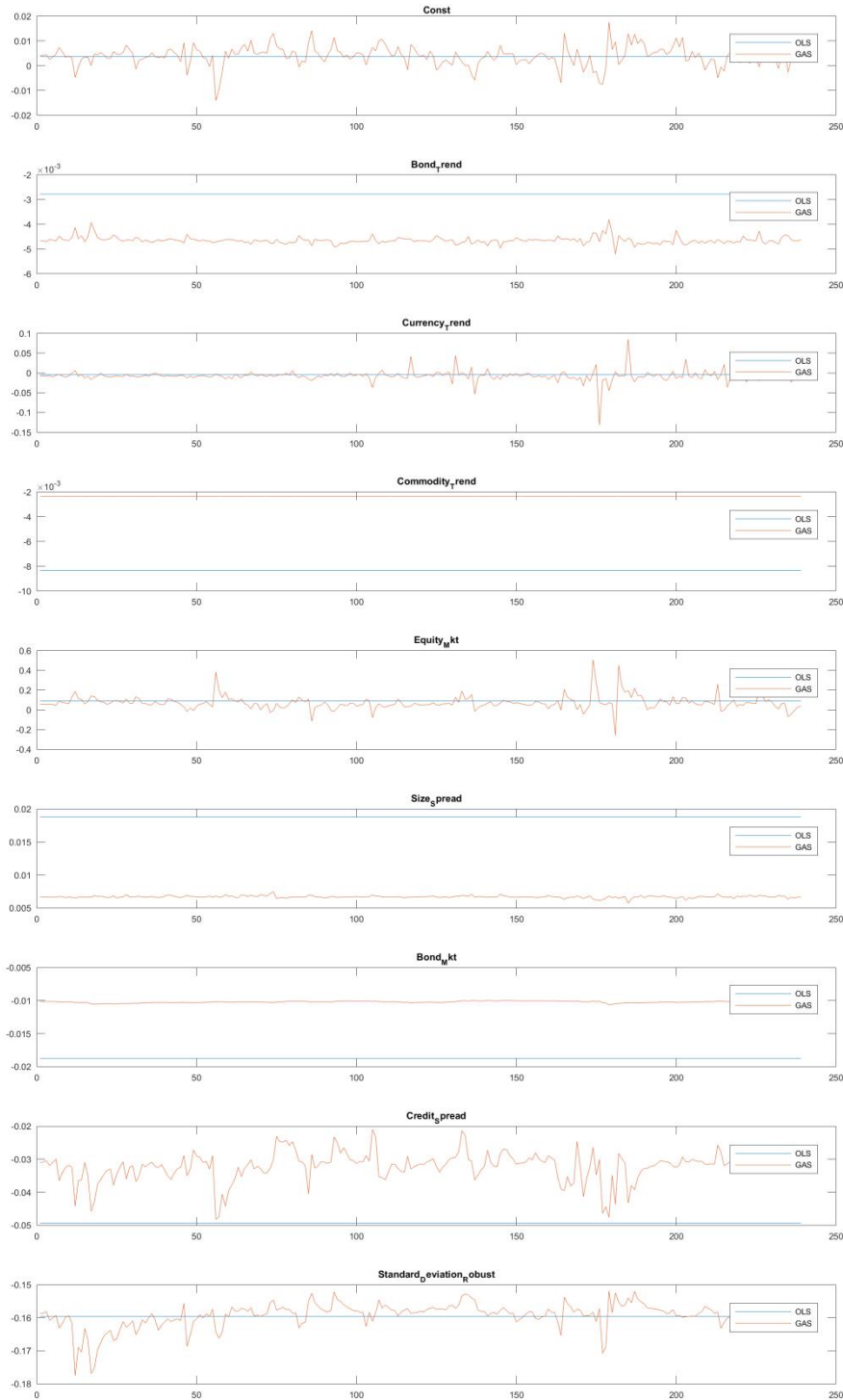
SDVIX Pearson correlation among factors

*, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels, respectively, for p-values. Values rounded to the 3rd decimal place.

Factors	<i>PTFSB</i>	<i>PTFSFX</i>	<i>PTFSCOM</i>	<i>SNPMRF</i>	<i>SCMLC</i>	<i>BD10RET</i>	<i>BAAMTSY</i>	<i>SDVIX</i>
<i>PTFSB</i>	1							
<i>PTFSFX</i>	0.273***	1						
<i>PTFSCOM</i>	0.211***	0.345***	1					
<i>SNPMRF</i>	-0.257**	-0.197**	-0.168**	1				
<i>SCMLC</i>	-0.074	-0.007	-0.061	0.215***	1			
<i>BD10RET</i>	-0.227**	-0.144*	-0.121**	0.199***	0.199***	1		
<i>BAAMTSY</i>	0.232***	0.296**	0.189**	-0.442**	-0.244**	-0.534**	1	
<i>SDVIX</i>	0.253***	0.171**	0.153**	-0.35 **	-0.172**	-0.242**	0.414***	1

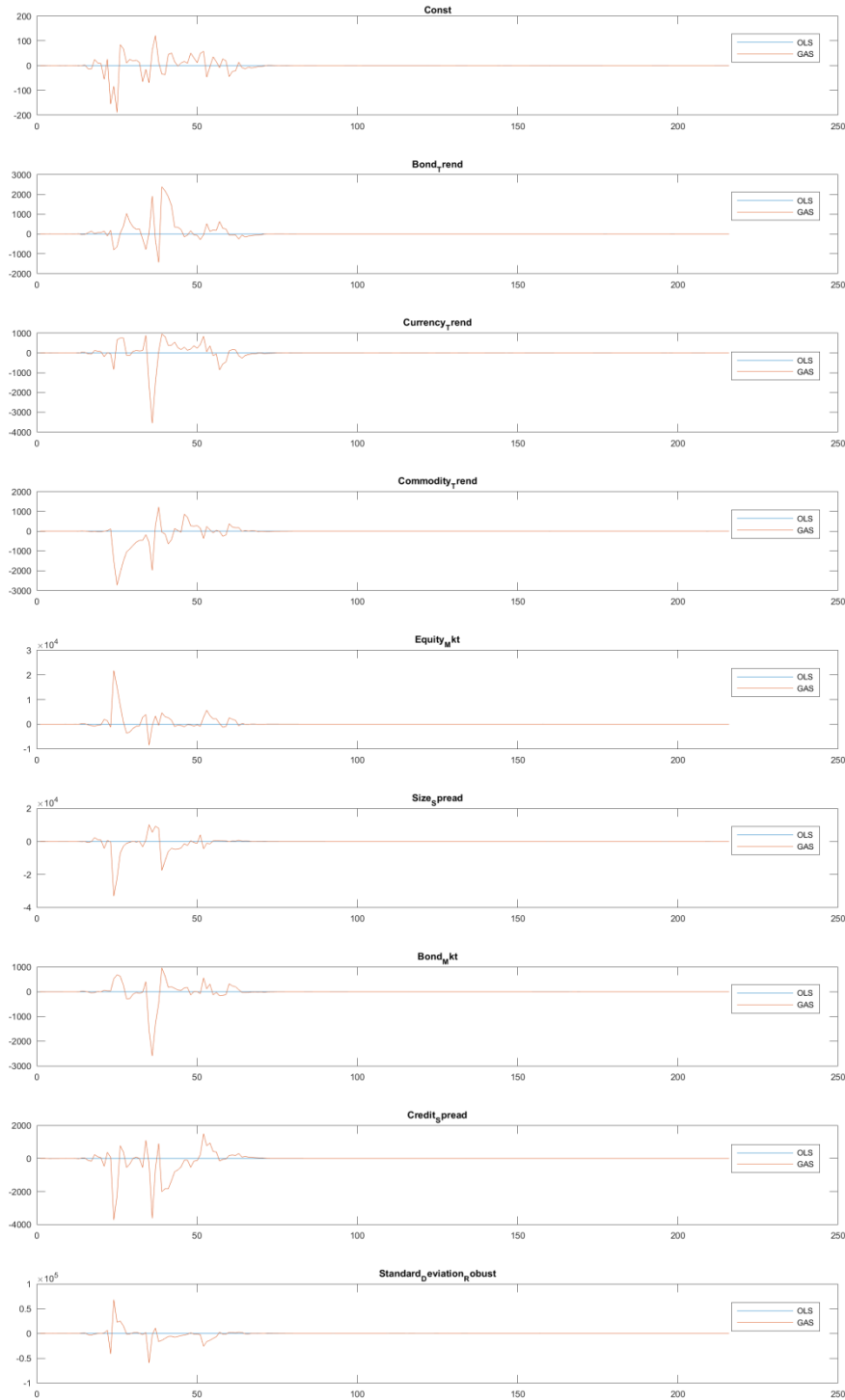
Supplementary Appendix figure 2.2: GAS All time-varying parameters vs. OLS for *SDVIX* Convertible Arbitrage Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 247 Convertible Arbitrage strategy funds between 1994 and 2013. Based on author's calculation.



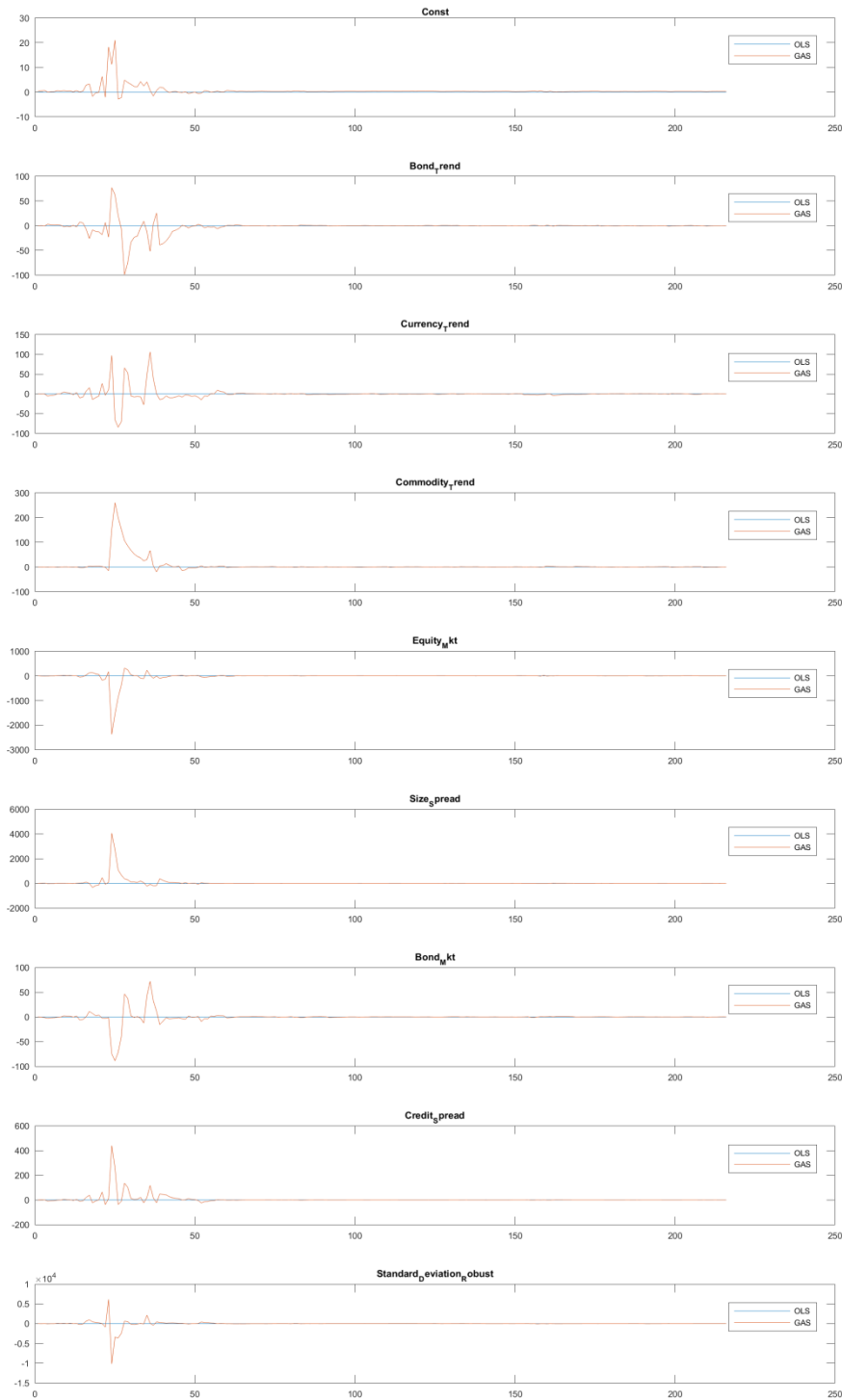
Supplementary Appendix figure 2.3: GAS All time-varying parameters vs. OLS for *SDVIX* Dedicated Short Bias Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 43 Dedicated Short Bias strategy funds between 1996 and 2013. Based on author's calculation.



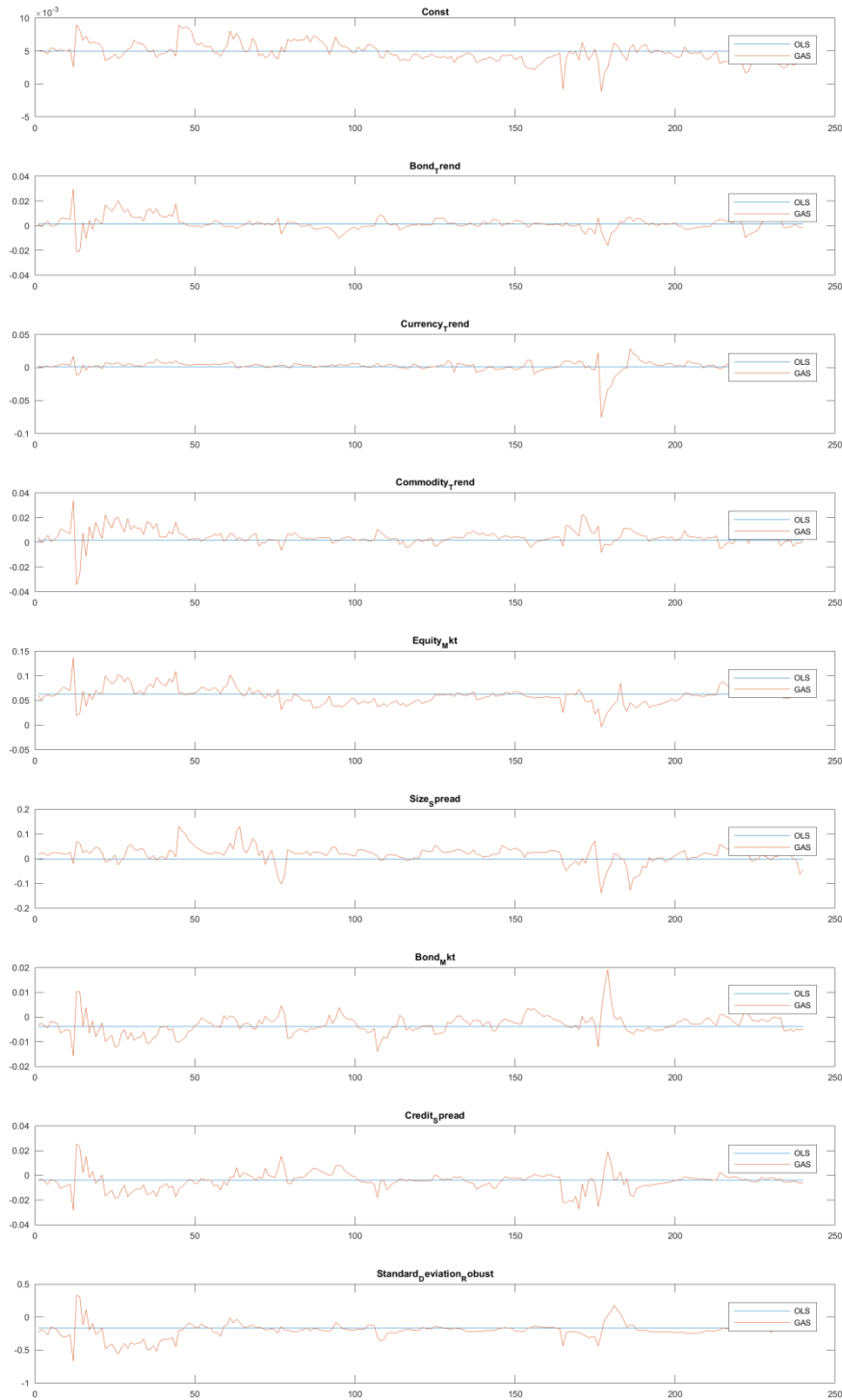
Supplementary Appendix figure 2.4: GAS All time-varying parameters vs. OLS for *SDVIX* Emerging Market Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 853 Emerging Market strategy funds between 1996 and 2013. Based on author's calculation.



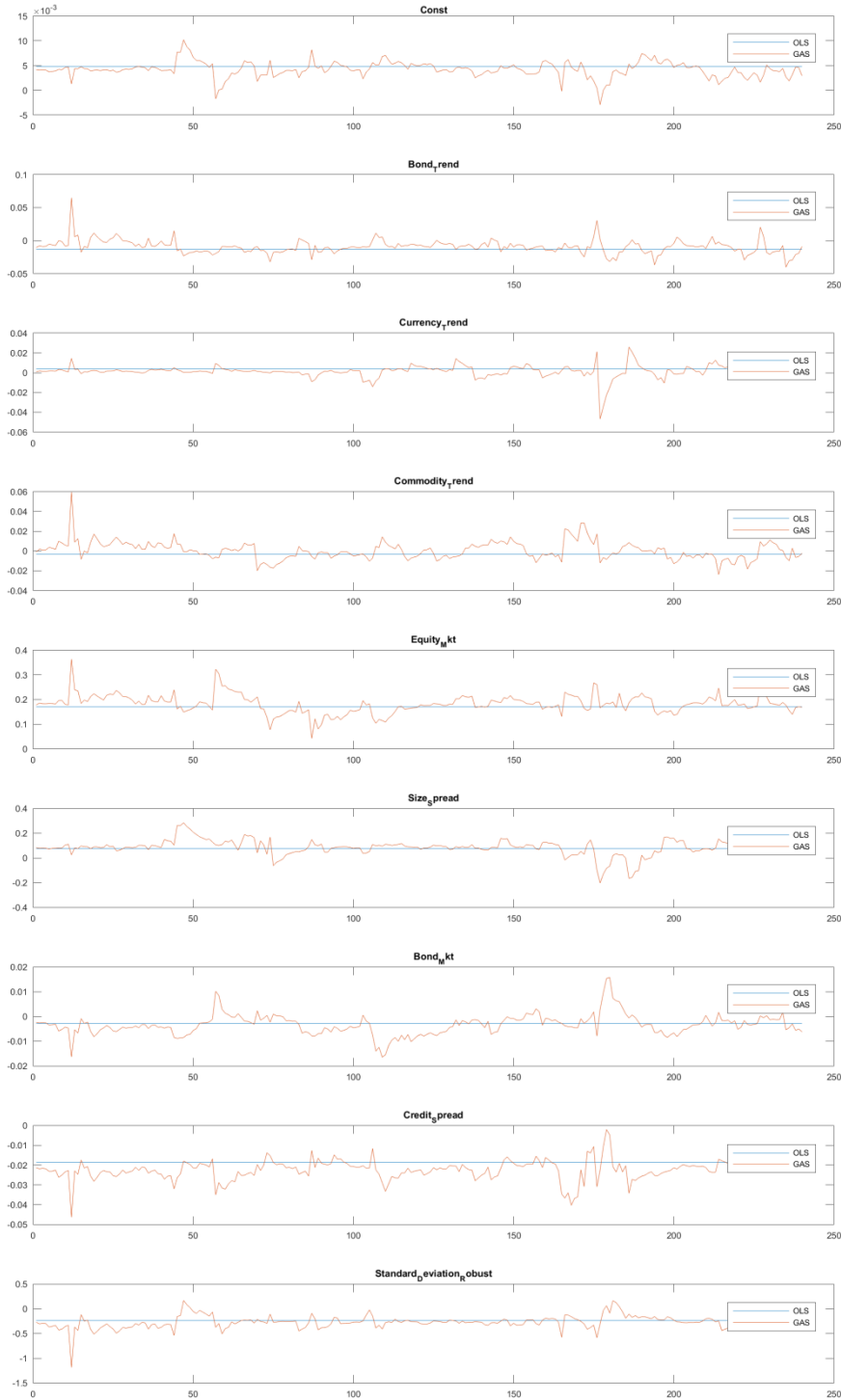
Supplementary Appendix figure 2.5: GAS All time-varying parameters vs. OLS for *SDVIX* Equity Market Neutral Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 579 Equity Market Neutral strategy funds between 1994 and 2013. Based on author's calculation.



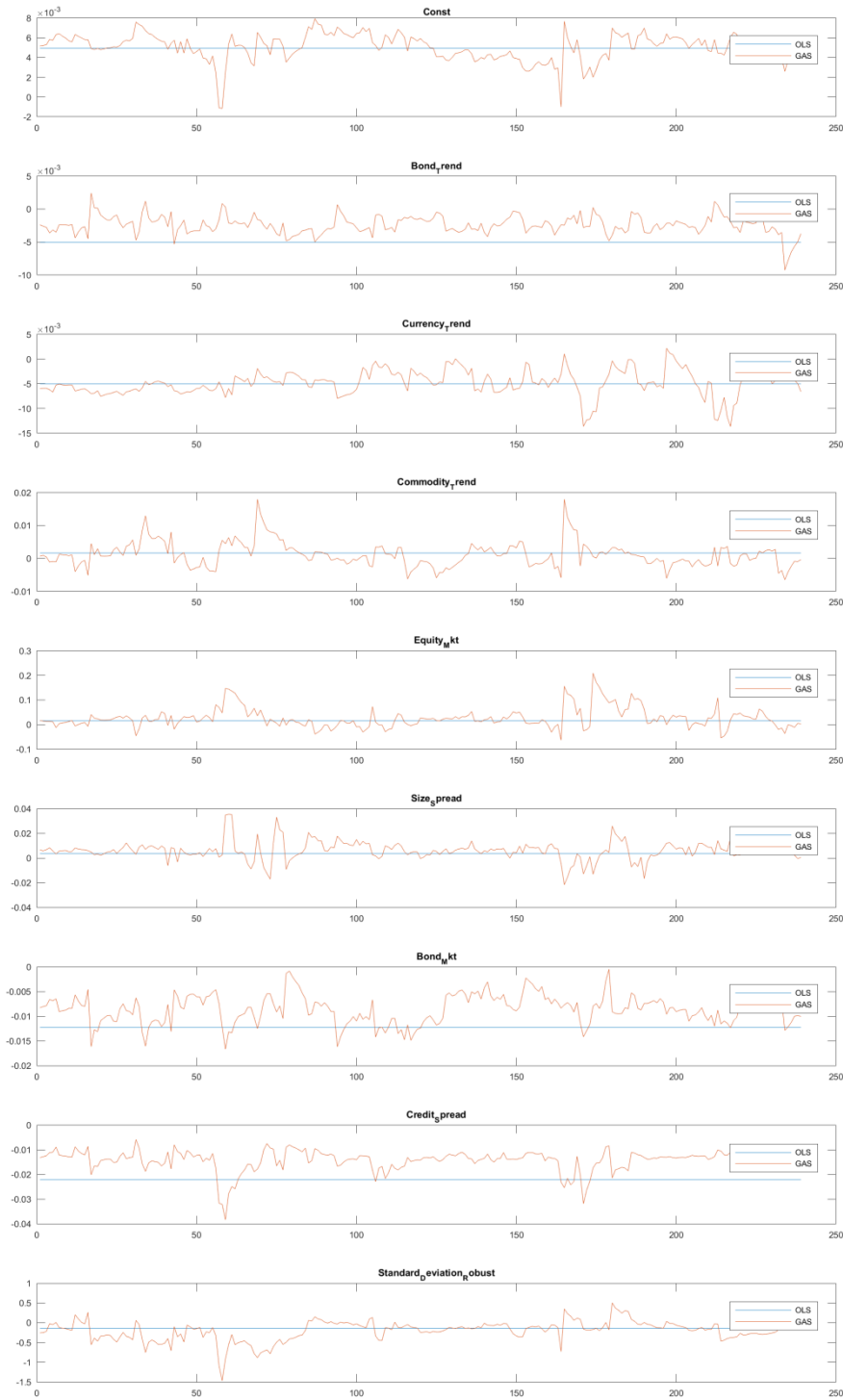
Supplementary Appendix figure 2.6: GAS All time-varying parameters vs. OLS for SDVIX Event Driven Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 658 Event Driven strategy funds between 1994 and 2013. Based on author's calculation.



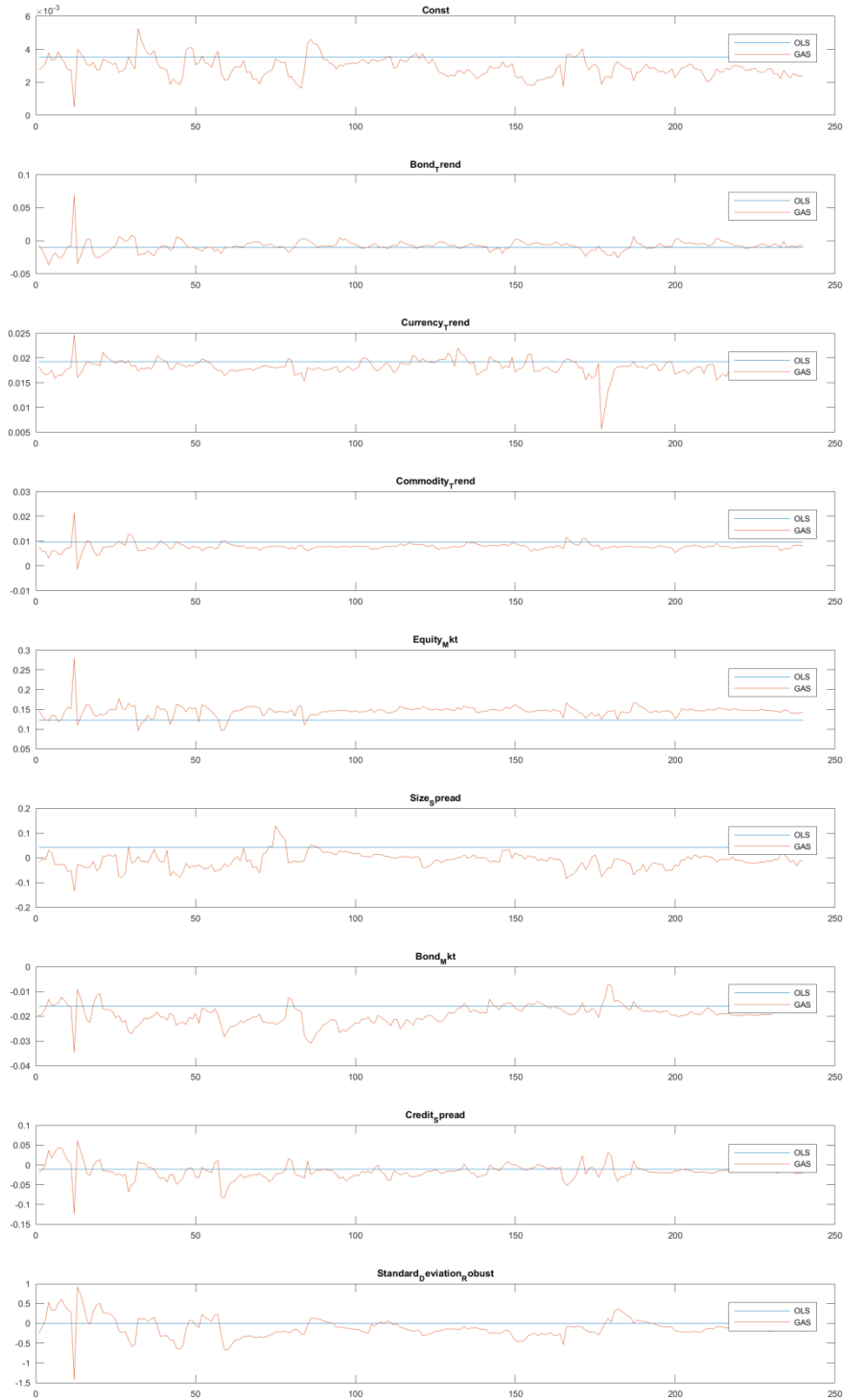
Supplementary Appendix figure 2.7: GAS All time-varying parameters vs. OLS for *SDVIX* Fixed-Income Arbitrage Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 394 Fixed-Income Arbitrage strategy funds between 1994 and 2013. Based on author's calculation.



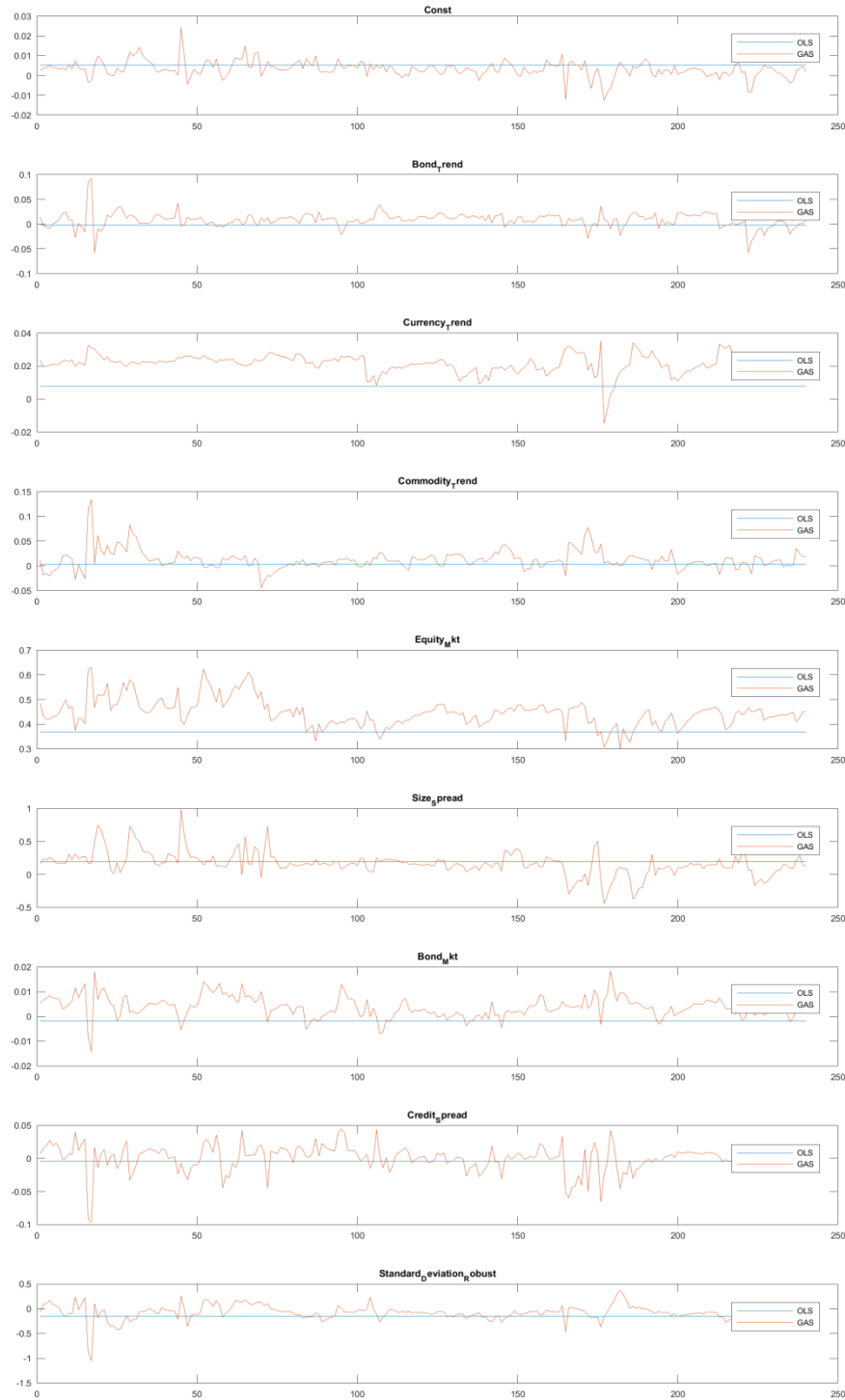
Supplementary Appendix figure 2.8: GAS All time-varying parameters vs. OLS for *SDVIX* Global Macro Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 718 Global Macro strategy funds between 1994 and 2013. Based on author's calculation



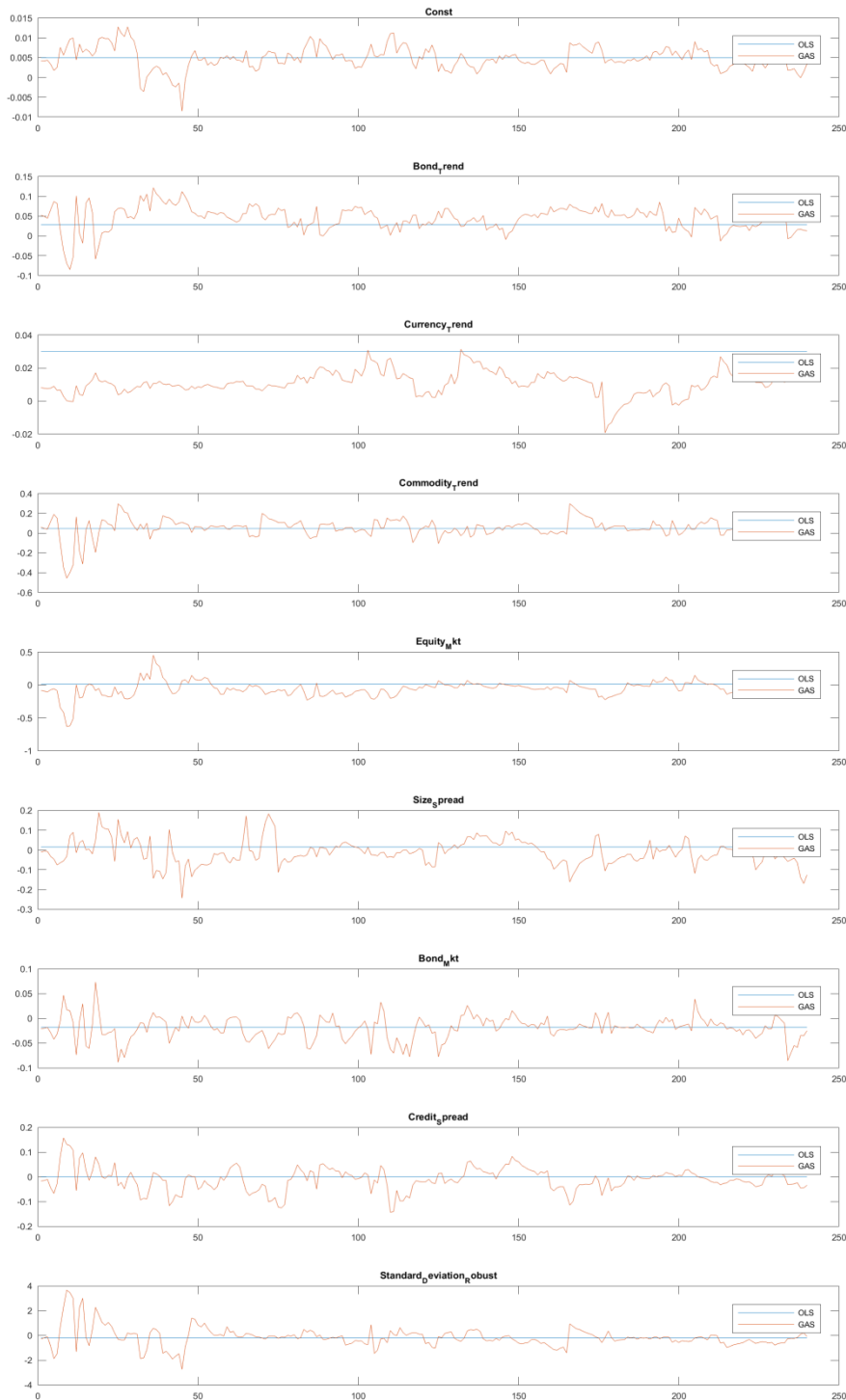
Supplementary Appendix figure 2.9: GAS All time-varying parameters vs. OLS for *SDVIX* Long/ Short Equity Hedge Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for 3,189 the Long/Short Equity Hedge strategy funds between 1994 and 2013. Based on author's calculation.



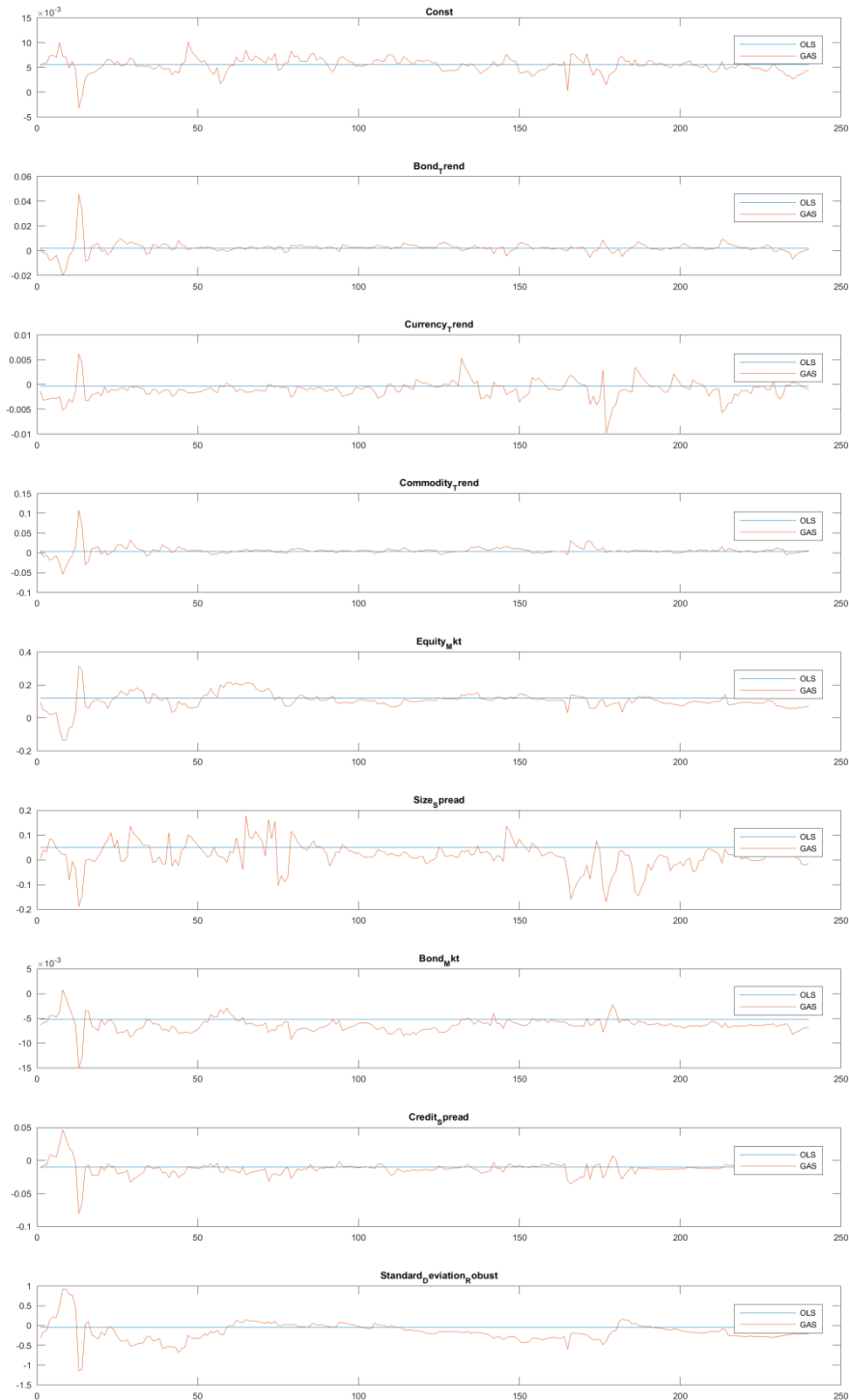
Supplementary Appendix figure 2.10: GAS All time-varying parameters vs. OLS for *SDVIX* Managed Futures Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 718 Global Macro strategy funds between 1994 and 2013. Based on author's calculation.



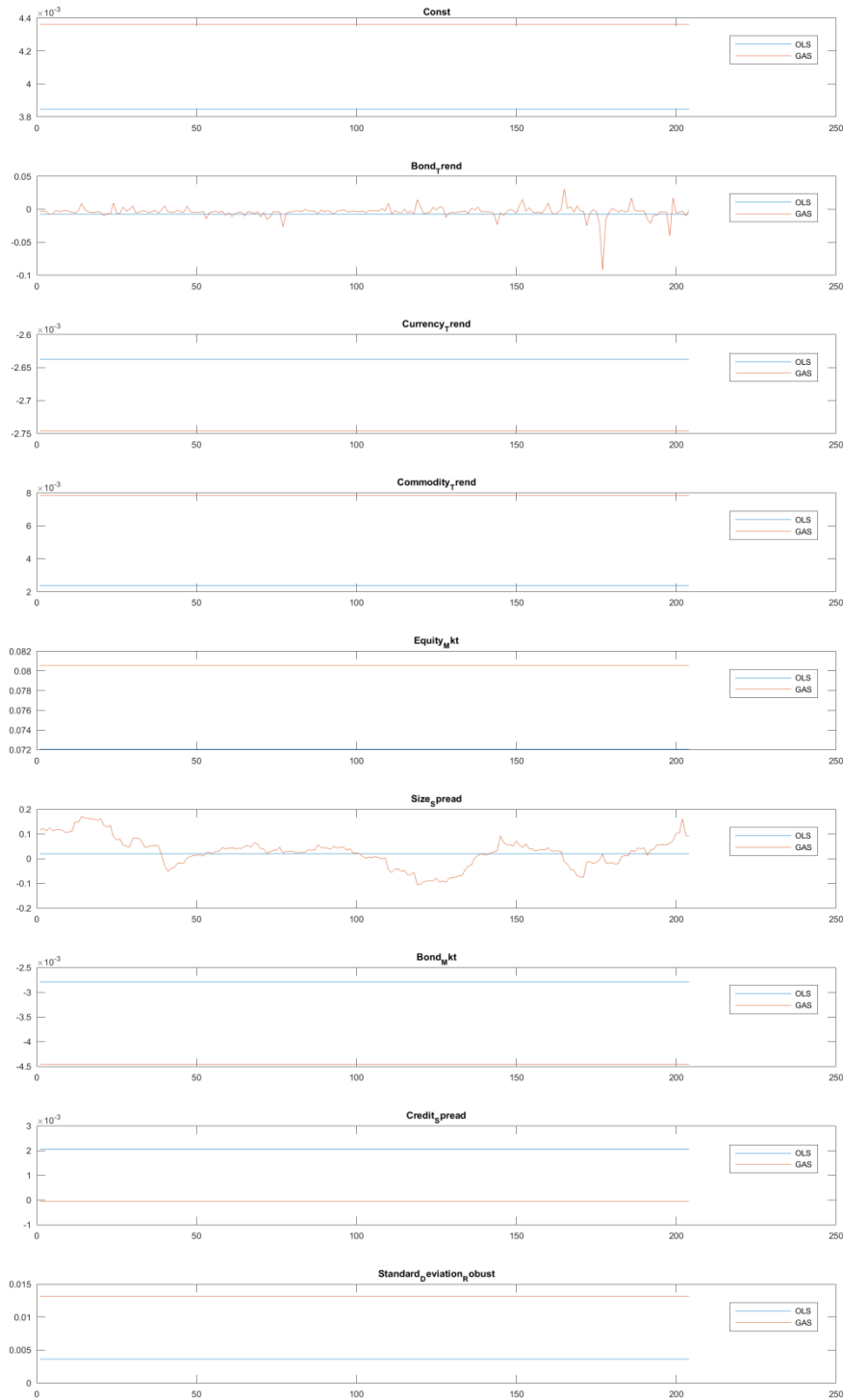
Supplementary Appendix figure 2.11: GAS All time-varying parameters vs. OLS for SDVIX Multi-Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 1,719 Multi-Strategy funds between 1994 and 2013. Based on author's calculation.



Supplementary Appendix figure 2.12: GAS All time-varying parameters vs. OLS for SDVIX Options Strategy

I show how the loadings for each risk factor have been changing over time. Data refer to the equally weighted portfolio for the 51 Options Strategy funds between 1994 and 2013. Based on author's calculation.



Supplementary Appendix table 2.2

SDVIX-only time-varying GAS parameters

This table displays the estimated ‘SDVIX-only time-varying’ GAS parameter values for each of the 11 equally weighted portfolio strategies. Their robust standard errors, which are reported in squared brackets. The strategies’ names are abbreviated as follows: CA stands for *Convertible Arbitrage*; DS for *Dedicated Short Bias*; EM for *Emerging Markets*; EN for *Equity Neutral*; FA for *Fixed Income Arbitrage*; GM for *Global Macro*; LS for *Long/Short Equity Hedge*; MF for *Managed Futures*; MS for *Multi-Strategy*; OS for *Options Strategy*. *, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels for the *A* t-test statistic, respectively. Robust standard errors are reported in squared brackets. Values rounded to the 3rd significant digit.

	CA	DS	EM	EN	ED	FA	GM	LS	MF	MS	OS
			*			**			***	***	***
<i>A</i>	0.01 [0.03]	0.01 [0.01]	0.01 [0.02]	0.01 [0.01]	0.01 [0.02]	0.05 [0.03]	0.01 [0.02]	0.01 [0.03]	0.01 [0.00]	0.01 [0.00]	0.01 [0.03]
<i>B</i>	0.99 [0.02]	0.99 [0.01]	0.99 [0.08]	0.99 [0.01]	0.99 [0.03]	0.95 [0.03]	0.99 [0.01]	0.99 [0.03]	0.99 [0.02]	0.99 [0.01]	0.97 [0.03]
α_{OLS}	0.004 [0.00]	0.004 [0.00]	0.005 [0.00]	0.005 [0.00]	0.005 [0.00]	0.005 [0.00]	0.003 [0.00]	0.005 [0.00]	0.005 [0.00]	0.006 [0.00]	0.004 [0.00]
β_{OLS}^{PTFSB}	-0.003 [0.00]	-0.012 [0.02]	-0.021 [0.01]	0.001 [0.00]	-0.013 [0.01]	-0.005 [0.00]	-0.01 [0.01]	-0.002 [0.01]	0.028 [0.01]	0.002 [0.00]	-0.007 [0.00]
β_{OLS}^{PTFSFX}	-0.004 [0.01]	-0.001 [0.02]	0.016 [0.02]	0.001 [0.00]	0.004 [0.01]	-0.005 [0.00]	0.019 [0.00]	0.008 [0.01]	0.03 [0.01]	-0.003 [0.00]	-0.003 [0.00]
$\beta_{OLS}^{PTFSCOM}$	-0.008 [0.01]	-0.013 [0.02]	0.002 [0.01]	0.002 [0.00]	-0.003 [0.01]	0.002 [0.00]	0.010 [0.01]	0.003 [0.01]	0.044 [0.01]	0.004 [0.00]	0.002 [0.01]
$\beta_{OLS}^{BD10RET}$	0.091 [0.02]	-0.611 [0.04]	0.382 [0.02]	0.063 [0.01]	0.170 [0.02]	0.016 [0.01]	0.123 [0.02]	0.369 [0.04]	0.012 [0.04]	0.121 [0.02]	0.072 [0.02]
$\beta_{OLS}^{BAAMTSY}$	0.018 [0.03]	-0.379 [0.08]	0.128 [0.07]	-0.002 [0.02]	0.076 [0.03]	0.004 [0.02]	0.043 [0.02]	0.192 [0.05]	0.015 [0.04]	0.051 [0.00]	0.026 [0.02]
β_{OLS}^{SNPMRF}	-0.018 [0.00]	-0.006 [0.01]	-0.008 [0.01]	-0.004 [0.00]	-0.003 [0.00]	-0.012 [0.00]	-0.016 [0.00]	-0.002 [0.01]	-0.018 [0.01]	-0.005 [0.00]	-0.003 [0.00]
β_{OLS}^{SCMLC}	-0.05 [0.01]	-0.011 [0.01]	-0.029 [0.01]	-0.004 [0.00]	-0.018 [0.01]	-0.022 [0.00]	-0.011 [0.00]	-0.004 [0.01]	0.002 [0.01]	-0.01 [0.00]	0.002 [0.00]
β_{OLS}^{SDVIX}	-0.161 [0.19]	-0.345 [0.10]	-0.379 [0.48]	-0.167 [0.09]	-0.237 [0.16]	-0.139 [0.11]	0.001 [0.12]	-0.153 [0.17]	-0.217 [0.22]	-0.047 [0.11]	0.004 [0.08]
ω	2.95 [0.60]	16.59 [5.21]	12.16 [3.44]	0.69 [0.09]	2.37 [0.31]	0.77 [0.41]	1.848 [0.26]	5.322 [0.66]	5.85 [0.83]	1.32 [0.25]	1.048 [0.12]

Supplementary Appendix table 2.3

Selected factors time-varying GAS SDVIX parameters

This table displays the estimated ‘SDVIX selected factors time-varying’ GAS parameter values for each of the 11 equally weighted portfolio strategies. Their robust standard errors, which are reported in squared brackets. The strategy name is abbreviated as follows: OS stands for *Options Strategy*. *, ** and *** denote significant differences from zero at the 90%, 95% and 99% levels for the *A* t-test statistic, respectively. Robust standard errors are reported in squared brackets. Values rounded to the 3rd significant digit.

	OS

<i>A</i>	0.157 [0.04]
<i>B</i>	0.843 [0.23]
α_{OLS}	0.004 [0.00]
β_{OLS}^{PTFSB}	-0.002 [0.00]
β_{OLS}^{PTFSFX}	-0.003 [0.00]
$\beta_{OLS}^{PTFSCOM}$	0.003 [0.00]
$\beta_{OLS}^{BD10RET}$	0.073 [0.02]
$\beta_{OLS}^{BAAMTSY}$	0.018 [0.02]
β_{OLS}^{SNPMRF}	-0.007 [0.00]
β_{OLS}^{SCMLC}	-0.003 [0.00]
β_{OLS}^{SDVIX}	-0.004 [0.05]
ω	0.87 [0.08]