Estimating Marketability Discounts in Sale Restricted Options Using Compound Option Pricing Theory

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Abstract

This thesis presents a method for estimating the discount for lack of marketability (DLOM) in call options which are restricted for sale. The DLOM is modeled as a put option on the restricted call option, known as a compound option, with two different approaches towards setting the strike price of the compound option.

The *Finnerty Approach* sets an average-strike price in order to reflect the lack of any special market timing by the holder of the restricted call option. The *Chaffe Approach* makes no assumption on the market timing of the holder and sets the strike price equal to the market value of the underlying call option.

The results show that the DLOM for a sample of four firms listed in the Swedish OMX30 index ranges from 53% to 82% with the *Chaffe Approach*. The *Finnerty Approach* predicts DLOMs from 72% to 136%. This implies the *Chaffe Approach* is the better method. The results supports the firm's choice of setting an implicit discount in the options they issue by valuing them below the market price. This is because the market price should be adjusted downwards to correctly price the risk the sale restrictions entails.

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1. Introduction

This thesis presents a method for estimating the discount for lack of marketability (DLOM) in call options which are restricted for sale using compound option pricing theory. This is an important contribution since call options are a common form of compensation for companies to give their employees. These options typically come with a restriction for sale, which makes it necessary to apply a DLOM in order to estimate their true value.

The reason for applying a DLOM for assets that are restricted for sale is because the liquidity of an asset affects the price. An illiquid asset is riskier than an equivalent liquid one since the holder cannot sell it if the market falls. Therefore the illiquid asset should be sold at a discount. The research for estimating DLOMs have been focused on sale restricted stocks in the U.S. market and there exists different approaches.

Hertzel and Smith (1993) looked at private equity placements in public firms. The private stock which cannot be traded publicly were typically sold at lower price, compared to the public stock. The price difference between the two is interpreted as the DLOM.

Emory et al. (2002) used IPO transactions to estimate the DLOM. They compared the price at which stock transactions where taking place before a company went public and compared it to the price the stock were trading at after they went public. If the price rose after the company went public it implies the stock was valued with a discount before the IPO.

Another approach is to model the DLOM as the value of a put option on the restricted asset. This is the method used in this thesis because it helps avoid some of the problems with the previously described methods. It can for example be difficult to determine the reason the stock price rises after an IPO. Which makes it hard to determine if there was a discount present before.

One of the fist papers that applied put option pricing techniques to estimate the DLOM in stocks was Chaffe (1993). He valued the DLOM as the price of a European[1] put option with a strike price equal to the underlying stock price at the point of issuance, using the Black-Scholes model (Black and Scholes, 1973). The DLOM is expressed as the value of the put option divided by stock price.
He based his study on avoiding the losses the sale restriction brings. He viewed the DLOM as the price of an insurance contract that would protect the holder of the restricted share from a decline in the underlying stock price. Since the strike price is equal to underlying stock price at the beginning of the restriction period, the put option protects from downside risk. However, the investor is prevented from realizing any gains if the underlying stock price appreciates. Therefore Chaffe concludes the DLOM estimated from this model should seen as a minimum bound.

Whereas Chaffe (1993) derived a minimum bound for the DLOM and focused on avoiding losses, Longstaff (1995) was interested in estimating the possible gains that could be lost due to the selling restriction, and deriving an upper limit for the DLOM. He chose a lookback put option which provides the holder with the right to choose at which price to exercise the option, in order to maximize the payoff. In the case of a put this means exercising when underlying stock price is at it's lowest.

The choice of a lookback put means the investor is assumed to have perfect market timing. In other words, the investor who receives the restricted share would know at which point in time during the restriction period it would have been optimal to sell the share. Therefore, estimating the DLOM as the value of lookback put is the same as estimating the maximum DLOM. With the lookback put option as the foundation, Longstaff (1995) derived a formula for the DLOM. This is in contrast to Chaffe (1993) who simply valued the put option.

Assuming investors have perfect market timing is an unrealistic assumption if one seeks to estimate the marketability discount in the real world. Finnerty (2012)

[1]European style options can only be exercised at the maturity of the option.

sought to provide a means to do this by modeling the DLOM as an average-strike put option[2]. He argues, under risk-neutrality, that an investor should be indifferent between having an unrestricted share, or a restricted share plus a short position in a forward contract. The forward contract is there as insurance to guarantee the investor can sell the restricted share for a certain price at the end of the restriction period.

If the investor cannot be assumed to possess any special market timing, she is assumed to be equally likely to sell the restricted share at any of the N discrete points in time. Since the investor is equally likely to sell at any point, there are N possible forward prices to choose as the delivery price. The rational thing to do is to choose an average, K, of these forward prices as the delivery price. This is because the investor cannot know which of these N forward prices is the optimal delivery price.

At maturity, the forward contract will have a value equal to $K - V(T)$, where the latter is the stock price at maturity. An investor holding this forward contract suffers an opportunity cost if K exceeds the stock price price at time T. In other words, the investor would have liked to sell the restricted share for the higher price K if she actually held the forward contract. This opportunity cost represents the value that's potentially lost due to the sale restriction. Therefore, the value of the opportunity cost represents the DLOM. The opportunity cost has the same payoff profile as an average-strike put, allowing the DLOM to modeled as such.

The average-strike put can be viewed as an agreement to exchange a forward contract on the underlying share for an unrestricted share itself at time T. This is fundamentally an agreement to exchange one asset, the forward contract for another asset, the unrestricted share. Finnerty then proceeds to combine his framework with the work of Margrabe (1978), who derived formulas for valuing options in which the parties have agreed to exchange one asset for another in order to derive his formula for estimating the DLOM[3]. He found the discounts predicted by his model were close

[2]See appendix A for a more comprehensive description of average-strike options, also known as Asian options.

[3]See appendix B for a more technical description of Finnerty's derivation and formula.

to the empirically observed discounts in private stock placements in 208 U.S. firms. This thesis presents a model called the *Compound Option DLOM Model* for estimating the DLOM in call options restricted for sale. This is done by modeling the DLOM as a compound put option[4]. That is, a put option on the sale restricted call option. I compare two different approaches towards setting the strike price of the compound option. In one case the strike price is set as the average market price of the underlying call option, following the methodology of Finnerty (2012). In the other case the strike price is set to the market value of the underlying call option, as suggested by Chaffe (1993). I will denote the two different approaches as the *Chaffe Approach* and the *Finnerty Approach*. The compound put option is valued using techniques developed by Geske (1979), who derived formulas for valuing compound options.

The option DLOM is calculated for a sample of four firms from the Swedish OMX30 index, to provide a real life example. The results show that the DLOM for the restricted call options ranges from 53% to 82% with the *Chaffe Approach* and between 72% to 136% with the *Finnerty Aapproach*. The results suggests the *Chaffe Approach* towards setting the strike price of the compound put option is the better method, since the discounts do not exceed 100%. The option DLOM is sensitive to the underlying call option being in-the-money or out-of-the-money, which is something the issuing firms can control, since they set the strike price.

The firms typically set a price for their call options which is below the market value, which I denote as the *implicit firm discount*. This thesis supports this practice. The market value of the call options should be adjusted downward in order to take the DLOM into account. The implicit discount the firms set is lower than the option DLOM predicted by the compound option DLOM model in all cases. This indicates the firms are still overvaluing the options they issue.

[4]See Appendix C for a description of compound options.

The rest of the thesis follows the following structure. Section 2 presents DLOM models for stocks and the compound option DLOM model used for call options. Section 3 presents the data used and how key inputs which are required in the valuation of the underlying call options and the compound option are calculated. Section 4 and 5 presents the results and discussion respectively. Section 6 summarizes the conclusions.

2. DLOM Models

The aim of a DLOM model is to estimate the size of the discount that should be applied to an asset restricted for sale. In other words an asset which lacks marketability. There exists different approaches to estimating the DLOM, this thesis focuses on put option based models. The common factor between these models is imagining the holder of the restricted asset purchases a put option on the restricted asset as a way of purchasing marketability.

The purpose of the put option is to protect the holder from the risk of not being able to sell the asset by guaranteeing it can be sold for a certain price at maturity. The value of the put option represents the cost of insuring the holder against the risk of not being able to sell the asset. The DLOM is presented as a percentage of the underlying asset value by dividing the price of the put option with the price of the underlying asset. This Section explains how to calculate the DLOM for stocks and presents a method to estimate the DLOM in call options.

2.1 DLOM Models for Stocks

Previous research on the area of estimating DLOMs for sale restricted assets have been focused on stocks. Put option models have been proposed by Chaffe (1993), Longstaff (1995) and Finnerty (2012), among others. The main difference between these models is the different approaches to setting the strike price of the put option.

Chaffe (1993) sets the strike price equal to the initial stock price, this approach protects against the downside risk of not being able to sell the asset. He values the put option using the Black-Scholes formula and then divides the option value with the stock price to calculate the DLOM. Longstaff (1995) assumes the holder has perfect market timing by modeling the DLOM as a lookback put and uses this foundation to derive a formula for the DLOM.

Longstaff (1995) derived the following formula for calculating the DLOM

$$F(S,T) = S\Big(2 + \frac{\sqrt{\sigma^2 T}}{2}\Big)N\Big(\frac{\sqrt{\sigma^2 T}}{2}\Big) + S\sqrt{\frac{\sigma^2 T}{2\pi}}e^{-\frac{\sigma^2 T}{8}} - S \tag{2.1}$$

$F(S,T)$ is the value of that should be subtracted from the price of the stock that is restricted for sale. When this value is divided by the stock price we get the DLOM as a percentage of the stock price. S is the stock price, T is the restriction period in years. σ is the volatility of the stock. $N(\cdot)$ is the cumulative normal distribution function.

Finnerty (2012) assumes the holder has no special market timing and sets the strike price equal to the average price of the underlying stock, thereby modeling the DLOM as a average-strike put option. He then proceeds to derive his formula for the DLOM.[1].

I will now present an example on how to calculate the DLOM for a fictive stock using the three put option models mentioned above.

Assume the stock of ABC has a sale restriction period of 2 years, an initial price of 100 SEK and a volatility equal to 30%. The risk-free rate is equal to 1% and the dividend yield is equal to zero.

The models suggests the DLOM for the stock of ABC lies between 9.6% and 38.6%. It is interesting to note that Finnerty's discount lies below the one predicted by Chaffe, even though the latter is seen as a minimum bound.

Table 1 below summarizes the information and presents the estimated DLOM for each respective model. Note that Chaffe (1993) use all the inputs. The formulas of Longstaff (1995) and Finnerty (2012) use only the stock price, restriction period and stock volatility.

[1]See appendix B for a more technical description of Finnerty's derivation and formula.

Table 1 - Example Illustrating the DLOM for a Fictive Stock

Inputs	ABC
Restriction Period	2
Stock Price	100
Strike Price	100
Stock Volatility	30%
Risk-Free Rate	1%
Dividend Yield	0%
DLOM	
Chaffe Model	17.6%
Longstaff Model	38.6%
Finnerty Model	9.6%

The stock and strike price are in SEK. The restriction period is in years.

2.2 Compound Option DLOM Model

This Subsection presents how a compound put option works and how it is valued. I then explain how the option DLOM is calculated using the Compound Option DLOM model. I also show a fictive example of a option DLOM. Finally, I discuss the limitations of the model.

2.2.1 Compound Put Option Valuation

If the asset restricted for sale is a call option we need to use compound option pricing theory in order to model the DLOM as a put option. This is because we are dealing with an option on an option. I model the DLOM for the sale restricted call options as a compound put option, that is a put option on a call option. I compare two different approaches towards setting the strike price of the compound option. In the first case I set the strike price equal to the average market value of the underlying call option, following the methodology of Finnerty (2012). In the other case I set the strike price equal to the market value of the underlying call option, as done by Chaffe (1993). This will give me two different values of the compound put option and therefore two different option DLOMs. I will denote the two appoaches as the *Finnerty Approach* and the *Chaffe Approach*. The purpose of this methodology is to compare which of the two approaches to the strike price produces the more realistic option DLOMs.

The value of the compound put option P_c, is calculated with the following formula[2] by Hull (2015)

$$\begin{aligned} P_c = K_2 e^{-rT_2} M(-a_2, b_2; -\sqrt{T_1/T_2}) - SM(-a_1, b_1; -\sqrt{T_1/T_2}) \\ + e^{-rT_1} N(-a_2) \end{aligned} \tag{2.2}$$

where

$$a_1 = \frac{\log(S/S^*) + (r + 0.5\sigma^2)T_1}{\sigma\sqrt{T_1}} \tag{2.3}$$

$$b_1 = \frac{\log(S/K_2) + (r + 0.5\sigma^2)T_2}{\sigma\sqrt{T_2}} \tag{2.4}$$

$$a_2 = a_1 - \sigma\sqrt{T_1} \tag{2.5}$$

$$b_2 = b_1 - \sigma\sqrt{T_2} \tag{2.6}$$

S is the initial underlying stock price, r is the risk-free rate. T_1 is the time to maturity for the compound put option and T_2 is the time to maturity for the underlying call option, where $T_1 < T_2$. The maturity of the compound put option, T_1, is $T_1 = T_2 - \frac{1}{252}$. That is, the life of the underlying call option minus one trading day. This is to reflect the compound put option's role as a marketability insurance, ensuring the underlying

[2]The dividends are not taken into account.

call option can be sold for a certain price at maturity, for as much of the of the life of the underlying call option as possible[3].

K_1 is the strike price of the compound put option for which the holder can sell the underlying call option at the maturity of the compound option. K_2 is the strike price for the underlying call option for which the holder can buy the underlying stock at the maturity of the call option. σ is the volatility of the underlying stock.

S^* is the underlying stock price that makes the underlying call option price equal to the strike price of the compound option, at the maturity of the compound put option. If the underlying stock price is *higher* than S^* the compound put option will not be exercised since the underlying call option's value will be higher than K_1. The compound put option will be out-of-the-money in this case. The compound put option will be exercised if the underlying stock price is *lower* than S^*.

$M(a, b; \rho)$ is the cumulative bivariate normal distribution function that the first variable will be less than a and that the second will be less than b, the correlation coefficient between them is ρ.

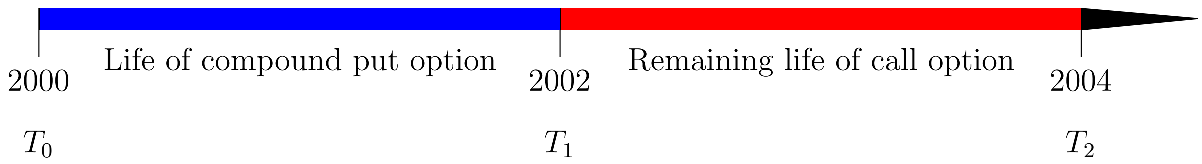
I will now present a simple timeline over how a compound put option works. Imagine a call option is issued 2000-01-01 with a maturity of four years.

On 2004-01-01 the holder has the right to purchase a stock at the agreed strike price, K_2. The call option therefore has a life that ranges between T_0 and T_2.

Now imagine there exists a put option on the call option, in other words a compound put option. This compound put is also issued 2000-01-01 and has a maturity of two years. On 2002-01-01 the holder of the compound put has the option of selling the underlying call option for the agreed strike price, K_1. The compound put option therefore has a life between T_0 and T_1.

[3]The valuation formulas for the compound option does not allow for $T_1 = T_2$.

Figure 1: Timeline Over a Compound Put Option



2.2.2 Option DLOM Calculation

The option DLOM represents the discount that should be applied to the *market value* of the underlying call option in order to calculate the value where the increased risk due to lack of marketability is accounted for. The option DLOM, presented as a percentage of the market value of the underlying call option is calculated as $\frac{P_c}{C_m}$. Where P_c is the value the compound put option and C_m is the market value of the underlying call option. The value of the compound put option represents the cost of insuring the underlying call option against the risk that comes with the sale restriction. Therefore, the DLOM is expressed as the ratio between the value of the compound put and the underlying call option.

For underlying call options that are in-the-money the call option will have a high value and the compound put will have a low value, this will drive the option DLOM downwards. This is because relative changes in the underlying call's value, due to changes in the underlying stock price will be fairly low. The value of the underlying call will therefore be less volatile. This reduces the value of the insurance for not being able to sell the underlying call option, in other words it reduces the value of the compound put.

When the underlying call option is out-of-the-money the situation is the opposite. The underlying call will have a low value and the compound put will have a high value. This will cause the option DLOM to increase. Since the underlying call option will have a low value, even small changes in the value will be a large relative change.

These relative changes increases the volatility for the underlying call option which increases the value of the compound put option.

I will visualize the process of calculating a option DLOM using the *Chaffe Approach* and the *Finnerty Approach* below.

- **Step 1: Calculate the market value of the underlying call option**

- **Step 2: Chaffe Approach**

  1. Set the strike price of the compound put option equal to the market value of the underlying call option.
  2. Calculate the value of compound put option.
  3. Divide the value of the compound put option with the market value of the underlying call to get the Chaffe option DLOM.

- **Step 3: Finnerty Approach**

  1. Estimate the average market value of the underlying call option.
  2. Set the strike price of the compound put option equal to the average market value of the underlying call option.
  3. Calculate a new value of compound put option.
  4. Divide the value of the compound put option with the market value of the underlying call to get the Finnerty option DLOM.

I will now present a simple example of how to calculate the DLOM for a fictive call option restricted for sale.

Imagine the *ABC* company issues a European call option on their own stock. The current stock price is 100 SEK, the volatility is 40%, the risk-free rate is 1% and the dividend yield is 0%. The call option has a time to maturity of 3 years. The call option cannot be sold during its life, meaning it has a sale restriction period of 3 years.

Table 2 on the next page shows the inputs for valuing the underlying call option.

Table 2 - Valuation of the Underlying Call Issued by ABC

ABC	
Stock Price	100
Strike Price	100
Stock Volatility	40%
Risk-Free Rate	1%
Dividend Yield	0%
Maturity of Underlying Call	3
Underlying Call Value	28

All prices and values are in SEK. The time to maturity is in years. I value the underlying call option using the Black-Scholes (1973) formula. I have rounded the prices to a whole number.

Since I compare the Chaffe approach and the Finnerty approach towards setting the strike price of the compound put option I will get two different values of the compound put option, and therefore two different option DLOMs. Table 3 shows the option DLOM estimated using the Chaffe approach and the Finnerty approach.

Table 3 - Calculation of DLOM for the Call Option Issued by ABC

ABC			
Chaffe Approach		**Finnerty Approach**	
Maturity of Compound Option	2.996	Maturity of Compound Option	2.996
Strike of Compound Option	28	Average-Strike of Compound Option	32
Chaffe Compound Value	19	Finnerty Compound Value	22
Chaffe Option DLOM	68%	Finnerty Option DLOM	79%

The strikes and values are in SEK. The time to maturity is in years. The average-strike price is not based on any calculation, it is my assumption for simplicity. I have rounded the prices to a whole number.

In this example the discount for the sale restricted call option issued by ABC would be 68% of the value of the underlying call option according the Chaffe approach. The Finnerty approach predicts a DLOM of 79%.

2.2.3 Model Limitations

A limitation of the Compound Option DLOM Model is that the DLOM can exceed 100% for underlying call options that are deep out-of-the-money when an average-strike price is used for the compound put option. This is not a realistic result since discounts cannot exceed 100%. This is due to the average-strike price of the compound put option being slightly higher than the market value of the underlying call option, at the time of issuance. For deep out-of-the-money calls which have a low value and are volatile this causes the value of the compound put option to exceed the value of the underlying call option. The model also does not include dividends[4].

[4]The dividend yield is included in the valuation of the underlying call option, using the Black-Scholes formulas. The dividend yield is *not* included when estimating the option DLOM, using the Compound Option DLOM Model.

3. Data and Methodology

This thesis aims to estimate the DLOM in call options restricted for sale in the Swedish market. 20 of 30 firms listed on the OMX30 index in Sweden offer some kind of option based remuneration to their employees and top management[1], at the time period I looked at, which was 2017-2019. Out of these 20, I selected a sample of four firms for more detailed analysis because they provide sufficient information on how they value their options. The four firms are ABB, Atlas Copco, Hexagon and Investor.

There are four values that needs to be calculated, the price at which the firms value their options, C_f, the market value of these call options, C_m, and the two values of the compound put option. One with with the average-strike price of the compound option, denoted the *Finnerty Compound Option Value*. The other value is denoted the *Chaffe Compound Option Value* where the strike price is set to the market value of the underlying call option. This is to be able to compare the implicit discount the firms set with the two discounts predicted by the *Chaffe Approach* and the *Finnerty Approach*.

3.1 Firm Pricing of the Underlying Call Option

I use the Black and Scholes (1973) formulas to value the underlying call options. This implies I assume the underlying call options are European. Even though it is more common to issue American options[2]. The firm value of the underlying call option is calculated using inputs the sample firms state in their financial statements, see ABB (2017), AtlasCopco (2019), Hexagon (2018) and Investor (2018).

[1]The other 10 offer some form of cash based compensation.

[2]American style options can be exercised at any time.

Table 4 below shows the inputs used the calculate the firm value of the underlying call option, C_f.

Table 4 - Inputs for Estimating the Firm Value of the Underlying Call Options

	ABB	**Atlas Copco**	**Hexagon**	**Investor**
Issuance day	2017-01-02	2018-01-01	2015-09-01	2018-01-01
Stock Price	192.90	275.60	269	380.51
Strike Price	201	264	347.80	456.60
Time to Maturity	6	4.4	4.4	5
Stock Volatility	19%	30%	24.3%	21%
Risk-Free Rate	-0.1%	1%	0.09%[3]	-0.09%
Dividend Yield	4.7%	6%	1.24%[4]	3.15%[5]
Firm Value of the Underlying Call	11.10	39.01	24.96	24.10

The prices and values are denoted in SEK. The time to maturity is in years. All info were taken from the financial statements of the firms unless otherwise stated. The date of issuance were not stated exactly by the firms and comes from my assumption based on the information given. The stock price reflects the price on the date of issuance, except for Atlas Copco, they used their own estimate of the stock price.

[3] The 5 year Swedish Government bond yield on 2015-09-02.

[4] The dividend of 0.35 EUR for 2014 converted with EUR/SEK rate of 9.523 divided by the stock price on the issuance day.

[5] The dividend of 12 SEK for 2017 by the stock price on the issuance day.

3.2 Market Pricing of the Underlying Call Option

In order to calculate the market value of the options new estimates are needed for the volatility and the risk-free rate. The market based stock volatility is estimated using historical implied volatility. I took the volatilities that were implied by the price of call options written on the firms that were trading in the market, at the time the firms issued their sale restricted options. They all had around a two year maturity and I matched their moneyness[6] to that of the sale restricted options the firms issue themselves.

Table 5 on the next page shows the inputs used to calculate the market value of the underlying call option, C_m.

[6] Moneyness refers to how much in-the-money or out-of-the-money an option is. For example, if a call option has a strike of 110 and the underlying stock price is at 100, the moneyness of the call would be $\frac{110}{100} = 110\%$.

Table 5 - Inputs for Estimating the Market Value of the Underlying Call

	ABB	**Atlas Copco**	**Hexagon**	**Investor**
Issuance day	2017-01-02	2018-01-02	2015-09-01	2018-01-01
Stock Price	192.90	313.50	269	373.80
Strike	201	264	347.80	456.60
Time to Maturity	6	4.4	4.4	5
Implied Volatility	19.7%	25.4%	28.5%	17.3%
Market Risk-Free Rate	0.15%	0.14%	-0.53%	-0.41%
Dividend Yield	4.7%	6%	1.24%	3.15%
Market Value of the Underlying Call	12.54	42.71	31.63	12.83

The prices and values are denoted in SEK. The time to maturity is in years. As a proxy for the risk-free rate I use the yield from the Swedish government bonds at the issuance date. For ABB and Atlas Copco it's the five-year yield and for Hexagon and Investor it's the two-year yield. The yields were taken from https://www.avanza.se and the implied volatilities from the Bloomberg Terminal.

3.3 Valuation of the Compound Put Option

This section describes how the two values of the compound put option is calculated. One value where the strike price of the compound option is set to the average market value of the underlying call option, using the *Finnerty Approach*.

The other when the strike price is set to the market value of the underlying call option, using the *Chaffe Approach*.

Table 6 below presents the common inputs that are used in calculating both the *Chaffe Compound Option Value* and the *Finnerty Compound Option Value*.

Table 6 - Inputs for Calculating the Compound Put Option Values

	ABB	**Atlas Copco**	**Hexagon**	**Investor**
Issuance day	2017-01-02	2018-01-02	2015-09-01	2018-01-01
Stock Price	192.90	313.50	269	373.80
Strike of the Underlying Call	201	264	347.80	456.60
Maturity of the Underlying Call	6	4.4	4.4	5
Maturity of the Compound Put	5.996	4.396	4.396	4.996
Implied Stock Volatility	19.7%	25.4%	28.5%	17.3%
Market Risk-Free Rate	0.15%	0.14%	-0.53%	-0.41%

The stock price and the strike price are denoted in SEK. The time to maturity is in years.

3.3.1 The Finnerty Approach

The strike price used in calculating the *Finnerty Compound Option Value* is an average strike in order to incorporate the lack of any special market timing, as suggested by Finnerty (2012). The average-strike price is equal to the average market value of the underlying call option during its life. I chose to take an average of the daily market price of the underlying call option. To do this I simulate the stock price of each firm at a daily interval using Geometric Brownian Motion[7] with the drift equal to the risk-free rate and volatility equal to the implied volatility. Each day I calculate the market value of the underlying call option using the Black-Scholes formula, in essence I simulate the life of the underlying call option.

I will get a market value for the underlying call option for each day during the life of the option. I then take the arithmetic average of the underlying call's market price over each day[8]. This is the average market value for one simulation. I do these simulations $1,000$ times. This gives me $1,000$ different values for the average market price for the underlying call. I then take the mean of these $1,000$ averages. This value is what I use as the average-strike price of the compound option, K_1. One disadvantage with simulating this way is that the risk-free rate is held constant over the life of the option, which is not a realistic assumption for longer time periods.

In Table 7 below I present the average-strike price of the compound option and the *Finnerty Compound Option Value* calculated with the inputs in Table 6 above.

Table 7 - Finnerty Approach

	ABB	**Atlas Copco**	**Hexagon**	**Investor**
Average-Strike of the Compound Option	18.71	56.39	43.18	21.55
Finnerty Compound Option Value	12.15	30.65	35.63	17.46

The strike and value are denoted in SEK.

[7] Geometric Brownian Motion is a stochastic process used for modeling stock prices as described by Hull (2015).

[8] If the underlying call has a life of 6 years, this means taking the average market price over $6 \cdot 252 = 1512$ days.

3.3.2 The Chaffe Approach

I calculate the *Chaffe Compound Option Value* with the strike price equal to the market value of the underlying call option, C_m. This is the approach suggested by Chaffe (1993) and makes no assumption of the market timing of the holder.

In Table 8 below I present strike price of the compound option and the *Chaffe Compound Option Value* calculated with the inputs in Table 6 above.

Table 8 - Chaffe Approach

	ABB	**Atlas Copco**	**Hexagon**	**Investor**
Strike of the Compound Option	12.54	42.71	31.63	12.83
Chaffe Compound Option Value	8	22.49	25.87	10.30

The strike and value are denoted in SEK. The compound option strike price is the market price of the underlying call option, as seen in Table 5.

4. Results

There are two values of the underlying call option that are of interest, the price at which the issuing firms value their call options and the market value of these options. Since issuing call options to their employees is a cost for the firms they have an incentive to set a low value. However, since the market value should be adjusted with the option DLOM in order to take the lack of marketability into account the *implicit discount* the firms set is justified as long as it does not exceed the the option DLOM.

The implicit firm discount is calculated as $\frac{C_m - C_f}{C_m}$ where C_m is the market value of the underlying call option and C_f is the firm value of the underlying call option. The implicit firm discount says how much lower the firms value the call options they issue, compared to the market. The implicit discount ranges from -88% to 21% for the four sample firms. The negative discount implies the firms value their options at a higher price compared to the market price.

The results show a DLOM should be applied when valuing call options that are restricted for sale and suggests the Chaffe (1993) approach of setting strike price of the compound option is better.

The Chaffe Option DLOM ranges from 53% to 82% of the market value of the underlying call option. The Finnerty Option DLOM ranges from 72% to 136%.

Table 9 below summarizes the results.

Table 9 - Option DLOMs for the Sample Firms

	ABB	**Atlas Copco**	**Hexagon**	**Investor**
Implicit Firm Discount	11%	9%	21%	-88%
Chaffe Option DLOM	64%	53%	82%	80%
Finnerty Option DLOM	97%	72%	113%	136%

The table shows the implicit firm discount and the Chaffe Option DLOM which is calculated as the Chaffe Compound Option price divided by C_m. *The Finnerty Option DLOM is the Finnerty Compound Option price divided by* C_m.

The option DLOMs the compound put DLOM model predicts are similar to the results of Chaffe (1993). He predicted high DLOMs, in excess of 75%, for stocks with a high volatility and longer terms to maturity. That is, maturities over three years and a volatility over 150%. Since options are derivative instruments they become more volatile than the underlying asset on which they are written. This helps explain the high DLOMs predicted by the Compound Option DLOM model.

The Chaffe Option DLOM are more realistic since they do not exceed 100%, which the Finnerty Option DLOM does for Hexagon and Investor. Both the *Chaffe Approach* and the *Finnerty Approach* predicts the highest DLOMs for Hexagon and Investor, which are the firms that issue their call options furthest out-of-the-money.

5. Discussion

The option DLOM is heavily influenced by the strike price of the underlying call option. This is something the firms that issue the options can control, since they set the strike price. The firms can therefore justify setting a low value for their options, especially when they are deep out-of-the-money. Although this is more interesting for in-the-money calls, since they have a high value and is therefore more costly for the firms to issue. The incentive to apply a discount becomes stronger.

It can also be argued that valuing the compound put option using an average-strike price to reflect the investors lack of market timing is misleading. This is because the holders of the restricted options are employees of the firms. They can therefore be assumed to possess some market timing ability, as suggested by Kahle (2000) and Clarke et al. (2001). In this case the option DLOM should be higher as a better market timing raises the opportunity cost of not being able to sell the restricted call option. However, since the Finnerty option DLOM is already unrealistically high, this might not be the correct approach.

The question of how the holders of the sale restricted options view the the risk that comes with not being able to sell the option is an aspect that this thesis has not explored. The fact that the restricted options are given to the holders, they are not bought, could have an impact on how much the holders care about the risk.

The *house money effect*, as described by Thaler and Johnson (1990), can be applied to this situation. The house money effect states that a person is less risk-averse with money when there has been a prior gain. In this case the restricted call options themselves, since they are given to the holders free of charge. This would have the effect of lowering the DLOM for the sale restricted options because the holders would care less about the risk. However, this would require taking the risk-preferences of the holders into account, which is tricky, since they vary from person to person.

A useful addition to the Compound Option DLOM Model would be a modification that takes the value of the underlying call option into account. This is account for the fact that deep out-of-the-money call options which are restricted for sale can exhibit DLOMs that exceed 100%. One way to do this is to include a coefficient which is tied to the value of the underlying option which suppresses the option DLOM when the value is low. This could be motivated by the fact that investors would not be very concerned with the risk the sale restriction brings for deep out-of-the-money call options, since they have a low value. The house-money effect could be a alternative argument.

A limitation of this thesis is the size of the data sample. This is because it is mostly larger companies that issue options as compensation for their employees. Few of these firms publish the information needed to make a useful analysis of the sale restricted options. It would therefore be interesting to expand this study and apply the compound option DLOM model to a large number of firms in order to gather more data and determine more conclusively what drives the option DLOM, and how it varies from firm to firm.

6. Conclusions

This thesis suggests a method called the Compound Option DLOM Model for estimating the marketability discount in call options that are restricted for sale. The DLOM is modeled as a compound put option with either an average-strike price or a strike price equal to the market value of the underlying call option. The average-strike price is based on the assumption that the investors does not possess any special market timing.

The analysis of sale restricted call options given out as compensation by four firms listed on the Swedish OMX30 index show that the option DLOM range between 53% and 82% for the *Chaffe Approach* and between 72% and 136% for the *Finnerty Approach*. The option DLOM differs from the implicit discount the firms apply to the options they issue, by setting a price lower than the market value. The results show the firms are justified to set a lower price for the call options they issue. A discount should be applied when valuing sale restricted call options in order to accurately price the increased risk the sale restriction brings.

The compound option DLOM model is less realistic in the sense that it produces option DLOMs that exceed 100% for underlying call options that are deep out-of-the-money when an average-strike price of the compound put option is used. This suggest using the approach of Chaffe (1993), which means setting the strike price equal to the market value of the underlying call option is the better method.

Suggestions for future research is to question the assumption that the holder of the restricted options possess no special market timing. Since at least some of the investors who receive option remuneration can be assumed to have above average market timing due to their position as insiders in the firms.

Appendix A - Asian Option Valuation

Asian options is collective name for two types of options, average-*strike* or average-*price* options[1]. Average-strike options use an average of the underlying asset as the strike and a average-strike put has a payoff of $max(0, S_{Ave} - S_T)$, where S_{Ave} is the average-strike price and S_T is the stock price at the maturity of the option. An average price option has a fixed strike K which is compared to an average of the underlying asset price. The payoff of a average price put is $max(0, K - S_{Ave})$.

Asian options are less volatile than other options since large fluctuations in the underlying asset price are averaged out which make them useful for markets where the liquidity is lower and that suffer from larger price jumps. Typical markets where averaged values are used include oil and currency. Asian options are often cheaper than regular options since the risk is lower, the potential up-and downsides are mitigated due the averaging. The probability of making a large profit or loss are reduced.

The main issue with valuing Asian options, as mentioned by Levy (1992), lie in the distribution of the averaged price. For geometric means the lognormality assumption is fulfilled and analytical solutions can be derived. For arithmetic means it gets more complicated, since they cannot be assumed to be lognormal.

Different routes have been taking to deal with the issue of arithmetic means. Kemna and Vorst (1990) value the options numerically with Monte Carlo and use the value of a geometric mean option as a control variate, to better estimate the value of the arithmetic average options. Vorst (1992) modified the analytical solution for geometric mean options to provide an estimate of the arithmetic mean option. Levy (1992) and Turnbull and Wakeman (1991) instead attempt to derive analytical formulas for arithmetic mean options by approximating a lognormal distribution to the arithmetic mean.

[1]The averaging can be both arithmetic and geometric.

Appendix B - Finnerty (2012) Model

John D. Finnerty presented the first iteration of his average-strike put option model in a 2002 paper. Finnerty (2002) called his model a "Transferability Discount Model" and chose an Asian type option in order to estimate the DLOM in the restricted stock. This is to reflect that an investor does not have a perfect market timing, in contrast to the assumption by Longstaff (1995). Finnerty presented a more complete version of his average-strike put model in 2012. Finnerty (2012) sought to estimate the marketability discounts in private stock placement in the United States that fell under the Rule 144 restriction, which limits the resale of private stock placements.

Finnerty (2012) starts his model description by stating a set of assumptions:

I The shares with trading restrictions are equal to the unrestricted shares, and trade continuously in a frictionless market.

II The selling restrictions prevent the investor from selling the shares for a period of T.

III The underlying share price $V(t)$ follows Geometric Brownian Motion.

$$dV = \mu V dt + \sigma V dZ \tag{6.1}$$

Where μ and is a constant and Z is a standard Wiener process.

IV The risk-less rate is denoted r and is constant and identical for all maturities between time $[0, T]$.

V The investor has no special market timing and would be equally likely to sell the restricted shares at any point during the restriction period.

He then argues that if an investor purchases a share in a risk-neutral world. Assuming the investor can sell the share at any time t, where $0 < t < T$, and invest the cash in an asset earning the risk-free rate r. The investor should be indifferent between selling the share immediately for a price of $V(t)$ and selling it forward with a price of $V(t)e^{r(T-t)}$ with delivery at time T.

This indifference holds in a risk-neutral world where all assets have an expected return equal to the risk-free rate and an investor should therefore be indifferent between holding the restricted share and investing the proceeds at the risk-free rate. The investor should be indifferent between having an unrestricted share, and a restricted share plus a short position in a forward contract expiring at time T, which guarantees the restricted share can be sold for a certain price.

Since the investor does not have perfect market timing, Finnerty assumes she is equally likely to sell the share at any point in time during the restriction period. She is equally likely to sell at time $t = 0, \frac{T}{N}, \frac{2T}{N} ... \frac{TN}{N} = T$. Since there are N possible forward prices to choose from the rational thing to do is to choose an average of the forward prices as the delivery price. This is because the investor cannot know which of the forward prices is the optimal one. The average forward price is equal to

$$\frac{1}{N+1}\sum_{j=0}^{N}\left[e^{rT(N-j)/N}V(jT/N)\right] \tag{6.2}$$

The value f of the short position in a forward contract is

$$f = (K - F_0)e^{-rT} \tag{6.3}$$

The forward contract will have a value at maturity equal to $K - V(T)$, since the forward price at maturity equals the underlying asset spot price.

Due to the trading restrictions the investor suffers an opportunity cost if the following inequality occurs.

$$\frac{1}{N+1}\sum_{j=0}^{N}\left[e^{rT(N-j)/N}V(jT/N)\right] > V(T) \tag{6.4}$$

In other words, if the average forward delivery price is greater than the stock price at time T, the investor would have liked to sell the restricted share and make a profit equal to $K - V(T)$, if she actually held the forward contract. The profit is the potential value an investor loses by not being able to sell the restricted share. The opportunity cost has the same payoff profile as a put option with the strike price equal to the average forward price, namely

$$max\Big(0, \frac{1}{N+1}\sum_{j=0}^{N}\Big[e^{rT(N-j)/N}V(jT/N)\Big] - V(T)\Big) \tag{6.5}$$

Therefore, if we value the average-strike put option we implicitly determine the size of the DLOM.

Finnerty views the average-strike put option as an option to exchange a forward contract on the underlying share for the unrestricted share, since it fundamentally is an agreement to exchange one asset, the forward contract for another asset, the unrestricted share. Finnerty then applies the work of Margrabe (1978), who developed formulas for valuing options in which the parties agree to exchange one asset for another to derive his formula for the DLOM

$$D(T) = Se\Big[N\big(\frac{v\sqrt{T}}{2}\big) - N\big(-\frac{v\sqrt{T}}{2}\big)\Big] \tag{6.6}$$

$$v\sqrt{T} = \sqrt{\sigma^2 T + \log[2(e^{T\sigma^2} - \sigma^2 T - 1)] - 2\log(e^{T\sigma^2} - 1)]} \tag{6.7}$$

$D(T)$ is the value of the the DLOM. S is the stock price. When the DLOM is divided by the stock price, the DLOM is expressed as a percentage of the underlying stock price. T is the restriction period in years and σ is the volatility of the stock return. The variable v is the volatility of the ratio between the average forward price and the underlying stock price. This is a result of Finnerty applying Margrabe's formulas for the option of exchanging one asset for another. The DLOM formula can easily be modified to take dividends into account.

Appendix C - Compound Option Valuation

Compound options are exotic derivatives and means an investor holds an option on an option. The compound option gives the holder the right to buy/sell an underlying option, which in turn gives owner the right to buy/sell an underlying asset for example a stock or a currency. There is exists four kinds of compound options, a call on a call (CoC), a call on a put (CoP), a put on a call (PoC) and a put on a put (PoP). (Hull, 2015).

In the seminal paper by Black and Scholes (1973) they mention the existence of compound options. They put fourth the idea that common stock can be seen as an option on the value of a firm, thereby making a call option on the company stock a compound option. They mention the fact that these compound options cannot be valued using their formulas for European options since the variance of a compound options cannot be assumed to be constant since it depends on several variables.

The valuation of compound options becomes more complicated than for vanilla options because there are more parameters to take into account, for example, two time to maturities, two exercise prices and two underlying assets. One also has to take account how these parameters interact with each other. Geske (1977) showed how to value coupon bonds by seeing them as compound options. Geske (1979) presented formulas for valuing compound options and showed that the Black and Scholes (1973) formulas can be seen as a specific case of his formula.

Geske (1979) based his arguments by seeing a call option on a stock as a compound option. The reasoning for this is that one can view a stock as an option on the firm assets, thereby making the call option a compound option, as suggested by Black and Scholes (1973). He assumes the underlying firm value follow Geometric Brownian

Motion and that the variance of the underlying option is proportional to the firm value, thereby deviating from the BSM framework of constant variance rate.

The value of a call on a call, denoted C, is calculated as follows

$$
\begin{aligned}
C = & VN_2(h + \sigma\sqrt{t_1}, k + \sigma\sqrt{t_2}; \sqrt{t_1/t_2}) \\
& - Me^{-rt_2}N_2(h, k; \sqrt{t_1/t_2} - Ke^{-rt_1})N_1(h)
\end{aligned}
\tag{6.8}
$$

where

$$
h = \frac{\log(V/\bar{V}) + (r - 0.5\sigma^2)t_1}{\sigma\sqrt{t_1}} \tag{6.9}
$$

$$
k = \frac{\log(V/M) + (r - 0.5\sigma^2)t_2}{\sigma\sqrt{t_2}} \tag{6.10}
$$

V is the value of the firm and M is the face value of the debt in the firm. σ is the volatility of the underlying firm price. r is the risk-free rate. t_1 is the time to maturity of the compound option and t_2 is the maturity of the underlying call option. $N_2(\cdot)$ is the bivariate cumulative normal distribution function with h and k as upper integral limits and the correlation coefficient between them is $\sqrt{t_1/t_2}$.

$\bar{V}$ is the value of the underlying firm that makes the underlying call option's value equal to the strike price of the compound option. If $V < \bar{V}$ the compound call option will not be exercised.

In the 1980s, Geske and Johnson (1984) were able to derive an analytical formula for valuing American put options. They realized that an American put can be seen as an infinite sequence of options on options. With the application of compound option pricing theory they were able to value the American put. Since an American option gives the holder the right to exercise at any point on the time interval $[0, T]$, they saw each exercise opportunity as a discrete point on the time interval $\frac{N}{T}$ where $N \to T$ with arbitrarily small increments. Each time interval is a European put with the choice of exercising or waiting to receive a new option at the next time interval. That is, an option on an option.

Carr (1988) developed a formula for valuing exchange opportunities as compound options by combining results from papers by Fischer (1978) and Margrabe (1978). The framework is built on the fact that many situations in which one asset is exchanged for another can be modeled as options. When there are several layers of these choices on top of each other compound option theory is applicable. Examples of these situations include performance incentive fees, investment decisions for firms or the value of the equity in a firm where the debt consists of a coupon bond.

Liu et al. (2018) built on the work of Gukhal (2004) and Kou (2002) to improve option pricing theory to take into account a more modern understanding of the distribution of returns. Whereas previous models such as Geske (1979) assumed a lognormal distribution, Liu et al. (2018) accounted for the leptokurtic [2] and skewed distributions of returns in real life by modeling the underlying asset with a jump-diffusion process.

[2] Distributions with fat tails which means extreme outcomes occur with a higher probability.

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