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by

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The Stability and Volatility of Electricity Prices: An Illustration of (λ, σ^2)-Analysis*

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Abstract

The aim of this letter is to discuss and illustrate what we call (λ, σ^2)-analysis, which is a method to distinguish between the stability of a stochastic dynamic system and the volatility of a variable generated by this system. It is also emphasized that this method is able to generate new research questions for economic theory. The data set used in an empirical illustration is spot electricity prices from Nord Pool.

JEL codes: C14; C22.
Keywords: Smooth Lyapunov Exponents; Stability; Stochastic Dynamic System; Volatility.

1 Introduction

We argue in this letter that one should contrast the stability of a stochastic dynamic system with the volatility of a variable generated by this system in what we call (λ, σ^2)-analysis.

For example, think of macroeconometric models with the aim of explaining observed features of aggregate fluctuations. At the heart of these models, there is an impulse-propagation mechanism in which impulses are shocks to

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the economy, while the propagation mechanism is the means by which these shocks lead to persistence over time of the cycle. Thus, a less stable economy is associated with a higher persistence of the shocks, meaning that even a small shock would have a large effect on the economy.

To further clarify the idea, let σ^2 denote the conditional variance of a variable generated by a stochastic dynamic system, and let λ denote the stability of this system. Then,

$$\sigma^2 = \sigma^2 (\lambda, \varepsilon), \tag{1}$$

where ε is exogenous shocks to the dynamic system, meaning that the conditional variance (σ^2) is not only affected by the system's stability (λ), it is also affected by shocks to the system (ε). To be more precise, the conditional variance of a variable increases when the dynamic system is less stable, but also when the amplitude of the shocks increases. Thus, because of ε in (1), there is no one-to-one correspondence between σ^2 and λ, which motivates the proposed method. In Bask et al [2], the stability of a stochastic dynamic system is also thoroughly examined, and here we evolve the analysis by examine the relationship between stability and volatility.

To give a taste of the method, we will examine how the stability of electricity prices has evolved during the integration process at the Nordic power market and contrast it with the volatility of these prices. In connection with this analysis, we will also explain why (λ, σ^2)-analysis is able to generate new research questions for economic theory. However, before doing this, we have to explain why λ is a measure of stability and how it can be estimated from data.

2 Method: (λ, σ^2)-analysis

Definition of λ Bask and de Luna [1] argue that the spectrum of smooth Lyapunov exponents can be used in the determination of the stability of a stochastic dynamic system. Specifically, assume that the dynamic system, $f : \mathbb{R}^n \to \mathbb{R}^n$, generates

$$S_{t+1} = f(S_t) + \varepsilon^s_{t+1}, \tag{2}$$

where S_t and ε^s_t are the state of the system and a shock to the system, respectively. For an n-dimensional system as in (2), there are n Lyapunov exponents that are ranked from the largest to the smallest exponent:

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n, \tag{3}$$

and it is these exponents that provide information on the stability properties of the dynamic system f.

Assume temporarily that there are no shocks, and consider how the dynamic system f amplifies a small difference between the initial states S_0 and S_0':

$$S_j - S_j' = f^j(S_0) - f^j(S_0') \simeq Df^j(S_0)(S_0 - S_0'), \tag{4}$$

where $f^j(S_0) = f(\cdots f(f(S_0))\cdots)$ denotes j successive iterations of the system starting at state S_0, and Df is the Jacobian of the system:

$$Df^j(S_0) = Df(S_{j-1})\, Df(S_{j-2}) \cdots Df(S_0). \tag{5}$$

Then, associated with each Lyapunov exponent, λ_i, $i \in [1, 2, \ldots, n]$, there are nested subspaces $U^i \subset \mathbb{R}^n$ of dimension $n+1-i$ with the property that

$$\lambda_i \equiv \lim_{j\to\infty} \frac{\log_e \left\| Df^j(S_0) \right\|}{j} = \lim_{j\to\infty} \frac{1}{j} \sum_{k=0}^{j-1} \log_e \left\| Df(S_k) \right\|, \tag{6}$$

for all $S_0 \in U^i - U^{i+1}$. Due to Oseledec's multiplicative ergodic theorem, the limits in (6) exist and are independent of S_0 almost surely with respect to the measure induced by the process $\{S_t\}_{t=1}^{\infty}$. (See Guckenheimer and Holmes [5] for a careful definition of the Lyapunov exponents and their properties.) Then, allow for shocks, meaning that the aforementioned measure is induced by a stochastic process. In this case, the Lyapunov exponents have been named smooth Lyapunov exponents in the literature.

Motivation of λ According to Bask and de Luna [1], the reason why the spectrum of smooth Lyapunov exponents provides information on the stability properties of a stochastic dynamic system may be seen by considering two starting values of a system, where the difference is an exogenous shock at time $t = 0$. The largest smooth Lyapunov exponent, λ_1, measures the slowest exponential rate of convergence of two trajectories of the dynamic system starting at these starting values at time $t = 0$, but with identical exogenous shocks at times $t > 0$. (When $\lambda_1 > 0$, the trajectories diverge from each other, and for a bounded stochastic dynamic system, this is an operational definition of chaotic dynamics.) In fact, λ_1 measures the convergence of a shock in the direction defined by the eigenvector corresponding to this exponent. However, if the difference between the two starting values lies in another direction of $\mathbb{R}^n$, then the convergence is faster. Thus, λ_1 measures a "worst case scenario."

The average of the smooth Lyapunov exponents,

$$\lambda \equiv \frac{1}{n} \sum_{i=1}^{n} \lambda_i, \tag{7}$$

measures the exponential rate of convergence in a geometrical average direction. That is, the convergence of two trajectories of the dynamic system in the geometrical average of the directions defined by the eigenvectors corresponding to the different exponents. Thus, λ measures an "average scenario." We can, therefore, compare the stability of two stochastic dynamic systems via the smooth Lyapunov exponents since a one-time shock has a smaller effect on the dynamic system with a smaller λ than for the system with a larger λ. Since we are dealing with dissipative systems, meaning that $\lambda < 0$ by definition, a dynamic system is more stable than another system, if λ is more negative.

Estimation of λ Since the actual form of the stochastic dynamic system f is not known, it may seem like an impossible task to determine the stability of the system. However, it is possible to reconstruct the dynamics using only a scalar time series, and, thereafter, to measure the stability of the reconstructed system. Therefore, associate the dynamic system f with an observer function, $g : \mathbb{R}^n \to \mathbb{R}$, that generates the following scalar time series:

$$s_t = g(S_t) + \varepsilon_t^m, \tag{8}$$

where $s_t \in S_t$ and ε_t^m are an observation in the time series and a measurement error, respectively. That is, the time series $\{s_t\}_{t=1}^N$ is observed, where N is the number of observations.

Specifically, the observations in a scalar time series contain information about unobserved state variables that can be used to define a state in present time. Therefore, let

$$T = (T_1, T_2, \ldots, T_M)' \tag{9}$$

be the reconstructed trajectory, where T_t is the reconstructed state and M is the number of states on the trajectory. Each T_t is given by

$$T_t = \{s_t, s_{t+1}, \ldots, s_{t+m-1}\}, \tag{10}$$

where m is the embedding dimension. Thus, T is an $M \times m$ matrix and the constants M, m and N are related as $M = N - m + 1$.

Takens [11] proved that the map

$$\Phi(S_t) = \left\{ g\left(f^0(S_t)\right), g\left(f^1(S_t)\right), \ldots, g\left(f^{m-1}(S_t)\right) \right\}, \tag{11}$$

which maps the n-dimensional state S_t onto the m-dimensional state T_t, is an embedding if $m > 2n$. This means that the map is a smooth map that performs a one-to-one coordinate transformation and has a smooth inverse. A map that is an embedding preserves topological information about the unknown dynamic system, like the smooth Lyapunov exponents, and, in particular, the map induces a function, $h : \mathbb{R}^m \to \mathbb{R}^m$, on the reconstructed trajectory,

$$T_{t+1} = h(T_t), \tag{12}$$

which is topologically conjugate to the unknown dynamic system f. That is,

$$h^j(T_t) = \Phi \circ f^j \circ \Phi^{-1}(T_t). \tag{13}$$

Thus, h is a reconstructed dynamic system that has the same smooth Lyapunov exponents as the unknown dynamic system f.

Then, to be able to estimate the smooth Lyapunov exponents, one has to estimate h. However, since

$$h : \begin{pmatrix} s_t \\ s_{t+1} \\ \vdots \\ s_{t+m-1} \end{pmatrix} \longrightarrow \begin{pmatrix} s_{t+1} \\ s_{t+2} \\ \vdots \\ v(s_t, s_{t+1}, \ldots, s_{t+m-1}) \end{pmatrix}, \tag{14}$$

the estimation of h reduces to the estimation of v:

$$s_{t+m} = v(s_t, s_{t+1}, \ldots, s_{t+m-1}). \tag{15}$$

Moreover, since the Jacobian of h at the reconstructed state T_t is

$$Dh(T_t) = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial v}{\partial s_t} & \frac{\partial v}{\partial s_{t+1}} & \frac{\partial v}{\partial s_{t+2}} & \cdots & \frac{\partial v}{\partial s_{t+m-1}} \end{pmatrix}, \tag{16}$$

a feedforward neural network is recommended to estimate the above derivatives to derive the smooth Lyapunov exponents, and this is because Hornik et al [6] have shown that a map and its derivatives of any unknown functional form can be approximated arbitrarily accurately by such a network.

Measuring σ^2 We will not spend time here on how to measure the volatility of a variable generated by a stochastic dynamic system, because there are several techniques for this purpose that are well-known within the research community.

3 Illustration of (λ, σ^2)-analysis

Since the beginning of the 1990s, there has been an evolution in the Nordic countries from national markets to a multi-national electricity market. To be more precise, Norway, Sweden, Finland and Denmark have all reformed their electricity sectors and have today access to a common wholesale market that consists of bilateral trade of electricity contracts between operators and the non-mandatory Nordic power exchange, Nord Pool.

The data set used in the empirical illustration is spot electricity prices from Nord Pool. Specifically, it is the daily average of the hourly system price for the period January 1, 1993, to December 31, 2005. Moreover, the data set is split in parts with the natural breakpoints when a new country is joining the common market. However, since prices are non-stationary, we use the returns in the analysis.

We first estimate the smooth Lyapunov exponents for each time series using the neural network algorithm proposed in Gencay and Dechert [4] and Kuan and Liu [7].[1] Specifically, we estimate these exponents making use of 4, 8, 12, 16 and 20 inputs to the neural network, respectively, where the number of hidden units in each case runs from 1 unit to 20 units. Our estimate of λ that minimizes SIC in each subperiod is reported in Table 1.

[Table 1 about here.]

The general picture is that the integration process is associated with more stable electricity prices.

Thereafter, we estimate an EGARCH model for each time series, where our estimate of σ^2 is the persistent volatility parameter in the model. See Table 1 for these estimates, where the general picture is that the integration process is associated with a decrease in volatility of electricity prices. (The parameter is not significant for the period July 1, 1999, to September 30, 2000, when eastern Denmark is not part of the common market.) Thus, there seems to be a pattern in the change in stability and volatility of electricity prices, but without having a one-to-one correspondence between these changes.[2]

4 Discussion

First of all, we are not aware of any distributional theory for λ. However, Shintani and Linton [10] show that a neural network estimator of the smooth Lyapunov exponents of the type that we have used is asymptotically normal. Our conjecture is, therefore, that asymptotic normality holds for a neural network estimator of λ since the eigenvectors corresponding to the different exponents are pairwise orthogonal.

Secondly, in Bask and de Luna [1], it is argued that when the volatility of a variable modelled is of interest, one should also consider the stability properties of the *same* model. In the present letter, however, a non-parametric approach is used when estimating λ, whereas any kind of (good) volatility model may be used when estimating σ^2.

[1] We have used NETLE 3.01, a program developed by R. Gencay, C.-M. Kuan and T. Liu, when estimating the smooth Lyapunov exponents.

[2] Detailed estimation results are available on request from the authors.

Clearly, the appropriateness of the two approaches depend on the purpose of the analysis. For instance, when it comes to developing a theoretical model with the aim of explaining movements in, for example, asset returns, we believe that one should not only evaluate the out-of-sample performance of the model, but also its stability to match it with the stability properties of asset returns. However, when a successful risk management is in focus, it is necessary to measure the stability of the "true" stochastic dynamic system generating asset returns, and not the stability of the model fitted to these returns. The reason is that there is no guarantee that the smooth Lyapunov exponents for the "true" system and the model selected to measure volatility coincide with each other.

Thirdly, in Bask et al [2], the stability of electricity prices is also examined, but a difference is that they focus on the presence of chaotic dynamics, which is the case when $\lambda_1 > 0$, whereas we examine how sensitive the dynamic system generating electricity prices is to shocks, making use of λ. Potter [8] mentions that λ_1 can be used not only to categorize an observed time series as stable or unstable, but also to give a measure of the speed of convergence or divergence. Shintani [9] uses λ_1 when he examine the speed of convergence towards PPP.

Last but not least, our method is able to generate new research questions for economic theory. For example, Bask et al [3] examine how the degree of market power has evolved during the aforementioned integration process at the Nordic power market using a conjectural variation method, and they found that the degree of competition has increased when the common market has expanded. One might, therefore, ask: is there any theoretical justification for the finding that the degree of market power is inversely related to the stability of this market? To our knowledge, this type of question has never been posed in the literature.

To conclude, even though more work has to be done to be able to test for a change in stability of a stochastic dynamic system, we believe that the empirical illustration in this letter has made it clear that (λ, σ^2)-analysis can be a fruitful tool in both empirical and theoretical research. It is, therefore, our hope that future research can resolve the remaining questions to have this powerful tool.

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Countries	Stability	Stability change	Volatility	Volatility change
Norway (1/1/1993-12/31/1995)	−0.268 12 inputs 5 hidden units		0.975	
		Increase		Decrease
Norway and Sweden (1/1/1996-12/28/1997)	−0.359 8 inputs 2 hidden units		0.935	
		Decrease		Decrease
Norway, Sweden and Finland (12/29/1997-6/30/1999)	−0.168 12 inputs 3 hidden units		0.686	
		Increase		Decrease
Norway, Sweden, Finland and western Denmark (7/1/1999-9/30/2000)	−0.273 8 inputs 1 hidden unit		0.148	
		Increase		Increase
Norway, Sweden, Finland and Denmark (10/1-2000-12/31/2005)	−0.275 8 inputs 5 hidden units		0.616	

Table 1: The stability (λ) and volatility (σ^2) of electricity prices during the integration process at the Nordic power market.