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# Non-Life-Threatening Ailments and Rational Patience

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# Non-Life-Threatening Ailments and Rational Patience

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**Abstract**

The time at which a rational patient might choose an elective medical procedure for a non-life-threatening ailment is contemplated. The resulting model is purposely uncomplicated but general, and accounts for several basic factors that might affect such a decision. One such factor is that a patient cannot know with certainty the degree to which the medical procedure will be successful. Even so, patients have information about the expected outcome of the procedure and its risk, and about how the expected outcome and risk are affected by medical technological progress and surgeon experience. The effect of changes in exogenous variables on the timing of the medical procedure and on patient welfare are investigated. It is shown that risk averse and prudent patients behave in an unambiguous manner in response to changes in all of the exogenous variables.

Keywords: Health Behavior; Optimal Timing; Medical Decisions

JEL Codes: D15; I12

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## 1. Introduction

Mr. Jones suffered an injury to his knee more than one half-century ago while parachute training in the U.S. Army. His knee was repaired to the extent possible at the time. Years later Mr. Jones visited a Veterans Affairs physician to complain about the condition of his knee, as it was causing him considerable discomfort during routine activities. The physician informed him that his knee was in a continuous state of decline, and that short of total knee replacement, there was nothing either of them could do about the condition of his knee. In addition, the physician stated that the decision to undergo total knee replacement was entirely up to Mr. Jones, as he was in the best position—not the physician—to judge how his quality of life was being affected by the declining condition of his knee. Mr. Jones was also informed that the outcome of the surgery is expected to be better, and the risk associated with it lower, the longer he waited to undergo surgery, due to advancements in medical technology and accumulation of surgeon experience. For these reasons Mr. Jones waited more than a decade after the aforesaid visit before electing to undergo total knee replacement. As put by Bayliss et al. (2017, p. 1430), "Patients who are considering undergoing joint replacement should balance the potential benefits of an improvement in their quality of life against the potential risks of the intervention: death, medical complications, infection, poor functional outcome, and the need for revision surgery. Patients have indicated that they require improved information about these outcomes, particularly in relation to deciding on the correct time to have surgery."

Versions of this story are commonplace. According to Williams et al. (2015, p. 5), total knee replacement was among the five most frequent inpatient surgeries from 2000–2010 in the U.S., and it was the most frequent in 2008, 2009, and 2010, with nearly 700,000 total knee replacements performed among inpatients aged 45 and over in 2010. Similarly, Wolford et al. (2015, p. 1) reported that approximately 326,000 total hip replacements were performed in the U.S. in 2010, while Sloan et al. (2018, p. 1458) forecasted that there will be 635,000 to 909,900 total hip replacements and 1.26 to 1.68 million total knee replacements by 2030 in the U.S. What is more, these estimates understate the number of total joint replacements, seeing as they do not include ankle, elbow, and shoulder replacements, nor do they account for partial joint replacements.

Labek et al. (2011) have documented that the revision rates are positive but small for all joint replacements—the revision rate for total hip replacement is 6.45% after five years and 12.9% after 10, with slightly lower percentages for total knee replacement. They also documented considerable variation in revision rates across countries for the same type of joint replacement. In other words, joint replacement surgery entails at least some risk, seeing as an exogenous, uniform, non-stochastic rate of depreciation cannot fully rationalize the above. Accordingly, surgical risk plays a fundamental role in the proposed model of patient decision making. Moreover, Stavrakis and SooHoo (2016, p. 1453) argued " … that total ankle replacement would demonstrate increasing utilization and lower complication rates given advances in implant design and growth in surgeon experience." Indeed, over time, surgical outcomes have generally improved and the risks associated with them have declined because medical technologies and surgeons have improved too. These facts are taken into account in the model as well.

Another condition that exhibits the above flavor is cataract, whose numbers are considerable. According to Lindstrom (2015, p. 62), cataract surgery is the most common procedure performed by an ophthalmic surgeon, with an estimated 3.6 million cataract procedures performed in the U.S. in 2015 and more than 20 million performed worldwide. The intraocular lens used in cataract surgery is permanent and will never cloud, and unlike an artificial joint, there are no moving parts in the lens that could wear out over time. Hence revisions are rare, but if they do occur they are the result of less-than-perfect, i.e., risky, surgeries.

In order to more generally frame the contribution herein, consider a patient who is diagnosed with a non-life-threatening condition or ailment. The condition is known to worsen over time and there is little that can be done about it either by the medical community or the patient, other than for the patient to elect a medical procedure. The medical expertise and technology exists to correct the ailment and is known to yield better expected outcomes with less variability the longer the patient waits to undergo the procedure. The decision to undergo the procedure is entirely up to the patient, not the physician, as the patient is in the best position to determine the degree to which the ailment is affecting their quality of life. The procedure is fully covered by insurance, so out-of-pocket expenses are sufficiently small and fixed. When does a rational patient elect to undergo the medical procedure to correct a non-life-threatening ailment? This decision problem is formally modeled and its qualitative and welfare properties are studied.

The resulting model is purposely uncomplicated but uses general specifications for the primitive functions. As a result, the refutable implications derived from it are robust and do not rely on the specification of functional forms that are chosen because, say, they provide a closed-form solution. The main finding is that the time at which an elective medical procedure is chosen by a risk averse and prudent patient, and the welfare of said patient, respond to changes in the exogenous variables in an entirely predictable fashion. Accordingly, the refutable implications form the basis for a statistical test of the rationality of patients faced with decisions of this nature.

There is a small literature related to, but decidedly different from, the present work, most notably that by Whynes (1995), Driffield and Smith (2007), and Meyer and Rees (2012). All three model a condition that involves "watchful waiting." According to Whynes (1995, p. 478), "Watchful waiting refers to the provision of routine, supervisory care within a monitoring regime which will alert the practitioner to any change in the patient's condition." It may be prescribed for slow growing cancers such lymphoma or prostate cancer, or for diseases that progress slowly. Watchful waiting is also a recommended treatment option for benign prostatic hyperplasia, inguinal hernia, abdominal aortic aneurysm, and mild chronic hepatitis C, and is also a recommended treatment for conditions that have the potential to resolve themselves due to the body's curative capacity, such as the common cold, ear infections, and mild forms of depression. This stands in contrast to the class of ailments contemplated here, which are not life-threatening, do not resolve themselves on their own, and for which only a medical procedure can halt their progression. Moreover, the monitoring and testing that goes along with watchful waiting is used to inform the physician—the decision maker in such instances—as to when surgery or more aggressive medical intervention is required. In contrast, for the class of ailments contemplated herein, the patient knows fully well how the progression of the ailment is affecting their quality of life. Hence the patient is in the best position to make the decision as to when to elect a medical procedure, not the physician.

The salient and distinguishing features of the proposed model are that (i) it includes gradual improvements in medical technologies and accumulation of surgical experience, (ii) the ailment is not life threatening, (iii) the medical procedure is elective, (v) the decision to elect the procedure is that of the patient, not the physician, and (v) the medical procedure is risky. In contrast, Whynes (1995) employed a watchful waiting framework and contemplated a physician-made decision to minimize total expected treatment costs for a given population by choosing the optimal time to switch from a treatment that involves a strictly positive risk of relapse, to a treatment that entails a definite cure. Meyer and Rees (2012) considered the optimal timing of a patient undergoing a medical treatment as an optimal stopping problem when the state of health of a patient evolves stochastically. The decision maker in their framework is not the patient, as it is here, but rather an omniscient regulator who knows a patient's preferences, observes the patient's state of health at every point in time, and knows the stochastic process governing the evolution of the patient's state of health. Driffield and Smith (2007) were the first to employ a real-options approach to the optimal timing of a medical treatment. Their model is essentially a special case of that considered by Meyer and Rees (2012).

## 2. The Model and Assumptions

At time $t = 0$ a patient visits a physician with a health complaint. The physician informs the patient that they have an ailment or condition that defines the patient's initial state of health, say $H(0) = H_0 \in \mathbb{R}_{++}$. Although the ailment is not life threatening, it grows worse over time unless the patient elects a medical procedure, as there is nothing the physician or patient can do about it otherwise. Consequently, before electing a medical procedure a patient's health is governed by the differential equation $\dot{H}_1(t) = -\delta(t;q) < 0$, where the depreciation function $\delta(\cdot)$ determines the rate at which the ailment worsens over time and $q \in \mathbb{R}_{++}$ is a shift parameter. It therefore follows that $H_1(t) = H_0 - \int_0^t \delta(s;q)ds$ for all $t \in [0,\tau)$. It is important to note that health is defined narrowly, as it is a reference to the state of a particular ailment. Observe too that the depreciation function $\delta(\cdot)$ cannot be influenced by a patient, i.e., a patient cannot influence the pace at which their health declines prior to the medical procedure, which is largely the case for candidates facing cataracts surgery and those contemplating joint replacement. In effect, this feature of the model defines the types of ailments under consideration.

Let $\tau \in [0,T]$ be the time at which a patient elects to undergo a medical procedure, which is completed instantaneously seeing as time is continuous, and let $T \in \mathbb{R}_{++}$ be the known time at which a patient dies. A patient's state of health post-procedure, say $H_2(t)$, $t \in [\tau,T]$, does not change over the remainder of their life, hence $H_2(t) = H_2(\tau)$ for all $t \in [\tau,T]$—adding exogenous depreciation does not change any of the results. However, the outcome of the medical procedure is not known with certainty, and therefore a patient's post-procedure state of health is $H_2(\tau) = \tilde{H}$, where $\tilde{H}$ is a random variable. Because the focus is on studying the effects of changes in exogenous variables on the time at which a patient elects the medical procedure and a patient's welfare, it is useful to parameterize the random variable in a simple manner that permits those goals to be achieved, to wit, $H_2(\tau) = \tilde{H} = \mu(\tau;m) + \sigma(\tau;v)\tilde{\varepsilon}$, where $m \in \mathbb{R}_{++}$ and $v \in \mathbb{R}_{++}$ are shift parameters and $\tilde{\varepsilon}$ is a random variable whose probability density function has domain $[\underline{\varepsilon},\bar{\varepsilon}]$, with $\underline{\varepsilon} < 0 < \bar{\varepsilon}$. The random variable $\tilde{\varepsilon}$ satisfies $\mathrm{E}[\tilde{\varepsilon}] = 0$ and $\mathrm{E}[\tilde{\varepsilon}^2] = 1$, which imply that $\mathrm{E}[\tilde{H}] = \mu(\tau;m) \in \mathbb{R}_{++}$ and $\mathrm{E}\left[[\tilde{H} - \mu(\tau;m)]^2\right] = \sigma(\tau,v)^2 \in \mathbb{R}_{++}$.

Instantaneous preferences are represented by $U\big(H_1(t)\big)$ before the medical procedure, i.e., for $t \in [0,\tau)$, and $U\big(H_2(\tau)\big)$ after the procedure, that is, for $t \in [\tau,T]$. A patient's intertemporal rate of time preference is $\rho \in \mathbb{R}_{++}$.

Pulling all of the preceding together, the decision problem can be stated as

$$V(\boldsymbol{\beta}) \stackrel{\text{def}}{=} \max_{\tau} \mathrm{E}\left[\int_0^{\tau} U\left(H_0 - \int_0^t \delta(s;q)ds\right)e^{-\rho t}\,dt + \int_{\tau}^{T} U\big(\mu(\tau;m) + \sigma(\tau;v)\tilde{\varepsilon}\big)e^{-\rho t}\,dt\right], \tag{1}$$

where $\boldsymbol{\beta} \stackrel{\text{def}}{=} (H_0,m,q,v,\rho,T)$ and $V(\boldsymbol{\beta})$ is the maximum expected present discounted value of lifetime utility given the parameter vector $\boldsymbol{\beta}$. The ensuing five assumptions are placed on the primitives of problem (1) and discussed subsequently. Two additional assumptions are relegated to the Appendix and discussed there so as to not break the flow with technical matters.

(A1) $U(\cdot):\mathbb{R}_+ \to \mathbb{R}_+$, $U(\cdot)\in C^{(2)}$, $U'(H)>0$ and $U''(H)<0$ for all $H\in\mathbb{R}_{++}$.

(A2) $\delta(\cdot):\mathbb{R}_+\times\mathbb{R}_{++}\to\mathbb{R}_{++}$, $\delta(\cdot)\in C^{(1)}$ and $\delta_q(t;q)>0$ for all $(t,q)\in\mathbb{R}_{++}\times\mathbb{R}_{++}$.

(A3) $\mu(\cdot):\mathbb{R}_+\times\mathbb{R}_{++}\to\mathbb{R}_{++}$, $\mu(\cdot)\in C^{(2)}$, $H_0<\mu(\tau;m)<H^*\in\mathbb{R}_{++}$, $\mu_\tau(\tau;m)>0$, $\mu_{\tau\tau}(\tau;m)<0$, $\mu_m(\tau;m)>0$, and $\mu_{\tau m}(\tau;m)\equiv 0$ for all $(\tau,m)\in(0,T)\times\mathbb{R}_{++}$.

(A4) $\sigma(\cdot):\mathbb{R}_+\times\mathbb{R}_{++}\to\mathbb{R}_{++}$, $\sigma(\cdot)\in C^{(2)}$, $\sigma_\tau(\tau;v)<0$, $\sigma_{\tau\tau}(\tau;v)>0$, $\sigma_v(\tau;v)<0$, and $\sigma_{\tau v}(\tau;v)\equiv 0$ for all $(\tau,v)\in(0,T)\times\mathbb{R}_{++}$.

(A5) $\mu(\tau;m)+\sigma(\tau;v)\underline{\varepsilon}<H_0<\mu(\tau;m)+\sigma(\tau;v)\overline{\varepsilon}\le H^*$ for all $(\tau,m,v)\in(0,T)\times\mathbb{R}_{++}\times\mathbb{R}_{++}$.

Assumption (A1) places prototypical restrictions on the function $U(\cdot)$ that represents instantaneous preferences, to wit, that (i) it is twice continuously differentiable, (ii) health is a good, and (iii) the patient is risk averse. Similarly, supposition (A2) asserts that the ailment depreciation function is once continuously differentiable and that an increase in its parameter increases the rate at which an ailment worsens over time.

Assumptions (A3), (A4), and (A5) specify the effects of medical technology progress and accumulation of medical experience on the expected state of health resulting from, and risk of, the medical procedure. Leaving the technical part of assumption (A3) aside, it asserts that the expected state of health after the medical procedure is higher than a patient's initial state of health but less than a perfect state of health $H^*$. It also stipulates that the expected state of health increases at a deceasing rate over time. Furthermore, an increase in the parameter $m$ increases the expected state of health at every point in time without affecting its slope, an assumption fully akin to a parallel shift in a demand curve.

Supposition (A4) places analogous stipulations on the standard deviation of a patient's state of health after the medical procedure. In particular, it supposes that the standard deviation decreases over time at a decreasing rate, and that an increase in the parameter $v$ decreases the standard deviation at each point in time without affecting its slope. Note that while the riskiness of the medical procedure declines over time, it is never eliminated.

Finally, assumption (A5) is required to fully capture the risk associated with any medical procedure, in that the worst outcome of the procedure results in a state of health below the initial state, i.e., the surgery can go tragically wrong. It also stipulates that the best outcome of the medical procedure can result in no more than perfect health. This rules out the implausible outcome in which a cataract patient who undergoes surgery has the eyes of an eagle.

Under the two additional assumptions stated and discussed in the Appendix, it is deduced in the Appendix that $\tau = \tau^*(\boldsymbol{\beta})$ is the locally unique and $C^{(1)}$ solution to problem (1). Thus the maximum expected present discounted value of lifetime utility $V(\boldsymbol{\beta})$ may be defined as

$$V(\boldsymbol{\beta}) \overset{\text{def}}{=} \int_0^{\tau^*(\boldsymbol{\beta})} U\left(H_0 - \int_0^t \delta(s;q)\,ds\right) e^{-\rho t}\,dt + \mathrm{E}\left\{ \int_{\tau^*(\boldsymbol{\beta})}^{T} U\left(\mu\left(\tau^*(\boldsymbol{\beta});m\right) + \sigma\left(\tau^*(\boldsymbol{\beta});v\right)\tilde{\varepsilon}\right) e^{-\rho t}\,dt \right\}, \quad (2)$$

and is at least a locally $C^{(1)}$ function in light of the foregoing deductions. Note that the first term on the right-hand side of Eq. (2) is the discounted lifetime utility of a patient up until the chosen time of the medical procedure $\tau = \tau^*(\boldsymbol{\beta})$, while the second is the expected discounted lifetime utility from that point in time until death.

As deduced in the Appendix, $\tau = \tau^*(\boldsymbol{\beta})$ satisfies the first-order necessary condition of problem (1), found by applying Leibniz's rule and the chain rule to it, to wit,

$$\begin{aligned} &e^{-\rho\tau} U\left(H_0 - \int_0^\tau \delta(s;q)\,ds\right) - e^{-\rho\tau}\mathrm{E}\left[U\left(\mu(\tau;m) + \sigma(\tau;v)\tilde{\varepsilon}\right)\right] \\ &+ \mathrm{E}\left[\int_\tau^T U'\left(\mu(\tau;m) + \sigma(\tau;v)\tilde{\varepsilon}\right)\left[\mu_\tau(\tau;m) + \sigma_\tau(\tau;v)\tilde{\varepsilon}\right] e^{-\rho t}\,dt\right] = 0. \end{aligned} \quad (3)$$

To get at the economic interpretation of Eq. (3), observe that it may be written in a more compact form using $H_1(t) = H_0 - \int_0^t \delta(s;q)\,ds$ and $H_2(\tau) = \mu(\tau;m) + \sigma(\tau;v)\tilde{\varepsilon}$ to arrive at

$$e^{-\rho\tau} U\left(H_1(\tau)\right) + \mathrm{E}\left[\int_\tau^T U'\left(H_2(\tau)\right)\left[\mu_\tau(\tau;m) + \sigma_\tau(\tau;v)\tilde{\varepsilon}\right] e^{-\rho t}\,dt\right] = e^{-\rho\tau}\mathrm{E}\left[U\left(H_2(\tau)\right)\right]. \quad (4)$$

The left-hand side of Eq. (4) is the expected present value of the marginal benefit of delaying the medical procedure. It is comprised of the (certain) present value of the utility gained from the pre- procedure state of health due to delaying the procedure, plus the expected present discounted value of the marginal utility over a patient's post-procedure lifetime resulting from the delay.

The right-hand side of Eq. (4) is the expected present value of the marginal cost of delaying the medical procedure. It consists of the present value of expected utility lost in the post-procedure state of health from delaying the medical procedure. With the above matters addressed, attention is turned to an examination of the qualitative properties of problem (1).

## 3. Qualitative Results

To begin, note that Eq. (3) is more useful than Eq. (4) for the purpose of comparative statics, as all of the parameters are explicitly indicated in it. To derive the comparative statics, substitute $\tau = \tau^*(\boldsymbol{\beta})$ in Eq. (3) to create an identity in the parameter vector $\boldsymbol{\beta}$, and then differentiate the identity with respect to the parameter of interest using Leibniz's rule and the chain rule. This is the approach followed to derive the ensuing comparative statics expressions. Furthermore, note that all such ensuing expressions are evaluated at $\tau = \tau^*(\boldsymbol{\beta})$, and that $D < 0$ by assumption (A7), where $D$ is defined in the Appendix.

*The Initial State of Health*

The effect of an improvement in a patient's initial state of health $H_0$ is given by

$$\frac{\partial \tau^*(\boldsymbol{\beta})}{\partial H_0} \equiv \frac{-e^{-\rho\tau} U'\big(H_1(\tau)\big)}{D} > 0 \,. \tag{5}$$

As health is a good, an increase in a patient's initial state of health, that is, a report from the physician that the ailment is not as bad as the patient might have imagined, results in the patient electing to undergo the medical procedure at a later date. Furthermore, a patient is better off because of the more favorable initial diagnosis. To see this, use Eq. (2) and the envelope theorem to arrive at

$$\frac{\partial V(\boldsymbol{\beta})}{\partial H_0} = \int_0^{\tau^*(\boldsymbol{\beta})} U'\big(H_1(t)\big) e^{-\rho t}\, dt > 0 \,, \tag{6}$$

the sign of which also follows from the assumption that health is a good.

*The Expected State of Health Resulting From the Medical Procedure*

The effect of an increase in the expected state of health resulting from the medical procedure, that is, an increase in the parameter $m$, is given by

$$\frac{\partial\tau^*(\boldsymbol{\beta})}{\partial m} \equiv \frac{e^{-\rho\tau}\mu_m(\tau;m)\mathrm{E}\left[U'\left(H_2(\tau)\right)\right]}{D} + \frac{-\mu_\tau(\tau;m)\mu_m(\tau;m)\mathrm{E}\left[U''\left(H_2(\tau)\right)\right]\left[\int_\tau^T e^{-\rho t}\,dt\right]}{D}$$
$$+\frac{-\mu_m(\tau;m)\sigma_\tau(\tau;v)\mathrm{E}\left[U''\left(H_2(\tau)\right)\tilde{\varepsilon}\right]\left[\int_\tau^T e^{-\rho t}\,dt\right]}{D}, \tag{7}$$

$$\frac{\partial V(\boldsymbol{\beta})}{\partial m} = \mathrm{E}\left[\int_{\tau^*(\boldsymbol{\beta})}^T U'\left(H_2\left(\tau^*(\boldsymbol{\beta})\right)\right)\mu_m\left(\tau^*(\boldsymbol{\beta});m\right)e^{-\rho t}\,dt\right] > 0\,. \tag{8}$$

Equation (8) shows that a patient is better off if the expected state of health resulting from the medical procedure increases, whereas Eq. (7) shows that the sign of $\partial\tau^*(\boldsymbol{\beta})/\partial m$ cannot be unequivocally determined under the given assumptions. The difficulty is that there are three effects present in the comparative statics. The first effect in Eq. (7) leads to an earlier medical procedure, as it reflects the increased cost of delay due to the better expected state of health that subsequently results. The second leads to an earlier medical procedure too. It arises because with an increase in the expected state of health there is also an additional cost to waiting that a risk averse patient prefers to avoid. Therefore, each of the first two effects result in the patient electing the medical procedure sooner. The third effect, on the other hand, is in general ambiguous, in as much as the sign of $\mathrm{E}\left[U''\left(H_2(\tau);a\right)\tilde{\varepsilon}\right] = \mathrm{Cov}\left[U''\left(H_2(\tau);a\right),\tilde{\varepsilon}\right]$ is not known under the given assumptions. That said, three additional subcases are worth mentioning.

For the first subcase, take the simplest instance in which a patient has risk neutral preferences, i.e., $U''(H) \equiv 0$. Under this stipulation only the first effect mentioned above remains, hence the patient elects the medical procedure sooner when the expected state of health from the procedure increases. A slightly more general subcase occurs if a risk averse patient had preferences that were representable by a quadratic utility function, that is, if $U''(H) < 0$ is a constant. In this instance the first two of the above-mentioned effects remain but the ambiguous third effect vanishes, again leading to the conclusion that $\partial\tau^*(\boldsymbol{\beta})/\partial m < 0$.

For the third more general subcase, assume that a patient is prudent, i.e., assume that $U(\cdot) \in C^{(3)}$ and $U'''(H) > 0$. Now observe that if there were an increase in the realizations of $\tilde{\varepsilon}$, the realizations of $H_2(\tau)=\mu(\tau;m)+\sigma(\tau;v)\tilde{\varepsilon}$ would increase too. And under the assumption of prudence, this would in turn increase the realizations of $U''\big(H_2(\tau)\big)$, from which it follows that $\mathrm{E}\big[U''\big(H_2(\tau)\big)\tilde{\varepsilon}\big]=\mathrm{Cov}\big[U''\big(H_2(\tau)\big),\tilde{\varepsilon}\big]>0$. Consequently, the third effect mentioned above is negative as well under prudence. Thus, under the additional stipulation that the patient is prudent, $\partial\tau^*(\boldsymbol{\beta})/\partial m<0$ and is larger in magnitude than either of the two previous subcases. Moreover, seeing as diminishing absolute risk aversion implies prudence, it follows that agents whose preferences exhibit diminishing absolute risk aversion elect the medical procedure sooner when the expected state of health that results from it improves.

*The Depreciation of Health Prior to the Medical Procedure*

The effect of an increase in the rate of depreciation prior to the medical procedure, i.e., an increase in the parameter $q$, is given by

$$\frac{\partial\tau^*(\boldsymbol{\beta})}{\partial q}\equiv\frac{e^{-\rho\tau}U'\big(H_1(\tau)\big)\Big[\int_0^\tau\delta_q(s;q)\,ds\Big]}{D}<0, \tag{9}$$

$$\frac{\partial V(\boldsymbol{\beta})}{\partial q}=-\int_0^{\tau^*(\boldsymbol{\beta})}U'\big(H_1(t)\big)\Big[\int_0^t\delta_q(s;q)\,ds\Big]e^{-\rho t}\,dt<0. \tag{10}$$

Equations (9) and (10) show that a patient elects to have the medical procedure sooner and is worse off if their health declines at a faster rate in the pre-procedure period. This is intuitive, seeing as if health declines at a faster rate before the medical procedure, then the expected benefit of an earlier surgery is higher and it can be enjoyed for a longer period of time.

*The Riskiness of the Medical Procedure*

The effect of a reduction in the risk associated with the medical procedure, that is, an increase in the parameter $v$, is given by

$$\begin{aligned}\frac{\partial\tau^*(\boldsymbol{\beta})}{\partial v}\equiv{}&\frac{e^{-\rho\tau}\sigma_v(\tau;v)\mathrm{E}\big[U'\big(H_2(\tau)\big)\tilde{\varepsilon}\big]}{D}+\frac{-\sigma_\tau(\tau;v)\sigma_v(\tau;v)\mathrm{E}\big[U''\big(H_2(\tau)\big)\tilde{\varepsilon}^2\big]\Bigg[\int_\tau^T e^{-\rho t}\,dt\Bigg]}{D}\\&+\frac{-\mu_\tau(\tau;m)\sigma_v(\tau;v)\mathrm{E}\big[U''\big(H_2(\tau)\big)\tilde{\varepsilon}\big]\Bigg[\int_\tau^T e^{-\rho t}\,dt\Bigg]}{D},\end{aligned} \tag{11}$$

$$\frac{\partial V(\boldsymbol{\beta})}{\partial v}=\sigma_v\left(\tau^*(\boldsymbol{\beta});v\right)\mathrm{E}\left[U'\left(H_2\left(\tau^*(\boldsymbol{\beta})\right)\right)\tilde{\varepsilon}\right]\left[\int_{\tau^*(\boldsymbol{\beta})}^{T}e^{-\rho t}\,dt\right]>0\,. \tag{12}$$

Equation (12) shows that a patient is better off when the variance of the medical procedure decreases. To see this, first note that $\mathrm{E}\left[U'\left(H_2(\tau)\right)\tilde{\varepsilon}\right]=\mathrm{Cov}\left[U'\left(H_2(\tau)\right),\tilde{\varepsilon}\right]$. Consequently, if it can be shown that $\mathrm{Cov}\left[U'\left(H_2(\tau)\right),\tilde{\varepsilon}\right]<0$, then $\partial V(\boldsymbol{\beta})/\partial v>0$ follows. If it were the case that the realizations of $\tilde{\varepsilon}$ increased, then so too would the realizations of $H_2(\tau)=\mu(\tau;m)+\sigma(\tau;v)\tilde{\varepsilon}$. But because of diminishing marginal utility, the realizations of $U'\left(H_2(\tau)\right)$ would decrease, thereby establishing that $\mathrm{Cov}\left[U'\left(H_2(\tau)\right),\tilde{\varepsilon}\right]<0$, and therefore that $\partial V(\boldsymbol{\beta})/\partial v>0$, as claimed.

In contrast, the sign of $\partial\tau^*(\boldsymbol{\beta})/\partial v$ cannot be determined in general because the sign of the third term on the right-hand side of Eq. (11) is ambiguous under the given assumptions. The first term, however, is negative seeing as $\mathrm{Cov}\left[U'\left(H_2(\tau)\right),\tilde{\varepsilon}\right]<0$, as demonstrated above. It reflects the direct reduction in the risk associated with the medical procedure by a risk averse patient. The second term is negative as well, in view of the fact that $\mathrm{E}\left[U''\left(H_2(\tau);a\right)\tilde{\varepsilon}^2\right]<0$, and captures the increased benefit from the reduction in risk over the post-procedure period. Under the additional stipulation that the patient is prudent, $\mathrm{E}\left[U''\left(H_2(\tau)\right)\tilde{\varepsilon}\right]=\mathrm{Cov}\left[U''\left(H_2(\tau)\right),\tilde{\varepsilon}\right]>0$, as previously demonstrated. As a result, a risk-averse and prudent patient will elect to undergo the medical procedure sooner if the risk associated with the medical procedure decreases. What is more, the same is true if a patient's preferences (i) were risk neutral, (ii) could be represented by a quadratic utility function, or (iii) exhibited decreasing absolute risk aversion.

*Time Preferences*

Now consider an increase in the intertemporal rate of discount $\rho$, that is,

$$\frac{\partial\tau^*(\boldsymbol{\beta})}{\partial\rho}\equiv\frac{\mu_\tau(\tau;m)\mathrm{E}\left[U'\left(H_2(\tau)\right)\right]\left[\int_{\tau}^{T}[t-\tau]e^{-\rho t}\,dt\right]}{D}+\frac{\sigma_\tau(\tau;v)\mathrm{E}\left[U'\left(H_2(\tau)\right)\tilde{\varepsilon}\right]\left[\int_{\tau}^{T}[t-\tau]e^{-\rho t}\,dt\right]}{D}<0\,, \tag{13}$$

$$\frac{\partial V(\boldsymbol{\beta})}{\partial\rho}=-\int_0^{\tau^*(\boldsymbol{\beta})}tU\left(H_1(t)\right)e^{-\rho t}\,dt-\mathrm{E}\left[\int_{\tau^*(\boldsymbol{\beta})}^{T}tU\left(H_2\left(\tau^*(\boldsymbol{\beta})\right)\right)e^{-\rho t}\,dt\right]<0\,, \tag{14}$$

where it should be noted that Eq. (4) was used to rewrite the numerator of Eq. (13). As demonstrated above, $\mathrm{E}\left[U'\left(H_2(\tau)\right)\tilde{\varepsilon}\right] = \mathrm{Cov}\left[U'\left(H_2(\tau)\right),\tilde{\varepsilon}\right] < 0$, hence a increase in the intertemporal rate of time preference makes a patient worse off and causes the medical procedure to be elected sooner.

*Length of Life*

Finally, consider an increase in the lifetime of a patient $T$, that is,

$$\frac{\partial \tau^*(\boldsymbol{\beta})}{\partial T} \equiv \frac{-e^{-\rho T}\mu_\tau(\tau;m)E\left[U'\left(H_2(\tau)\right)\right]}{D} + \frac{-e^{-\rho T}\sigma_\tau(\tau;v)E\left[U'\left(H_2(\tau)\right)\tilde{\varepsilon}\right]}{D} > 0\,, \tag{15}$$

$$\frac{\partial V(\boldsymbol{\beta})}{\partial T} = \mathrm{E}\left[U\left(H_2\left(\tau^*(\boldsymbol{\beta})\right)\right)e^{-\rho T}\right] > 0\,. \tag{16}$$

Again recalling that $\mathrm{E}\left[U'\left(H_2(\tau)\right)\tilde{\varepsilon}\right] = \mathrm{Cov}\left[U'\left(H_2(\tau)\right),\tilde{\varepsilon}\right] < 0$, an increase in the lifetime of a patient makes the patient better off and results in a delay of the medical procedure. The latter reflects the fact that by delaying the procedure, its expected outcome is improved and its riskiness is reduced, both of which are now captured over a longer period of time.

## 4. Summary and Conclusion

A model describing the time at which a patient will choose to undergo an elective medical procedure in order to address a worsening but non-life-threatening ailment was developed and analyzed for its refutable qualitative properties. It was purposely simple yet employed general functions, thereby making the refutable implications drawn from it robust, and importantly, not conditioned on ad hoc functional form assumptions. For a risk averse and prudent patient, the time at which an elective medical procedure is chosen, and the welfare of said patient, respond to changes in the exogenous variables in a wholly predictable fashion. Accordingly, the refutable implications form the basis for a statistical test of the rationality of patients faced with decisions of this ilk.

## 5. Appendix

In addition to assumptions (A1)–(A5), the following two suppositions are made:

(A6) There exists an interior solution to the first-order necessary condition of problem (1) when $\boldsymbol{\beta} = \boldsymbol{\beta}^0 \in \mathbb{R}^7_{++}$, denoted by $\tau^0 \in (0,T)$.

(A7) The second-order sufficient condition for problem (1) holds at $(\tau^0, \boldsymbol{\beta}^0) \in (0,T) \times \mathbb{R}^7_{++}$.

Assumption (A6) is employed because so little structure is placed on the four primitive functions $\left(U(\cdot), \delta(\cdot), \mu(\cdot), \sigma(\cdot)\right)$. In conjunction with supposition (A7), a well-known local sufficiency theorem is satisfied and thus may be invoked to conclude that the value $\tau^0 \in (0,T)$ is the unique local solution to problem (1). What is more, the conditions of the implicit function theorem are met under assumptions (A6) and (A7). Consequently, it follows from the implicit function theorem that the first-order necessary condition given in Eq. (3) may, in principle, be solved for $\tau$ as a locally $C^{(1)}$ function of $\boldsymbol{\beta}$ in an open neighborhood of the point $(\tau^0, \boldsymbol{\beta}^0)$, yielding, say, $\tau = \tau^*(\boldsymbol{\beta})$. Thus a differential comparative statics analysis of problem (1) may be carried out.

The second-order sufficient condition is given by

$$\begin{aligned}
D \stackrel{\text{def}}{=} & -\rho e^{-\rho\tau} U\left(H_1(\tau)\right) - \delta(\tau;q) e^{-\rho\tau} U'\left(H_1(\tau)\right) \\
& + \rho e^{-\rho\tau} \mathrm{E}\left[U\left(H_2(\tau)\right)\right] - 2e^{-\rho\tau} \mathrm{E}\left[U'\left(H_2(\tau)\right)\left[\mu_\tau(\tau;m) + \sigma_\tau(\tau;v)\tilde{\varepsilon}\right]\right] \\
& + \mathrm{E}\left[\int_\tau^T U''\left(H_2(\tau)\right)\left[\mu_\tau(\tau;m) + \sigma_\tau(\tau;v)\tilde{\varepsilon}\right]^2 e^{-\rho t}\, dt\right] \\
& + \mathrm{E}\left[\int_\tau^T U'\left(H_2(\tau)\right)\left[\mu_{\tau\tau}(\tau;m) + \sigma_{\tau\tau}(\tau;v)\tilde{\varepsilon}\right] e^{-\rho t}\, dt\right] < 0,
\end{aligned} \tag{17}$$

and is satisfied at $\tau = \tau^*(\boldsymbol{\beta})$. Observe that the second-order sufficient condition neither implies, nor is implied by, diminishing marginal utility of health. This is unusual, as the assumption of diminishing marginal utility typically implies that the second-order sufficient condition holds in an expected utility maximization problem. Even so, the added stipulation of diminishing marginal utility has value in deriving refutable comparative statics, as seen in §3.

## 6. References

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