



UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Testing the weak form EMH

An empirical study of the Swedish stock market

Joel Frank
Viktor Öhrström

Abstract:

This thesis investigates whether the Swedish stock market shows signs of weak form efficiency between January 2012 and January 2019. Weekly data is gathered from the OMXSPI and from three indices of different capitalization segments, namely Large cap, Mid cap and Small cap. The OMXSPI represents the whole market and the segments are used to see if there are possible patterns between size and efficiency. Each index is tested for the random walk hypothesis by employment of the Ljung-Box autocorrelation test, the runs test, the variance ratio test, the multiple variance ratio test, Wright's test, the wild bootstrap methodology and the joint rank and signs test. The tests are complementary to cover for possible weaknesses.

We find that the whole market is following a random walk, and this indicates that the Swedish market is weak form efficient. Contrasted with earlier studies, this study gives the strongest indications of efficiency. Our results, however, do not establish that the market is weak form efficient. Therefore, we propose testing if it is possible to set up trading rules that generate excess returns on the Swedish market.

There are indications of a relationship between size and efficiency on the Swedish market. It is not established if this is caused by an inefficiency in the Small cap segment or not. We propose further studies to investigate whether this is due to less efficiency for small stocks, spurious autocorrelations caused by infrequent trading or both.

Bachelors thesis (15hp)
Department of Economics
School of Business, Economics and Law
University of Gothenburg
Supervisor: Andreas Dzemski

Acknowledgements: We would like to express our gratitude to Andreas Dzemski for his useful feedback, comments and for contributing to valuable discussions.

1. Introduction	1
1.1 Background.....	1
1.2 Purpose and Research Question.....	2
1.3 Delimitations.....	2
2. Theory and Literature Review	3
2.1 Efficient Market Hypothesis.....	3
2.2 Martingale hypothesis.....	5
2.3 Random Walk Hypothesis.....	5
2.4 Literature Review.....	6
3. Data and Method	9
3.1 Data	9
3.2 Methods.....	10
3.2.1 Autocorrelation and Ljung-Box's Q.....	10
3.2.2 Runs test.....	11
3.2.3 Variance ratio.....	12
3.2.3.1 LM-test.....	12
3.2.3.2 CD-test.....	16
3.2.3.3 Wright's test.....	17
3.2.3.4 Belaire-Franch & Contreras test.....	19
3.2.3.5 Wild bootstrap method.....	19
4. Results.....	20
4.1 Descriptive Statistics.....	20
4.2 Autocorrelation tests.....	21
4.3 Runs test.....	23
4.4 LM-test.....	23
4.5 Wright's test.....	25
4.6 Multiple VR-tests.....	27
5. Analysis and Discussion	27
5.1 Comparison with other studies	29
6. Concluding remarks	30
References.....	32
Appendix A.1.....	37
A.1.1 Kim's Bootstrap	37
A.1.2 Wright's test critical values.....	37
Appendix A.2.....	38
A.2.1 Data for examples	38
A.2.2 Test statistics example.....	39
A.2.3 Wright's critical values – example	39

1. Introduction

1.1 Background

The efficient market hypothesis (hereafter EMH) is an old hypothesis, still relevant today. It states that information cannot be used to consistently make excess returns on asset markets, which has great implications for all members of society. The hypothesis affects investors, policy makers and portfolio managers. It most likely affects your pension. Therefore, this subject should be of interest to everyone.

Kendall (1953) suggested in an early study that asset prices behave like a wandering series. He suggested that price changes in a homogenous price series are independent of earlier changes, which is practically the definition of the random walk hypothesis (hereafter RWH). Fama (1970) reviewed, summarized and expanded on the existing literature. He also made the distinction between three types of the EMH, first mentioned by Roberts (1967), widely known. The distinctions are: weak form, semi-strong form and strong form efficiency. The weak form EMH answers if historical information is profitable, the semi-strong form answers if publicly available information is profitable and the strong form answers if private information is profitable for investors.

Many early studies were generally in favor of the EMH, leading the theory to become generally accepted in academic circles. Anomalies started to show up in later studies, evidence which was enough for some to reject the notion of efficient markets. Others did not see the results as evidence against market efficiency per se. They attributed the anomalies to rejections of the underlying theory used in making the EMH testable instead of rejections of the EMH itself. Those that reject the notion of efficient markets, generally ascribe the anomalies to different behavioral aspects, for example over- or underreaction to information by investors (De Bondt & Thaler, 1985; De Bondt & Thaler, 1987) or social dynamics in the society (Shiller, Fischer & Friedman, 1984).

There exists a lot of research on the EMH, especially on the weak form. The results are often contradictory (see the literature review), where time and methods are factors that lead to different conclusions. This is the case for the Swedish market, which this study covers. Some studies have concluded that the Swedish market is inefficient, while other studies show indications of efficiency. Since this is such an important topic and the studies on the Swedish

market give different conclusions, this study aims to bring newer insight on the efficiency of the Swedish market, by using more recent data and newer methods than previous studies.

1.2 Purpose and Research Question

This thesis aims to investigate whether the Swedish stock market shows evidence of weak form efficiency during the period 2012-01-01 – 2019-01-01. A set of statistical tests are used for this purpose.

The tests are applied to weekly data on the Swedish stock index OMXSPI, which is a value weighted price index of all stocks listed on the OMX Stockholm. Additionally, the tests are applied to indices of different capitalization from stocks on the OMXSPI: the OMXSSCPI (Small cap), the OMXSMCPI (Mid cap) and the OMXSLCPI (Large cap).

The question this paper seeks to answer is:

- Does the Swedish stock market and its three segments of different index capitalization show signs of weak form efficiency from 2012 to 2019?

The research question will be answered primarily by the tests on respective index. The RWH is the means by which we measure efficiency, and thus the null hypothesis is:

- *H₀: The indices follow a random walk, and are therefore assumed to be weak form efficient*

And the alternative hypothesis is:

- *H₁: The indices do not follow a random walk, and this implies that they are inefficient.*

1.3 Delimitations

There are many ways to test the EMH, which means that it is not possible to cover all angles in a limited period of time. Therefore, it is necessary to limit the scope of this study.

Since weak form efficiency is a condition for the stricter forms of efficiency and the literature about the Swedish market is not conclusive for tests of weak form efficiency, this study aims to test the weak form EMH on the Swedish stock market.

There are different ways to test the weak form EMH. It can be done either by mechanical trading rules or by statistical tests. The former investigates if it is possible to set up a strategy that generates excess returns and the latter tests if the RWH holds. The statistical tests can be divided into two groups: unit root tests and tests for independence. Campbell, Lo and

MacKinlay (1997, p.65) mention that even though unit roots are a condition for a random walk, the tests cannot answer if there is predictability in the time series. Under the null hypothesis of a unit root, it may be possible that the first difference of the time series is predictable. Therefore, since the EMH is primarily concerned with predictability, this study's authors choose to focus only on statistical test concerned with the RWH.

These tests will be employed on the whole market and on three segments differing in respect to capitalization size. A discussion will follow if there are some patterns regarding size and efficiency, but this study will not examine that relationship in a formal, statistical way.

2. Theory and Literature Review

2.1 Efficient Market Hypothesis

The EMH states that markets are efficient with respect to information. Fama (1970) defines efficiency as when a security fully reflects all available information. The implication is that trading based on an information set cannot consistently generate excess returns. This concept as stated by Fama is quite vague and needs some definitions in order to make testing of efficiency possible. It is done by using different levels of efficiency, with the information set distinguishing the levels. Building on the framework of Roberts (1967), Fama defines the levels as weak form, semi-strong form and strong form efficiency.

The information set in the weak form EMH is historical price information. This implies that stock market prices move randomly and only react to new price information. Therefore, this form is often associated with the random walk hypothesis and unit roots. If a market is weak form efficient it is impossible to consistently make excess returns based on historical information alone. The practical implication is that weak form efficient markets make technical analysis useless (technical analysis is the practice of analyzing signals and patterns from the market in order to make high returns). However, if a market is weak form efficient, it might still be possible to earn excess returns by using other sources of information, and the practice of fundamental analysis might still be profitable (fundamental analysis is the practice of analyzing assets in order to find mispriced assets, thus yielding excess returns).

The information set in the semi-strong form EMH is public information, which includes announced earnings, dividends, mergers, scandals et cetera. It also includes all sources of

information contained in the weak form EMH. If a market is semi-strong efficient, fundamental analysis is not profitable.

The strictest form of efficiency has not gained much attention. The information set in this category is all information from the weaker forms and private information. If a market is strong-form efficient it is impossible to make excess returns, even with monopoly access to information.

It is necessary to give the optimal market conditions for an efficient market. In an efficient market there should be no transaction costs, all information should be available to all investors for free and all investors should interpret the information exactly in the same way. Fama (1970) states that these conditions are sufficient for market efficiency but not necessary, because violations of the sufficient conditions do not necessarily violate market efficiency. He notes, however, that violations of these conditions are potential threats to the EMH and are something to be aware of. Related to the market conditions is a note by Lo and MacKinlay (1988). They mention that smaller stocks may be falsely rejected because of a correlation that is introduced in the time series because of non-trading. Large cap stocks incorporate information first, but Small cap stocks may need longer time for prices to adjust, because the information is incorporated through trading and small stocks are less frequently traded. This could result in a lag being introduced that could cause spurious correlations, which could lead to a false rejection of the RWH.

Since this study is focused on the weak form EMH, some things must be noted. It is difficult to test the EMH and conclude that a market is either efficient or inefficient. First, there is a difficulty called the “joint hypothesis problem” which arises due to the fact that all tests of the EMH need an equilibrium model to be able to calculate excess returns. Therefore, the tests jointly test the EMH and the equilibrium model. It is thus difficult to be certain if the reason for rejection lies in a market inefficiency or in the equilibrium model. Second, tests of the EMH concentrated on testing the RWH can be misleading because the RWH is stricter than necessarily implied by the EMH. A solution to this will be given in the section about the RWH, where the RWH is divided into three different levels. Even though the assumptions of the RWH are relaxed, it is nonetheless in harmony with tests of the EMH (Campbell et al., 1997, p.33).

2.2 Martingale hypothesis

A martingale process can be described in terms of a fair game. Consider a game of dice between two players. Given that the dice are not manipulated, both players have the same odds at receiving a high score. No one has an advantage over the other, it is a fair game.

If P_t represents a stochastic process, a martingale process is as follows:

$$E(P_{t+1}|P_t, P_{t-1}, \dots) = P_t. \quad (2.1)$$

This expression says that the best estimate of the future is the value today. In terms of asset prices, Equation (2.1) states that the best prediction of the future price, conditional on all historical prices, is the present price. In a world where the only available information is historical price data, (2.1) constitutes a fair game for all investors. It does not matter if rich people could buy all historical information available and the poor could only know the price of today. Given that the martingale hypothesis holds, these conditions would still constitute a fair game.

The martingale hypothesis is often tested instead of the RWH when testing the weak form EMH. One objection raised is that the martingale hypothesis does not account for risk. This might be a weakness, because Leroy (1973) demonstrates that the martingale property is violated when investors are risk averse. However, he emphasizes that there are uncertainties as to how important the violation is. Thus, Leroy concludes that the martingale hypothesis can be a good approximation of the behavior of returns in an efficient market despite existing risk aversion among investors. Campbell et al. (1997, p.31) mention the study above when noticing the flaws with the martingale hypothesis. However, they point out that the hypothesis led to the formulation of the RWH, implying its usefulness.

2.3 Random Walk Hypothesis

The random walk hypothesis states that consecutive price changes are independent and identically distributed (hereafter iid). The RWH is a form of the martingale hypothesis but stricter, because the martingale hypothesis states that the expected price of tomorrow is the price of today while the RWH demands that the distribution is identical for all lags.

A random walk with drift is defined as:

$$p_t = \mu + p_{t-1} + \epsilon_t \quad (2.2)$$

where p is the natural logarithm of the price, the subscript t denotes time, μ is a drift parameter and ϵ_t is the error term which is iid with a mean of 0 and a variance of σ^2 . Equation (2.2) can be rewritten as a first difference to illustrate the concept of a random walk:

$$\Delta p_t = \mu + \epsilon_t. \quad (2.3)$$

As Equation (2.3) shows, the only thing that changes the price is the drift parameter (μ) and random shocks (ϵ_t). The series will therefore wander randomly and unpredictably.

Because the consensus is that financial data rarely has an identical distribution over time, the RWH in its original form is too strict (Campbell et al., 1997, p.53). However, this does not make the concept of random walks as tests for market efficiency useless. It is possible to formulate a random walk model with less restrictive assumptions. Campbell et al. (1997, pp.28-33) divide the random walk model into three levels: RW1, RW2 and RW3. RW1 is the pure random walk as described above, following an iid. RW2 is independent but not identically distributed, which allows the error terms to be heteroscedastic, a weaker condition than the RW1. RW3 is the weakest form of random walk, allowing dependencies in the error term, except for linear dependence. For example, $Cov(\epsilon_t, \epsilon_{t-q}) \neq 0$ is not allowed but $Cov(\epsilon_t^2, \epsilon_{t-q}^2) \neq 0$ is. Hence, the RW3 is neither strictly independent nor identically distributed but it is linearly independent.

The RW3 is the form of the RWH mostly tested in the literature, and the one most tests in this study assumes.

2.4 Literature Review

There exists an extensive literature about testing the EMH, especially concerning the weak form. Most recent studies are concerned with emerging markets, but there are also studies about developed markets. To get a broad picture about the inconsistencies in the conclusions that can be found both in emerging and developed markets, this review presents studies conducted on both kinds of markets. Studies on emerging markets are followed by some studies of developed European markets before covering the studies in Sweden.

The tests in Latin America show conflicting results. Urrutia (1995) and Charles and Darné (2009) show inefficiencies in the Brazilian, Argentinian, Mexican and Chilean stock markets. Grieb and Reyes (1999) only test the Brazilian and Mexican markets but reject efficiency in both. Chadhuri and Wu (2003) test many emerging markets from different continents. They include the countries mentioned above: Brazil, Argentina, Mexico and Chile. The tests show

inefficiencies in the Brazilian and Argentinian markets but fail to reject efficiency for the Mexican and Chilean markets. Ojah (1999), in contrast, fails to reject efficiency for all the markets.

The studies of Asian markets neither give a unanimous answer as to which markets are efficient and inefficient. Huang (1995) shows that the markets of Japan, Indonesia and Taiwan follow a random walk while Hong Kong, South Korea, Malaysia, the Philippines, Singapore and Thailand do not. Hoque, Kim and Pyun (2007) reject efficiency for Indonesia, Malaysia, the Philippines, Singapore, Thailand and Hong Kong, while they do not reject efficiency for South Korea and Taiwan. Kim and Shamsuddin (2008) show that generally Hong Kong, Japan, South Korea, Singapore and Taiwan are efficient and Indonesia, Malaysia and the Philippines are inefficient. Thailand varies between sub-samples and it seems to become more efficient after 1997. To conclude, there are some countries (Malaysia, the Philippines etc.) that consistently reject efficiency, while other countries vary between studies (Hong Kong, Indonesia etc.) and some are consistently unable to reject efficiency (South Korea, Taiwan).

There are many tests of European emerging markets (see, for example Smith and Ryoo, 2003; Smith, 2009; Guidi, Gupta & Maheshwari, 2011; Dragota and Tilica, 2014), but few that focus on developed countries in Europe. Of the few that focus on the developed countries, Worthington and Higgs (2004) test the weak form EMH on twenty European markets, of whom sixteen are considered developed. They perform an autocorrelation test, the runs test, unit root tests (ADF, PP and KPSS) and the variance ratio (hereafter VR) test by Lo and MacKinlay (1988) (hereafter LM). The results from the autocorrelation test and runs test show that Germany, Ireland, the Netherlands, and Portugal fail to reject. Concerning the LM-tests, Worthington and Higgs find that Germany, Ireland, Sweden, Portugal and the UK fail to reject while France, Finland, the Netherlands, Norway and Spain satisfy some requirements of a random walk. Specifically, they are able to reject the homoscedastic statistic but not the heteroscedastic one.

Borges (2010) is the author of the other test on developed European markets. She tests the random walk hypothesis on the markets of UK, France, Germany, Spain, Portugal and Greece, by focusing on the martingale properties of the time series. The tests she employs are the runs test, the LM-test, Chow and Denning's (1993) multiple variance ratio test (hereafter CD), the wild bootstrap methodology (hereafter WBM) by Kim (2006), and the joint sign statistic by Belaire-Franch and Contreras (2004) (hereafter BFC). Greece and Portugal show significant positive autocorrelations, but these become less severe when considering a subperiod from

2003-2007, in which the two countries had become developed instead of emerging as opposed to earlier periods (1993-2003). UK and France are also shown to be inefficient, and even more inefficient in the later subperiod (2003-2007). All countries reject the EMH by some method, but Spain is the most effective country overall, and Germany the second most effective among the countries investigated.

Urquhart (2014) tests the impact of the introduction of the Euro on ten developed European countries, of which three countries have their own currencies (UK, Sweden and Norway). He uses the autocorrelation test, runs test, LM-test, CD-test, the rank and signs test by Wright (2000) (hereafter Wright's) and the non-parametric BDS-test. The data is from 1988-2012, with sub-samples from 1988-1998 and 1999-2012. Generally, the Netherlands and Germany show high efficiency while Ireland and Italy show inefficiency for all time periods. France becomes inefficient after the introduction of the Euro, while Finland and Spain become efficient. Sweden and Norway move towards being more efficient, but with mixed results from different tests, which means that at least some tests reject the RWH. The tests of the European developed markets show that developed countries are not necessarily efficient. Worthington and Higgs (2004) and Urquhart include Sweden, but there are more studies that likewise do.

Much has happened since the 70-90s when the first studies on the Swedish market was conducted, but it can be profitable to have some knowledge about the results from those anyways. Jennergren and Korsvold (1974) study the random walk properties on some Swedish and Norwegian stocks by using the runs test and autocorrelation test. Their conclusion is that although most stocks reject a random walk, it does not necessarily mean that they are inefficient. Berglund, Wahlroos and Örnmark (1983) test the Finnish, Swedish, Danish and Norwegian markets by using the autocorrelation test, the runs test and a technical trading strategy called "filter rules". All countries show serial correlations in the time series. However, the results using the filter rules indicate that the inefficiencies are not large enough for an investor to make excess returns. Thus, the authors conclude that they cannot reject efficiency. Frennberg and Hansson (1993) conducted an extensive study on a Swedish stock index ranging from 1919 to 1990. They were the first ones to use the LM-test on the Swedish market. The results show a rejection of the RWH. However, they note the low R^2 value and conclude that it might not be possible to gain excess returns, even though the data is serially correlated. Metghalchi, Chang and Marcucci (2008) answer that question by testing some trading rules on the Swedish stock index between 1986 and 2004. They find that their simple methods could be used during this period to gain excess returns.

There are, to our knowledge, two more recent studies on the Swedish market. Shaker (2013) tests the efficiency of the Swedish and Finnish stock indices OMXS30 and OMXH25 between 2003 and 2013. Using the autocorrelation test, a unit root test and the LM-test, he finds that neither market follows a random walk. The most recent study (Graham, Peltomäki & Sturludóttir, 2015) is primarily focused on the Icelandic market, but it includes the remaining Nordic countries for comparison. The primary focus is on how capital controls affect market efficiency, but nevertheless, the same methods that are relevant for weak form efficiency are used. They are the following: the LM-test, the CD-test, the WBM and the BFC-test. This study's data is the most recent, covering a period from 1993 to 2013. The results indicate that for the full period, the Swedish market is efficient, while it is not in the subsample of 2008-2013.

As demonstrated, there is contradictory evidence concerning the efficiency of the Swedish market. Some studies (Jennergren & Korsvold, 1974; Frennberg & Hansson, 1993; Metghalchi et al., 2008; Shaker, 2013) seem to suggest that the Swedish market is inefficient, other studies (Worthington & Higgs, 2004; Graham et al., 2015) seem to point to the opposite conclusion while some (Berglund et al., 1983; Urquhart 2014) are not clear or conclusive in their conclusions. This study aims to contribute to the conversation by using more recent techniques and data than many studies.

3. Data and Method

3.1 Data

The data selected for this paper is weekly data covering the period from 2012-01-01 to 2019-01-01. Following Lo and MacKinlay (1988) we use closing prices from Wednesdays. If data from Wednesday is missing Thursday is chosen. If Thursday is missing, Tuesday is used, and if all three are missing an average between the prior observation and the following observation is used. The reason why we choose weekly data is to eliminate biases associated with daily prices, for example the day-of-the-week effect and non-trading. These biases are less prominent with weekly data (Lo and MacKinlay, 1988). Weekly data is preferred over monthly because monthly data would lose more observations than desirable.

The indices chosen are the OMXSPI, which is a price index of all stocks listed on the Nasdaq Stockholm and the indices of Large cap, Mid cap and Small cap companies on the

OMXSPI (OMXLSCPI, OMXSMCPI and OMXSSCPI). The Large cap segment includes companies with a market value over €1 billion, the Mid cap segment includes companies with a market value between €150 million and €1 billion and the Small cap segment includes companies with a market value less than €150 million. All data is collected from Nasdaq OMX Nordic's web page (Nasdaq, 2019).

Log returns are calculated as:

$$x_t = p_t - p_{t-1} \quad (3.1)$$

where x_t is the return at time period t , p_t and p_{t-1} are the natural logarithm of the prices at t and $t-1$.

The statistical software "R" is used in all calculations (R Core Team, 2019). All tests that fall under section 3.2.3 are employed by a package developed by Kim (2014). Other packages used are by Komsta and Novometsky (2015) and Caero and Mateus (2014) in addition to the standard packages.

3.2 Methods

3.2.1 Autocorrelation and Ljung-Box's Q

If a market is weak form efficient, there is by definition no significant serial correlation present in the data. For this reason, this paper tests the return series for autocorrelations. The portmanteau test by Ljung and Box (1978) (hereafter Ljung-Box) is used to formally detect autocorrelation in the return series. Specifically, it answers whether the return series follows a RW1.

The autocorrelation coefficient ρ_q is given by:

$$\rho_q = \frac{Cov(x_t, x_{t-q})}{Var(x_t)}, \quad (3.2)$$

where x_t denotes the present return and x_{t-q} denotes the return at lag q . If the correlation coefficients are statistically different from zero, there is a dependency between the present return and the return of the previous period. The implication is that the return series is dependent and it might be possible to predict future returns based on it.

The Ljung-Box portmanteau test is used to test the joint hypothesis that all correlations are simultaneously zero for all lags ($\rho_1 = \rho_2 = \dots = \rho_m = 0$). It is based on the Q-statistic by Box and Pierce (1970) but is superior for small samples as it more precisely follows a chi-square

distribution. If the null hypothesis is rejected, serial correlation exists in the data which implies that the market violates a random walk. The test statistic, Q , is expressed as follows:

$$Q = n(n + 2) \sum_{q=1}^m \frac{\hat{\rho}_q^2}{(n - q)}, \quad (3.3)$$

where n denotes the number of observations, m is the maximum number of lags, q is the present lag and $\hat{\rho}_q^2$ is the squared correlation coefficient at lag q .

Care should be taken, according to Campbell et al. (1997, p.47), when choosing m . If the maximum level is too low, high order correlations will be missed. If it is too high, the test loses power. Other studies use around 10-12 lags, and since this study uses weekly data which includes fewer observations than daily data, the choice falls on the lowest number, 10.

3.2.2 Runs test

The runs test is a non-parametric test designed to detect serial independency in a return series. In other words, it tests if the change in returns are random. The definition of a run is the sequence of successive changes in returns. These can be either positive or negative. For example, the sequence “+ + - + - - - + +” consists of five runs, three of which are positive and two negative.

The test assumes that the data series is random if the number of observed runs R is close to the number of expected runs m . If the test rejects, then there is non-randomness in the return-series. The expected number of runs m are computed by the following equation:

$$m = \frac{2N_+N_-}{N} + 1 \quad (3.4)$$

where N_+ is the number of positive runs, N_- is the number of negative runs and N is the sum of N_+ and N_- . The standard deviation is:

$$\sigma_M = \sqrt{\frac{2N_+N_-(2N_+N_- - N)}{N^2(N - 1)}}. \quad (3.5)$$

The standard normal Z-statistic is calculated as:

$$Z = \frac{R - m}{\sigma_M}. \quad (3.6)$$

If the Z-statistic is greater than |1.96| the test is rejected at a 5% level and if it is greater than |2.58| the test is rejected at a 1% level. The strict RW1 is evaluated in the runs test.

3.2.3 Variance ratio

A variance ratio test is often used when testing a stock return series for randomness. The most widely used VR-test is the LM-test by Lo and MacKinlay (1988). Variants of the VR-test can be employed to test a price series or the first difference (i.e. returns) for an RW1, RW3 or a martingale difference sequence. Following sections deal with the variants of the VR-test. They are quite technical, but it is critical to at least grasp the intuition behind the variance ratio test. Read carefully.

3.2.3.1 LM-test

Lo and MacKinlay (1988) designed a test for studying the random walk hypothesis using variance ratios. The test exploits the fact that if a market follows the RWH, the variance of the return series (or price series) is a linear function of time, increasing by q for each interval. This means that the variance at the q :th lag is q times greater than the variance of the first lag. For example, if the RWH holds, the variance of two day relative log prices should be twice as large as the variance of daily returns. Equally, the variance of weekly relative log-prices should be five times larger than the variance of daily relative log-prices. A formal expression is given in Equation (3.7):

$$\text{Var}(p_t - p_{t-q}) = q * \text{Var}(p_t - p_{t-1}). \quad (3.7)$$

Expressed in terms of returns, Equation (3.7) becomes:

$$\text{Var}(x_t + x_{t-1} + \dots + x_{t-q+1}) = q * \text{Var}(x_t). \quad (3.8)$$

If Equation (3.7) and (3.8) holds, the ratio between the left side and right side should be equal to one. This is what the VR-test uses to formally test the RWH: if the variance ratio statistically differs from one, the RWH is rejected.

The variance ratio can be defined as:

$$V(q) = \frac{\text{Var}(x_t + x_{t-1} + \dots + x_{t-q+1})}{q * \text{Var}(x_t)} \text{ or } \frac{\text{Var}(p_t - p_{t-q})}{q * \text{Var}(p_t - p_{t-1})}, \quad (3.9)$$

which can be rewritten as:

$$V(q) = 1 + 2 \sum_{i=1}^{q-1} \frac{(q-i)}{q} \hat{\rho}_i, \quad (3.10)$$

where ρ_i is an autocorrelation coefficient for the time-series and q is the lag chosen. The sequence is uncorrelated if $\hat{\rho}_i = 0$. This makes $V(q)$ equal to one, which is assumed under the

null hypothesis. Reversely, correlation in the time series makes the variance ratio differ from one.

The unbiased estimator for $V(q)$ is as follows:

$$VR(q) = \left\{ Tq^{-1} \sum_{t=q}^T (x_t + x_{t-1} + \dots + x_{t-q+1} - q\hat{\mu})^2 \right\} \div \left\{ T^{-1} \sum_{t=1}^T (x_t - \hat{\mu})^2 \right\} \quad (3.11)$$

where,

$$\hat{\mu} = \frac{1}{T} \sum_{t=1}^T x_t. \quad (3.12)$$

Lo and MacKinlay (1988) developed two different test statistics to test the null hypothesis that the variance ratio is equal to one. The first test statistic, $M_1(q)$, assumes iid returns (RW1) while the other, $M_2(q)$, allows for conditional heteroscedasticity (RW3) in the time series. Under the assumption of iid returns, the test statistic $M_1(q)$ is given by:

$$M_1(q) = \frac{(VR(q) - 1)}{\phi(q)^{\frac{1}{2}}} \quad (3.13)$$

where the variance ratio follows the asymptotic variance ϕ :

$$\phi(q) = \frac{2(2q - 1)(q - 1)}{3qT}. \quad (3.14)$$

$M_1(q)$ follows a standard normal distribution asymptotically, which means that $M_1(q)$ is normally distributed as q is fixed and T moves towards infinity. This is convenient because the critical values are standardized and can be obtained from a Z-table.

When the return series displays conditional heteroscedasticity, Lo and MacKinlay (1988) suggested the robust test statistic $M_2(q)$, which also follows a standard normal distribution asymptotically. It is defined as:

$$M_2(q) = \frac{(VR(q) - 1)}{\phi^*(q)^{\frac{1}{2}}} \quad (3.15)$$

where $\phi^*(q)$ is the asymptotic variance given by:

$$\phi^*(q) = \sum_{j=1}^{q-1} \left[\frac{2(q-j)}{q} \right]^2 * \delta(j) \quad (3.16)$$

and

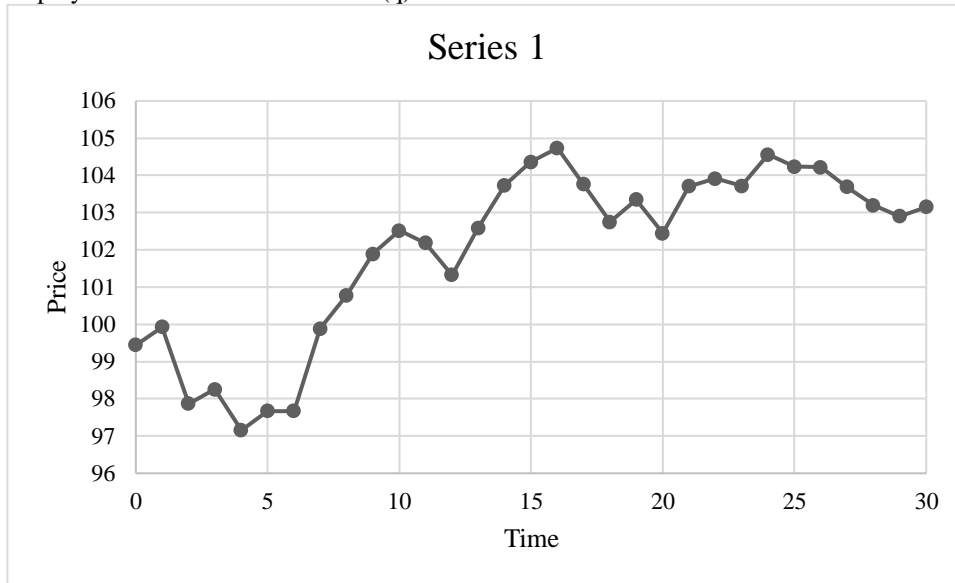
$$\delta(j) = \left\{ \sum_{t=j+1}^T (x_t - \hat{\mu})^2 (x_{t-j} - \hat{\mu})^2 \right\} \div \left\{ \left[\sum_{t=1}^T (x_t - \hat{\mu})^2 \right]^2 \right\}. \quad (3.17)$$

If the RWH holds, the variance ratio must equal one for all values of q . It is therefore not sufficient to test only one value of q , say $q = 2$, because even if $V(2)$ equals one, it does not automatically mean that, say $V(4)$, does. This study follows Lo and MacKinlay (1988) and uses four values of q : 2, 4, 8 and 16. The benefit of this is that multiple horizons are tested, and the result of the test is more reliable. A weakness is, however, that multiple hypotheses are tested simultaneously, which can lead to an over rejection of the composite null hypothesis; that the time series follows a random walk.

Lo and MacKinlay (1989) showed empirically that the distributions of the test statistics, $M_1(q)$ and $M_2(q)$, are rightly skewed. Moreover, they pointed out that the test is weak when q is large relative to T . In fact, when $\frac{q}{T} = 0.5$, the lower bound of the test statistics is -1.73. Since the critical value is -1.96 for the left tail, the test will never reject under these circumstances if the test statistic is drawn from the left side of the distribution. Generally, Lo and MacKinlay showed that as q increases, the reason for rejection successively becomes more due to the right tail and less due to the left tail. Overall, the rejection rate decreases because the left tail becomes less and less able to reject as q increases. The findings, that the underlying distributions of the test statistics are skewed, mean that the asymptotic approximations are not representative. However, this is only true when q is large relative to T , otherwise the test statistics are reliable. In the case of this study, q is far from large relative to T , so this should not be a problem.

For illustrative purposes on how the test statistics work, consider a random walk series as illustrated in Figure 1, called Series 1. Figure 2 depicts a non-random series with a VR above 1 and Figure 3 depicts a non-random series with a VR below 1. All useful information for the coming examples is found in Appendix A.2. Table A.2 gives the price series to the examples.

Figure 1 – Display of random walk series as $V(q) = 1$



Excel is used to simulate a random walk series with a variance ratio around 1. The base price is 100 and the error term is normally distributed. Returns are calculated using Equation (3.1) and the VR estimated by Equations (3.11) and (3.12). Finally, the test statistics are computed using Equations (3.13) and (3.14) for $M_1(q)$ and Equations (3.15-3.17) for $M_2(q)$. This is done for lag 2.

Figure 2 – Display of random walk series as $V(q) > 1$

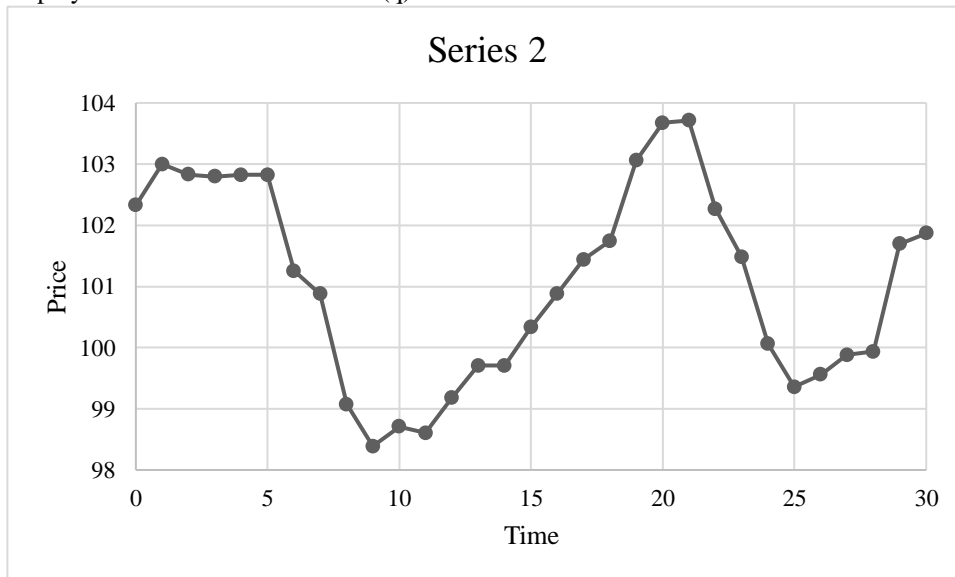
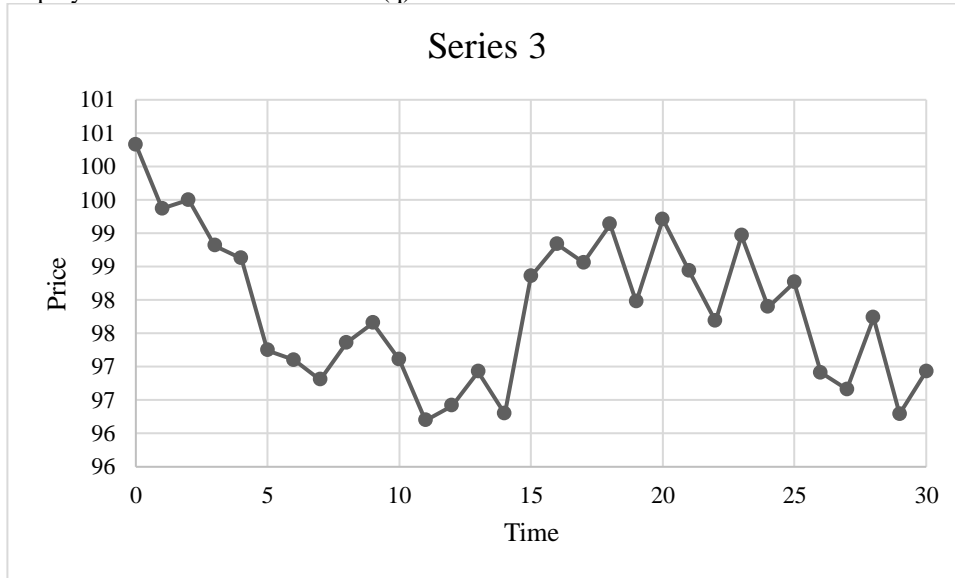


Table A.3 gives the values of the variance ratio and test statistics. Series 1 has a VR of 1, Series 2 of 1.40 and Series 3 of 0.59. $M_1(2)$ takes the value 0.04 for Series 1, 2.21 for Series 2 and -2.25 for Series 3 and $M_2(2)$ takes the values 0.06 for Series 1, 2.91 for Series 2 and -2.53

for Series 3. These results mean that Series 1 is not rejected while Series 2 and 3 are rejected by both test statistics. Thus, the series with a variance ratio of one does not reject while the series with variance ratios different from one reject, as expected from the method.

Figure 3 – Display of random walk series as $V(q) < 1$



3.2.3.2 CD-test

Chow and Denning (1993) proposed an improvement of the LM-test in their article. The problem with the LM-test is that it must stay true for all values of q , and therefore several tests need to be employed to test all those values. This leads to an over rejection of the null hypothesis. To correct for this, Chow and Denning proposed a Sidak (1967) correction of the alpha values. This approach sets the significance level of each individual test so that the significance of the whole test is 5% instead of having a 5% significance level for each individual test. Thus, the following null hypothesis is tested:

$$H_0: V(q_i) = 1, \text{ for } i = 1, \dots, m$$

$$H_1: V(q_i) \neq 1 \text{ for any } i.$$

Since the null hypothesis is rejected if one variance ratio differs from one, Chow and Denning proposed the following test statistics:

$$CD_1 = \max_{1 \leq i \leq m} |M_1(q_i)| \quad (3.20)$$

$$CD_2 = \max_{1 \leq i \leq m} |M_2(q_i)| \quad (3.21)$$

where $M_1(q_i)$ and $M_2(q_i)$ are the test statistics of the LM-test. Equations (3.20) and (3.21) state that only the highest absolute value of the LM-test statistics is tested. Because the test statistics

correspond to the LM-test, the CD test statistics test the same form of random walk as the respective LM test statistics: RW1 for CD_1 and RW3 for CD_2 .

The test statistics follow a studentized maximum modulus distribution for a given alpha, the number of q and the sample size ($SMM(\alpha, q, T)$). Applying the results of Sidak (1967), Hochberg (1974) and Richmond (1982), Chow and Denning (1993) use the fact that when T moves towards infinity, the SMM distribution becomes a standard normal distribution with a Sidak corrected alpha value: $\alpha^* = 1 - (1 - \alpha)^{\frac{1}{q}}$. If the test statistic is greater than the $(1 - \frac{\alpha^*}{2})$ th percentile of the standard normal distribution, the test rejects. For a 5% significance level and four levels of q , $\alpha^* = 0.01274$. This means that the test statistic rejects if it is greater than the 99.363th percentile of the standard normal distribution instead of the 97.5th. Consequently, this will make the critical values larger and rejections stricter.

Returning to the example equations, Table A.3 gives the values of CD_1 and CD_2 . If Table A.3 had presented the LM test statistics for all lags 2, 4, 8 and 16, the CD statistics would be the maximum values of those. Only lag 2 is included but the CD-statistics are calculated as if the lags were 2, 4, 8 and 16.

The new critical value for 5% significance is 2.49, which is higher than for the LM-test. Series 1 has CD_1 and CD_2 values of 0.93 and 1.02 respectively, Series 2 has the values 2.52 and 2.91 and Series 3 has the values 2.25 and 2.53. The result of this is that Series 1 is not rejected, Series 2 is rejected by both statistics and Series 3 is rejected by CD_2 .

3.2.3.3 Wright's test

Wright (2000) developed and modified the original test by Lo & MacKinlay (1988). The test he proposes is a non-parametrical test using ranks and signs.

Wright (2000) provides two advantages his test has over the LM-test. First, it is possible to calculate the distribution. It does not suffer from size distortions, because the test does not rely on asymptotic theory. Second, the test handles non-normal data better than the LM-test.

The rank based tests can be evaluated under the assumptions of RW1 or RW3. The test statistics are exact under the assumptions of an RW1. If the test allows for conditional heteroscedasticity, assuming an RW3, the tests are not exact. However, the size distortions are small (Wright, 2000). The signs test statistic is exact, even under conditional heteroscedasticity.

$r_{1,t}$ and $r_{2,t}$ are defined as:

$$r_{1,t} = \frac{r(x_t) - \frac{T+1}{2}}{\sqrt{\frac{(T-1)(T+1)}{12}}} \quad (3.22)$$

$$r_{2,t} = \phi^{-1} \frac{r(x_t)}{T+1} \quad (3.23)$$

where $r(x_t)$ is the rank of x_t in the series $x_1, x_2 \dots x_T$ and ϕ^{-1} is the inverse standard normal cumulative distribution.

The s_t in the signs test is defined as follows: $s_t = 2u(x_t, 0)$ where $u(x_t, 0) = 0.5$ if x_t is positive and -0.5 otherwise. This means that s_t can only take the values 1 and -1 with the probability of 0.5 for each instance. The series s_t is iid with zero mean and a variance of 1. If allowing for conditional heteroscedasticity the series becomes a martingale with zero mean without restrictions on the variance.

The ranks and signs test statistics, given a set of T observations of x_t , are as follows:

$$R_1(q) = \left(\frac{Tq^{-1} \sum_{t=q}^T (r_{1,t} + \dots r_{1,t-q+1})^2}{T^{-1} \sum_{t=q}^T r_{1,t}^2} - 1 \right) \left(\frac{2(2q-1)(q-1)}{3qT} \right)^{-\frac{1}{2}} \quad (3.24)$$

$$R_2(q) = \left(\frac{Tq^{-1} \sum_{t=q}^T (r_{2,t} + \dots r_{2,t-q+1})^2}{T^{-1} \sum_{t=q}^T r_{2,t}^2} - 1 \right) \left(\frac{2(2q-1)(q-1)}{3qT} \right)^{-\frac{1}{2}} \quad (3.25)$$

and

$$S_1 = \left(\frac{Tq^{-1} \sum_{t=q}^T (s_t + s_{t-1} \dots + s_{t-q+1})^2}{T^{-1} \sum_{t=1}^T s_t^2} - 1 \right) \left(\frac{2(2q-1)(q-1)}{3qT} \right)^{-\frac{1}{2}}. \quad (3.26)$$

The observant reader will notice that these equations are equal to the $M_1(q)$ statistic. But the difference is that the returns are substituted by ranks or signs.

For illustrative purposes, the time series from the examples can be used to illustrate how the ranks and signs are computed. If x_t is the largest log return it receives the rank $r(x_t) = 1$. Likewise, the smallest log return receives the rank $r(x_t) = 30$. Substituting T with 30 and the value of $r(x_t)$ in Equation (3.22) yields the value of $r_{1,t}$ for the specific observation. The same is done for $r_{2,t}$. Depending on which test statistic one chooses, the series of $r_{1,t}$ or $r_{2,t}$ is used in Equations (3.24) and (3.25) for calculating $R_1(q)$ or $R_2(q)$. The series s_t consists of the numbers 1 and -1 depending on if the return is positive or negative. Following Equation (2.26) gives the value of the S_1 .

The computed test statistics for Series 1, at lag 2 are: 0.35, 0.35 and 0 for the statistics $R_1(q)$, $R_2(q)$ and S_1 . For Series 2 they are 2.13, 1.96 and 1.46 and for Series 3 they are -2.79 ,

–2.59 and –1.83. The 5% critical values are found in Table A.4. According to Table A.4, Series 1 is not rejected and Series 2 and 3 are rejected by $R_1(q)$ and $R_2(q)$ but not by S_1 .

3.2.3.4 Belaire-Franch & Contreras test

Wright's test suffers from the same problem that Chow and Denning found with the LM-test, namely that the use of individual VR-tests leads to an over rejection of the null hypothesis. Therefore, using the strategy of Chow and Denning (1993), Belaire-Franch and Contreras (2004) designed a multiple variance ratio test based on the Wright's test. The test statistics are defined as follows:

$$JR_1 = \max_{1 < i < m} |R_1(q_i)|$$

$$JR_2 = \max_{1 < i < m} |R_2(q_i)|$$

$$JS_1 = \max_{1 < i < m} |S_1(q_i)|,$$

where $R_1(q_i)$, $R_2(q_i)$ and $S_1(q_i)$ are the test statistics from Wright's test. Like the CD-test, the BFC-test rejects if the maximum value of the largest test statistic is greater in absolute terms than the critical values. Also, because the test is based on Wright's test JR_1 tests the RW1, JR_2 tests the RW3 and JS_1 tests the martingale model.

Returning to our example, the maximum values of the BFC test statistics are given in Table A.3. For Series 1, these are 0.86 for JR_1 , 0.89 for JR_2 and 0.49 for JS_1 . In the same order they are 2.73, 2.46 and 1.85 for Series 2 and 2.79, 2.59 and 1.83 for Series 3. The BFC-test has larger critical values than Wright's test, which makes rejections more difficult. However, this does not change the conclusions with the series used in this example.

3.2.3.5 Wild bootstrap method

The LM-test has some potential distributional issues, as discussed in section 3.2.3.1. Wright (2000) proposed the use of ranks and signs in order to deal with this, while Kim (2006) proposes another way.

The wild bootstrap method was suggested by Kim (2006) as an alternative method of conducting the VR-test. The WBM approximates and resamples the distribution of the data, which gives new critical values and a new confidence interval. A benefit with this approach is that the distribution of an unknown form can be approximated. The WBM allows for conditional heteroscedasticity and therefore tests the RW3.

Kim (2006) presented the applications of the wild bootstrap on the $M_2(q)$ and CD_2 . In his paper, he showed by simulations that the use of the bootstrap method has great power and superiority over the traditional LM- and CD-tests. The steps in conducting the bootstrap is explained in section A.1.1.

In plain words, the test works as follows: a bootstrap sample is formed, the original LM or CD test statistic is calculated based on the bootstrapped sample and this is repeated many times (this study uses 1000) in order to get a distribution of the test statistic. The statistical significance is obtained by the proportion of the distribution of the bootstrap test statistic ($M_2(q)^*$ or CD_2^*) larger than the original test statistic ($M_2(q)$ or CD_2).

Continuing with the example equations, Table A.3 displays the p-values for the test statistics simulated by the WBM. According to the bootstrapped sample, the conclusions are the same as those from the LM- and CD-tests: Series 1 is not rejected but Series 2 and 3 are rejected.

4. Results

4.1 Descriptive Statistics

The descriptive statistics show the attributes of each index. An interesting note is that the OMXSSCPI and the OMXSMCPI have a lower standard deviation and higher mean returns than the OMXSPI and the OMXSLCPI. Further, the first two have lower maximum and minimum values. This is inconsistent with economic theory, which predicts that stocks either have higher returns and higher volatility or lower returns and lower volatility.

All indices are negatively skewed and have an excess kurtosis larger than zero. A negatively skewed distribution in terms of returns signalizes that the tails are long to the left, and thus large negative returns are more common than large positive returns. An excess kurtosis greater than zero signifies that the tails are fat, so that extreme returns are more common than in a normal distribution. Note should be given to the fact however, that the excess kurtosis is not that much greater than zero, so the magnitude of the fat tails need not have such a great impact.

Table 1 – Descriptive Statistics

	<i>OMXSPI</i>	<i>OMXSSCPI</i>	<i>OMXSMCPI</i>	<i>OMXSLCPI</i>
Observations	365	365	365	365
Mean	0.0014	0.0029	0.0030	0.0013
SD	0.0199	0.0184	0.0195	0.0203
Max	0.0625	0.0568	0.0686	0.0621
Min	-0.0619	-0.0597	-0.0638	-0.0628
Skewness	-0.2063	-0.3684	-0.3504	-0.1875
Excess Kurtosis	0.6488	0.7122	0.691	0.6541

Note: Skewness and Excess Kurtosis are normal at 0.

4.2 Autocorrelation tests

The autocorrelation coefficients and Ljung-Box's Q-statistic are reported in Table 2, where ρ_q denotes the autocorrelation coefficient and Q-stat denotes the statistic for the Ljung-Box test. One interesting note is that the Ljung-Box test is unable to reject the RW1 for all indices, except for the OMXSSCPI, which rejects the RW1 at lags 1 through 6. The first correlation coefficient is 0.155 and is rejected at a 1% significance level. This means that the rejection is strong, and the size of the correlation coefficient is quite large. The remaining correlation coefficients for the OMXSSCPI are smaller and reject at a 5% significance level. Even if there is small but significant autocorrelation in the series, it is possible that the correlation would be too small for an investor to make excess returns based on that correlation.

Table 2 – Autocorrelation and Ljung-Box's Q-statistic

	q	1	2	3	4	5	6	7	8	9	10
<i>OMXSPI</i>	ρ_q	-0.034	-0.065	-0.071	0.043	0.035	-0.054	-0.037	0.021	0.017	-0.036
	Q-Stat	0.4265	1.9813	3.8675	4.548	5.0159	6.1218	6.6304	6.8026	6.9064	7.3888
<i>OMXSSCPI</i>	ρ_q	0.155	-0.016	-0.017	0.043	0.085	0.036	0.03	0.036	0.034	-0.041
	Q-Stat	8.8803*	8.978**	9.0896**	9.7713**	12.471**	12.967**	13.308	13.783	14.209	14.832
<i>OMXSMCPI</i>	ρ_q	0.093	-0.028	-0.04	0.025	0.057	-0.036	-0.022	0.018	0.000	-0.052
	Q-Stat	3.1714	3.4593	4.0631	4.2954	5.65167	5.9998	6.1806	6.3029	6.3029	7.3054
<i>OMXSLCPI</i>	ρ_q	-0.044	-0.068	-0.072	0.047	0.033	-0.054	-0.037	0.023	0.017	-0.035
	Q-Stat	0.7286	2.4563	4.3916	5.2130	5.6294	6.7306	7.2501	7.4508	7.5599	8.0255

Note: ρ_q signifies the autocorrelation coefficient at lag q and Q-Stat signifies the Ljung-Box test statistic. ** and * signify a rejection of independence at a 5% and 1% level respectively.

4.3 Runs test

The results from the runs test are reported in Table 3. N_+ is the number of positive returns, N_- is the number of negative returns and N is the sum of both. Note that positive returns are more common than negative during this period.

The Z-statistic is a standard normal variable that rejects the null hypothesis of independence if Z is greater than the absolute value of 1.96 and 2.58 respectively for 5% and 1% significance levels. According to Table 3 the OMXSSCPI and OMXSMCPI reject the runs test. Their Z-statistics are greater than the absolute value of 2.58, which means that both the OMXSSCPI and OMXSMCPI reject randomness at a 1% significance level. A rejection of the runs test is a rejection of the RW1, so it can be said that the OMXSSCPI and the OMXSMCPI reject the RW1. The OMXSPI and OMXSLCPI, on the other hand, cannot be rejected by the runs test. This means that it is not possible to reject that these series follow a RW1 and have an iid.

The results from the runs test are different to the results of the autocorrelation tests in one way: in the autocorrelation test, only the OMXSSCPI is rejected but in the runs test both the OMXSSCPI and the OMXMCPI are rejected.

Table 3 – Runs Test

	<i>OMXSPI</i>	<i>OMXSSCPI</i>	<i>OMXSMCPI</i>	<i>OMXSLCPI</i>
N_+	206	216	222	204
N_-	159	149	143	161
N	365	365	365	365
R	177	147	151	173
m	180.4740	177.3507	174.9507	180.9671
Z-statistic	-0.3703	-3.2929	-2.6345	-0.8470
P-value	0.7111	0.0009*	0.0084*	0.397

Note: N_+ signifies the number of positive returns, N_- signifies the number of negative returns and N the total number of observations. R signifies the observed number of runs and m signifies the expected number of runs.

* signifies a rejection of independence at a 1% level.

4.4 LM-test

Table 4 shows the results from the LM-test. VR signifies the variance ratio, $M_1(q)$ is the test statistic under homoscedastic assumptions and $M_2(q)$ is the test statistic that assumes heteroscedasticity. The tests are performed with lags 2, 4, 8 and 16.

The OMXSSCPI is the only index that is rejected by the LM-test. The $M_1(q)$ rejects the RW1 at a 1% level at lag 2, and at a 5% level at lag 4 and 8. Similarly, the $M_2(q)$ also rejects, but only at a 5%-level at lag 2. This rejection is a rejection of the less restrictive RW3. The shift to fewer rejections and smaller significance by the $M_2(q)$ might be an indicator that the OMXSSCPI series suffers from heteroscedasticity. This does not change any conclusions, though, since the $M_2(q)$ also rejects.

Using the WBM to approximate the sample distribution, new critical values are obtained. However, it is still only the OMXSSCPI that is rejected by this approach. The “w’s” indicate a 5% level rejection of the null hypothesis using the approximated critical values. Lags 2 and 8 are rejected by the bootstrap distribution. Even when the distribution is simulated by a wild bootstrap, the OMXSSCPI violates the RW3.

Table 4 – Variance Ratio

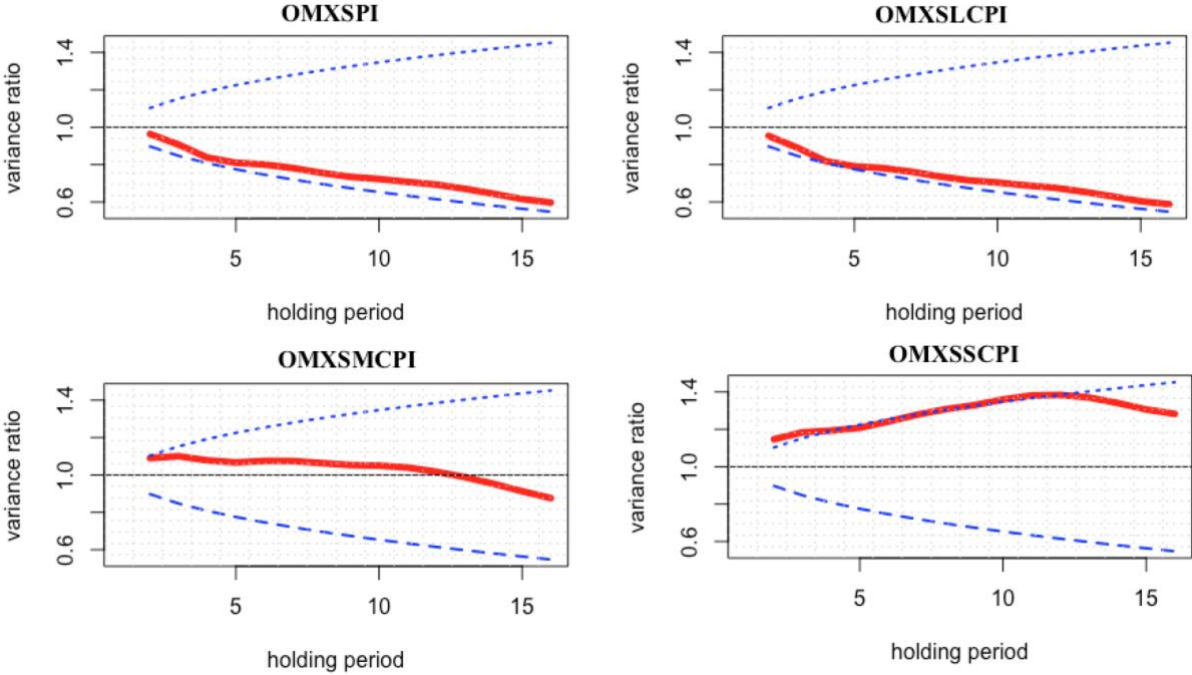
q	2	4	8	16
<i>OMXSPI</i>				
$VR(q)$	0.9647	0.8400	0.7565	0.5992
$M_1(q)$	-0.6747	-1.6341	-1.5724	-1.7394
$M_2(q)$	-0.5650	-1.3712	-1.3475	-1.5340
<i>OMXSSCPI</i>				
$VR(q)$	1.1484	1.1976	1.3121	1.2878
$M_1(q)$	2.8342*	2.0182**	2.0155**	1.2490
$M_2(q)$	2.2493***^w	1.6314	1.6996^w	1.1116
<i>OMXSMCPI</i>				
$VR(q)$	1.0902	1.0800	1.0656	0.8769
$M_1(q)$	1.7234	0.8169	0.4235	-0.5341
$M_2(q)$	1.5237	0.7194	0.3814	-0.49711
<i>OMXSLCPI</i>				
$VR(q)$	0.9544	0.8206	0.7368	0.5902
$M_1(q)$	-0.8711	-1.8315	-1.6998	-1.7788
$M_2(q)$	-0.7334	-1.5432	-1.4571	-1.5648

Note: $VR(q)$ is the variance ratio, $M_1(q)$ is the regular test statistic and $M_2(q)$ is robust for heteroscedasticity. ** and * signify a rejection at the 5% and 1% level respectively. “w” signifies a 5% rejection using critical values computed by Kim’s (2006) wild bootstrap methodology.

Figure 4 presents a plot over the respective variance ratio for the different indices. The red line is the variance ratio from lag one through sixteen and the blue dotted lines are 95% confidence intervals. Thus, if the red line lies outside of the blue dotted line, there is a 5% rejection of the test statistics.

The OMXSPI and the OMXLCPI have quite similar variance ratios, with a negative trend over increasing lags. This could indicate that the series suffer from mean reversion, i.e. that bad periods are followed by compensating good periods. The OMXSMCPI is the index with the VR closest to one. The OMXSSCPI has a positive long-term trend, which could indicate price persistence. That is a term describing that the price series is moving away from the mean, in the long term. Note however, that the variance ratios are only significant for the OMXSSCPI. Therefore, the OMXSPI and the OMXLSCPI cannot be said to be mean reverting. However, the OMXSSCPI which is rejected by the LM-test is possibly price persistent.

Figure 4 – Variance Ratio plots



Note: The red line represents the variance ratio for lag 2 to 16. The blue dotted line represents the 95% confidence interval.

4.5 Wright’s test

Table 5 presents the results from Wright’s test, based on ranks and signs. $R_1(q)$ and $R_2(q)$ are the test statistics based on ranks and S_1 is the statistic based on signs. The critical values are presented in Table A.1. They are simulated using 10.000 iterations, as in Wright (2000).

From Table 5, it is possible to read that the OMXSPI and the OMXSLCPI seem to follow a random walk, because neither is rejected by the ranks or signs statistics. The OMXSSCPI index seems to be heavily rejected by all test statistics, with at least one 1% rejection by all ranks and signs. This means that independence is rejected, and movements are not random in the Small cap index. Some less evident results come from the OMXSMCPI. There, the $R_1(q)$ rejects at a 1% level at the second lag, the $R_2(q)$ fails to reject and the S_1 rejects at 1% level for all lags. This is more interesting, since the results indicate that OMXSMCPI does not follow a RW1 or a martingale difference sequence, but it may follow a RW3.

These results are in harmony with the results from the runs test, where the OMXSMCPI is also rejected, but in contrast with the LM-test and autocorrelation tests where only the OMXSSPCI is rejected.

Table 5 – Wright’s ranks and signs test.

q	2	4	8	16
<i>OMXSPI</i>				
$R_1(q)$	0.0315	-1.4531	-1.6494	-1.6455
$R_2(q)$	-0.4284	-1.6320	-1.6448	-1.7766
S_1	0.5758	-0.1119	-0.1681	-0.1070
<i>OMXSSCPI</i>				
$R_1(q)$	3.2833*	2.6183**	2.7967*	2.4484**
$R_2(q)$	2.8153*	2.0297**	2.0311**	1.4854
S_1	3.7163*	3.1895*	3.9725*	4.9171*
<i>OMXSMCPI</i>				
$R_1(q)$	2.4440**	1.3980	0.7536	0.0666
$R_2(q)$	1.7136	0.7058	0.2948	-0.4367
S_1	3.2976*	3.1336*	3.2293*	3.7190*
<i>OMXSLCPI</i>				
$R_1(q)$	-0.2245	-1.7316	-1.8927	-1.8122
$R_2(q)$	-0.7239	-1.9335	-1.8903	-1.9021
S_1	0.9945	-0.0839	0.0708	0.0684

Note: $R_1(q)$, $R_2(q)$ and S_1 are the ranks and signs test statistics. ** and * signify rejections at the 5% and 1% level respectively.

4.6 Multiple VR-tests

The problem of multiple hypothesis testing with the original variance ratio as pointed out by Chow and Denning (1993) is tested in this section. Since Belaire-Franch and Contreras (2004) pointed out the same issue for Wright's test, we present the findings in the same table.

Table 6 presents the observed values of the test statistics. CD_1 and CD_2 represent the multiple variance ratio test statistics based on Lo and MacKinlay's (1988) statistics $M_1(q)$ and $M_2(q)$. CD_2^* is the modified version of CD_2 , but with new critical values based on the distribution by using Kim's (2006) WBM. Further, JR_1 , JR_2 and JS_1 are the joint statistics of the BFC test,

Table 6 shows that the OMXSPI and the OMXSLCPI fail to reject any test at 5%. The OMXSMCPI is rejected at 5% by the JR_1 and at 1% by the JS_1 and OMXSSCPI is rejected by all statistics, except for CD_2 . This means that the OMXSMCPI, as the case for the individual Wright's test rejects the RW1 and the martingale difference sequence but not the RW3 and the OMXSSCPI rejects both types of random walks and the martingale difference sequence.

Table 6 – Joint tests

	CD_1	CD_2	CD_2^*	JR_1	JR_2	JS_1
<i>OMXSPI</i>	1.7394	1.5340	0.392	1.6494	1.7766	0.5758
<i>OMXSLCPI</i>	1.8315	1.5648	0.309	1.8927	1.9335	0.9945
<i>OMXSMCPI</i>	1.7234	1.5237	0.269	2.4440**	1.7136	3.7190*
<i>OMXSSCPI</i>	2.8342**	2.2493	0.035**	3.2833*	2.8153**	4.9171*

Note: CD_1 and CD_2 signify the test statistics for the CD-test, JR_1 , JR_2 and JS_1 signify the test statistics for the BFC test and CD_2^* signify the p-value for the tests statistics based on the WBM. Critical values for CD_1 and CD_2 are 2.4909 and 3.0222 for a 5% and 1% rejection. Critical values for JR_1 , JR_2 and JS_1 are 2.2870, 2.2907 and 2.2942 for 5% significance and 2.9048, 2.8934 and 2.9835 for 1% significance. ** and * signify a rejection at 5% and 1% respectively.

5. Analysis and Discussion

Table 7 gives an overview of the results previously presented. The OMXSPI and the OMXSLCPI are not rejected by any test, the OMXSMCPI is rejected by some and the OMXSSCPI is rejected by the majority of the tests. This means that the market as a whole and the Large cap segment seem to follow a random walk, while the Mid- and Small cap segments do not. Note that it is necessary to remember the joint hypothesis problem when drawing

conclusions. This means that a rejection of the RWH for the Mid- and Small cap segments are not necessarily rejections of the weak form EMH.

Table 7 – Overview of test results

	<i>OMXSPI</i>	<i>OMXSSCPI</i>	<i>OMXSMCPI</i>	<i>OMXSLCPI</i>
Ljung-Box	A	R	A	A
Runs test	A	R	R	A
$M_1(q)$	A	R	A	A
$M_2(q)$	A	R	A	A
CD_1	A	R	A	A
CD_2	A	A	A	A
CD_2^*	A	R	A	A
$R_1(q)$	A	R	A	A
$R_2(q)$	A	R	A	A
S_1	A	R	R	A
JR_1	A	R	R	A
JR_2	A	R	A	A
JS_1	A	R	R	A

Note: R (Reject) denotes rejection and A (Accept) denotes failure to reject.

The results above allow us to answer our research question. The answer is that the market as a whole and the Large cap index show signs of weak form efficiency while the Mid- and Small Cap indices show signs of inefficiency. Since the market as a whole shows indications of weak form efficiency, investors have reasons to believe that the market conditions are fair, like a game of dice. However, the failure to reject the RWH does not necessarily mean that the market is weak form efficient, because the null hypothesis is not proven to be true. A suggestion for further studies is to test the OMXSPI for different trading techniques corresponding to the RW2, to see if strategies that beat the market can be used. This would in turn give a broader picture of whether the Swedish market is weak form efficient.

Another thing Table 7 indicates is that there seems to be a pattern concerning size and efficiency. The greater the capitalization of the index, the more efficient it seems to be. We do not present any statistical method to test the difference, but we may suggest why Small cap stocks seem to be less efficient than other stocks. It could be that the Small cap stocks are less

liquid than other stocks, and trade less frequently. This would imply that the rejection may not be attributed to inefficiency but to non-trading (Lo & MacKinlay, 1988). This is another suggestion for further studies: to test if non-trading is solely responsible for the OMXSSCPI rejection of weak form efficiency.

5.1 Comparison with other studies

The results from this study can be contrasted with the other studies that have been conducted on the Swedish market. Note should be given that most studies have been conducted on the OMXS30 as a proxy for the Swedish market while this present study uses the whole market; the OMXSPI. Another thing to keep in mind is that there are differences in the methods used and periods studied, which can be a reason why there are differences in results and conclusions. In this analysis we will limit our results to the OMXSPI, to enable comparisons with other studies.

Jennergren and Korsvold (1974) and Berglund et al. (1983) found, by using autocorrelation tests and the runs test, that there is autocorrelation and non-randomness in the return series. This study does not find any significant autocorrelation with the same tests. The latter authors also used filter rules, a technical trading rule designed to earn excess returns, and found no signs of inefficiency in the return series. This study does not use a similar approach, but our results tilt towards the same conclusion.

Frennberg and Hansen (1993) and Shaker (2011) rejected the RWH based on the LM-test while this study fails to reject. Frennberg and Hanssen (1993) commented about how the rejection does not necessarily mean that the market is inefficient. This was not, however, tested in a formal way. Metghalchi et al. (2008) is another study that used trading rules, a method that is not similar to ours. This also makes a comparison difficult. However, the fact that their trading rules were able to generate excess returns is intriguing, especially compared to our results that indicate the opposite conclusion.

Worthington and Higgs (2004), Urquhart (2014) and Graham et al. (2015), have more similarities with this study because they were more in favor of the RWH than the previously mentioned studies. However, these results are not unambiguous, because some tests rejected the RWH. Worthington and Higgs (2004) failed to reject the LM-test and autocorrelation test for Sweden, while the runs test was strongly rejected. Even if this is a difference compared with this study, note should be given that the runs test corresponds to the stricter RW1. A rejection

of the RW1 does not necessarily mean that the market is inefficient, because the RW1 is stricter than necessarily implied by the weak form EMH. Therefore, it is not certain that Worthington and Higgs could reject the weak form EMH for the Swedish market.

Urquhart (2014) had similar results as Worthington and Higgs (2004), but they used additional methods. The CD test statistic did not reject, but the joint sign and joint rank statistics did, while this study could not reject either test. Graham et al. (2015) did not find a rejection by the CD-test, neither by the joint rank test statistics but by the joint sign test statistic. These comparisons demonstrate that this study seems to have a stronger case than earlier studies for efficiency in the Swedish market.

When comparing all studies on the Swedish market, it is evident that no result has been in favor of the RWH as strongly as this study. A suggestion would be that markets might not always be efficient. In fact, markets might evolve between periods, as Lo (2004) proposed. This means that markets can be efficient in some periods and inefficient in others. Lo called his hypothesis the “Adaptive market hypothesis” (AMH) and it uses biological reasoning to reconcile the EMH with its behavioral critics. Since the same methods at the same market have shown different results over time, it is possible that the concept of market efficiency is better captured by the AMH than the EMH.

6. Concluding remarks

This study has tested the weak form EMH on the Swedish stock market by applying tests of the RWH and the martingale model. The results have shown that the market as a whole is following a random walk between 2012 and 2019. Furthermore, the OMXSLCPI shows signs of efficiency while the OMXSMCPI and the OMXSSCPI show signs of inefficiency. The implication of the results is that it gives a clue that the market as a whole is weak form efficient, but that need not necessarily be the case. A suggestion was made to test the efficiency on the Swedish market by using some trading rules. This could give a broader picture as to whether the signals of weak form efficiency in this study are true or if there are other factors that cannot be captured by the RWH that lead to inefficiencies in the market.

Another thing this study gave signals to is that even though the whole market seems to be efficient, the Small cap segment of the market is not. However, we pointed out that there might

be other factors that contribute to the rejections, such as a spurious autocorrelation caused by illiquidity in small stocks. This was suggested a topic for further studies.

A final observation was that the results of the weak form tests on the Swedish market are conflicting, but the market seems to have become more efficient recently. The AMH was briefly discussed as a possible solution to reconciling the concept of efficient markets with the conflicting results from previous studies.

References

- Belaire-Franch, J., & Contreras, D. (2004). *Rank and signs-based multiple variance ratio tests*. Unpublished manuscript. Retrieved from https://www.researchgate.net/profile/Jorge_BELAIRE-FRANCH/publication/228583649_Ranks_and_signs-based_multiple_variance_ratio_tests/links/00b4951920895f04f6000000.pdf
- Berglund, T., Wahlroos, B., & Örnmark, A. (1983). The Weak-Form Efficiency of the Finnish and Scandinavian Stock Exchanges: A Comparative Note on Thin Trading. *The Scandinavian Journal of Economics*, 85(4), 521-530. <https://www.jstor.org/stable/3439949>
- Borges, M. R. (2010) Efficient market hypothesis in European stock markets, *The European Journal of Finance*, 16(7), 711-726, doi: [10.1080/1351847X.2010.495477](https://doi.org/10.1080/1351847X.2010.495477)
- Box, G., & Pierce, D. (1970). Distribution of Residual Autocorrelations in Autoregressive-Integrated Moving Average Time Series Models. *Journal of the American Statistical Association*, 65(332), 1509-1526. doi:10.2307/2284333
- Caeiro, F., & Mateus, A. (2014). randtests: Testing randomness in R (Version 1.0). Retrieved from <https://CRAN.R-project.org/package=randtests>
- Campbell, J. Y., Lo, A. W. & MacKinlay, A. C. *The Econometrics of Financial Markets* (3rd ed.). Princeton, NJ: Princeton University Press.
- Charles, A., & Darné, O. (2009). Variance-Ratio Tests of Random Walk: An Overview. *Journal of Economic Surveys*, 23(3), 503–527. <https://doi-org.ezproxy.ub.gu.se/10.1111/j.1467-6419.2008.00570.x>
- Chaudhuri, K., & Wu, Y. (2003). Random walk versus breaking trend in stock prices: Evidence from emerging markets. *Journal of Banking & Finance*, 27(4), 575-592. [https://doi-org.ezproxy.ub.gu.se/10.1016/S0378-4266\(01\)00252-7](https://doi-org.ezproxy.ub.gu.se/10.1016/S0378-4266(01)00252-7).
- Chow, W. K., & Denning, K. C. (1993). A simple multiple variance ratio test. *Journal of Econometrics*, 58(3), 385-401. [https://doi-org.ezproxy.ub.gu.se/10.1016/0304-4076\(93\)90051-6](https://doi-org.ezproxy.ub.gu.se/10.1016/0304-4076(93)90051-6)
- De Bondt, W. F., & Thaler, R. (1985). Does the Stock Market Overreact? *The Journal of Finance*, 40(3), 793-805. doi:10.2307/2327804
- De Bondt, W. F., & Thaler, R. (1987). Further Evidence on Investor Overreaction and Stock Market Seasonality. *The Journal of Finance*, 42(3), 557-581. doi:10.2307/2328371

- Dragotă, V., & Țilică, E. (2014). Market efficiency of the Post Communist East European stock markets. *Central European Journal of Operations Research*, 22(2), 307–337. <https://doi-org.ezproxy.ub.gu.se/10.1007/s10100-013-0315-6>
- Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance*, 25(2), 383-417.
- Frennberg, P., & Hansson, B. (1993). Testing the random walk hypothesis on Swedish stock prices: 1919-1990. *Journal of Banking & Finance*, 17(1), 175-191. [https://doi-org.ezproxy.ub.gu.se/10.1016/0378-4266\(93\)90087-T](https://doi-org.ezproxy.ub.gu.se/10.1016/0378-4266(93)90087-T)
- Graham, M., Peltomäki, J., & Sturludóttir, H. (2015). Do capital controls affect stock market efficiency? Lessons from Iceland. *International Review of Financial Analysis*, 41(1), 82-88. <https://doi-org.ezproxy.ub.gu.se/10.1016/j.irfa.2015.05.009>
- Grieb, T., & Reyes, M. G. (1999). Random walk tests for Latin American equity indexes and individual firms. *Journal of Financial Research*, 22(4), 371-383. <https://doi-org.ezproxy.ub.gu.se/10.1111/j.1475-6803.1999.tb00701.x>
- Guidi, F., Gupta, R., & Maheshwari, S. (2011). Weak-form Market Efficiency and Calendar Anomalies for Eastern Europe Equity Markets. *Journal of Emerging Market Finance*, 10(3), 337-389. doi:10.1177/097265271101000304
- Hochberg, Y. (1974). Some generalizations of the T-method in simultaneous inference. *Journal of Multivariate Analysis*, 4(2), 224-234. [https://doi.org/10.1016/0047-259X\(74\)90015-3](https://doi.org/10.1016/0047-259X(74)90015-3)
- Hoque, H. A., Kim, J. H., & Pyun, C. S. (2007). A comparison of variance ratio tests of random walk: A case of Asian emerging stock markets. *International review of Economics and Finance*, 16(4), 488-502. <https://doi-org.ezproxy.ub.gu.se/10.1016/j.iref.2006.01.001>
- Huang, B.-N. (1995). Do Asian stock market prices follow random walks? Evidence from the variance ratio test. *Applied Financial Economics*, 5(4), 251–356. <https://doi-org.ezproxy.ub.gu.se/10.1080/758536875>
- Jennergren, P. L., & Korsvold, P. E. (1974). Price Formation in the Norwegian and Swedish Stock Markets: Some Random Walk Tests. *The Swedish Journal of Economics*, 76(2), 171-185. <https://www.jstor.org/stable/3439199>
- Kendall, M. G. (1953). The Analysis of Economic Time-Series-Part I: Prices. *Journal of the Royal Statistical Society*, 116(1), 11-34. doi:10.2307/2980947

- Kim, J. H. (2006). Wild bootstrapping variance ratio tests. *Economics Letters*, 92(1), 38-43. <https://doi-org.ezproxy.ub.gu.se/10.1016/j.econlet.2006.01.007>
- Kim, J. H. (2014). vrtest: Variance Ratio tests and other tests for Martingale Difference Hypothesis (Version 0.97). Retrieved from <https://CRAN.R-project.org/package=vrtest>
- Kim, J. H. & Shamsuddin, A. (2008). Are Asian stock markets efficient? Evidence from new multiple variance ratio tests. *Journal of Empirical Finance*, 15(3), 518-532. <https://doi-org.ezproxy.ub.gu.se/10.1016/j.jempfin.2007.07.001>
- Komsta, L., & Novometsky, F. (2014). moments: Moments, cumulants, skewness, kurtosis and related tests (Version 0.14). Retrieved from <https://CRAN.R-project.org/package=moments>
- LeRoy, S. (1973). Risk Aversion and the Martingale Property of Stock Prices. *International Economic Review*, 14(2), 436-446. doi:10.2307/2525932
- Ljung, G., & Box, G. (1978). On a Measure of Lack of Fit in Time Series Models. *Biometrika*, 65(2), 297-303. doi:10.2307/2335207
- Lo, A. W. (2004). The Adaptive Markets Hypothesis: Market Efficiency from an Evolutionary Perspective. *Journal of Portfolio Management*, 30(5), 15-29. <https://doi.org/10.3905/jpm.2004.442611>
- Lo, A. W., & MacKinlay, A. C. (1988). Stock Market Prices do not Follow Random Walks: Evidence from a Simple Specification Test. *The Review of Financial Studies*, 1(1), 41-66. <http://www.jstor.org.ezproxy.ub.gu.se/stable/2962126>
- Lo, A. W., & MacKinlay, A. C. (1989). The size and power of the variance ratio test in finite samples: A Monte Carlo investigation. *Journal of Econometrics*, 40(2), 203-238. [https://doi-org.ezproxy.ub.gu.se/10.1016/0304-4076\(89\)90083-3](https://doi-org.ezproxy.ub.gu.se/10.1016/0304-4076(89)90083-3)
- Lucas, R. (1978). Asset Prices in an Exchange Economy. *Econometrica*, 46(6), 1429-1445. doi:10.2307/1913837
- Metghalchi, M., Chang, Y. -H., & Marcucci, J. (2008). Is the Swedish stock market efficient? Evidence from some simple trading rules. *International Review of Financial Analysis*, 17(3), 475-490. doi:0.1016/j.irfa.2007.05.001
- Nasdaq. (2019). *Nasdaq OMX Nordic*. [Online]. Retrieved 2019 March 26 from <http://www.nasdaqomxnordic.com>
- Ojah, K. (1999). Random Walks and Market Efficiency Tests of Latin American Emerging Equity Markets: A Revisit. *Financial Review*, 34(2), 57. <https://doi-org.ezproxy.ub.gu.se/10.1111/j.1540-6288.1999.tb00454.x>

- R Core Team (2019). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria (Version 1.1.463). Retrieved from <https://www.R-project.org/>.
- Richmond, J. (1982). A General Method for Constructing Simultaneous Confidence Intervals. *Journal of the American Statistical Association*, 77(378), 455-460. doi:10.2307/2287267
- Roberts, H. V. (1967). *Statistical versus Clinical Prediction of the Stock Market*, Unpublished manuscript.
- Shaker, A. T. (2013). TESTING THE WEAK-FORM EFFICIENCY OF THE FINNISH AND SWEDISH STOCK MARKETS. *European Journal of Business and Social Sciences*, 2(9), 176-185. https://www.researchgate.net/profile/Abu_Towhid_Shaker2/publication/259493181_Testing_the_Weak-Form_Efficiency_of_the_Finnish_and_Swedish_Stock_Markets/links/0046352c3cca8afc11000000.pdf
- Shiller, R., Fischer, S., & Friedman, B. (1984). Stock Prices and Social Dynamics. *Brookings Papers on Economic Activity*, 1984(2), 457-510. doi:10.2307/2534436
- Sidak, Z. (1967). Rectangular Confidence Regions for the Means of Multivariate Normal Distributions. *Journal of the American Statistical Association*, 62(318), 626-633. doi:10.2307/2283989
- Smith, G. (2009). Martingales in European emerging stock markets: Size, liquidity and market quality, *The European Journal of Finance*, 15(3), 249-262, doi:10.1080/13518470802423262
- Smith, G., & Ryoo, H. -J. (2003). Variance ratio tests of the random walk hypothesis for European emerging stock markets, *The European Journal of Finance*, 9(3), 290-300, doi:[10.1080/1351847021000025777](https://doi.org/10.1080/1351847021000025777)
- Urquhart, A. (2014). The Euro and European stock market efficiency. *Applied Financial Economics*, 24(19), 1235-1248. doi:10.1080/09603107.2014.924292
- Urrutia, J. L. (1995). Tests of Random Walk and Market Efficiency for Latin American Emerging Equity Markets. *Journal of Financial Research*, 18(3), 299-309. doi:10.1111/j.1475-6803.1995.tb00568.x.
- Worthington, A. C., & Higgs, H. (2004) Random walks and market efficiency in European equity markets. *Global Journal of Finance and Economics*, 1(1), 59-78.

Wright, J. (2000). Alternative Variance-Ratio Tests Using Ranks and Signs. *Journal of Business & Economic Statistics*, 18(1), 1-9. doi:10.2307/1392131

Appendix A.1

A.1.1 Kim's Bootstrap

The WBM is employed as explained by Kim (2006):

1. Form a bootstrap sample of T observations $X_t^* = \eta_t X_t$ ($t = 1, \dots, T$) where η_t is a random sequence with $E(\eta_t) = 0$ and $E(\eta_t^2) = 1$.
2. Calculate $MV^* \equiv MV_2(X^*, k_i)$, the $MV_2(X, k_i)$ statistic obtained from the bootstrap sample
3. Repeat (1) and (2) sufficiently many, say m , times to form a bootstrap distribution of the test statistic $\{MV(X^*, k_j; j)\}_{j=1}^m$.

A.1.2 Wright's test critical values

Table A.1 – Critical values for Wrights test

q	2	4	8	16
$R_1(q)$				
1%	-2.6617; 2.4783	-2.5390; 2.6261	-2.3975; 2.6357	-2.2647; 2.6067
5%	-2.0404; 1.8693	-2.0383; 1.8866	-1.9833; 1.8400	-1.9237; 1.7441
$R_2(q)$				
1%	-2.6739; 2.4344	-2.5808; 2.5710	-2.4255; 2.64915	-2.2828; 2.5894
5%	-2.0735; 1.8666	-2.0281; 1.8791	-1.9832; 1.8645	-1.9435; 1.7367
S_1				
1%	-2.5648; 2.5648	-2.4341; 2.6299	-2.3534; 2.7516	-2.1494; 2.9342
5%	-1.9367; 1.9367	-1.9305; 1.9305	-1.8934; 1.9641	-1.8224; 1.9769

Note: 10.000 iterations are used in the simulations, as in Wright (2000)

Appendix A.2

A.2.1 Data for examples

Table A.2 – Price series for examples

Series 1	Series 2	Series 3
99.45	102.33	100.33
99.93	103.00	99.37
97.87	102.83	99.50
98.25	102.79	98.82
97.16	102.82	98.63
97.67	102.82	97.24
97.67	101.26	97.10
99.88	100.88	96.81
100.78	99.08	97.36
101.89	98.39	97.66
102.52	98.71	97.11
102.18	98.60	96.19
101.33	99.18	96.41
102.58	99.71	96.93
103.73	99.70	96.30
104.36	100.34	98.35
104.74	100.88	98.83
103.76	101.44	98.56
102.75	101.74	99.13
103.35	103.06	97.97
102.45	103.68	99.21
103.71	103.71	98.44
103.91	102.27	97.68
103.72	101.48	98.96
104.56	100.06	97.89
104.23	99.36	98.27
104.22	99.56	96.91
103.69	99.88	96.66

103.20	99.94	97.73
102.90	101.69	96.29
103.15	101.87	96.93

A.2.2 Test statistics example

Table A.3 – Test statistics example

Series 1		Series 2		Series 3	
$VR(2)$	1.00	$VR(2)$	1.40	$VR(2)$	0.59
$M_1(2)$	0.04	$M_1(2)$	2.21	$M_1(2)$	-2.25
$M_2(2)$	0.06	$M_2(2)$	2.91	$M_2(2)$	-2.53
CD_1	0.93	CD_1	2.52	CD_1	2.25
CD_2	1.02	CD_2	2.91	CD_2	2.53
$R_1(2)$	0.35	$R_1(2)$	2.13	$R_1(2)$	-2.79
$R_2(2)$	0.35	$R_2(2)$	1.96	$R_2(2)$	-2.59
S_1	0.00	S_1	1.46	S_1	-1.83
JR_1	0.86	JR_1	2.73	JR_1	2.79
JR_2	0.89	JR_2	2.46	JR_2	2.59
JS_1	0.49	JS_1	1.85	JS_1	1.83
$M_2^*(2)$	0.95	$M_2^*(2)$	0.00	$M_2^*(2)$	0.01
CD_2^*	0.90	CD_2^*	0.00	CD_2^*	0.02

Note: The variance ratio and all test statistics are presented with lag 2, but calculations are made using lags 2, 4, 8 and 16. This is important for the joint hypothesis tests. One period joint tests are the same as individual tests. M_2^* and CD_2^* show the p-values of the bootstrapped LM and CD statistics. The JR_1 , JR_2 and JS_1 critical values are 2.11, 2.09 and 2.19 at the 5% significance level.

A.2.3 Wright's critical values – example

Table A.4 – Wright's critical values

	Negative	Positive
$R_1(2)$	-2.24	1.53
$R_2(2)$	-2.21	1.52
S_1	-2.19	1.83

Note: The critical values at the 5% level.