

**ECONOMIC STUDIES**  
**DEPARTMENT OF ECONOMICS**  
**SCHOOL OF ECONOMICS AND COMMERCIAL LAW**  
**GÖTEBORG UNIVERSITY**  
**150**

---

**ESSAYS ON THE SKILL PREMIUM**

**Klas Sandén**

**ISBN 91-85169-09-9**  
**ISBN 978-91-85169-09-2**  
**ISSN 1651-4289 print**  
**ISSN 1651-4297 online**



Doctoral Thesis  
Department of Economics  
Göteborg University

---

# Essays on the Skill Premium

Klas Sandén



September 11, 2007  
Last Changed Rev: 158

Blank page... text to be printed starts on next page...

*To my parents and brother,  
Siv, Per-Eli, and Karl-Axel Sandén*



# Abstract

This thesis focuses on the wage distribution among individuals with different skills and skill levels. It consists of three essays after a short introduction.

## Essay I

### **Risk, Occupational Choice, and Inequality**

This essay presents a new theory explaining increased wage inequality. A standard endogenous growth model is augmented with occupational choice of high-skill workers. Depending on the occupational choice, high-skill workers earn either a certain or uncertain income. Wage inequality, measured by the average wage of high-skill workers divided by the average wage of low-skill workers, can increase or decrease due to an *increased* supply of high-skill workers.

## Essay II

### **Market Imperfections and Wage Inequality**

This essay investigates, theoretically, the relationship between various market imperfections and the skill premium. As opposed to other models relating market imperfections to wage inequality, the model in this paper assumes perfectly competitive labor markets but distorted product and financial markets. The paper predicts that the skill premium is positively correlated with consumer preference for variety, because preference for variety leads to greater market power and thereby higher profits. In addition, shorter product cycles increase the skill premium.

## **Essay III**

### **Firm Fragmentation and the Skill Premium**

This essay investigates the interaction between demand uncertainty and non-competitive labor markets where firm owners have the option to shut down and relocate. Workers cannot find new jobs instantly and therefore accept wage reductions to avoid unemployment, if firm owners credibly threaten to shut down.

The analysis shows that the expected wage rate is a mix of a competitive wage rate and a bargained wage rate and that this lowers the skill premium. Further, the option of firms to shut down and relocate increases the average size of firms. The analysis also shows that outsourcing or contracting out is more likely if demand is more uncertain, if market power is smaller, and if the markets for intermediate goods are more competitive.

Fragmentation increases the skill premium because it leads to more homogenous firms, with respect to workers' skills. With more homogenous firms, low-skill workers cannot compensate their inferior productivity in wage bargains with high-skill workers.

# Acknowledgment

In writing this thesis I have benefited from several people. My supervisor Prof. Douglas A. Hibbs, Jr. has supported me throughout the process. His struggle with early drafts of each paper must have required quite some effort and, given some of his email responses, also a lot of frustration. However, his comments and criticisms greatly improved them all. I have definitely learned a lot from your comments and our discussions Douglas, and your respect for honesty and openness in doing research is something I will carry along for the future.

I would also like to take the opportunity to thank Senior Lecturer Wlodek Bursztyn. His sincere interest in and devotion to economics inspired me to apply for the PhD program and to carry it through. I owe you Wlodek, not only for your very careful reading of my papers, never missing out on a single indistinct formulation or argument, but also for your encouragement during the years.

I am also grateful that Prof. Karl Ove Moene signed up as opponent for my licentiate seminar. He not only provided valuable comments and insights, but also promoted open and entertaining discussions. My discussions with Senior Lecturer Håkan Locking has greatly improved and added to my thesis, and naturally I am grateful for your help easing my financial situation. I'm also grateful for the encouragement and support from Dr. Daniela Andrén. Prof. Henry Ohlsson always took the time to listen to and discuss ideas and papers, as well as to clutter my drafts with language corrections which was much appreciated.

The thesis would not have materialized without the financial support of Stiftelsen Fru Mary von Sydow född Wijk's donationsfond, and Wallander-Hedelius' stiftelse. Once again I'm truly grateful for the help with financial support from Prof. Lennart Hjalmarsson and Prof. Arne Bigsten. Eva Jonasson and Eva-Lena Neth have helped me with all the various administrative issues.

In writing the thesis I have experienced times of weariness and doubts. I'm sincerely grateful for the support of my friends in restoring my confidence and helping put things in perspective. Finally, Jens and Pål, thanks for all the much appreciated stimulating discussions on the widest



variety of topics during our lunches.

*Göteborg, February 2006*

# Contents

<b>Abstract</b>	<b>i</b>
<b>Acknowledgment</b>	<b>iii</b>
<b>Introduction</b>	<b>1</b>
1 Inequality in Different Countries . . . . .	1
2 Inequality over Time . . . . .	3
3 Theoretical Explanations . . . . .	8
4 Contributions . . . . .	20
Bibliography . . . . .	21
<b>I Risk, Occupational Choice, and Inequality</b>	<b>I.1</b>
1 Introduction . . . . .	I.1
2 Model . . . . .	I.5
3 Results . . . . .	I.16
4 Conclusions . . . . .	I.23
Appendix I.A Record of Notation . . . . .	I.24
Appendix I.B Proofs and Derivations . . . . .	I.24
Bibliography . . . . .	I.30
<b>II Market Imperfections and Wage Inequality</b>	<b>II.1</b>
1 Introduction . . . . .	II.1
2 Model . . . . .	II.3
3 Equilibrium . . . . .	II.8
4 Households . . . . .	II.10
5 Conclusions . . . . .	II.17

Appendix II.A	Record of Notation . . . . .	II.19
Appendix II.B	Demand for Variations . . . . .	II.19
Appendix II.C	Supply of Variations . . . . .	II.21
Bibliography	. . . . .	II.23
<b>III Firm Fragmentation and the Skill Premium</b>		<b>III.1</b>
1	Introduction . . . . .	III.1
2	Model . . . . .	III.7
3	Intermediate Results . . . . .	III.12
4	Firms Revisited . . . . .	III.20
5	Equilibrium results . . . . .	III.32
6	Conclusions . . . . .	III.43
Appendix III.A	Record of Notation . . . . .	III.44
Appendix III.B	Profit Maximization . . . . .	III.46
Appendix III.C	Equilibrium Conditions . . . . .	III.50
Bibliography	. . . . .	III.53

# Introduction

Economics is a fascinating subject. Modern economics aims at using the intrinsic logic of mathematics to explain an ever growing amount of real life phenomena in society. This thesis revolves around one of the most fundamental issues in every society, namely the distribution of what is commonly produced. The positive and normative discussion of distribution has always been a central subject in economics, from the writings of the classical economists, such as Marx, Smith, and Ricardo, to present writings in economics.

However, distribution is, given the vast amount of work on the subject, a much too broad a subject for a thesis. This thesis focuses mainly on industrialized countries and differences in wages across individuals with different skill levels. In what follows, a short overview of trends in distribution of income in some industrialized countries is presented, then the existing theoretical literature on wage inequality is briefly surveyed before the contribution of this thesis is summarized.

## 1 Inequality in Different Countries

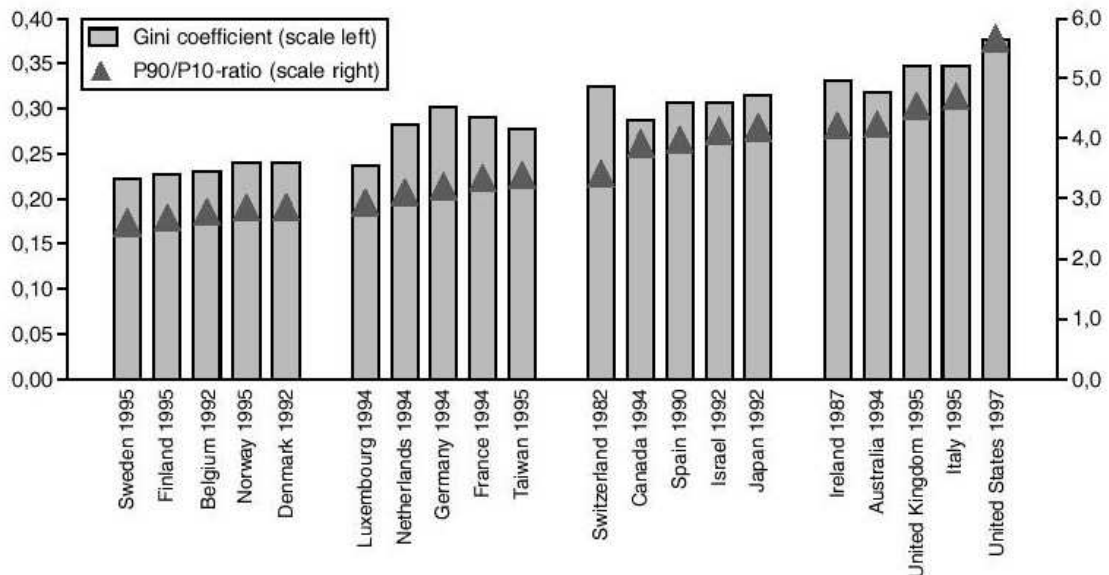
The distribution of income differs considerably among different industrialized countries. For example, as is shown in Figure 1, the disposable income of the person at the 90th percentile was almost 6 times the disposable income of the person at the 10th percentile in the U.S. in 1997.<sup>1</sup> The corresponding figure for Sweden was less than 3.5 in 1995.

Percentile ratios are attractive because of their intuitive simplicity, but they can be misleading since only two points in the distribution are used. The Gini coefficient may appear more appealing since all data points contribute to the summary statistic. However, it is difficult to interpret and also weights the observations arbitrarily. Fortunately, as seen in Figure 1, using

---

<sup>1</sup>The disposable income is adjusted using an equivalence scale.

Figure 1: Distribution of Disposable Income



The figure illustrates the distribution of disposable income for a subset of industrialized countries. Source: Caminada and Goudswaard (2001)

the Gini-coefficient or the 90/10 percentile ratio generally does not alter the ranking of the countries. In general, the variation in disposable income is lowest in the Scandinavian countries (Sweden, Norway, and Finland), while the greatest variation in disposable income is found in the English speaking countries (United States, Great Britain, and Australia).

## 1 Explaining Differences

Labor markets are complicated and the evidence that the standard competitive supply and demand framework is insufficient is overwhelming. For example, in the competitive framework, the wage rate for a specific type of worker should be uncorrelated with firm characteristics such as firm location, profit rate, and size. However, it is well known that this is not the case. Slichter (1950) notes that wages are correlated with for example the value added. Several later scholars have verified systematic differences of wages across industries, taking into account differences in worker characteristics and workplace conditions (Krueger and Summers 1988; Hildreth and Oswald 1997; Edin and Zetterberg 1992; Hibbs and Locking 2000)

Once the simple supply and demand framework is abandoned, there seems to be an endless variation of theories explaining wage and income inequality. In general, the variation in wages is greater than the variation in disposable income. This is because every industrialized country

provides some income security for its citizens (Ervik 1998). Notice that this is true for the U.S. as well as for Sweden, even though the U.S. and Sweden often are considered to represent the polar cases with respect to government intervention.<sup>2</sup> While the redistributive role of the government is interesting in itself and certainly very important for the variation in individual disposable income levels, this thesis focuses on the variation in wages rather than income.

In explaining cross country differences in the variation in wages, Blau and Kahn (1996) and Wallerstein (1999) show that after controlling for union concentration and coverage of collective bargaining agreements, various other potential explanatory variables can be discarded. Interestingly, even the supply of high skill workers seems to have only a minor impact on the distribution of wages, which is certainly at odds with the standard competitive theory of the labor market. The link from centralized wage bargains to lower inequality is supported by the evolution of wage inequality in Sweden during the 1980s. Prior to 1980 the Swedish labor market was characterized by a high degree of centralized wage bargaining, but beginning in the 1980s the wage setting process gradually became more decentralized. The decentralization coincides with increased wage inequality in Sweden (Hibbs 1991; Hibbs and Locking 1996; Edin and Topel 1997).

## **2 Inequality over Time**

Describing the changes in income inequality during the period 1970 – 2000 among developed countries is not a straightforward exercise. The U.S. is typically used as the benchmark case for comparison with other countries, since in most respects the U.S. shows an early and clear cut case of increased inequality. While U.S. wage inequality increased most rapidly during the 1980s, a majority of industrialized countries experienced increased wage inequality during the 1990s. Within the family of European countries, the U.K. stands out by exhibiting the most dramatic increase in inequality.

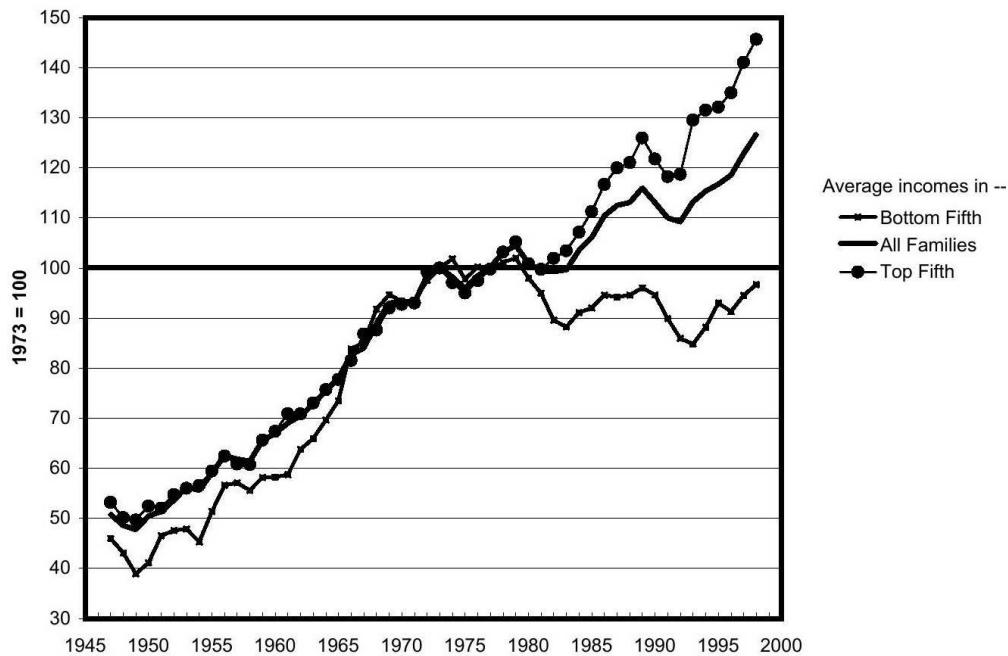
### **1 The U.S. Experience**

Looking only at the distribution of wages without reference to any worker characteristics such as education, experience etc. reveals a growing dispersion of wages between the lowest paid and the highest paid workers in the U.S. (Juhn et al. 1993, p. 415, Fig. 1). During the 1973

---

<sup>2</sup>It might be argued that governmental redistribution changes the distribution of wages because individuals respond by changing their behavior. If so, the effect of redistributive policies are less clear.

Figure 2: Trend in the U.S. Real Average Family Income



The family income of rich and poor families started to diverge in the 1980s as the income of poor families declined while the income of rich families increased. Source: Smeeding (2002)

to 1989 period the median wage increased by 5 percent. The wages of the 90th percentile in the wage distribution increased by approximately 20 percent between 1975 and 1989. On the other hand, wages *decreased* by about 25 percent for the 10th percentile during the 1970 – 1989 period.

The pattern in the early 1970s and after contrasts sharply with the pattern from the 1960s when wages increased in both group. "After about two and one-half decades [1963 – 1989] workers in the top 10 percent of the wage distribution have gained almost 40 percent, whereas workers in the bottom 10 percent have lost over 5 percent in real terms" (Juhn et al. 1993, p. 416). Gottschalk (1997) reports that the real income ratio between the 80th and 20th percentiles in the distribution shows a clear upward trend in the 1968 to 1992 period (Gottschalk 1997, p. 23). The findings for the 1970 – 1990 period are confirmed by Juhn et al. (1993) and by Figure 2. It is also clear that during the 1990s the trend towards increasing income inequality was muted because the income levels of those with the lowest income started to increase. In fact, the U.S. inequality increase in the 1985 – 1995 period does not stand out relative to many other countries (Förster and Pellizzari 2000; Smeeding 2002).

Inspecting and comparing wages among and between individuals with different years of

schooling show that the skill premium increased dramatically between 1980 and 1990. Gottschalk estimates standard (log) wage regressions in each year from 1970 – 1990. The dummy variable for a college degree shows a decreasing trend during the first part of the period, 1970 – 1980, and an increasing trend during the 1980s. Hence, changes in the skill premium cannot explain the increased inequality during the 1970s. The pattern over the entire period shows a significant increase in the skill premium (Gottschalk 1997, p. 30). This finding is verified by Juhn et al. (1993), but Gottschalk (1997, p. 30) points out that “... it is important to remember that the increases in the college premium are being driven more by the decline in real earnings of high school graduates than by the increase in earnings of college workers.” Any full explanation of the changes in the U.S. skill premium is therefore obligated to present a plausible case for an absolute *decrease* in earnings of workers with less education. In summary, U.S. earnings data point unambiguously to a trend of increased income inequality and a growing skill premium over the last 20 years.

## **2 The European Experience**

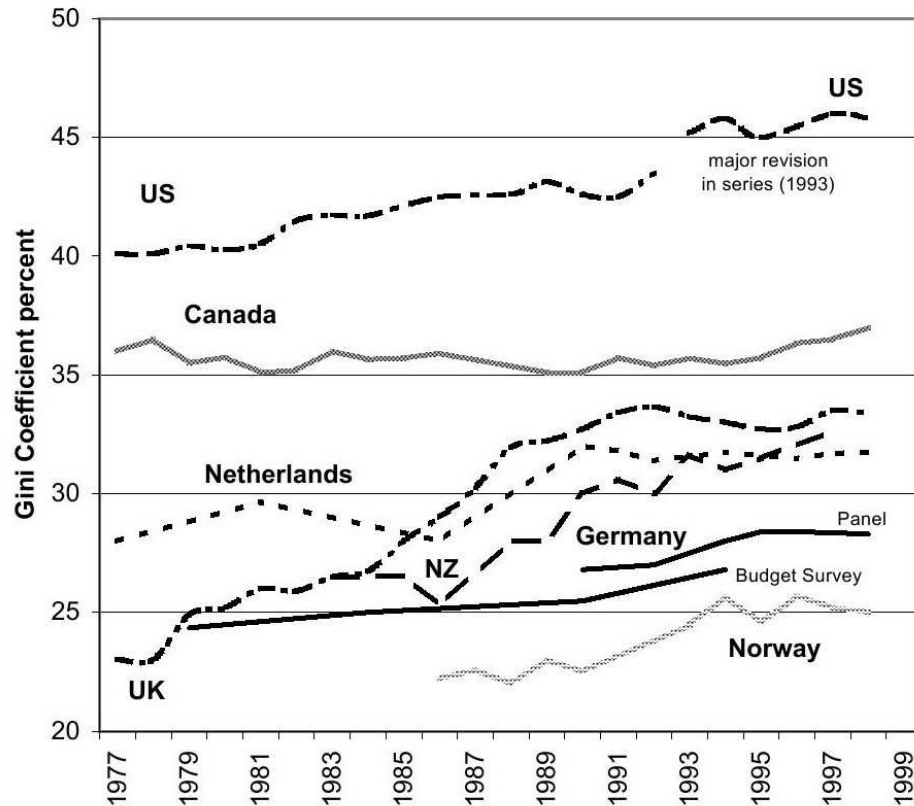
The rapid increase in U.S. wage inequality was unmatched by most European countries in the 1980s. Gottschalk and Smeeding (1997) summarize the changes in Europe. While the U.K. stands out in the European family by experiencing large increases in earnings inequality during the 1980 – 1990 period, the European experience is in general mixed. Most countries but not all experienced some increases in earnings inequality. In Figure 3 the changes in the Gini coefficient over disposable income is graphed for a subset of countries.

The U.K., Canada, and Austria all have experiences quite similar to the changes in the U.S., according to Gottschalk and Smeeding. Other countries such as Germany, Italy, Finland, and the Netherlands experienced either no or only slight increases in inequality during the 1980s (Gottschalk and Smeeding 1997, p. 654, Table 2).

If one extends the period by including 1990 – 2000, several scholars point out a weak trend towards increased inequality in disposable income; see for example Förster and Pearson (2002), Caminada and Goudswaard (2001), and Smeeding (2002). However, focusing instead on earnings inequality, Gottschalk and Joyce (1999, Figure 1) report increased earnings inequality for a subset of industrialized countries mainly during the 1980s. For the period starting in the mid 1980s and ending in the mid 1990s, Förster and Pellizzari (2000, Table 3.1) report the change in the share of market earnings obtained by individuals in the bottom deciles and the top deciles. Remarkably, the market income share for the bottom deciles *decreased* in 17 of 18 countries,



Figure 3: Changes in Income Inequality



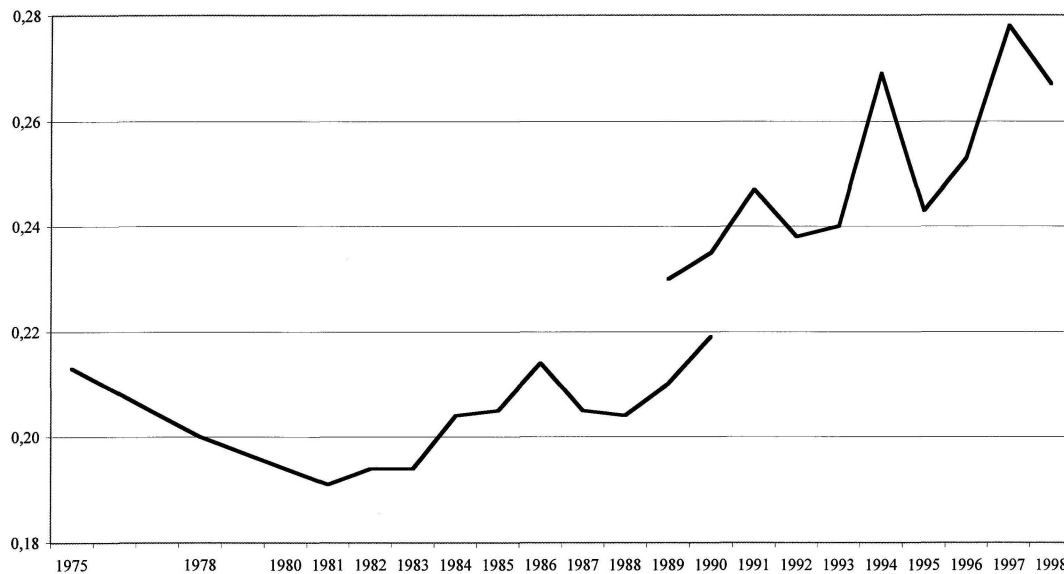
The figure graphs the trend in the Gini coefficient over the period 1980 – 1990, showing a general trend toward increased inequality in most countries. Source: Smeeding (2002)

while the income share of the top deciles increased in 14 of 18 countries.

### 3 The Swedish Experience

Cross-country studies of changes in inequality in Sweden during the 1980s and early 1990s give a mixed picture. While for example Caminada and Goudswaard (2001) and Gottschalk and Smeeding (2000) report dramatic changes relative to other industrialized countries, Smeeding (2002) and Förster and Pearson (2002) report only modest changes. However, looking at studies focusing on inequality trends in Sweden, a general trend towards more variation in income appears during the 1990s. Figure 4 shows the evolution of inequality in disposable income measured by the Gini coefficient for the 1975 – 1998 period. The figure is taken from Gustafsson and Palmer (2001) and is confirmed by for example Nelander and Goding (2004). During the

Figure 4: Gini Coefficient for Disposable Income in Sweden



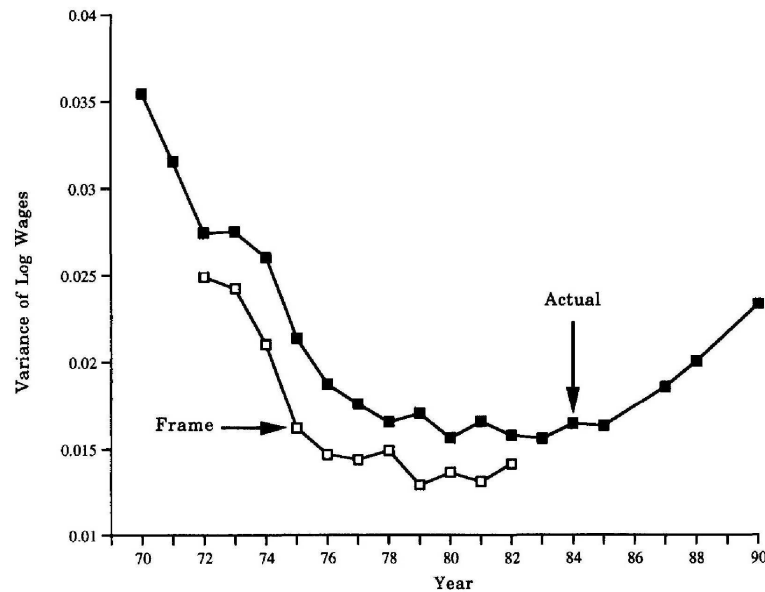
The figure displays the Gini coefficient for disposable income, adjusted using an equivalence scale, in Sweden over 25 years, starting in 1975. While the Gini coefficient decreased during the 1970s, it has clearly increased during the last 20 years. Due to the tax reform in 1990, implying that more income sources was included in the tax base, there is discontinuity in the graph. Source: Gustafsson and Palmer (2001)

1975 – 1982 period, inequality in disposable income decreased (Gustafsson and Palmer 1997; Nelander and Goding 2004), while it increased during 1983 – 2000 (Gustafsson and Palmer 2001; Fritzell 2001; Jansson 2000).

More interesting for the theoretical analysis in this thesis is the changes in earnings, or more specifically wage, inequality. According to Gustafsson and Palmer (1997, Figur 13.5) and Nelander and Goding (2004, Diagram 6), there is no apparent trend in the Gini coefficient for 1980 – 1990 earnings inequality. This is also confirmed by inspecting the changes in earnings of different decile groups (Gustafsson and Palmer 2001, Tabell 2). However, Hibbs and Locking (1996, 2000) find a significant increase in wage inequality for blue-collar workers after the breakdown of the centralized wage setting process in Sweden. Prior to 1983, the wage setting process for blue-collar workers was extremely centralized. In principle, every blue-collar worker wage was covered by the negotiations between LO, an association consisting of several unions, and SAF, the Swedish employer's association. After 1983, this arrangement was significantly weakened. Hibbs and Locking (1996) document an increase in the wage dispersion after 1983, illustrated in Figure 5.

Several studies point out increased earnings inequality during the 1990s in Sweden. Fritzell

Figure 5: Wage inequality for blue-collar workers in Sweden



The Actual curve displays the variation for blue-collar workers in Sweden during 1970 – 1990. The Frame curve depicts the wage inequality implied by the centralized bargaining outcome, which became gradually less important after 1983. For both curves, inequality is measured by the variance of log wages. Source: Hibbs and Locking (1996)

(2001) presents estimates of the Gini coefficient for factor payments (see Tabell 6). Over the 1991 – 1999 period the Gini coefficient increases by 10%, with the major change occurring during the deep recession starting in 1991. The same observation is found in Nelander and Goding (2004) and is confirmed in Gustafsson and Palmer (2001)'s decomposition of the Gini coefficient.

To summarize, there is ample evidence of a trend towards greater dispersion in the distribution of earnings. It is not confined to a small set of countries, but appears to be present in most industrialized countries. Naturally, scholars have been inspired to develop theories, trying to explain this trend. Below these theories are briefly summarized and categorized.

### 3 Theoretical Explanations

This section provides a brief introduction to theories aimed at explaining wage inequality, and in particular the changes in wage inequality during the last decades. A more formal survey is provided by Acemoglu (2002). There are several proposed explanations to the increased inequality observed over the 1980 – 2000 period. The first explanation at hand for an economist

would probably suggest that increased wage differences must be the result of supply and demand shifts. However, the supply of high-skill workers shows a clearly increasing trend, see Gottschalk (1997, p. 30) and Laitner (2000, p. 808). Hence supply and demand theory must explain either a substantial demand shift in favor of high-skill workers, a substantial demand shift working against low-skill workers, or both.

It is possible to identify at least three broad categories of explanations: skill biased technological change, increased trade with developing countries, and institutional change. The skill bias technological change argument focuses on a demand shift increasing the demand for high-skill workers, while the trade argument focuses on a demand shift decreasing the demand for low-skill workers. The most commonly mentioned institutional changes are de-unionization and the decreased real value of the minimum wage. In addition, there are quite a number of more or less ad hoc explanations. It is, however, difficult to find one theory that alone can explain all the empirical variations discussed above.

## 1 Technological Change and Inequality

The first and seemingly, at least initially, the most popular theory is usually referred to as *skill biased technological change*, proposing that the driving forces behind the changes in earnings inequality are shifts in the supply and demand of workers with different observable and non-observable skills and experience. According to those theories, the supply of high-skill and experienced workers decelerated during the 1980 – 1990 period while the demand for skill and experience increased, leading to a neat textbook supply and demand explanation of the changes in inequality.

Autor et al. (1998) test this hypothesis. The relative supply and relative wage of high-skill workers are known for the 1940 – 1996 period. Hence, it is possible to compute the shift in relative demand for high-skill workers that is necessary to generate the observed changes in the relative wage rate of high-skill workers.<sup>3</sup> Autor et al. conclude that given a steady increase in the relative demand for high-skill workers (i.e. a skill-biased technological change), the changes in the supply of high-skill workers do fairly well in explaining the changes in the relative wage of high-skill workers until the 1980s.

In the 1980s the steady increase in the relative demand for high-skill workers is not sufficient to explain the surge in the relative wage of high-skill workers. This verifies the earlier results of Katz and Murphy (1992). The question arises whether the 1980s marks the start of a new

---

<sup>3</sup>Autor et al. use a CES production function in high-skill and low-skill labor.

era in which the skill bias is significantly higher than during the post World War II period. The analysis by Autor et al. for the early 1990s casts some doubt on this interpretation, since the relative demand for high-skill workers seems to have decelerated. Hence, it seems that the drastic changes in the skill premium during the 1980s are likely to be explained by factors more specific to the same period.

The key assumption in the more naive theories is that technological change is always skill-biased. This reasoning can be traced back to Griliches (1969) who found that skilled labor is more complementary to capital than unskilled labor. Galor and Moav (2000) motivate this inherent bias in favor of high-skill workers in a theoretical model where high-skill labor has a comparative advantage, relative to low-skill labor, in adapting to new technologies. While the *level* of technological progress is skill neutral, technological progress is biased in favor of high-skill labor. This claim is supported, i.e. not rejected, by the empirical analysis in Bartel and Lichtenberg (1987) who show that plants with older machines generally employ more low-skill workers.

Krusell et al. (2000) present a model where technological improvements embodied in capital equipment affect high-skill and low-skill workers differently. Further they distinguish capital structures from capital equipment.<sup>4</sup> Following (Griliches 1969), differences in complementarity to capital equipment between high-skill and low-skill workers imply that benefits from increased accumulation and use of capital equipment in the production process are captured mostly by high-skill workers. While the growth rate of capital structures has been low, the growth rate of capital equipment started to increase around 1975 (Krusell et al. 2000, p. 1031). Hence, complementarity of high-skill workers with respect to capital equipment tended to, *ceteris paribus*, push the skill premium upwards during the period following 1975.

Even though Krusell et al. show that complementarity of high-skill workers and capital equipment can explain a large part of the increased wage premium during the 1980s, it is somewhat of a black box explanation, since the complementarity itself is not explained. Acemoglu (1998) and Kiley (1999) derive two different models where this complementarity is endogenous. The key assumption is that technological change is not exogenously given, but is driven by choices of profit maximizing agents.

In slightly more detail, Acemoglu assumes that unskilled and skilled workers use different designs of new technologies. Hence, production of new technology must be directed against either unskilled or skilled workers. The increasing supply of skilled workers decreases their

---

<sup>4</sup>In Krusell et al. 's Appendix, capital equipment includes computers and peripherals; communications; instruments, photocopiers and other equipment; general industrial equipment; transportation; and others.

wages by the usual supply and demand process. However, as the market size for a new technology designed for skilled workers grows, it becomes more profitable to invest in new technologies designed for skilled workers. The labor demand curve for skilled workers shifts outward, tending to increase wage rates for skilled workers. A priori, either of the two effects can dominate. Supposedly, during the 1970s the increased supply of high-skill workers depressed wages due to the first effect, but during the 1980s the market size effect became dominant and the wage premium increased and surpassed its early 1970s level.

In Caselli (1999) technological change is neutral but the workers face different costs using the new technology. Technological change is skill biased if the cost of the new technology implies that fewer workers find it optimal to invest in learning to use the new technology. The technological change is de-skilling if more workers find it optimal to invest in learning the new technology. Caselli argues that the increased use of computers and the development of information technology is a skill-biased technological revolution. Inequality increases because not all workers, i.e. not high cost workers, find it optimal to invest in learning to use the new technology.

It should be emphasized that Caselli's model can explain the drop in wage rates of some workers. New technologies are complementary to capital, and learning to use new technologies implies a fixed learning cost. Because the market for capital is competitive, the marginal product of capital must be the same across workers. Since the marginal product of capital is diminishing, the capital intensity must be higher for workers using the latest technology. This endogenous difference in capital intensity can explain the absolute wage losses for some workers.

It is worth pointing out that both Acemoglu and Caselli depart from the idea that technological change is always skill biased. For example, Acemoglu refers to the industrialization process where several factors contributed to increasing the supply of unskilled workers in the cities. The large increase in unskilled workers made industrialized production profitable, where skilled craftsmen were displaced by machines and unskilled labor (Acemoglu 2002). Caselli argues that the car industry was de-skilled as Henry Ford replaced skilled artisans with unskilled workers.

It should also be noted that neutral technological change can generate non-symmetric outcomes for high-skill and low-skill workers. Auerbach and Skott (2005) derive a static model where low-skill workers can apply only for low-skill jobs but where high-skill workers can apply for both high-skill and low-skill jobs. High-skill workers prefer high-skill jobs, but apply for low-skill jobs if they fail to find a high-skill job. Auerbach and Skott consider the effect of an *adverse* but *skill neutral* productivity shock. The demand for high-skill and low-skill workers

decrease proportionally due to the adverse shock. However, the option for displaced high-skill workers to find employment at low-skill jobs creates an additional setback for low-skill workers, because the competition for low-skill jobs becomes more fierce.

Auerbach and Skott claim that the 1970s witnessed a productivity slowdown. Their analysis shows that such an adverse skill neutral productivity shock not only can explain the increase in wage inequality among high-skill workers, but also replicate the changes in the skill-premium both in the U.S. and in Germany, given reasonable parameter values.

Laitner (2000) proposes a simple model based on innate differences in the abilities of individuals, explaining some of the variations in the empirical data. Individuals endogenously decide how much to invest in education, i.e. human capital. Individuals have different levels of *ability* that monotonically map into different levels of human capital investments. Due to the model's *unbiased* technological change, each subsequent generation can afford to invest in more human capital. This implies that each level of ability maps into a higher level of education in each subsequent generation. Therefore, in each subsequent generation, the average level of ability decreases in each educational cohort. As a result, earnings in each educational cohort grow more slowly than the economy's average earnings (Laitner 2000, p. 818–819).

The model predicts that the most highly educated group's wages will outgrow the least educated group's wages (Laitner 2000, p. 818–819) while the variance of log earnings will be constant (Laitner 2000, p. 818). Thereby, Laitner's model can explain the growing difference in high school dropout and college graduate earnings, but not the growing distance between the right and left tail of the wage distribution.

The essay in Chapter I of this thesis, "Risk, Occupational Choice, and Inequality," analyzes a model where technology is biased towards high-skill workers in the short run. High-skill workers have an advantage in developing and producing new inputs in the production process. More rapid technological change implies that more high-skill workers are employed by research firms where workers are paid a risk premium due to the uncertain return to new inputs.

Technological change is induced by an increase in the relative supply of high-skill workers, which causes a decline in the wage rate for high-skill production workers. However, more high-skill workers seek employment by research firms and earn a risk premium, thereby increasing the average wage rate for all high-skill workers.

**The Product Cycle and Inequality** In recent years several scholars have picked up the product cycle hypothesis (Vernon 1966). Development of new goods and production of goods early in the product cycle, non standardized goods, is assumed to be high-skill labor intensive. Pro-

duction of goods later in the product cycle, standardized goods, is assumed to be relatively more low-skill labor intensive.

In Ranjan (2001), a higher rate of unbiased technological progress implies that more resources are devoted to development of new goods. The skill-premium increases for two reasons: (1) the development process is high-skill intensive, and (2), at every moment in time, the number of non-standardized goods relative to the number of standardized goods increases. Therefore the relative demand for high-skill workers increases, and so does the skill premium.

Mendez (2002) introduces efficiency wages in a product life cycle model. Firms producing non-standardized goods pay efficiency wages while firms producing standardized goods pay competitive wages. This dual labor market setting generates wage inequality among workers with identical characteristics that can be traced back to the different stages in the product's life cycle. Mendez also shows that the skill premium is related to the product cycle by assuming that non-standardized goods are more skill intensive.

The essay in Chapter II of this thesis, "Market Imperfections and Wage Inequality", develops a model lending support to the idea that a shorter product cycle is associated with a higher skill premium. However the product cycle in Chapter II consists of a developing phase and a production phase, where the former is high-skill intensive and the latter low-skill intensive. While a shorter product cycle on the one hand decreases the expected profit from developing a new good, it on the other hand decreases the number of competing goods on the market, increasing the profitability of developing a new good. The latter effect dominates the former and a shorter product cycle increases the demand for high-skill workers, and thereby their relative wage.

## 2 Trade and Inequality

A second set of proposed explanations falls under the *trade* category. During the 1960 – 1990 period, the U.S. manufacturing imports in relation to the U.S. GDP grew from 2.1 percent to 7.3 percent (Sachs and Shatz 1994). Increased imports from developing countries exporting goods that are low-skill labor intensive lowers the prices such goods, thereby affecting the wage rate of low-skill workers domestically (Wood 1995; Borjas and Ramey 1995; Aghion et al. 1999).

The trade argument in its most simple form hinges on the Stolper and Samuelson theorem (Stolper and Samuelson 1941). To illustrate the implication of the theorem, suppose that two countries with different factor endowments but similar technology agree on lowering the tariff rates. The factor used most intensely in the export sector benefits relative to the factor used



most extensively in the import sector, within each country.

Within this framework industrialized countries are supposed to be high-skill labor abundant and to export high tech goods, while developing countries are low-skill labor abundant and export low-tech goods. Low-skill workers in industrialized countries are therefore hurt by more liberal trade policies.

A slightly different trade argument also exist where mainly intermediate goods are traded (Aghion et al. 1999). It is assumed that intermediates and low-skill labor are substitutes in producing final goods. Hence, increased trade and import of cheap raw materials and intermediate goods tend to push the wages for low-skill workers downward. This theory is complementary to the theory proposed in Krusell et al., mentioned above, where high-skill workers complement capital equipment to a larger degree than low-skill workers do. Increased trade in intermediate goods combined with increased use of capital equipment both weaken the position of low-skill workers on the labor market.

Borjas and Ramey (1995) emphasize the importance of increased imports of durable goods. According to Borjas and Ramey, durable goods producers earn rents, pay higher wages conditioned on observable characteristics, and employ mainly unskilled labor. Hence, increased imports of durable goods hurts unskilled workers in two ways: First, increased competition decreases profits and hence wages, directly affecting wages of unskilled workers. Second, if foreign competition reduces employment in the durable goods sector, less low-skill workers find high wage employment, reducing the number of unskilled workers earning wages above their marginal productivity (Borjas and Ramey 1995, p. 1079-1080).

The link between outsourcing and globalization are investigated in among others Helpman (1984) and Feenstra and Hanson (1999). Outsourcing of low skill intensive activities obviously hurts low-skill workers in industrialized countries. The analysis in Chapter III of this thesis, "Firm Fragmentation and the Skill Premium," shows that a general trend towards outsourcing or contracting out domestically also benefits high-skill workers relative to low-skill workers. The idea is that with more outsourcing or contracting out, firms become more specialized, and thereby more homogenous with respect to employee skill levels. With more specialized firms, high-skill and low-skill workers are sorted into different firms and low-skill workers cannot make up for their inferior productivity in wage negotiations with high-skill workers.

**Trade Induced Technological Change** Wood (1998, p. 1466) puts forward the idea of *defensive innovations*. By defensive innovations, Wood proposes that industries characterized by low-skill intensive production, facing increased competition by low-skill intensive imports, de-

velop new low-skill labor saving technologies in order to compete, thereby further eroding the labor market position for low-skill workers.

Wood's defensive invitation hypothesis is formalized by Neary (2002). In Neary's analysis, incumbent firms respond to entry threats by strategic investments which lower the variable production cost, thereby keeping potential entrants outside the market. Assuming that the investment increases demand for high-skill workers and that the reduction in variable costs decreases the demand for low-skill workers, it is easy to see that defensive innovations are likely to increase the skill-premium.

Several scholars note that globalization increases market size. In the paper by Dinopoulos and Segerström (1999) lower tariff rates motivate more development, via higher temporary Schumpeterian profits. Since development is high-skill labor intensive, high-skill workers benefit, relative to low-skill workers. In Ekholm and Midelfart (2005), entering firms choose technologies characterized by a higher fixed to variable cost ratio as the market size increases. Supposedly fixed costs are paid to high-skill workers thereby connecting market size and technology choice with the skill premium.

One of the main criticisms of the trade theory is that the prices of high-skill goods have not increased relative to low-skill goods, as predicted by the Stolper-Samuelson theorem. In the paper by Acemoglu (2003b), lower barriers to trade increase the price of skill intensive goods which in turn spurs research directed towards reducing the cost of producing high-skill goods, which in turn creates a feedback, tending to lower the price of high-skill goods.

Thoenig and Verdier (2003) elaborate on the market size idea by assuming that innovations can be skill neutral or skill biased. While skill biased innovations are less cost reducing, the duration of the monopoly from the innovation is longer. In smaller economies, prior to trade, the benefits from cost reduction outweigh the benefits from longer spells of monopoly profits, while in larger economies, ex post trade, the benefits from longer monopoly spells outweigh the benefits from cost reduction. Therefore, lowering the barriers to trade induces skill-biased technological change, thereby increasing the skill premium.

In the paper by Andersen (2005), a range of goods are produced. A country exports and imports goods according to its comparative advantages. Lower trading costs, i.e. globalization, polarizes the economy. On the one hand, foreign producers competing with domestic exporters are sheltered by trade costs. As trade costs diminish, the profit rates for exporting firms increase, and thereby also the wage rates in the export sector in the domestic country. On the other hand, domestic producers not exporting but competing with foreign firms on the domestic market face stiffer competition, and profit and wage rates therefore deteriorate. In addition, Andersen shows

that the number of goods produced in more than one country decreases due to lower trading costs, consequently decreasing the scope for unions to extract rents. While some workers are hurt by lower trading costs they are welfare enhancing and most workers benefit them.

The connection between market size and profitability of research resembles the analysis in the essay “Market Imperfections and Wage Inequality”, in Chapter II of this thesis, where instead of market size, more market power creates incentives for more development of new goods. In both cases research and development is high-skill intensive and greater a incentive for research and development benefits high-skill workers.

### 3 Institutions and Inequality

The third category of explanations focuses on *institutional changes*. Several studies point out the importance of institutions in explaining cross-country differences in wage inequality.<sup>5</sup> The degree of centralization in the wage setting process appears to be of significant importance (Blau and Kahn 1996; Wallerstein 1999). If so, it seems that institutional changes during the 1980s cannot be overlooked as explanations of changes in wage inequality.

The two most common factors discussed are minimum wage legislation and union membership rates. During the 1980 – 1990 period the real value of the U.S. minimum wage eroded, and interestingly the change in the minimum wage coincides with the change in inequality. Falling union membership rates is another potential explanation, since union membership is positively correlated with wages (Freeman 1982). A third category within the institutional explanations concerns deregulation and privatization (DiNardo et al. 1996; Fortin and Lemieux 1997; DiNardo and Lemieux 1997).

On theoretical grounds it is a priori impossible to determine whether unions increase or decrease wage inequality. If unionized workers are paid a union premium, a wage rate discrepancy is created *between* unionized and non-unionized workers. However, unions are also known to decrease the variation of wages *among* unionized workers (Freeman 1980). The latter effect always tends to decrease wage inequality, while the former tends to decrease wage inequality only if the likelihood of unionization is greater among low pay workers.

It is commonly accepted that unionized workers earn higher wages (Freeman 1982). How-

---

<sup>5</sup>Gottschalk and Joyce (1999) is an exception. They point out that countries that have experienced smaller changes in overall inequality often have changes in the education or skill premium or within group inequality that offset each other, leaving overall inequality unchanged (Gottschalk and Joyce 1999, p. 497). Further changes in the supply of different skill groups and age groups explain a large share of the difference of these offsetting changes in inequality (Gottschalk and Joyce 1999, p. 497-498).

ever, to what degree this union premium is due to a pure union effect or is the result of a selection bias where workers with some unobservable characteristic are more frequently unionized, is still an open question, since it has been difficult to reconcile the results using cross section and longitudinal estimation techniques (Robinson 1989).

Card (1998) concludes that changes in unionization patterns can explain 10 to 20 percent of the changes in wage inequality in the U.S. during the first half of the 1980s because unionization rates increased for higher paid workers, and decreased for lower paid workers. Because the union premium did not change during the period, this increased the dispersion in wages.

Deregulation in the early 1980s is a potential factor in explaining changes in inequality. Rose (1987) provides evidence that unionized workers in the regulated trucking industry earn an above average union premium in the U.S. Both Rose and Hirsch (1988) conclude that deregulation decreased rents in the trucking industry and thereby reduced the rents and wages captured by unionized workers.

Studies have pointed out the strong correlation between changes in the real value of the minimum wage and the wage dispersion among low paid workers, both in the U.S. (Lee 1999) and Great Britain (Machin and Manning 1994). The minimum wage can effect the wage distribution in different ways. If different workers are perfect substitutes, workers with insufficient productivity become unemployed. However, if different workers are perfect complements, the employer has no choice but to increase the wage rate for those workers paid wages below the minimum wage. In both cases the variation in wages must decrease. Also, changing the minimum wage might have spill-over effects on the wages of higher paid workers (Grossman 1983; Teulings 2000).

If a higher minimum wage decreases the variation in wages by increasing the unemployment rate, the problem faced by low pay workers is not solved. Instead poor employed workers become poor unemployed workers. However several studies in the U.S. have refuted the severe negative employment effects predicted by Stigler (1946), instead finding that the employment effects of the minimum wage are very small, see Katz and Krueger (1992); Card (1992); Card and Krueger (1994). The findings of Machin and Manning (1994) confirm these results for Great Britain .

In addition, Acemoglu (2003a) shows that it is possible that a higher minimum wage not only increases the wage of the lowest paid worker but also increases the productivity of low-skill workers. In Acemoglu's equilibrium unemployment model, a higher minimum wage decreases the rents captured by firms, but investing in new technology directed towards unskilled workers makes firms the residual claimant of productivity gains. As a result, minimum wages create

incentives for a higher rate of biased technology change, directed towards *unskilled* workers. However, the decreased wage inequality is not without cost. There is indeed some extra unemployment associated with a higher minimum wage.

It seems that for men, de-unionization and to some lesser extent the decreasing value of the minimum wage are important factors in explaining increasing male wage inequality (DiNardo et al. 1996; Fortin and Lemieux 1997; DiNardo and Lemieux 1997; Card 1998). For U.S. women the unionization rate was fairly stable during 1980s (Card 1998), and the falling real value of the minimum wage stands out as the primary institutional factor explaining increased inequality (DiNardo et al. 1996; Fortin and Lemieux 1997).

#### 4 Other Explanations

The presence of rents and its impact on wages are well documented,<sup>6</sup> (Katz and Summers 1989; Abowd and Lemieux 1993; Arai and Heyman 2001; Blanchflower et al. 1996). If rents affect the wage level, then it is reasonable to assume that the distribution of rents also affects the distribution of wages.

As mentioned above, Borjas and Ramey (1995) combine the bargaining idea with the trade argument. Abraham and Taylor (1996) briefly discuss the possibility that within larger and more heterogenous firms, equity motives play an important role in the wage determination process. Abraham and Taylor relate quite closely to the analysis in the essay “Firm Fragmentation and the Skill Premium” in Chapter III of this thesis, but they do not discuss how and by what mechanisms these equity considerations work.

Machin and Manning (1997) and Acemoglu (1999) present the hypothesis that increasing the supply of high-skill workers can increase the wage rate of high-skill workers by a change in the composition of jobs. The argument is straightforward. The optimal amount and or type of capital complementing high-skill and low-skill workers differ. It is assumed that firms are required to make investments in capital before they start searching for workers. Given an increase in the supply of high-skill workers, the probability of matching a high-skill vacancy with a high-skill worker increases. If the supply of high-skill workers is small, all firms make the same investment, optimal for low-skill workers.

---

<sup>6</sup>By noting that the wage-industry correlation in Sweden is much lower than in the U.S, using data from 1984, Edin and Zetterberg (1992) draw the conclusion that centralized wage bargaining decreases the impact of rents on wages. Using matched employer-employee data from 1991-1995, Arai and Heyman (2001) find evidence for rent sharing in Sweden. This shift in the importance of rents is intuitive, taking into account the breakdown of centralized bargaining in Sweden (Hibbs and Locking 1996).

If the supply of high-skill workers increases, the probability of matching a vacancy with a high-skill worker increase. If the supply of high-skill workers is large enough, some firms open vacancies for low-skill workers and some firms open vacancies for high-skill workers. This increases the productivity and hence the wage rate for high-skill workers.

Acemoglu et al. (2001) focus on the distribution of rents. They propose a model where high-skill and low-skill workers bargain over rents. Further, Acemoglu et al. assume that skill biased technological change increases high-skill workers gains from switching to specialized firms. This increases the outside option for high-skill workers, and also decreases the possibility for low-skill workers to specify redistributive wage contracts, leading to a lower unionization rate. Hence, the drop in unionization rates does not explain the increased skill-premium. Instead, both the increase in the skill premium *and* the drop in unionization rates are a consequence of skill biased technological change.

Rosén and Wasmer (2002) develop an unemployment equilibrium model where an increased supply of high-skill workers increases wage differentials. Since wages are set by Nash-bargaining, the firm's outside option is an important factor for the outcome. As the supply of high-skill workers increases, the firm's outside option increases, due to the increased value of a vacancy. Since the wage is proportional to output minus the firm's outside option, increases in the firm's outside option hurt unskilled workers relatively more than high-skill workers, and the skill premium increases.

Rosén and Wasmer obtain another appealing result by introducing firing costs into the model. Firing costs work in the opposite direction of the firm's outside option, i.e. the wage is proportional to the total output minus the firm's outside option *plus* the firing cost. Hence, with large firing costs, wage inequality decreases as the supply of high-skill workers increases. Countries in Europe are generally considered to have higher firing costs than the U.S., and Rosén and Wasmer's model indicates that differences in firing costs contribute to the milder changes in inequality in most European countries.

The models just mentioned above are based on non-competitive wage rates derived from bargains, usually between workers and firm owners. The "Firm Fragmentation and the Skill Premium" essay in Chapter III of this thesis moves the focus to wage bargains between high-skill and low-skill workers due to shut down threats by firm owners. Low-skill workers can partially make up for their relatively low marginal productivity by within-firm bargaining with high-skill workers. Outsourcing and contracting out cause high-skill and low-skill workers to be sorted into different firms and as a result, the possibility for low-skill workers to make up for low productivity via wage sharing bargains with high-skill workers in the same firm diminishes.

Consequently, fragmentation of production, even domestically, increases the skill premium.

## 4 Contributions

Several studies have exploited the relatively high skill intensity of research and development to explain the increasing skill premium. Globalization has increased the demand for high-skill workers because lower barriers to trade have increased market size and competition. Firms in industrialized countries have responded either with defensive innovations or by increasing the effort to develop new or improved goods, which in turn has increased the demand for and wage level of high-skill workers.

The essay in Chapter I of this thesis, “Risk, Occupational Choice, and Inequality”, relates to those explanations but exploits another property of research and development, namely its inherent uncertainty regarding future profits. In economics individuals are in general, assumed to be risk averse. This implies that if firms engaged in the risky activity of developing new products share risk with their employees, they must pay employees a premium to take on risk. Therefore, workers employed in the research/development sector are paid a risk premium relative to workers employed by firms producing an already existing good. Any exogenous change that increases the expected profitability of developing new goods will increase development employment and the average wage rate of high-skill workers, since a larger number of those workers are paid a risk premium.

In the analysis in Chapter I, increasing the supply of high-skill workers stimulates development, thereby increasing the share of high-skill workers doing development work. This has two immediate implications: First, more high-skill workers earn a risk premium. It is therefore indeed possible that the skill premium increases as the supply of high-skill workers increases. Second, as more high-skill workers work in the development sector where profits are uncertain, more high-skill workers earn a stochastic wage rate, and thereby the wage inequality among high-skill workers increases. The essay in Chapter I proposes a novel theory unifying the increase in the supply of high-skill workers with the increase in the skill premium and increased wage inequality among high-skill workers, which has been observed during the last two decades.

In Chapter II of this thesis, “Market Imperfections and Wage Inequality,” a continuous time framework is developed to investigate the impact of various market imperfections on the skill premium. In this analysis, consumer preferences for variety provide the market power necessary for development firms to cover the sunk costs associated with developing new variations of

goods. While other studies have used similar models to analyze separately a more narrow set of questions, this analysis simultaneously investigates the impact of consumer preferences for variety (which provide firms with market power), shorter product cycles, development externalities and capital market distortions on the skill premium. In addition, the model derives analytically tractable steady state equilibrium results. It also offers the possibility of easily adding more sophisticated representations of consumer intertemporal choices.

The analysis concludes that greater market power and shorter product cycles increase the skill premium. The results for the effect of capital market distortions are ambiguous, but improving heavily distorted capital markets, increases the skill premium. Further, the less rivalrous and less excludable the development, the lower the skill premium, since every development firm tries to free ride on every other development firm, which tends to decrease employment of high-skill development workers.

Globalization, or more specifically increased trade, was early recognized as a candidate for explaining the surge in wage inequality, and numerous models have been developed to formalize the arguments. Within this strand of the literature, papers concerning outsourcing to low-wage countries, or more generally, the disintegration of the production chain globally, are numerous. The essay, "Firm Fragmentation and the Skill Premium," in Chapter III of this thesis complements those papers by considering specialization and fragmentation of domestic firms, where the term fragmentation spans outsourcing as well as contracting out. The idea is simple. High-skill workers benefit from wages based on marginal productivity, because high-skill workers have a higher marginal productivity than low-skill workers. Low-skill workers benefit from wage bargaining where the outcome depends on several factors, and not only on marginal productivity.

Demand faced by each firm's good is stochastic. Firm owners occasionally threaten to shut down the firm, and workers re-negotiate wages to motivate the firm owner not to shut down the firm. If the production process becomes more fragmented, a larger fraction of firms employ only one type of worker, either high-skill or low-skill. This implies that even if the firm owner threatens to shut down the firm, high-skill and low-skill workers do not bargain with each other over wage rates. The possibility for low-skill workers to make up for low marginal productivity by bargaining with high-skill workers vanishes. It is further shown that if firm owners have limited ability to adjust employment due to long term wage contracts, yet have the opportunity to shut down the firm, increased demand uncertainty increases the size of the firm, in the steady state equilibrium.



## Bibliography

- 1 Abowd, J. A. and Lemieux, T. (1993). The effect of product market competition on collective bargain agreements: The case of foreign competition in Canada, *The Quarterly Journal of Economics* pp. 983–1014.
- 2 Abraham, K. G. and Taylor, S. K. (1996). Firm's use of outside contractors: Theory and evidence, *Journal of Labor Economics* 14(3): 394–424.
- 3 Acemoglu, D. (1998). Why do new technologies complement skills? Directed technological change and wage inequality, *The Quarterly Journal of Economics* 113(4): 1055–1089.
- 4 Acemoglu, D. (1999). Changes in unemployment and wage inequality: An alternative theory and some evidence, *The American Economic Review* 89(5): 1259–1278.
- 5 Acemoglu, D. (2002). Technical change, inequality, and the labor market, *Journal of Economic Literature* XL: 7–72.
- 6 Acemoglu, D. (2003a). Cross-country inequality trends, *The Economics Journal* 113(485): 121–149.
- 7 Acemoglu, D. (2003b). Patterns of skill premia, *Review of Economics Studies* 70: 199–230.
- 8 Acemoglu, D., Aghion, P. and Violante, G. L. (2001). Deunionization, technical change and inequality, *Carneige-Rochester Conference Series of Public Policy* 55(1): 229–264.
- 9 Aghion, P., Caroli, E. and García-Peñalosa, C. (1999). Inequality and economic growth: The perspectives of new growth theories, *Journal of Economic Literature* XXXVII: 1615–1660.
- 10 Andersen, T. M. (2005). Product market integration, wage dispersion and unemployment, *Labour Economics* 12: 379–406.
- 11 Arai, M. and Heyman, F. (2001). Wages, profits and individual unemployment risk: Evidence from matched worker-firm data, *Working Paper 172*, Trade Union Institute for Economic Research (FIEF).
- 12 Auerbach, P. and Skott, P. (2005). Wage inequality and skill asymmetries, in M. Setterfield (ed.), *Interactions in Analytical Political Economy: Theory, Policy and Applications*, The University of Chicago Press, pp. 27–54.

- 13 Autor, D. H., Katz, L. F. and Krueger, A. B. (1998). Computing inequality: Have computers changed the labor market?, *Quarterly Journal of Economics* 113(4): 1169–1214.
- 14 Bartel, A. P. and Lichtenberg, F. R. (1987). The comparative advantage of educated workers in implementing new technology, *The Review of Economics and Statistics* LXIX(1): 1–11.
- 15 Blanchflower, D. G., Oswald, A. J. and Sanfey, P. (1996). Wages, profits, and rent-sharing, *The Quarterly Journal of Economics* 111(1): 227–251.
- 16 Blau, F. D. B. and Kahn, L. M. (1996). International differences in male wage inequality: Institutions versus market forces, *The Journal of Political Economy* 104(4): 791–837.
- 17 Borjas, G. J. and Ramey, V. A. (1995). Foreign competition, market power, and wage inequality, *The Quarterly Journal of Economics* pp. 1075–1110.
- 18 Caminada, K. and Goudswaard, K. (2001). International trends in income inequality and social policy, *International Tax and Public Finance* 8: 395–415.
- 19 Card, D. (1992). Do minimum wages reduce employment, *Industrial and Labor Relations Review* 46(1): 38–54.
- 20 Card, D. (1998). Falling unionism and rising wage inequality: What’s the connection?, *Working Paper 6520*, NBER, 1050 Massachusetts Avenue Cambridge, MA 02138.
- 21 Card, D. and Krueger, A. B. (1994). Minimum wages and employment: A case study of the fast-food industry in New Jersey and Pennsylvania, *The American Economic Review* 84(4): 772–793.
- 22 Caselli, F. (1999). Technological revolutions, *American Economic Review* 89(1): 78–102.
- 23 DiNardo, J. and Lemieux, T. (1997). Diverging male wage inequality in the United States and Canada, 1981-1988: Do institutions explain the differences?, *Industrial and Labor Relations Review* 50(4): 629–651.
- 24 DiNardo, J., Fortin, N. M. and Lemieux, T. (1996). Labor market institutions and the distribution of wages, 1973–1972: A semiparametric approach, *Econometrica* 64(5): 1001–1014.
- 25 Dinopoulos, E. and Segerström, P. (1999). A Schumpeterian model of protection and relative wages, *The American Economic Review* 89(3): 450–472.

- 26 Edin, P.-A. and Topel, R. (1997). Wage policy and restructuring: The Swedish labor market since 1960, in R. B. Freeman, R. Topel and B. Swedenborg (eds), *The Welfare State in Transition*, The University of Chicago Press, chapter 4, pp. 155–201.
- 27 Edin, P.-A. and Zetterberg, J. (1992). Interindustry wage differentials: Evidence from Sweden and a comparison with the United States, *The American Economic Review* 82(5): 1341–1349.
- 28 Ekholm, K. and Midelfart, K. H. (2005). Relative wages and trade-induced changes in technology, *European Economic Review* 49: 1637–1663.
- 29 Ervik, R. (1998). The redistributive aim of social policy. A comparative analysis of taxes, tax expenditure transfers and direct transfers in eight countries, *LIS Working Paper Series 184*.
- 30 Feenstra, R. C. and Hanson, H. G. (1999). The impact of outsourcing and high-technology on wages: Estimates for the United States, 1979-1990, *Quarterly Journal of Economics* 114(3): 907–1003.
- 31 Förster, M. and Pearson, M. (2002). Income distribution and poverty in the OECD area: Trends and driving forces, *OECD Economic Studies* I(34): 7–39.
- 32 Förster, M. F. and Pellizzari, M. (2000). Trends and driving factors in income distribution and poverty in the OECD area, *Labour Market and Social Policy - Occasional Papers 42*, OECD.
- 33 Fortin, M. N. and Lemieux, T. (1997). Institutional change and rising wage inequality: Is there a linkage?, *Journal of Economic Perspectives* 11(2): 75–96.
- 34 Freeman, R. B. (1980). Unionism and the dispersion of wages, *Industrial and Labor Relations Review* 34(1): 3–23.
- 35 Freeman, R. B. (1982). Union wage practices and wage dispersion, *Industrial and Labor Relations Review* 36(1): 3–21.
- 36 Fritzell, J. (2001). Inkomstfördelningens trender under 1990-talet, *SOU 2001:57*, chapter 3, pp. 131–187.
- 37 Galor, O. and Moav, O. (2000). Ability-biased technological transition, wage inequality, and economic growth, *The Quarterly Journal of Economics* 115(2): 469–407.
- 38 Gottschalk, P. (1997). Inequality, income growth, and mobility: The basic facts, *Journal of Economic Perspectives* 11(2): 21–40.

- 39 Gottschalk, P. and Joyce, M. (1999). Cross-national differences in the rise in earnings inequality: Market and institutional factors, *The Review of Economics and Statistics* LXXX(4): 489–502.
- 40 Gottschalk, P. and Smeeding, T. M. (1997). Cross-national comparisons of earnings and income inequality, *Journal of Economic Literature* XXXV: 633–687.
- 41 Gottschalk, P. and Smeeding, T. M. (2000). Empirical evidence on the income inequality in industrialized countries, in A. Atkinson and F. Bourguignon (eds), *Handbook of Income Distribution*, first edn, Vol. 1, Elsevier Science, chapter 5.
- 42 Griliches, Z. (1969). Capital-skill complementarity, *The Review of Economics and Statistics* 51(4): 465–468.
- 43 Grossman, J. B. (1983). The impact of the minimum wage on other wages, *The Journal of Human Resources* 18(3): 359–378.
- 44 Gustafsson, B. and Palmer, E. (1997). Changes in Swedish inequality, in P. Gottschalk, B. Gustafsson and E. Palmer (eds), *Changing Patterns in the Distribution of Economic Welfare: An Economic Perspective*, Cambridge University Press, pp. 293–325.
- 45 Gustafsson, B. and Palmer, E. (2001). 90-talets inkomstklyftor – hur och varför de ökade, *Ekonomisk Debatt* 29(7): 487–498.
- 46 Helpman, E. (1984). A simple theory of international trade with multinational corporations, *Journal of Political Economy* 92(3): 451–471.
- 47 Hibbs, Jr., D. A. (1991). Market forces, trade union ideology and trends in Swedish wage dispersion, *Acta Sociologica* 34: 89–102.
- 48 Hibbs, Jr., D. A. and Locking, H. (1996). Wage compression, wage drift and wage inflation in Sweden, *Labour Economics* 3(2): 109–141.
- 49 Hibbs, Jr., D. A. and Locking, H. (2000). Wage dispersion and productive efficiency: Evidence for Sweden, *Journal of Labor Economics* 18(4): 755–782.
- 50 Hildreth, A. K. G. and Oswald, A. J. (1997). Rent-sharing and wages: Evidence from company and establishment panels, *Journal of Labor Economics* 15(2): 318–337.

- 51 Hirsch, B. T. (1988). Trucking regulation, unionization, and labor earnings: 1973-85, *The Journal of Human Resources* 23(3): 296–319.
- 52 Jansson, K. (2000). Inkomstfördelningen under 1990-talet, *SOU 2000:40 Valfärd och Försörjning*, Socialdepartementet, chapter 3, pp. 15–55.
- 53 Juhn, C., Murphy, K. M. and Brooks, P. (1993). Wage inequality and the rise in returns to skill, *Journal of Political Economy* 101(3): 410–442.
- 54 Katz, L. F. and Krueger, A. B. (1992). The effect of minimum wage on the fast-food industry, *Industrial Labor Relations Review* 46(1): 6–21.
- 55 Katz, L. F. and Murphy, K. M. (1992). Changes in the relative wages, 1963-1987: Supply and demand factors, *The Quarterly Journal of Economics* 107(1): 35–78.
- 56 Katz, L. F. and Summers, L. H. (1989). Industry rents: Evidence and implications, *Brookings Papers: Microeconomics* pp. 209–275.
- 57 Kiley, M. T. (1999). The supply of skilled labour and skill-biased technological progress, *The Economic Journal* 109(459): 708–724.
- 58 Krueger, A. B. and Summers, L. H. (1988). Efficiency wages and the inter-industry wage structure, *Econometrica* 56(2): 259–293.
- 59 Krusell, P., Ohanian, L. E., Rìos-Rull, J.-V. and Violante, G. L. (2000). Capital-skill complementary and inequality: A macroeconomic analysis, *Econometrica* 68(5): 1029–1053.
- 60 Laitner, J. (2000). Earnings within education groups and overall productivity growth, *Journal of Political Economy* 108(4): 807–832.
- 61 Lee, D. S. (1999). Wage inequality in the United States during the 1980s: Rising dispersion or falling minimum wages?, *The Quarterly Journal of Economics* 114(3): 977–1023.
- 62 Machin, S. and Manning, A. (1994). The effects of minimum wages on wage dispersion and employment: Evidence from the U.K. wage councils, *Industrial and Labor Relations Review* 47(2): 319–329.
- 63 Machin, S. and Manning, A. (1997). Can supply create its own demand? Implications for rising skill differentials, *European Economic Review* 41(3-5): 507–516.

- 64 Mendez, R. (2002). Creative destruction and the rise of inequality, *Journal of Economic Growth* 7: 259–281.
- 65 Neary, J. P. (2002). Foreign competition and wage inequality, *Review of International Economics* 10(4): 680–693.
- 66 Nelander, S. and Goding, I. (2004). *Inkomster och Förmögenheter*, Landsorstorganisationen i Sverige. ISBN 91-566-2050-0.
- 67 Ranjan, P. (2001). Productivity growth and wage inequality: The role of the product cycle and international trade, *Working Paper*, University of California, Irvine.
- 68 Robinson, C. (1989). The joint determination of union status and union wage effects: Some tests of alternative models, *The Journal of Political Economy* 97(3): 639–668.
- 69 Rose, N. L. (1987). Labor rent sharing and regulation: Evidence from the trucking industry, *The Journal of Political Economy* 95(6): 1146–1178.
- 70 Rosén, Å. and Wasmer, E. (2002). Higher education levels, firms' outside options and the wage structure, *Discussion Paper 420*, The Institute for the Study of Labor (IZA). Forthcoming in *Review of Labor Economics and Industrial Relations*.
- 71 Sachs, J. D. and Shatz, H. J. (1994). Trade and jobs in U.S. manufacturing, *Brookings Papers on Economic Activity* (1): 1–84.
- 72 Slichter, S. H. (1950). Notes on the structure of wages, *The Review of Economic and Statistics* 32(1): 80–91.
- 73 Smeeding, T. M. (2002). Globalization, inequality and the rich countries of G-20: Evidence from the Luxembourg income study(LIS), in D. Gruen, and T. O'Brien (eds), *Globalization, Living Standards, and Inequality: Recent Progress and Continuing Challenges*, Reserve Bank of Australia, pp. 27–54.
- 74 Stigler, G. J. (1946). The economics of minimum wage legislation, *The American Economic Review* 36(3): 358–365.
- 75 Stolper, W. F. and Samuelson, P. A. (1941). Protection and real wages, *Review of Economic Studies* 9(1): 58–73.

- 76** Teulings, C. N. (2000). Aggregation bias in elasticities of substitution and the minimum wage paradox, *International Economic Review* 41(2): 359–398.
- 77** Thoenig, M. and Verdier, T. (2003). A theory of defensive skill-biased innovation and globalization, *The American Economics Review* 93(3): 709–728.
- 78** Vernon, R. (1966). International investment and international trade in the product cycle, *The Quarterly Journal of Economics* 80(2): 190–207.
- 79** Wallerstein, M. (1999). Wage-setting and pay inequality in advanced industrial societies, *American Journal of Political Science* 43(3): 649–680.
- 80** Wood, A. (1995). How trade hurt unskilled, *Journal of Economic Perspectives* 9(3): 57–80.
- 81** Wood, A. (1998). Globalisation and the rise in the labour market inequalities, *The Economic Journal* 108: 1463–1482.

# Essay I

## Risk, Occupational Choice, and Inequality

### 1 Introduction

There is a growing consensus that over the last 25 years, the dispersion of income and wages have increased in developed countries. The U.S. and the U.K experienced a larger dispersion earlier than most other countries but later several other countries fell in line. Those changes are well documented by among others Förster and Pearson (2002). The dispersion of wages can be decomposed into dispersion among individuals with similar characteristics (residual wage inequality) and dispersion between individuals with different characteristics, such as for example skill, experience or gender.

In order to briefly exemplify the changes, consider the changes in the U.S. during the period 1973 to 1989. The wage rate for the 10 percent at the top of the wage distribution increased by approximately 20%, greatly surpassing the growth rate of wages at the median, which was approximately 5%. Even more strikingly, the wage rate for the poorest 10% *decreased* by about 25% during the same period (Juhn et al. 1993). Those divergent trends for the wage rates of the lowest paid and the highest paid workers are likely to have a profound impact on income. This is confirmed by Gottschalk, reporting that the real income ratio between the 80th and 20th percentiles in the distribution shows a clear upward trend in the period 1968 to 1992 (Gottschalk 1997, p. 23).

This paper focuses on two components of inequality. On the one hand, the distribution of wages *between* individuals with different skill levels. On the other hand, the distribution of wages *among* high-skill workers.

The dispersion of wages between high-skill workers and low-skill workers in the U.S. has



increased. Gottschalk's annual (log) wage regressions, including a dummy variable for college graduates shows a decreasing trend during the first part of the period, 1970 – 1980, and there after an increasing trend during the 1980s (Gottschalk 1997). However, as the difference in wages *between* college graduates and high school graduates increased, wage dispersion *among* both groups' members also widened. There is a very small fraction at the bottom of the wage distribution of college graduates that experienced decreased real wages whereas college graduates at the top of the distribution gained more than 20 percent in real terms (Juhn et al. 1993, p. 422, fig. 6).

## 1.1 Contribution

The central idea in this paper is that workers can make an active choice concerning risk exposure. Exposing oneself to more risk in the model is considered a substitute for decreased wage earnings. Endogenous choices of occupation, i.e. whether or not to expose oneself to risk, changes the economy's distribution of wages. Workers characterized by relative risk aversion would never substitute lower earnings for increased risk unless paid a risk premium. Hence there must exist a sector in the economy to which workers can switch and which is characterized by higher but more uncertain returns. The research sector is assumed to be such a sector in this paper.

By this approach, this paper draws heavily on the literature on entrepreneurship, which can be traced back to the writings of Knight (1921). Knight's ideas have been formalized, at least partially, by Kanbur (1979) and Kihlstrom and Laffont (1979). A fundamental property of those models is that entrepreneurs bear risk. Workers chose to become entrepreneurs only if the *expected* utility of being an entrepreneur exceeds the *certain* utility of ordinary work with a certain wage rate. The difference in this paper is that entrepreneurs are high-skill workers that form co-operatives, and hence do not employ workers.

A second strand of literature which this paper relies on is the endogenous growth models where growth is driven by development of new intermediate goods. This idea is formalized by Romer (1990). The model presented in this paper augments Romer's model by introducing stochastic development of new intermediate goods.

The model postulates to two main characteristics of research activity. First, it is assumed that only high-skill workers can work in the research sector. This is a crude enforcement of the assumption that research is human capital intensive (Barro and Xavier 1995, p. 179) and hence high-skill workers have an advantage over low-skill workers.

Second, research is stochastic. Researchers directly face uncertainty concerning the products ex post productivity or ex post capacity to generate utility and therefore its value. It is common to model firms as risk neutral. Risk neutral firms maximize expected profits and, hence, Pareto optimality implies that risk averse agents negotiate wage contracts with no uncertainty. This paper models research firms as co-operatives. The members share the revenues and the firm's decisions are determined by the representative member, trying to maximize his or her utility.

The choice to model research firms as co-operatives is not to be taken literally. Co-operatives are used in order to keep the analysis simple and emphasize research firms' need to share risk with their employees. Risk sharing between firms and employees can take on several shapes ranging from for example flexible working hour arrangements to profit bonuses and options programs where employees are offered stock shares.

The model abstracts from all kinds of risk sharing by financial markets. Not allowing any insurance possibilities via financial markets unrealistic but this assumption is made to simplify the model. The key assumption is that firms and workers can benefit from risk sharing. It is however too important to recognize that precluding risk-sharing can have strong implications. As is shown by Newman (1999), combining the standard theory of entrepreneurship (Kanbur 1979; Kihlstrom and Laffont 1979) with moral hazard considerations and some risk sharing can reverse some of the standard results. Since the analysis is very similar to the standard entrepreneur theory some caution should be applied.

Within the framework in this paper, increasing the supply of high-skill workers shifts high-skill workers into research co-operatives, paying a stochastic wage rate. On the one hand, more high-skill workers earn a stochastic wage rate, increasing the residual wage inequality among high-skill workers. On the other hand more high-skill workers earn a risk premium, tending to increase the average wage rate for high-skill workers. To summarize, the two main hypotheses investigated in this paper are:

1. An increased supply of high-skill workers *increases* the wage dispersion *among* high-skill workers.
2. An increased supply of high-skill workers *increases* the wage dispersion *between* high-skill and low-skill workers.

The first hypothesis refers to the residual wage inequality for high-skill workers, while the second hypothesis concerns the skill premium.

## 1.2 Related Literature

The analysis in this paper does not fit into any of the three broad categories generally used to explain changes in the distribution, namely skill-biased technological change (Acemoglu 1998; Krusell et al. 2000), increased trade (Aghion et al. 1999; Borjas and Ramey 1995; Wood 1995, 1998), or institutional change (DiNardo and Lemieux 1997; Fortin and Lemieux 1997).

The idea that an increased supply of high-skill workers can *increase* the skill premium is not new. In Acemoglu (1998), and also Kiley (1999), an increased supply of high-skill workers can increase the skill premium due to an increased market size for inventions, *directed towards* high-skill workers, possibly increasing the skill premium in the long run. Acemoglu's paper is remotely connected to this paper in the sense that the skill premium increases due to changes in the research process.

In Machin and Manning (1997) and Acemoglu (1999), increasing the supply of high-skill workers motivates firms to open vacancies tailored to high-skill workers, thereby increasing the productivity of high-skill workers relative to low-skill workers. This in turn increases the relative wage for high-skill workers.

In the paper by Rosén and Wasmer (2002), the skill premium is positively correlated with the relative supply of high-skill workers due to the increased outside option of firms in their wage negotiations. In Rosén and Wasmer's paper wages are determined in Nash-bargains and increases in firms' outside option hurts low paid (i.e. low-skill) workers more than high-skill workers, increasing the skill premium.

It is reasonable to assume that there is an asymmetry between high-skill and low-skill workers. While high-skill workers can occupy low-skill jobs, low-skill workers cannot occupy high-skill jobs. Auerbach and Skott (2005) investigate the impact of a skill neutral productivity slow down given this asymmetry. As productivity decreases, more high-skill workers occupy low-skill jobs and residual wage inequality of high-skill workers increases.

A somewhat similar argument is presented in Mendez (2002). For incentive reasons, workers producing goods in the early stage of the product cycle are paid efficiency wages, while workers in the later stage are paid competitive wages. This creates a wage gap between workers in the early and late stage of the product cycle, and workers that fail to find work producing products in the early stage earn a lower wage rate.

The paper is organized as follows. Section 2 describes the formal model. Section 3 summarizes the results obtained. Section 4 concludes and summarizes the findings. Appendix I.A

contains a full record of notation. The subsequent appendix contain proofs and derivations.

## 2 Model

The model used to study the occupational choices of high-skill workers originates from Romer (1990)'s endogenous growth model. Romer's model is characterized by horizontal innovations, that is new innovations do not replace innovations made earlier but complement them. A vertical innovation process is modeled in, for example, Aghion and Howitt (1992). In their model inventions lead to monopoly power, but by creative destruction, any new invention erodes the previous monopolist's profit, whereas in Romer's model, monopoly profits last forever. This paper simplifies Romer's model by assuming that monopoly profits are eroded exogenously after one period and leaving out the distinction between designs and intermediate goods.

Leaving out the distinction between intermediate goods and designs simplifies the presentation of the model by reducing the number of concepts but can also be confusing. It is important to realize that already produced intermediate goods can not be stored and used in the subsequent period. Therefore, at the beginning of each period the number of intermediate goods ready to be used in production is zero. However there exists a variety of old intermediate goods that can be produced, i.e. in Romer's words, there exists a variety of designs for intermediate goods.

Workers live for one period and consume their entire wage. They derive utility from consuming  $w$  units of the single consumption good. Preferences over consumption are described by expected utility under the CRRA utility function:

$$u(w) = \frac{w^{1-\theta}}{1-\theta} \quad \theta > 0.$$

At the expense of realism but for the benefit of simplicity all forms of non fully depreciating capital are excluded from the model. However, the model includes capital that is fully used up in the production process and therefore called *intermediate goods*.

The economy's total endowment of labor is normalized to unity. Workers can be divided into two categories depending on the worker's level of human capital, high-skill workers and low-skill workers. The fraction of high-skill workers is denoted  $\phi$ , and hence  $1 - \phi$  denotes the fraction of low-skill workers. The fraction of the labor force being considered high-skill is taken to be exogenous. While in reality,  $\phi$  is endogenous in this analysis  $\phi$  is exogenous. This is a reasonable assumption as long as the changes in the skill composition of the work force is slow relative to other responses. This seems reasonable in the present context, since switching

occupation can be done several times during a lifetime while human investment in general are made once at young age.

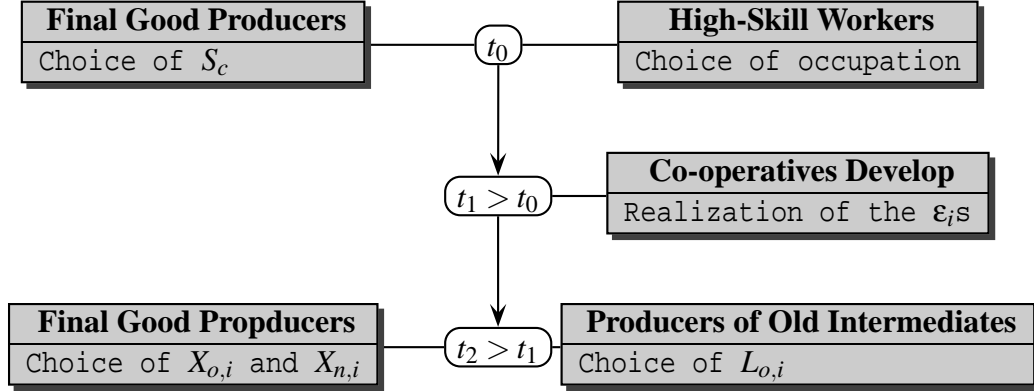
It will be assumed that high-skill workers can replace low-skill workers, but not the opposite. This implies that high-skill workers never earn less than low-skill workers in equilibrium. If wages for low-skill workers were higher than wages for high-skill workers, some high-skill workers would switch to the low-skill occupation until both groups' wages were equalized. It will be assumed that there are enough low-skill workers, to ensure that wages of high-skill workers are higher than wages of low-skill workers.

The sectors in the economy can be divided into two categories depending on what they produce. One of the sectors produce the single consumption good by employing high-skill workers and purchasing a variety of fully depreciating intermediate goods. The other sector produces the set of intermediate goods. While low-skill workers only have the possibility to work in the intermediate goods sector, high-skill workers have the possibility to work in either sector, either as a worker in the consumption good sector or as a member of a co-operative in the intermediate goods sector. Co-operatives invent and produce new intermediate goods. A larger variety of intermediate goods increases the economy's total output, i.e. generates growth. Research co-operatives invent new intermediate goods and sell these to consumption good producers. Therefore the number of intermediate goods is endogenous. The gains from inventing a new intermediate good arise due to the one period monopoly profit derived from selling it to the producers of final goods. The share of high-skill workers doing research is denoted  $\mu$ .

Intermediate goods are categorized as *old* or *new*. A new intermediate good is considered as an old intermediate good the subsequent period. Hence a research co-operative have monopoly for one period. The number of old intermediate goods, denoted by  $k_o$ , is predetermined while the number of new intermediate goods, denoted by  $k_n$ , is endogenous. Old intermediate goods are available at the beginning of the period and produced by low-skill labor while new intermediate goods are invented and produced by high-skill workers in the research sector.  $X_{o,i}$  denotes the quantity of the  $i$ th old intermediate good and  $X_{n,i}$  denotes the quantity of the  $i$ th new intermediate good.

Since high-skill workers can choose to work in the stochastic research sector and invent and produce new intermediate goods, at the time of production the set of intermediate goods is extended by those newly invented intermediate goods, i.e.  $k_n$  newly invented intermediated goods can be used by consumption good producers. Hence, at the time of production, consumption good producers can combine  $k_o + k_n$  different intermediate goods to produce the consumption

Figure I.1: Choice Sequence



The figure shows the sequence of choices, within a single period.

good.

## 2.1 The Consumption Good Sector

The model's single consumption good is produced by combining high-skill labor, and intermediate goods. Hence each firm producing the consumption good must decide upon hiring a certain *quantity* of high-skill labor, denoted  $S_c$ , and purchase a certain *quantity* of each available intermediate good, i.e. chose values for all members of the set  $\mathcal{X} = \{X_{o,i} \mid i = 1, 2, \dots, k_o\} \cup \{X_{n,i} \mid i = 1, 2, \dots, k_n\}$ . The production function obeys to the standard properties, such as diminishing marginal productivity in each input and constant returns to scale. These properties eliminate profits in the consumption good sector. The old intermediate good  $i$ 's productivity is measured by  $\gamma_i$  and the new intermediate good  $i$ 's productivity is measured by  $\epsilon_i$ . Formally the technology is described by:

$$Y = S_c^\alpha \left[ \sum_{i=1}^{k_o} \gamma_i X_{o,i}^{1-\alpha} + \sum_{i=1}^{k_n} \epsilon_i X_{n,i}^{1-\alpha} \right] \quad \alpha \in (0, 1). \quad (\text{I.1})$$

The constant returns to scale property and perfect competition implies that firms in the consumption good sector can be modeled as a single price taking firm. In what follows, let  $P_{o,i}$  denote the price of the  $i$ th old intermediate good, let  $P_{n,i}$  denote the price of the  $i$ th new intermediate good, and let  $w_c$  denote the wage rate for high-skill workers employed in the consumption good sector. Competitive behavior implies that prices of intermediate goods and the wage rate

are taken as given by producers of the final good.

In every period, a sequence of choices are made by different agents. The order in which choices are made is illustrated in Figure I.1. At the start of every period, high-skill workers choose either to work for firms producing the final good or to start a co-operative and final good producers choose the amount of high-skill workers,  $S_c$ , to employ.

Next, every co-operative develops a new intermediate good and its productivity is revealed, i.e. the value of  $\varepsilon_i$  is revealed. Finally, final good producers choose the quantity of every intermediate input to use, and producers of old intermediate goods hire low-skill labor.

The objective function for the competitive final good producer is

$$\begin{aligned} \pi &= S_c^\alpha \left[ \sum_{i=1}^{k_o} \gamma_i X_{o,i}^{1-\alpha} + \sum_{i=1}^{k_n} \varepsilon_i X_{n,i}^{1-\alpha} \right] \\ &- w_c S_c - \sum_{i=1}^{k_o} P_{o,i} X_{o,i} - \sum_{i=1}^{k_n} P_{n,i} X_{n,i}, \end{aligned} \quad (I.2)$$

where the price of the consumption good is normalized to unity. Profit maximizing behavior yields the following inverse factor demand functions:

$$P_{o,i} = (1 - \alpha) \gamma_i \frac{S_c^\alpha}{X_{o,i}^\alpha} \quad (I.3a)$$

$$P_{n,i} = (1 - \alpha) \varepsilon_i \frac{S_c^\alpha}{X_{n,i}^\alpha} \quad (I.3b)$$

$$w_c = \frac{\alpha Y}{S_c} = \alpha S_c^{\alpha-1} \left[ \sum_{i=1}^{k_o} \gamma_i X_{o,i}^{1-\alpha} + \sum_{i=1}^{k_n} E \varepsilon_i X_{n,i}^{1-\alpha} \right]. \quad (I.3c)$$

To obtain those first order conditions, first maximize the objective function in (I.2), given the realizations of every  $\gamma_i$  and  $\varepsilon_i$ , with respect to every  $X_{o,i}$  and every  $X_{n,i}$ , taking  $S_c$  as given. Those first order conditions are the profit maximizing choices corresponding to node  $t_2$  in Figure I.1, for *any* choice of  $S_c$  and *any* realization of the productivity variables, i.e. the different  $\varepsilon$ 's.

At the  $t_0$  node in Figure I.1, the price taking producer of the final good maximizes the expected value of the objective function in (I.2), where every occurrence of  $X_{o,i}$  and  $X_{n,i}$  have been replaced, using the first order conditions in (I.3a) and (I.3b). This first order condition implies a zero profit condition, but by, again, using the first order conditions in (I.3a) and (I.3b) to eliminate every  $P_{o,i}$  and  $P_{n,i}$ , the first order condition in (I.3c) is obtained. Those first order

conditions define the cost minimizing mix of high-skill labor and intermediate goods. Since the technology is characterized by constant returns to scale and the firm act as a price taker, the scale of production is not determined by the first order conditions but from a set of equilibrium market clearing conditions.

The choice of technology ensures that high-skill workers in the consumption good sector share a constant fraction,  $\alpha$ , of total output. The inverse demand functions are intuitive, a more productive intermediate good,  $\gamma$  or  $\varepsilon$  large, increases the expenditure on the intermediate good.

## 2.2 The Intermediate Goods Sector

The knowledge necessary to produce old intermediate goods is freely available to all workers and hence it is most appropriate to model the market for old intermediate goods as a perfectly competitive market with zero profits. The knowledge necessary to produce new intermediate goods is only available for the workers that developed the new intermediate. Hence, a co-operative that develops a new intermediate good becomes the sole producer of that good. Therefore the market for a new intermediate good is characterized by monopoly.

### Old Intermediate Goods

Old intermediate goods are produced by low-skill labor. To keep the analysis simple it is assumed that one unit of low-skill labor produces one unit of the intermediate good. Formally:

$$X_{o,i} = L_{o,i}. \quad (\text{I.4})$$

The linear technology and the competitive market implies that the size of the firm is indeterminate but the industry can be modeled as if there is a single competitive firm. Hence the production of a specific old intermediate good  $X_{o,i}$  is modeled as such. The profit maximization problem for a competitive firm producing an old intermediate  $i$  is:

$$\max_{L_{o,i}} P_{o,i}L_{o,i} - w_oL_{o,i}. \quad (\text{I.5})$$

The price taking firm takes  $P_{o,i}$  and  $w_o$  as given but chooses the quantity of low-skill labor,  $L_{o,i}$ , to employ. The first order condition for this problem ensures zero profit, but does not, as noted



above, pin down the number of employees in the firm. The first order/zero profit condition is:

$$w_o = P_{o,i}. \quad (\text{I.6})$$

In order to find the number of employees engaged in producing an old intermediate good  $i$ , combine the zero profit equation (I.6) with the inverse demand function for old intermediates, given by (I.3a). Total output and total employment in a industry producing the old intermediate good  $X_{o,i}$  become:

$$X_{o,i} = L_{o,i} = S_c \left[ \frac{(1-\alpha)\gamma_i}{w_o} \right]^{\frac{1}{\alpha}}. \quad (\text{I.7})$$

Wages for low-skill workers must be equal across all firms producing intermediate goods since there is no stickiness in the economy. Therefore, it is not necessary to index the wage rate for low-skill workers by the specific old intermediate good they produce. As seen by (I.6), all old intermediate goods are sold at the same price, but more productive intermediate goods are sold in larger quantities, see (I.7). Hence it is not necessary to index the price of old intermediates. Henceforth  $P_o$  will denote the price of any old intermediate good.

### New Intermediate Goods

A co-operative producing a new intermediate good must spend a fixed amount,  $l$ , of labor units in order to develop the new intermediate good. Ex ante, research co-operatives cannot perfectly foresee the ex post productivity of the intermediate good they plan to develop. Hence there is some uncertainty concerning future revenues. Formally the uncertainty is modeled by the log-normal random variable  $\varepsilon_i$ . For shorter notation,  $\sigma^2$  will be used to denote the variance of  $\varepsilon_i$ , i.e.  $\text{var}(\varepsilon_i)$ . Hence:

$$\ln \varepsilon_i \sim N \left( \ln \left[ \{E\varepsilon\}^2 \right] - \frac{1}{2} \ln \left[ \{E\varepsilon\}^2 + \sigma^2 \right], \ln \left[ 1 + \frac{\sigma^2}{\{E\varepsilon\}^2} \right] \right). \quad (\text{I.8})$$

Besides being non-negative, the log-normal distribution is chosen because its mathematical properties makes it easy to work with. The intuitive assumption that development requires some high-skill labor,  $l > 0$  introduces the fixed cost necessary to assure that co-operatives do not produce an infinitely small output.

Once the new intermediate good is developed it takes one unit of high-skill labor to produce

one unit of the new intermediate good. Formally the technology for research firms are:

$$X_{n,i} = S_{n,i} - l. \quad (\text{I.9})$$

It is assumed that research firms are managed as co-operatives where total revenue is distributed uniformly among its members. Since research firms hire only high-skill workers, and all high-skill workers are identical, each member has the same objective. The objective of the co-operative is described by the maximization of the expected utility of the representative member. The co-operative's revenue is the quantity produced times the price. The price is given by the inverse demand function in (I.3b)

The maximization problem for the representative co-operative member is:

$$\begin{aligned} \max_{S_{n,i}, P_{n,i}, X_{n,i}} \quad & Eu \left[ \frac{P_{n,i} \times X_{n,i}}{S_{n,i}} \right] \\ \text{s.t.} \quad & (\text{I.3b}), (\text{I.9}). \end{aligned} \quad (\text{I.10})$$

The first constraint, (I.3b), ensures that the research co-operative's price-quantity combination lies somewhere on the demand schedule of final good producers. The second constraint, (I.9), ensures that the research firm uses the only feasible production technology. Solving problem (I.10) defines the optimal co-operative size for research co-operatives,  $S_{n,i}$ , the optimal quantity to produce,  $X_{n,i}$ , the monopoly price,  $P_{n,i}$  and the wage for the co-operative member,  $w_{n,i}$ :

$$S_{n,i} = \frac{l}{\alpha} \quad (\text{I.11a})$$

$$X_{n,i} = \frac{1-\alpha}{\alpha} \times l \quad (\text{I.11b})$$

$$P_{n,i} = (1-\alpha)^{1-\alpha} \left[ \frac{\alpha S_c}{l} \right]^\alpha \varepsilon_i \quad (\text{I.11c})$$

$$w_{n,i}(\varepsilon_i) = \alpha^\alpha (1-\alpha)^{2-\alpha} \left[ \frac{S_c}{l} \right]^\alpha \varepsilon_i. \quad (\text{I.11d})$$

The derivations are shown in Appendix B.1. Note that the co-operative's size,  $S_{n,i}$ , and quantity produced,  $X_{n,i}$  is independent of  $i$ . This is intuitive, all research firms are identical ex-ante. Hence, the size of the co-operative, which is determined before the productivity of the new intermediate is realized, is equal across all research co-operatives. Further, since the co-operative

members share the revenues all members are engaged in the production process, unconditioned on the ex-post productivity,  $\varepsilon_i$ . Hence, the quantity produced is equal across all research co-operatives. However, depending on the productivity, each co-operative sell their intermediate good at a different price and hence earn idiosyncratic revenues.

### 2.3 Occupational Choice

Based on the assumption that research is a human capital intensive activity, the model precludes low-skill workers to take any part in the economy's research process. High-skill workers, on the other hand, have the opportunity to work in the consumption sector earning a certain wage or to work in the research sector as a member of a research co-operative. A rational worker chooses the occupation that yields the highest expected utility. Since all high-skill workers have identical endowments and identical preferences all high-skill workers make the same occupational choice unless they are indifferent between the two choices.

It is possible to have an equilibrium where all high-skill workers are employed in the consumption good sector. With no research sector this model becomes a standard two sector model and nothing new is added. Hence, it is assumed that if all high-skill workers are employed in the consumption good sector the expected utility of forming a research co-operative will exceed the expected utility of working in the consumption good sector, formally:

$$u(w_c) \Big|_{S_c = \phi} < Eu[w_{n,i}(\varepsilon_i)] \Big|_{S_c = \phi} . \quad (\text{I.12})$$

With this additional assumption it is possible to show that a feasible equilibrium requires that the expected utility of working in either the research sector or in the consumption good sector is equal. *It is very important to note that this implies that every result derived later, must be checked against this condition, or equivalently, that the share of high-skill workers doing research,  $\mu$ , is greater than zero.*

If the share of high-skill workers doing research work, which is determined endogenously, is solved for and turns out negative, i.e.  $\mu < 0$ , then the utility of working in the consumption good sector exceeds the expected utility working in the research sector.

If the expected utility of working in the consumption good sector exceeds the expected utility of working in the research sector more high-skill workers choose to work in the consumption good sector. This increases  $S_c$ , and given (I.3c) and (I.11d) decreases the wage rate in the con-

sumption good sector but increases the expected wage in the research sector. Hence  $S_c$  must become larger as long as the expected utility of working in the consumption good sector exceeds the expected utility of working in the research sector. In equilibrium the expected utility of working in the consumption good sector can not exceed the expected utility of working in the research sector unless all high-skill workers are employed in the consumption good sector. That scenario is discarded due to the assumption stated in (I.12).

If the expected utility of working in the research sector exceeds the expected utility of working in the consumption good sector all high-skill worker will choose to work in the research sector. To see why that is impossible in equilibrium see equations (I.3c) and (I.11d). If all high-skill workers are employed in the research sector  $S_c$  equals zero. From (I.3c) it is clear that the wage rate in the consumption good sector equals  $+\infty$  and from (I.11d) it is clear that the expected income from research work equals zero. Hence the expected utility of research work can not exceed the expected utility of working in the consumption good sector in equilibrium.

Hence, a feasible equilibrium requires that the utility of working in the consumption sector equals the expected utility of working in the research sector,  $u(w_c) = Eu(w_{n,i})$ . Elaborating on this condition, see Appendix B.2, implies the following equilibrium condition, from now on called *the high-skill arbitrage condition*:

$$w_c = \alpha^\alpha (1 - \alpha)^{2-\alpha} \left[ \frac{S_c}{l} \right]^\alpha \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (\text{I.13})$$

## 2.4 Equilibrium

Table I.1 reviews the important equilibrium variables. The endogenous variables of interest are the income of low-skill workers, high-skill workers in the consumption sector, high-skill workers in the research sector and the fraction of the high-skill workers that choose to become researchers, denoted  $w_o$ ,  $w_c$ ,  $w_{n,i}(\varepsilon_i)$  and  $\mu$  respectively. In Appendix B.3 it is shown how to derive expressions for those endogenous variables by combining full employment assumptions and the high-skill arbitrage condition.

$$\mu = \frac{(1 - \alpha) \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} - \left[ \frac{\alpha}{1-\alpha} \right]^{1-\alpha} [k_o l]^\alpha \frac{(1-\phi)^{1-\alpha}}{\phi} \left[ \overline{\gamma_i^\alpha} \right]^\alpha}{(1 - \alpha) \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} + \alpha E\varepsilon_i} \quad (\text{I.14a})$$

Table I.1: Equilibrium Variables

Producing	Wage	Skill Level	Employees/Members	Fraction
$X_{o,i}$	$w_o$	Low-Skill	$L_{o,i}$	$(1 - \phi)$
Y	$w_c$	High-Skill	$S_c$	$\phi(1 - \mu)$
$X_{n,i}$	$w_{n,i}(\varepsilon_i)$	High-Skill	$S_{n,i}$	$\phi\mu$

$$w_o = (1 - \alpha) \left[ \frac{\phi(1 - \mu)}{1 - \phi} \right]^\alpha \left[ \gamma_i^\alpha \right]^\alpha k_o^\alpha \quad (\text{I.14b})$$

$$w_c = \alpha^\alpha (1 - \alpha)^{2 - \alpha} \left[ \frac{\phi(1 - \mu)}{l} \right]^\alpha \left[ E\varepsilon_i^{1 - \theta} \right]^{\frac{1}{1 - \theta}} \quad (\text{I.14c})$$

$$w_{n,i}(\varepsilon_i) = \alpha^\alpha (1 - \alpha)^{2 - \alpha} \left[ \frac{\phi(1 - \mu)}{l} \right]^\alpha \varepsilon_i. \quad (\text{I.14d})$$

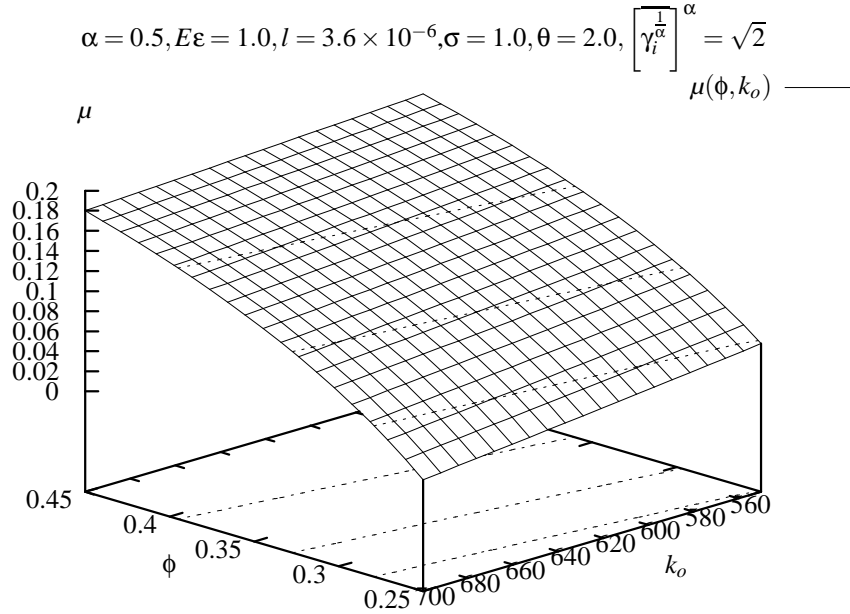
### Properties of $\mu$

Equation (I.14a) gives an expression for the fraction of the high-skill workers in the research sector. Hence this expression is constrained to be greater than zero but less than one. Since the denominator is greater than the numerator  $\mu$  can never exceed one. It is, however, quite possible that  $\mu$  falls below zero. This is likely to happen if the variety of old intermediate goods possible to produce in the period is large, i.e.  $k_o$  large. A large variety of old intermediate goods depresses the profitability of research co-operatives producing new intermediate goods.

At a corner, solution, i.e.  $\mu$  constrained to zero, both hypotheses investigated in this paper can be rejected. It is immediately clear that an increase in the fraction of high-skill workers in the economy,  $\phi$  greater, increases the fraction high-skill workers that choose to work in the research sector. Figure I.2 plots  $\mu$ , the fraction high-skill workers that choose to work in the research sector.

Figure I.2 verifies that as  $\phi$  increases, the fraction of high-skill workers that choose to work in the research sector increases. More old intermediate goods tend to decrease the fraction of high-skill workers choosing to work in the research sector, by depressing the profitability of new intermediate goods. Also, Figure I.2 is important because for the given parameter values it shows that  $\mu$  is positive, thereby verifying the assumption in (I.12).

Figure I.2: Share of High-Skill Workers in Research Sector



The figure shows the fraction of high-skill workers employed by research co-operatives, for different combinations of the relative supply of high-skill workers,  $\phi$ , and the number of old intermediate goods,  $k_o$ . For every combination of  $\phi$  and  $k_o$ ,  $\mu > 0$ .

### Properties of $w_o$

Equation (I.14b), describing the wage rate of low-skill workers, have some interesting properties. Increasing the fraction of high-skill workers,  $\phi$ , has several effects. The term,  $\left( \frac{\phi(1-\mu)}{1-\phi} \right)^\alpha$ , captures the supply and demand effects. Note that since more high-skill workers choose to work in the research sector the effect is hampered, as  $\phi$  increases,  $1 - \mu$  decreases. The term  $k_o$  measures the higher productivity gains from more intermediate goods, obtained by low-skill workers.

In a static setting, low-skill workers do not benefit from a larger variety of intermediate goods due to more research, i.e.  $k_n$  higher. However, in the long run, old intermediate goods must be the result of new intermediate goods developed in earlier periods. Hence, in the long run both high-skill and low-skill workers benefit from a larger variety of intermediate goods.

**Properties of  $w_c$  and  $w_{n,i}$** 

Together equations (I.14c) and (I.14d) show that on average  $Ew_{n,i}$  is proportional to  $w_c$ , that is:

$$\frac{w_c}{Ew_{n,i}(\varepsilon_i)} = \frac{\left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}}{E\varepsilon_i}. \quad (\text{I.15})$$

Hence the wage levels for high-skill workers in the consumption sector and high-skill workers in the research sector move together and the log differential is determined solely by the relative risk aversion,  $\theta$ , and the mean and variance of the log-normal distribution,  $E\varepsilon$  and  $\sigma^2$ . As in the case of  $w_o$ ,  $w_c$  and  $w_{n,i}$  are affected by supply and demand effects, via the term  $(\phi(1-\mu))^\alpha$ . However, wages in the consumption sector are only indirectly dependent on the supply of low-skill workers, via  $\mu$ .

**3 Results**

The following section makes use of the previously defined and solved model to draw conclusions about wage differences in the growth economy. Recall that the paper's two hypotheses are:

1. Increasing the supply of high-skill workers *increases* the wage dispersion *among* high-skill workers
2. Increasing the supply of high-skill workers *increases* the wage dispersion *between* high-skill workers and low-skill workers

**3.1 High-Skill Workers Wage Distribution**

Defining the income dispersion among high-skill workers, i.e. residual wage inequality for high-skill workers, as the ratio of expected wage rate for workers in the research sector to the certain wage rate of workers in the consumption sector implies a measure of inequality which, for this model, is independent of the relative supply of high-skill workers. This claim is easily verified by the equilibrium relation between  $w_c$  and  $Ew_{n,i}$ , given by (I.14c) and (I.14d). The expected income levels of researchers and high-skill workers are linked and their relative magnitude depends only on the properties of  $E\varepsilon_i / \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}$ .

Table I.2: Distribution of  $W^S$ 

Realization, $w^S$ , of $W^S$	Probability/Share
$w^S < w_c$	$\mu P(w_{n,i}(\varepsilon_i) < w^S)$
$w^S \geq w_c$	$(1 - \mu) + \mu [1 - P(w_{n,i}(\varepsilon_i) < w^S)]$

Inspecting the expression  $\frac{Ew_{n,i}}{w_c} = \frac{E\varepsilon_i}{[E\varepsilon_i^{1-\theta}]^{\frac{1}{1-\theta}}}$  more closely, see Appendix B.2, confirms some standard economic results concerning the risk premium:

$$\frac{\partial}{\partial \theta} \frac{Ew_{n,i}}{w_c} = \left[ \ln \frac{\sqrt{\{E\varepsilon\}^2 + \sigma^2}}{E\varepsilon} \right] \frac{E\varepsilon}{[E\varepsilon_i^{1-\theta}]^{\frac{1}{1-\theta}}} > 0 \quad (\text{I.16a})$$

$$\frac{\partial}{\partial \sigma} \frac{Ew_{n,i}}{w_c} = \theta \sigma \frac{[\{E\varepsilon\}^2 + \sigma^2]^{\frac{\theta}{2}-1}}{E\varepsilon^\theta} > 0 \quad (\text{I.16b})$$

$$\frac{\partial}{\partial E\varepsilon} \frac{Ew_{n,i}}{w_c} = -\theta \sigma^2 \{E\varepsilon\}^{-(1+\theta)} \left( \{E\varepsilon\}^2 + \sigma^2 \right)^{\frac{\theta-2}{2}} < 0. \quad (\text{I.16c})$$

Income inequality among high-skill workers increases with stronger relative risk aversion, (I.16a), and more uncertain returns to research, (I.16b). All else equal, increasing the expected productivity of a new intermediate good invented by a co-operative makes research more profitable, which tends to increase inequality. However, at the same time, in equilibrium, the higher profitability in research firms implies that more high-skill workers choose to work in the research sector. An increasing number of workers in the research sector decreases the number of high-skill workers in the consumption good sector, increasing wages in the consumption good sector. Hence in equilibrium an increase in the expected productivity of new intermediate goods have two counteracting effects. As (I.16c) shows, the net effect is decreased inequality.

The independence between the income dispersion *among* high-skill workers and the relative supply of high-skill workers is fragile. Measuring inequality among high-skill workers by the variance wages for all high-skill workers shows that inequality can increase or decrease with an increased relative supply of high-skill workers. Let  $W^S$  denote the wage rate of a high-skill worker *unconditional* on being employed in the consumption good sector or in the research sector. Formally the distribution of  $W^S$  is summarized in Table I.2. The variance of  $W^S$  can be



decomposed as:

$$\text{var}(W^S) = (1 - \mu)\sigma_c^2 + \mu\sigma_n^2 + \mu(1 - \mu)(Ew_{n,i}(\epsilon_i) - Ew_c)^2. \quad (\text{I.17})$$

Equation (I.17) decomposes the variance of *all* high-skill workers into three parts, variance *among* workers in the consumption good sector,  $\sigma_c^2$ , variance *among* workers in the research sector,  $\sigma_n^2$ , and lastly the part of total variance due to the wage differential *between* consumption good workers and research workers,  $(Ew_{n,i}(\epsilon_i) - Ew_c)^2$ . High-skill workers producing the consumption good are paid non-stochastic wages, therefore  $Ew_c = w_c$  and  $\sigma_c^2 = 0$ . Increasing the proportion of high-skill workers has several implications for the overall variance. First, from the second term of (I.17) more high-skill workers earn a stochastic income,  $d\mu/d\phi > 0$ , increasing the overall variance. Second, the weight of the third term,  $\mu(1 - \mu)$  changes. The weight of the this term is maximized for  $\mu = 1/2$  so it can increase or decrease depending on the number of high-skill workers that already choose to do research work.

Apart from changing the weights in (I.17), increasing the supply of high-skill workers also changes the variance among research workers,  $\sigma_n^2$ , and the wage difference between research and consumption workers,  $Ew_{n,i}(\epsilon_i) - w_c$ . Since the expression for  $w_{n,i}(\epsilon_i)$  is multiplicative separable and the sign of  $dw_{n,i}(\epsilon_i)/d\phi$  is ambiguous,  $\sigma_n^2$  can either increase or decrease due to increases in  $\phi$ . The same is true for the wage difference between high-skill workers working with research and high-skill workers employed by final good producers. Hence, increasing the share of high-skill workers,  $\phi$ , can either increase or decrease the variance of all high-skill workers' wages.

To avoid this ambiguity and simplify the decomposition of total variance for high-skill workers it is useful to investigate the distribution of  $\ln W^S$ . This also brings the analysis closer to the empirical literature concerning wage inequality which concentrates on the distribution of the natural logarithm of wages. Let  $\hat{\sigma}_n^2$  denote the variance of the natural logarithm of wages in the research sector, i.e.  $\text{var}\{\ln[w_{n,i}(\epsilon_i)]\}$ . The expression decomposing total variance becomes:

$$\text{var}(\ln W^S) = \mu\hat{\sigma}_n^2 + \mu(1 - \mu)[E \ln(w_{n,i}(\epsilon_i)) - \ln(w_c)]^2. \quad (\text{I.18})$$

The multiplicative separability of the wage expressions turns into additive separability for the natural logarithm of the wage expressions. This in turn implies that:

$$\frac{d\hat{\sigma}_n^2}{d\phi} = 0 \quad , \quad \frac{d[E \ln(w_{n,i}(\epsilon_i)) - E \ln(w_c)]}{d\phi} = 0.$$

Hence the effect of changes in the supply of high-skill workers,  $\phi$ , on the variance of the distribution of the natural logarithm of wages for all high-skill workers operates only via changes in the fraction,  $\mu$ , of high-skill workers choosing to work in the research sector. Proposition I.1 summarizes how the variance of high-skill wages changes with the relative supply of high-skill workers. The proof can be found in Appendix B.4.

**Proposition I.1 (Residual Wage Inequality)** *The variance of the natural logarithm of wages for high-skill workers, derived from the theoretical distribution of  $\ln W^S$  is*

$$\text{var}(\ln W^S) = \mu \ln \left( 1 + \frac{\sigma^2}{\{E\varepsilon\}^2} \right) + \mu(1-\mu)(1-\theta)^2 \left( \ln \frac{E\varepsilon}{\sqrt{\{E\varepsilon\}^2 + \sigma^2}} \right)^2 \quad (\text{I.19a})$$

and its derivative with respect to  $\phi$  is:

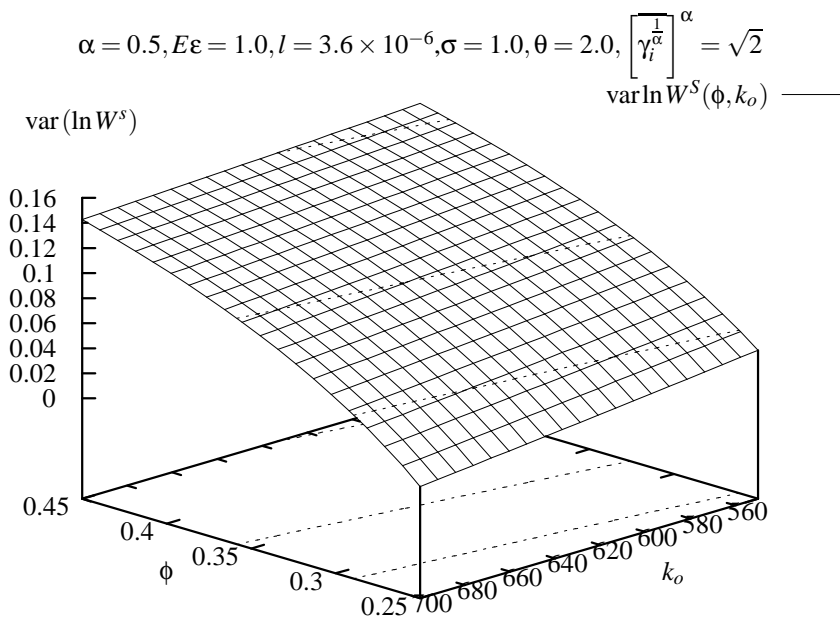
$$\begin{aligned} \frac{d\text{var}(\ln W^S)}{d\phi} &= \left[ \ln \left( 1 + \frac{\sigma^2}{\{E\varepsilon\}^2} \right) + (1-2\mu)(1-\theta)^2 \left( \ln \frac{E\varepsilon}{\sqrt{\{E\varepsilon\}^2 + \sigma^2}} \right)^2 \right] \\ &\times \frac{d\mu}{d\phi}. \end{aligned} \quad (\text{I.19b})$$

Hence for sufficient low shares of researchers among high-skill workers,  $\mu < 1/2$ , an increased supply of high-skill workers increases wage dispersion among high-skill workers. For larger shares of researchers among high-skill workers,  $\mu > 1/2$ , the wage dispersion might increase or decrease with an increased supply of high-skill workers.

On the one hand, increasing the number of high-skill workers doing research always increases residual wage inequality, due to the fact that more high-skill workers earn a stochastic wage rate. On the other hand, the discrepancy between the expected research wage and the certain wage rate paid to workers employed by final good producers, i.e. the risk premium, also increases residual wage inequality. However, the latter contribution is maximized as number of workers in the research sector and the number of workers employed by final good producers are equalized. Therefore this latter effect can increase or decrease the residual wage inequality as the number of research workers increases.

By inspecting the expression for  $\mu$ , given by (I.14a), it is immediately clear that  $\mu$  is more likely to be less than  $1/2$  if the number of old intermediate goods,  $k_o$ , is large, the time necessary to develop a new intermediate good,  $l$ , is large, or the fraction of high-skill workers,  $\phi$ , is small.

Figure I.3: Residual Wage Inequality for High-Skill Workers



The figure graphs the variance of wages, for high-skill workers, given different combinations of the relative supply of high-skill labor,  $\phi$ , and the number of old intermediate goods,  $k_o$ .

Note that a low fraction of high-skill workers,  $\phi$ , is associated with a low fraction of high-skill workers choosing to work in the research sector,  $\mu$ . Hence, economies starting with a low fraction of high-skill workers, but increasing the fraction, are likely to experience increased wage inequality for high-skill workers.

Figure I.3 plots the residual wage inequality, as defined in Proposition I.1 for the same parameters values as in Figure I.2. As is clear from the latter figure,  $\mu$  is less than  $1/2$  and consequently, the residual wage inequality for high-skill workers increases with the relative supply of high-skill workers.

### 3.2 The Skill Premium

Let  $\tau$  denote the ratio of high-skill workers' expected wage rate and low-skill workers' wage rate, that is  $\tau = \frac{EW^S}{w_o}$ . After some algebraic manipulations, see Appendix B.5, the expression for

$\tau$  turns out to be:

$$\tau = \frac{\alpha}{1-\alpha} \times \frac{1-\phi}{\phi} \times \frac{(1-\mu) \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} + \mu E\varepsilon_i}{(1-\mu) \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} - \frac{\alpha}{1-\alpha} \mu E\varepsilon_i}. \quad (\text{I.20})$$

The following lemma is useful for comparing the impact of augmenting a standard equilibrium model with a stochastic research sector, the proof is given in Appendix B.6.

**Lemma I.1** *The expression*

$$\frac{\alpha}{1-\alpha} \times \frac{1-\phi}{\phi}.$$

*describes the wage dispersion between high-skill workers and low-skill workers if there is no research sector.*

To investigate the second hypothesis, that an increased supply of high-skill workers increases wage dispersion between high-skill workers and low-skill workers, it is necessary to investigate how  $\tau$  changes with  $\phi$ . That is, it is necessary to find the derivative of  $\tau$  with respect to  $\phi$ . Proposition I.2 summarizes the results on wage dispersion between high-skill and low-skill workers. The proof is found in Appendix B.7.

**Proposition I.2 (The Skill Premium)** *Define the skill-premium as the ratio of high-skill workers' expected wage rate to low-skill workers' wage rate, then in equilibrium:*

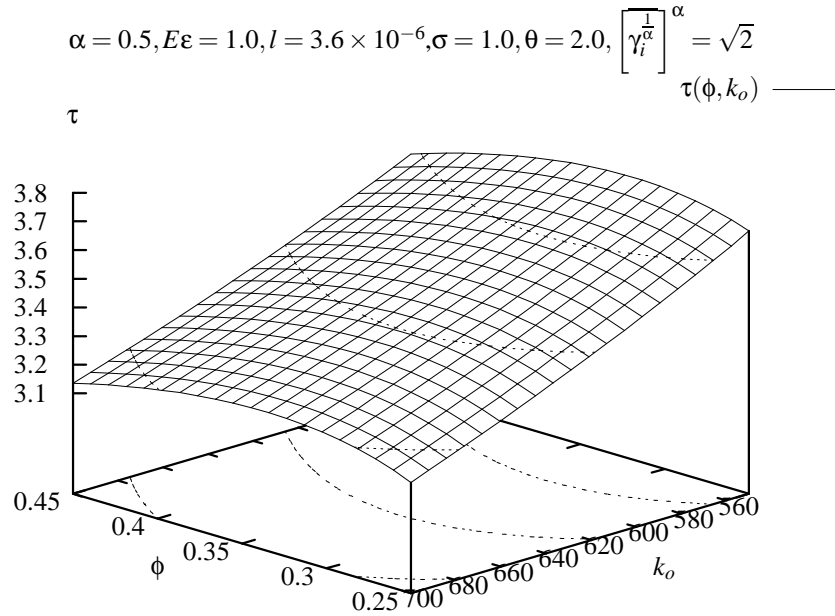
1. *The skill-premium increases with the number of high-skill workers choosing to work in the research sector.*
2. *Increased supply of high-skill workers, relative to the supply of low-skill workers, increases the skill-premium if and only if:*

$$\left[ \frac{\alpha}{lk_o} \right]^\alpha \times \left[ \frac{1-\alpha}{1-\phi} \right]^{1-\alpha} \times \frac{E\varepsilon_i \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}}{\left[ \gamma_i^\alpha \right]^\alpha \left( E\varepsilon_i - \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} \right)} \times \phi^2 < 1 \quad (\text{I.21a})$$

and

$$(1-\alpha) \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} - \left[ \frac{\alpha}{1-\alpha} \right]^{1-\alpha} [k_{ol}]^\alpha \frac{(1-\phi)^{1-\alpha} \left[ \gamma_i^\alpha \right]^\alpha}{\phi} \geq 0. \quad (\text{I.21b})$$

Figure I.4: Skill Premium



The figure illustrates the skill premium for different combinations of relative supply of high-skill workers,  $\phi$ , and the number of old intermediate goods,  $k_o$ .

**Corollary I.3** *If there is no uncertainty,  $\sigma = 0$ , or high-skill workers are not risk averse,  $\theta = 0$ , increased relative supply of high-skill workers decreases the skill premium.*

The first condition in the second part of proposition ensures that the skill premium increases as the relative supply of high-skill workers increase. The second condition in the second part of the proposition assures that in equilibrium, the number of research workers is non negative, see (I.14a). Both those conditions must be satisfied, but it is not easy to prove that such an equilibrium exist.

To prove the existence of such an equilibrium, Figure I.4 plots the skill premium for the same parameter values as in Figure I.2. From Figure I.2 it is clear that for all those parameter values there is a non negative number of research workers and the second condition in Proposition I.2 is fulfilled. Further as is seen in Figure I.4, for some range of values the skill premium increases as the fraction of high-skill workers increases.

In general, large values of  $l, \left[ \gamma_i^\alpha \right]^\alpha$ , and large a number of old intermediate goods,  $k_o$ , clearly ensures that expression (I.21a) is less than unity, and hence the wage dispersion increases as the

number of high-skill workers increases.

Corollary I.3 highlights the importance of risk and risk aversion for the results to hold. If agents are not risk averse or there is no risk, an increased relative supply of high-skill workers decreases the wage inequality between high-skill and low-skill workers, even though high-skill workers have the opportunity to make an occupational choice.

## 4 Conclusions

This paper presents a stylized general equilibrium model, augmented by a profit driven research sector. The labor force is divided into two categories, low-skill workers and high-skill workers. Capital is excluded and the time frame is collapsed into a single period during the entire analysis. The research sector is characterized by uncertain pay-offs. Due to the model's limited insurance possibilities for research workers, they have to bear all risk associated with inventing new intermediate goods, their opportunity cost being a foregone certain wage, paid by producers of consumption goods.

In a standard general equilibrium model, without a risky research sector, increased supply of a specific production factor, *ceteris paribus*, tends to lower the returns to that factor. If the research sector is excluded, research is non-stochastic or workers are risk neutral, the model presented in this paper also predicts that increasing the supply of high-skill workers lowers the average wage rate for high-skill workers.

However, since high-skill workers can choose to work in the research sector, as their wages tend to fall due to an increased supply of high-skill workers, there is a reallocation such that the fraction of high-skill workers choosing to work in the research sector increases. This shift implies that more high-skill workers are paid a risk premium for bearing risk. The reduction in the wage rate for high-skill workers, due to the increased supply of high-skill workers, is partly counteracted by a flow of high-skill workers from the consumption goods sector to the research sector. Due to the flow of workers to the research sector, more high-skill workers earn a risk premium. Therefore the average wage rate for high-skill workers can increase due to an increased supply of high-skill workers.

The formal analysis shows that the intuition outlined above is correct. There is no simple relationship between an increased supply of high-skill workers and reduction in wages for the same group. The comparative advantage of high-skill workers in producing knowledge, a good that is human capital intensive, combined with the uncertainty associated with research activity blurs the standard increased supply, decreased wage argument.

Table I.3: List of Symbols

Symbol	Meaning
$\alpha$	Cobb-Douglas exponent.
$\varepsilon$	Productivity of some new intermediate good.
$\gamma$	Productivity of some old intermediate good.
$\mu$	Fraction of high-skill workers in developing co-operatives.
$k$	Number of intermediate goods.
$l$	Hours necessary to develop a new intermediate good.
$L$	Quantity of low-skill labor.
$P$	Price of some good.
$\phi$	Share of high-skill workers.
$\pi$	Profit rate.
$\sigma^2$	Variation in productivity for a new intermediated good.
$S$	Quantity of high-skill labor.
$\theta$	Relative risk aversion.
$\tau$	Skill premium.
$w$	Wage rate.
$X$	Quantity of an intermediate good.
$Y$	Quantity of the consumption good.

## Appendix

### I.A Record of Notation

In Table I.3 the symbols used in the paper are listed and briefly explained.

### I.B Proofs and Derivations

The following section contains the derivations and proofs of various results in the paper.

#### B.1 The Co-operative Problem

A co-operative aiming at producing a new intermediate good  $X_{n,i}$ , must spend a fixed amount of labor units,  $l$ , in order to invent the new good. Once the new intermediate good is invented it takes one unit of high-skill labor to produce one unit of the intermediate good. Let  $S_{n,i}$  denote of size of the  $i$ th co-operative, i.e. the total number of high-skill labor units supplied by all its

members. The co-operatives technology is described formally by (I.9).

The co-operative's revenue is given by the price times the quantity produced. Since the co-operative's intermediate good is unique, co-operatives face a monopoly situation. Hence, co-operatives exploit the price-quantity relation given by the consumption good producer's demand function, given by (I.3b).

The earnings are shared uniformly across the co-operative's members. Hence the earning of each member equals total revenues divided by the number of members. Since the co-operative members are identical they all share the same objective; maximizing the expected utility of total revenue per member. By using (I.9) and (I.3b) to substitute out  $X_{n,i}$  and  $P_{n,i}$ . Given that the utility function is monotonically increasing and multiplicative separable, the following simplifying steps are feasible:

$$\begin{aligned}
 & \max_{P_{n,i}, X_{n,i}, S_{n,i}} Eu \left[ \frac{P_{n,i} \times X_{n,i}}{S_{n,i}} \right] \\
 & \quad s.t. (I.3b) \text{ and } (I.9) \\
 & \max_{S_{n,i}} Eu \left[ (1 - \alpha) S_c^\alpha \frac{(S_{n,i} - l)^{1-\alpha}}{S_{n,i}} \varepsilon_i \right] \\
 & \max_{S_{n,i}} \frac{(S_{n,i} - l)^{1-\alpha}}{S_{n,i}}. \tag{I.22}
 \end{aligned}$$

The last optimization problem is simple to solve. Substituting back gives the results for the size of the co-operative, the quantity produced, the price, and the wage, as listed in (I.11a), (I.11b), (I.11c), and (I.11d) respectively.

## B.2 The High-Skill Arbitrage Condition

The derivation and simplification of the arbitrage condition, ensuring equalization of high-skill workers' expected utility from co-operative research work and working for a certain wage in the consumption good sector, follows (note that  $w_{n,i}$  is given by (I.11d)):

$$\begin{aligned}
 Eu[w_c] &= E_{\varepsilon_i} u[w_{n,i}(\varepsilon_i)] \\
 w_c^{1-\theta} &= E_{\varepsilon_i} [w_{n,i}(\varepsilon_i)^{1-\theta}] \\
 w_c &= \alpha^\alpha (1 - \alpha)^{2-\alpha} \left[ \frac{\phi(1-\mu)}{l} \right]^\alpha [E \varepsilon_i^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{I.23}
 \end{aligned}$$



If  $\varepsilon_i$  is lognormally distributed with mean  $E\varepsilon$  and variance  $\sigma^2$ . Then  $\ln\varepsilon_i$  is normally distributed with mean  $m$  and variance  $s^2$ , where

$$\begin{aligned} m &= \ln \left[ \{E\varepsilon\}^2 \right] - \frac{1}{2} \ln \left[ \{E\varepsilon\}^2 + \sigma^2 \right] \\ s^2 &= \ln \left[ 1 + \frac{\sigma^2}{\{E\varepsilon\}^2} \right] \end{aligned} \quad (\text{I.24})$$

and  $\left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}$  can be written as:

$$\left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} = e^{m + \frac{(1-\theta)s^2}{2}}. \quad (\text{I.25})$$

Substituting out  $m$  and  $s^2$  by use of (I.24) the final expression for  $\left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}$  become:

$$\left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} = \frac{\{E\varepsilon\}^{1+\theta}}{\sqrt{\{E\varepsilon\}^2 + \sigma^2}^\theta}. \quad (\text{I.26})$$

To verify that  $E\varepsilon > \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}$  note that if  $\sigma^2 = 0$  then  $\left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} = E\varepsilon$ . Since  $\left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}$  is monotonically decreasing in  $\sigma^2$  and  $\sigma^2 > 0$  the statement is verified.

### B.3 Equilibrium

In equilibrium there is full employment and hence the market for low-skill workers must clear. Since the economy's total labor endowment is normalized to unity,  $(1 - \phi)$  is the share and the total number of low-skill workers. The number of high-skill workers employed in the consumption good sector,  $S_c$ , equals  $\phi(1 - \mu)$ . That is,  $\phi$  is the share of high-skill workers,  $\mu$  is the share of high-skill workers that are members of research co-operatives and the economy's total labor endowment is normalized to unity. Low-skill labor market equilibrium imply:

$$(1 - \phi) = \sum_{i=1}^{k_o} L_{o,i}.$$

Substituting the low-skill labor demand, (I.7), and simplifying gives:

$$\begin{aligned}
(1 - \phi) &= \sum_{i=1}^{k_o} S_c \left[ \frac{(1 - \alpha)\gamma_i}{w_o} \right]^{\frac{1}{\alpha}} \\
w_o &= (1 - \alpha) \left[ \frac{\phi(1 - \mu)}{1 - \phi} k_o \right]^{\alpha} \left[ \sum_{i=1}^{k_o} \frac{\gamma_i^{\frac{1}{\alpha}}}{k_o} \right]^{\alpha} \\
w_o &= (1 - \alpha) \left[ \frac{\phi(1 - \mu)}{1 - \phi} k_o \right]^{\alpha} \left[ \gamma_i^{\frac{1}{\alpha}} \right]^{\alpha}. \tag{I.27}
\end{aligned}$$

Using the inverse demand function for high-skill workers in the consumption good sector, (I.3c), is equivalent to clearing the market for high-skill workers in the consumption good sector. By substituting the quantity of each intermediate good used, (I.7) and (I.11b), this condition becomes:

$$w_c = \alpha \left[ \frac{1 - \alpha}{w_o} \right]^{\frac{1 - \alpha}{\alpha}} k_o \left[ \gamma_i^{\frac{1}{\alpha}} \right] + \alpha^{\alpha} \left[ \frac{(1 - \alpha)l}{\phi(1 - \mu)} \right]^{1 - \alpha} k_n E \varepsilon_i. \tag{I.28}$$

The number of new intermediate goods,  $k_n$  is endogenous. Each research co-operative employs  $l/\alpha$  high-skill labor units, see (I.11a), and there are  $\phi\mu$  high-skill labor units in the research sector. Hence there are

$$\alpha \frac{\phi\mu}{l} = k_n. \tag{I.29}$$

research co-operatives and new intermediate goods.

Using the arbitrage condition for high-skill workers, (I.13), the expression for low-skill workers' wages, (I.27), and the expression for the number of new intermediate goods, (I.29), to substitute out  $w_c$ ,  $w_o$  and  $k_n$ , respectively, after simplification  $\mu$  is the only unknown:

$$\begin{aligned}
&\alpha^{\alpha} (1 - \alpha)^{2 - \alpha} \left[ \frac{1}{l} \right]^{\alpha} \left[ E \varepsilon_i^{1 - \theta} \right]^{\frac{1}{1 - \theta}} \phi - \alpha (1 - \phi)^{1 - \alpha} \left[ \gamma_i^{\frac{1}{\alpha}} \right]^{\alpha} k_o^{\alpha} \\
&= \\
&\alpha^{\alpha} (1 - \alpha)^{2 - \alpha} \left[ \frac{1}{l} \right]^{\alpha} \left[ E \varepsilon_i^{1 - \theta} \right]^{\frac{1}{1 - \theta}} \phi \mu + \alpha^{1 + \alpha} (1 - \alpha)^{1 + \alpha} (1 - \alpha)^{1 - \alpha} l^{\alpha} \phi \mu E \varepsilon_i \\
&\mu = \frac{(1 - \alpha) \left[ E \varepsilon_i^{1 - \theta} \right]^{\frac{1}{1 - \theta}} - \left[ \frac{\alpha}{1 - \alpha} \right]^{1 - \alpha} [k_o l]^{\alpha} \frac{(1 - \phi)^{1 - \alpha}}{\phi} \left[ \gamma_i^{\frac{1}{\alpha}} \right]^{\alpha}}{(1 - \alpha) \left[ E \varepsilon_i^{1 - \theta} \right]^{\frac{1}{1 - \theta}} + \alpha E \varepsilon_i}. \tag{I.30}
\end{aligned}$$

By substituting this expression into (I.27) and (I.28) the wage for low-skill workers and high-

skill workers in the consumption sector can be obtained as a function of exogenous variable and parameters, only. However, the results are quite complex therefore it is better to keep  $\mu$  and remember that  $\mu$  is endogenous.

Finally to find the earnings for a co-operative member of the co-operative indexed by  $i$ , replace  $S_c$  by  $\phi(1-\mu)$  in (I.11d). Again substituting out  $\mu$  by use of (I.14a) makes the expression unnecessary complicated.

## B.4 Proof of Proposition I.1

Let  $\ln W^S$  denote the random variable describing the natural logarithm of the wage rate for any high-skill worker. The variance of  $\ln W^S$  can be decomposed into three parts: the variance among high-skill consumption workers, the variance among high-skill research workers and the difference between the average research and consumption wage. The consumption worker wage is non-stochastic so the first part vanishes from the decomposition, hence:

$$\text{var}(\ln W^S) = \mu \hat{\sigma}_n^2 + \mu(1-\mu) [E \ln w_{n,i}(\varepsilon_i) - \ln w_c]^2. \quad (\text{I.31})$$

$\hat{\sigma}_n^2$  denotes the variance of the natural logarithm of the wage of research workers, which by (I.14d) equals  $\text{var}(\ln \varepsilon_i)$ . By (I.14d) and (I.14c):

$$E \ln w_{n,i}(\varepsilon_i) - \ln w_c = E \ln \varepsilon_i - \ln \left[ E \varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

By using (I.8), describing the parameterization of the log-normal distribution of  $\varepsilon_i$  and (I.26) the variance decomposition can be shown, by simple substitution and straight forward simplification, to equal the expression in (I.19a). The differentiation is straightforward.

## B.5 Wage Dispersion

First note that the average (i.e. expected) level of income for high-skill workers,  $EW^S$ , can be written in terms of  $w_c$ :

$$EW^S = \frac{w_c \left[ (1-\mu) \left[ E \varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} + \mu E \varepsilon_i \right]}{\left[ E \varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}}. \quad (\text{I.32})$$

Defining wage dispersion as the ratio between the average high-skill worker's wage,  $EW^S$ , and the wage of low-skill workers,  $w_o$ , implies:

$$\frac{EW^S}{w_o} = \frac{\alpha^\alpha (1-\alpha)^{1-\alpha} (1-\phi)^\alpha}{l^\alpha k_o^\alpha \left[ \gamma_i^\alpha \right]^\alpha} \times \frac{(1-\mu) \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} + \mu E\varepsilon_i}{\left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}}. \quad (\text{I.33})$$

Using the equilibrium value of  $\mu$  given by (I.14a) to find an expression for  $k_o^\alpha l^\alpha \left[ \gamma_i^\alpha \right]^\alpha$  as:

$$k_o^\alpha l^\alpha \left[ \gamma_i^\alpha \right]^\alpha = \left[ \frac{1-\alpha}{\alpha} \right]^{1-\alpha} \frac{\phi}{(1-\phi)^{1-\alpha}} \left[ (1-\alpha)(1-\mu) \left[ E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}} - \alpha \mu E\varepsilon_i \right]. \quad (\text{I.34})$$

Substituting this expression into (I.33) and performing some straightforward algebraic manipulations, results in the expression for  $\tau$  given by (I.20).

## B.6 Proof of Lemma I.1

By the inverse demand function for high-skill labor, (I.3c), the total wages payment for all high-skill workers in the consumption sector,  $w_c S_c$ , are:  $\alpha Y$ . The total earnings of *all* low-skill workers,  $L_o$  must equal the remaining part:

$$w_o L_o = Y - w_c S_c = (1-\alpha)Y.$$

Now, given that  $S_c = \phi$  and  $L_o = 1 - \phi$ :

$$\frac{w_c}{w_o} = \frac{\alpha}{1-\alpha} \times \frac{1-\phi}{\phi}. \quad (\text{I.35})$$

## B.7 Proof of Proposition I.2

The expression for the income dispersion,  $\tau$ , given by (I.20), depends on  $\mu$  and  $\mu$  depends on  $\phi$ . Hence to find the derivative  $\frac{d\tau}{d\phi}$  it is necessary to replace  $\mu$  by the expression (I.14a) or use the

chain rule. Replacing  $\mu$  by (I.14a) gives:

$$\begin{aligned} \tau &= \frac{\alpha}{\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}} \times \frac{E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\alpha E\varepsilon_i + (1-\alpha) \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}} \times \\ &\times \left[ \frac{E\varepsilon_i \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}} \left[\frac{1-\alpha}{\alpha}\right]^{1-\alpha} \left[\frac{(1-\phi)}{k_o l}\right]^{\alpha} - \frac{1-\phi}{\phi} \left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha} \right]. \end{aligned} \quad (\text{I.36})$$

Simple derivation and simplification of (I.36) gives

$$\begin{aligned} \frac{d\tau}{d\phi} &= \frac{\alpha}{\phi^2} \times \frac{E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\alpha E\varepsilon_i + (1-\alpha) \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}} \times \\ &\times \left[ 1 - \left[\frac{\alpha}{lk_o}\right]^{\alpha} \left[\frac{1-\alpha}{1-\phi}\right]^{1-\alpha} \frac{E\varepsilon_i \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} \phi^2}{\left(E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}\right) \left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}} \right]. \end{aligned} \quad (\text{I.37})$$

which is positive only if (I.21a) is fulfilled. Condition (I.21b) is easily obtained by simplifying  $\mu \geq 0$  using (I.14a).

To prove Corollary I.3 note that if  $\sigma = 0$  or  $\theta = 0$ :

$$E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} = 0.$$

De-factorizing  $E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}$  in (I.37) and imposing  $E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} = 0$  gives:

$$\begin{aligned} \frac{d\tau}{d\phi} &= -\frac{\alpha}{\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}} \times \frac{E\varepsilon_i \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\alpha E\varepsilon_i + (1-\alpha) \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}} \times \left[\frac{\alpha}{lk_o}\right]^{\alpha} \left[\frac{1-\alpha}{1-\phi}\right]^{1-\alpha} \\ &= -\frac{\alpha}{\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}} \times E\varepsilon_i \times \left[\frac{\alpha}{lk_o}\right]^{\alpha} \left[\frac{1-\alpha}{1-\phi}\right]^{1-\alpha} < 0. \end{aligned}$$

## Bibliography

- 1 Acemoglu, D. (1998). Why do new technologies complement skills? Directed technological change and wage inequality, *The Quarterly Journal of Economics* 113(4): 1055–1089.
- 2 Acemoglu, D. (1999). Changes in unemployment and wage inequality: An alternative theory and some evidence, *The American Economic Review* 89(5): 1259–1278.
- 3 Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction, *Econometrica* 60(2): 323–351.
- 4 Aghion, P., Caroli, E. and García-Peñalosa, C. (1999). Inequality and economic growth: The perspectives of new growth theories, *Journal of Economic Literature* XXXVII: 1615–1660.
- 5 Auerbach, P. and Skott, P. (2005). Wage inequality and skill asymmetries, in M. Setterfield (ed.), *Interactions in Analytical Political Economy: Theory, Policy and Applications*, The University of Chicago Press, pp. 27–54.
- 6 Barro, J. R. and Xavier, S.-I.-M. (1995). *Economic Growth*, international edn, McGraw-Hill Inc.
- 7 Borjas, G. J. and Ramey, V. A. (1995). Foreign competition, market power, and wage inequality, *The Quarterly Journal of Economics* pp. 1075–1110.
- 8 DiNardo, J. and Lemieux, T. (1997). Diverging male wage inequality in the United States and Canada, 1981-1988: Do institutions explain the differences?, *Industrial and Labor Relations Review* 50(4): 629–651.
- 9 Förster, M. and Pearson, M. (2002). Income distribution and poverty in the OECD area: Trends and driving forces, *OECD Economic Studies* I(34): 7–39.
- 10 Fortin, M. N. and Lemieux, T. (1997). Institutional change and rising wage inequality: Is there a linkage?, *Journal of Economic Perspectives* 11(2): 75–96.
- 11 Gottschalk, P. (1997). Inequality, income growth, and mobility: The basic facts, *Journal of Economic Perspectives* 11(2): 21–40.
- 12 Juhn, C., Murphy, K. M. and Brooks, P. (1993). Wage inequality and the rise in returns to skill, *Journal of Political Economy* 101(3): 410–442.

- 13 Kanbur, S. (1979). Of risk taking and the personal distribution of income, *Journal of Political Economy* 87(4): 769–797.
- 14 Kihlstrom, R. E. and Laffont, J.-J. (1979). A general equilibrium entrepreneurial theory of firm formation based on risk aversion, *Journal of Political Economy* 87(4): 719–748.
- 15 Kiley, M. T. (1999). The supply of skilled labour and skill-biased technological progress, *The Economic Journal* 109(459): 708–724.
- 16 Knight, F. (1921). *Risk, Uncertainty, and Profit*, Boston: Houghton-Mifflin.
- 17 Krusell, P., Ohanian, L. E., Rìos-Rull, J.-V. and Violante, G. L. (2000). Capital-skill complementary and inequality: A macroeconomic analysis, *Econometrica* 68(5): 1029–1053.
- 18 Machin, S. and Manning, A. (1997). Can supply create its own demand? Implications for rising skill differentials, *European Economic Review* 41(3-5): 507–516.
- 19 Mendez, R. (2002). Creative destruction and the rise of inequality, *Journal of Economic Growth* 7: 259–281.
- 20 Newman, A. F. (1999). Risk-bearing, entrepreneurship and the theory of moral hazard, *Working paper*, University College London and CEPR.
- 21 Romer, P. M. (1990). Endogenous technological change, *Journal of Political Economy* 98(5): 71–102.
- 22 Rosén, Å. and Wasmer, E. (2002). Higher education levels, firms' outside options and the wage structure, *Discussion Paper 420*, The Institute for the Study of Labor (IZA). Forthcoming in *Review of Labor Economics and Industrial Relations*.
- 23 Wood, A. (1995). How trade hurt unskilled, *Journal of Economic Perspectives* 9(3): 57–80.
- 24 Wood, A. (1998). Globalisation and the rise in the labour market inequalities, *The Economic Journal* 108: 1463–1482.

# Essay II

## Market Imperfections and Wage Inequality

### 1 Introduction

After several decades of decreasing wage inequality most industrialized countries have experienced substantial increases in the dispersion of wages. The U.S. and the U.K. witnessed the change in the early 1980s while many other industrialized countries have seen similar changes during the second half of the 1980s or early in the 1990s (Juhn et al. 1993; Gottschalk 1997; Förster and Pellizzari 2000). The aim of this paper is to provide a theoretical model describing returns to skill that can be applied in a variety of economic contexts. Therefore the model allows for varying degrees of product market power, capital market distortion and fixed to variable cost ratios.

The main result in the paper is that consumer preference for variety increases the skill premium, shorter product cycles increase the skill premium, while capital taxation has an ambiguous impact on the skill premium. A fundamental characteristic of the model in this paper is the division of labor tasks into two distinct categories, production and development, which have different skill requirements. The model postulates that only high-skill workers do development work while only low-skill workers do production work, a crude implementation of the hypothesis that development is human capital intensive.

Further, the model postulates that development must always precede production. Development is costly and financed by the households via ownership. Product markets are not perfectly competitive, implying that, in equilibrium, the profit rate is sufficiently high to motivate house-



holds to invest in owner shares. A key insight necessary to understand the predictions of the model is that while production employment increases with competitiveness, development employment decreases because lower profits imply less incentive to develop new products. Therefore the skill premium is closely related to market power.

## 1.1 Related Literature

The connection between market power, via the ability to pay, and wage premia is well documented by, among others, Blanchflower et al. (1996), Nickell et al. (1994). and Nickell (1999). The discussion generally concerns the distribution of labor market rents among workers and owners via collective bargaining between firm and union representatives. This paper on the other hand assumes perfectly competitive labor markets, thereby departing from the assumptions of most labor economists.

This paper has similarities with Mendez (2002), which studies the relation between product life cycles and wage inequality. In Mendez's dual labor market setting, efficiency wages are paid to workers producing goods in the early stage of the product cycle, while competitive wages are paid to workers producing goods in the later stage of the product cycle. In Mendez setting, shorter product cycles affect wage inequality, but in an ambiguous direction.<sup>1</sup>

In Glazer and Ranjan (2001) preference for variety contributes to increased wage differences between high and low-skill workers. However, in Glazer and Ranjan's paper, the main assumption is that high-skill workers prefer consuming goods produced by high-skill labor, while low-skill workers prefers consuming goods produced by low-skill labor. Preference for variety is a necessary assumption because, in the Dixit and Stiglitz (1977) framework, increasing the number of variations of a good generates a positive externality, increasing the utility of every other variation of the good.

The paper by Dinopoulos and Segerström (1999) is somewhat similar to this paper. Both papers connect the profitability of development, labeled research in Dinopoulos and Segerström, with the demand for high-skill workers. However, in Dinopoulos and Segerström lower tariff rates motivate more development, via higher temporary Schumpeterian profits. In both papers, high-skill workers benefit, relative to low-skill workers, from higher profits. Other studies where high-skill workers do "fixed cost work" and low-skill workers do "production-work" are Ranjan (2001), Ekholm and Midelfart (2005), and Burda and Dluhosch (2002). None of those papers

---

<sup>1</sup>Mendez is primarily concerned with residual wage inequality, i.e. wage inequality between workers with similar observable characteristics, but he also briefly discusses the skill premium, which is shown to be positively correlated with residual wage inequality.

investigate the impact of changing the preference for variety, the length of the product cycle, financial market distortions, or externalities in the development process.

An integral part of the model is preference for variety in consumption, modeled using the same setup as in Dixit and Stiglitz (1977). The preference for variety provides firms with some market power. Without market power firms would not be able mark up prices above marginal cost, which is necessary to recapture development costs.

## 1.2 Plan of the Paper

The production side of the model is laid out in Section 2. Section 3 gives the various market clearing conditions. In Section 4 households are introduced, the model is closed, and the results are presented. Section 5 summarizes and discusses the results. Appendix II.A supplies a record of notation used in the paper. Lengthy derivations of key results are presented in Appendices II.B and II.C.

## 2 Model

Consider an economy consisting of  $L$  households, each with a single divisible labor unit. A fraction  $\phi L$  of the households supply high-skill labor and  $(1 - \phi)L$  supply low-skill labor. There are two types of goods in the economy, a consumption good and a capital good. There are  $\bar{n}L$  different variations of the consumption good, where  $\bar{n}$  denotes the number of variations per household. There are  $\bar{n}L$  different *production* firms producing variations of the consumption good. Hence, every production firm produces a single variation of the consumption good. New variations are developed by *development* firms.

Let  $\bar{k}$  denote the amount of capital per household. The amount of capital available for use in production,  $\bar{k}L$ , is determined endogenously. Capital depreciates and must constantly be reproduced. Capital is chosen as the numeraire good and its price is normalized to unity. It is assumed that households supply firms with capital via financial markets, but households are subject to a capital tax or some other distortion.

The consumption good is more attractive to produce because households care for variety, which provide firms with some market power. However, the lifetime of any variation of the consumption good is limited and uncertain. If a variation of the consumption good becomes obsolete, the firm can not sell any output and the firm is shut down. The market for real capital is perfectly competitive with zero profits.

## 2.1 Demand

Let  $y$  denote household net income,  $c$  household consumption,  $s$  household saving in capital, and  $m$  household development saving. Let  $\bar{y}$ ,  $\bar{c}$ ,  $\bar{s}$  and  $\bar{m}$  denote the corresponding averages over all households.

Consider any household in the economy with a net income of  $y$  and let consumption be given by  $c = y - s - m$ . Total saving by a household is  $s + m = y - c$ . Total saving falls into two different categories, real capital and owner shares (development saving). By purchasing  $s$  worth of newly produced capital, households add new capital to its existing stock of capital. By providing development firms with  $m$  worth of financial capital households can increase its stock of owner shares in production firms,  $n$ .

The household devotes  $c$  for consumption of the single consumption good. Instantaneous utility is characterized by:

$$u(c) = u[v(c)]. \quad (\text{II.1})$$

The auxiliary  $v(c)$  function is defined by optimal allocation of consumption over the different variations of the consumption good, *given* the household's choice of consumption spending,  $c$ :

$$v(c) = \max_{\vec{x}} \left[ \sum_{i=1}^{\bar{n}L} x_i^{1-\beta} \right]^{\frac{1}{1-\beta}} \quad (\text{II.2})$$

$$s.t. \quad \sum_{i=1}^{\bar{n}L} p_i x_i = c.$$

The variable  $x_i$  denotes the household's consumption of the  $i$ th variation of the consumption good.  $\beta \in [0, 1)$  parameterizes household demand for variety, and thereby also contributes to market power of production firms. The solution to this problem (see Appendix II.B for a derivation) is easily obtained:

$$x_i(c) = \frac{c}{p_i^{1/\beta} \hat{p}} \quad (\text{II.3a})$$

$$v(c) = c \hat{p}^{\frac{\beta}{1-\beta}} \quad (\text{II.3b})$$

$$\hat{p} = \sum_{i=1}^{\bar{n}L} p_i^{\frac{\beta-1}{\beta}}. \quad (\text{II.3c})$$

Since the demand function is linear in  $c$ , aggregate demand is consistently analyzed using a representative agent with average consumption spending. Therefore let  $\bar{c}$  denote the average of all households' consumption spending:

$$x_i(\bar{c}) = \frac{\bar{c}L}{p_i^{1/\beta} \hat{p}}. \quad (\text{II.4})$$

Relation (II.4) together with (II.3c) defines the demand function for any consumption good.

## 2.2 Capital Producers

The price of capital is normalized to unity, and the technology for capital production is given by a Cobb-Douglas production function in low-skill labor and capital. Capital producing firms operate on a perfectly competitive market, which is a logical assumption since capital produced by different firms are perfect substitutes in all production activities. The constant return to scale technology and the zero profit condition implies that the number of firms competing is indeterminate, but production of capital can be modeled as if there is a single price taking firm. The firm manager solves the following problem:

$$\max_{K_k, L_k} a_l K_k^\alpha L_k^{1-\alpha} - rK_k - w_l L_k.$$

The capital producer hires low-skill labor,  $L_k$ , and capital  $K_k$ . The  $K_k$  units of capital are rented from households. The wage rate of low-skill workers is denoted  $w_l$  and  $r$  denotes the interest rate for capital. Overall productivity is denoted by  $a_l$ , the marginal rate of technical substitution between capital and labor is given by  $\alpha L_k [(1 - \alpha)K_k]^{-1}$ .

Each household saves  $s$  in capital and thereby demands  $s$  new capital units. Aggregate demand for new capital therefore equals  $\bar{s}L$ . Combing the first order conditions for the problem above with the aggregate demand for capital, i.e.  $\bar{s}L = a_l K_k^\alpha L_k^{1-\alpha}$ , yields the factor demand functions for firms producing capital:

$$K_k(\bar{s}, r) = \frac{\alpha \bar{s}L}{r} \quad (\text{II.5a})$$

$$L_k(\bar{s}, w_l) = \frac{(1 - \alpha) \bar{s}L}{w_l}. \quad (\text{II.5b})$$

### 2.3 Consumption Good Producers

The production technology used by consumption good producers is the same Cobb-Douglas technology used by capital producers. Let  $b_c$  denote the corresponding cost function. A firm producing a variation of the consumption good can alter the employment of low-skill production workers instantly. Therefore, given production during a short interval of time, the firm solves the following problem:

$$\begin{aligned} \max_{p_i} \quad & p_i x_i - b_c(x_i) & (II.6) \\ \text{s.t.} \quad & x_i = \frac{\bar{c}L}{p_i^{1/\beta} \hat{p}} \\ & b_c(x_i) = \frac{x_i}{a_l} \left[ \frac{r}{\alpha} \right]^\alpha \left[ \frac{w_l}{1-\alpha} \right]^{1-\alpha}. \end{aligned}$$

The firm maximizes revenues minus cost under the demand and technology constraint. The demand constraint is given by relation (II.4) and the technology constraint is given by the cost function, corresponding to the Cobb-Douglas production function. The firm treats all variables, except the price of the firm's own variation,  $p_i$ , as given, i.e.  $\partial \hat{p} / \partial p_i = 0$ . This is perfectly consistent with rational behavior only if the number of competing firms is infinite, i.e.  $\bar{n}L \rightarrow \infty$ . Solving the maximization problem (see Appendix II.C) implies:

$$p_i = \frac{r^\alpha w_l^{1-\alpha}}{a_l (1-\beta) \alpha^\alpha (1-\alpha)^{1-\alpha}} \quad (II.7a)$$

$$x_i(\bar{c}, \bar{n}) = \frac{a_l (1-\beta) \alpha^\alpha (1-\alpha)^{1-\alpha} \bar{c}}{\bar{n} r^\alpha w_l^{1-\alpha}} \quad (II.7b)$$

$$\pi_{ci}(\bar{c}, \bar{n}) = \frac{\beta \bar{c}}{\bar{n}}. \quad (II.7c)$$

It is immediately clear that zero profits can only occur in two ways; either households do not care for variety and firms have no market power, i.e.  $\beta = 0$ , or the number of firms producing variations is infinite, i.e.  $nL \rightarrow \infty$ . Labor and capital demand functions conditioned on the quantity produced,  $x_i$ , materialize in the process of deriving the cost function,  $b_c$ . Inserting the quantity given by (II.7b) yields the factor demand functions for firms producing the consump-

tion good:

$$k_{ci}(\bar{c}, \bar{n}, r) = \frac{\alpha(1-\beta)\bar{c}}{\bar{n}r} \quad (\text{II.8a})$$

$$l_{ci}(\bar{c}, \bar{n}, w_l) = \frac{(1-\alpha)(1-\beta)\bar{c}}{\bar{n}w_l}. \quad (\text{II.8b})$$

## 2.4 Development Firms

New variations can be developed by combining high-skill labor and capital. More formally, a firm hiring  $k_{dj}$  units of capital and  $h_{dj}$  units of high-skill labor produces the “development intensity”  $z_j$ . Depending on the model,  $z_j$  can have different interpretations.

In a continuous time setting, it is logical for  $z_j$  to represent a firm-specific Poisson process intensity, where a development event implies that the firm succeeds in development a new variation of the consumption good. During a short period of length  $dt$ , the probability that a single development event occurs is  $z_j dt$ .

The logical equivalence in a discrete time setting is that  $z_j$  represents the mean of a Poisson distributed random variable, where  $z_j dt$  is the expected number of successful developments events during a given time period,  $dt$ . Alternatively in the discrete time setting  $z_j$  can represent some index increasing in the expected number of successful developments.

The technology available for producing the “development intensity” is:

$$\begin{aligned} z_j &= \left[ a_h k_{dj}^\gamma h_{dj}^{1-\gamma} \right]^\sigma \bar{z}^{1-\sigma} \\ \bar{z} &= \frac{\sum_j^L z_j}{L} \\ \sigma &\in (0, 1). \end{aligned}$$

Hence, by hiring more high-skill labor and capital, a development firm increases the probability developing a new variation, or the expected number of new successful developments. If  $\sigma$  equals unity there are no externalities and the development function reduces to a standard Cobb-Douglas production function. However as  $\sigma$  approaches zero, the incentive to free ride increases as a given firm’s development effort become less important relative to the average effort, denoted by  $\bar{z}$ .

The parameter  $a_h$  parameterizes overall development efficiency and  $\gamma$  denotes the relative importance of capital compared to high-skill labor. If a Poisson event occurs, the firm succeeds in development a new variation of the consumption good. The associated cost function,  $b_d$ , and

factor demand functions for development firms are:

$$b_d(z_j) = \frac{1}{a_h} \left[ \frac{r}{\gamma} \right]^\gamma \left[ \frac{w_h}{1-\gamma} \right]^{1-\gamma} \left[ \frac{z_j}{\bar{z}^{1-\sigma}} \right]^{1/\sigma} \quad (\text{II.9a})$$

$$k_d(z_j, r, w_h) = \frac{1}{a_h} \left[ \frac{\gamma w_h}{(1-\gamma)r} \right]^{1-\gamma} \left[ \frac{z_j}{\bar{z}^{1-\sigma}} \right]^{1/\sigma} \quad (\text{II.9b})$$

$$h_d(z_j, r, w_h) = \frac{1}{a_h} \left[ \frac{(1-\gamma)r}{\gamma w_h} \right]^\gamma \left[ \frac{z_j}{\bar{z}^{1-\sigma}} \right]^{1/\sigma}. \quad (\text{II.9c})$$

Those functions are easily derived noting that the production function, see II.9, is a standard Cobb-Douglas function with productivity  $a_h \bar{z}^{1-\sigma}$  and exponents  $\gamma\sigma$  and  $(1-\gamma)\sigma$ .

### 3 Equilibrium

The previous section described the overall economy and the behavior of every firm. The following section imposes market clearing conditions. At every moment in time the market for high-skill and low-skill labor must clear; every unit of newly produced capital must be sold and every existing unit of capital must be rented by a development or production firm.

#### 3.1 The Market for Low-Skill Labor

Low-skill workers can be employed either by a firm producing capital or any of  $\bar{n}L$  firms producing different variations of the consumption good. Full employment implies:

$$L_k(\bar{s}, w_l) + \bar{n}L l_{ci}(\bar{c}, \bar{n}, w_l) = (1 - \phi)L.$$

Low-skill labor demand for capital production,  $L_k(\bar{s}, w_l)$ , can be replaced by the factor demand function in (II.5b).  $\bar{n}$  and  $l_{ci}(\bar{c}, \bar{n}, w_l)$  are eliminated replacing the factor labor demand function using (II.8b). Solving for  $w_l$ :

$$w_l = \frac{(1 - \alpha) [(1 - \beta)\bar{c} + \bar{s}]}{1 - \phi}. \quad (\text{II.10a})$$

Low-skill workers benefit both from increased consumption and increased capital savings. Both increase aggregate production, to the advantage of low-skill workers. Stronger preference for variety provides production firms with some market power, which decreases supply and thereby

the demand for production workers, i.e. low-skill workers.

### 3.2 The Market for High-Skill Labor

Given that a high-skill or low-skill worker save  $m$  by financing development of a new product, it is clear from the cost function (II.9a) and the factor demand function (II.9c), that the household employs  $m(1 - \gamma)/w_h$  high-skill labor units. Total high-skill labor demand therefore equals:

$$\sum_{i=1}^{\phi L} \frac{1 - \gamma}{w_h} m_{hi} + \sum_{i=1}^{(1-\phi)L} \frac{1 - \gamma}{w_h} m_{li} = \bar{m}L.$$

Assuming full employment, simplifying, and solving for  $w_h$  yields:

$$w_h = \frac{1 - \gamma}{\phi} \bar{m}. \quad (\text{II.10b})$$

It is immediately clear that high-skill workers benefit from more development saving. Realizing that consumption, real capital saving, and development saving are rival, high-skill and low-skill workers' wages are clearly driven by very different underlying forces.

The wage rate of low-skill workers is adversely affected by preference for variety directly via the  $1 - \beta$  term, as seen by (II.10a). The effect of preference for variety is likely to be the opposite for high-skill workers. A larger  $\beta$  increases the profit rate of firms producing variations of the consumption good, increasing the incentives to invest in development firms, i.e. increasing  $\bar{m}$ .

### 3.3 The Markets for Capital

Households supply production and development firms with capital. Since there is no alternative usage for capital, aggregate capital supply equals  $\bar{k}L$ . Demand for capital by firms producing capital,  $K_k(\bar{s}, r)$ , is given by (II.5a). There are  $\bar{n}L$  production firms. Each production firm's demand for capital is given by the factor demand function in (II.8a).

Aggregate capital demand by development firms is obtained by summing over every household's development investment. By the cost function and factor demand functions in (II.9a) and (II.9b), any household investing  $m$  in development hires  $m\gamma/r$  units of capital. Aggregating over all households is straightforward, and equalizing aggregate capital supply with aggregate



capital demand implies:

$$r = \frac{\alpha [(1 - \beta)\bar{c} + \bar{s}] + \gamma\bar{m}}{\bar{k}}. \quad (\text{II.10c})$$

The interest rate increases if consumption, capital investments or development increase, since capital is used in production as well as development.

New capital is produced by capital production firms and bought by households. Aggregate household spending on, and thereby demand for, new capital equals  $\bar{s}L$ . The aggregate supply is given by the production function of capital producers, i.e.  $a_l K_k^\alpha L_k^{1-\alpha}$ , together with the factor demand functions in (II.5a) and (II.5b). Clearing the market for new capital implies:

$$1 = \left[\frac{\alpha}{r}\right]^\alpha \left[\frac{1 - \alpha}{w_l}\right]^{1-\alpha}. \quad (\text{II.10d})$$

## 4 Households

This section closes the model by adding households. Adding households is, in principle, necessary to determine average consumption,  $\bar{c}$ , average capital saving,  $\bar{s}$ , and average development saving,  $\bar{m}$ . Several possible configurations are possible. For example, the model can be set in either continuous or discrete time, or in infinitely lived or overlapping generations of households. The configuration used here is a continuous time setting with infinitely lived households.

### 4.1 A Simple Household Model

The income of any household in the economy can be written as:

$$y = w + (1 - \tau)(rk + \pi n) + \tau(r\bar{k} + \pi\bar{n}), \quad (\text{II.11})$$

where  $w$  denotes the wage rate;  $w_l$  for a household that supplies low-skill labor and  $w_h$  for a household that supplies high-skill labor.  $k$  denotes the amount of capital owned by the household,  $n$  denotes the number of production firms, i.e. shares, the household owns, and  $\tau$  is a tax on financial income or more general a capital market distortion. The second term,  $(1 - \tau)(rk + \pi n)$  is the financial income from owning capital and production firms.  $rk$  captures interest payments by firms renting the household's capital, and  $\pi$  captures dividend payments.

The parameter  $\tau \in [0, 1)$  has two interpretations. Either it parameterizes a financial market

imperfection, i.e. a transaction cost paid by households collected by financial market intermediaries. In this case, the fourth term,  $\tau r(\bar{k} + \bar{\pi})$  distributes the profits earned by financial intermediaries uniformly over all households. Alternatively  $\tau$  can be viewed as a tax on savings, paid by households, where the tax revenues are uniformly distributed as a lump sum to each household.

### Consumption and Saving

Households maximize the discounted value of lifetime utility of consumption:  $\int_0^\infty e^{-\rho t} u(c) dt$ . At every moment in time the household must obey the instantaneous budget constrain  $c = y - s - m$ , i.e. divide its income into consumption,  $c$ , saving in real capital,  $s$ , and saving by financing development,  $m$ . Let  $k'$  denote the next period's capital holding. The law of motion for capital is  $k' = (1 - \delta dt)k + sdt$ , given that the price of capital is normalized to unity and the depreciation rate is  $\delta$ .

Development saving by some household is  $m$ . The probability that the development firm succeeds is  $z(m)dt$ . Clearly, development saving is risky. To simplify, it is assumed that households cross-insure their savings in individual development firms, thereby completely eliminating risk. The law of motion for shares in production firms is:  $n' = (1 - qdt)n + z(m)dt$ .  $qdt$  parameterizes the probability that the variation produced by a specific production firm becomes obsolete, i.e. that a shut down shock occurs with probability  $qdt$ . The decisions of a rational household satisfies:

$$\begin{aligned}
 V(k, n) = \max_{s, m} & \quad u(c)dt + \frac{1}{1 + \rho dt} EV(k', n') & \text{(II.12)} \\
 \text{s.t.} & \quad c = y - s - m \\
 & \quad k' = (1 - \delta dt)k + sdt \\
 & \quad n' = (1 - qdt)n + z(m)dt.
 \end{aligned}$$

Differentiating the value function and using the first order conditions result in the following characterization of optimal consumption and development saving:

$$(1 - \tau)r = \rho + \delta - \frac{cu''(c)}{u'(c)} E\dot{c}/c \quad \text{(II.13a)}$$

$$(1 - \tau)z'(m)\pi = \rho + q - \frac{cu''(c)}{u'(c)} E\dot{c}/c + \frac{mz''(m)}{z'(m)} E\dot{m}/m. \quad \text{(II.13b)}$$

Those relations form a no-arbitrage relation between the returns from capital and development saving.

### Steady State

If the economy is in steady state, the change in consumption and development saving is zero, i.e.  $\dot{c} = 0$ , and  $\dot{m} = 0$ . Further, the household's holdings of capital and owner shares does not change. From the laws of motion,  $k' = k \rightarrow s = \delta k$  and  $n' = n \rightarrow z(m) = qn$ . Therefore in steady state:

$$r = \frac{\rho + \delta}{1 - \tau} \quad (\text{II.14a})$$

$$\pi = \frac{\rho + q}{(1 - \tau)z'(m)} \quad (\text{II.14b})$$

$$s(k) = \delta k \quad (\text{II.14c})$$

$$z(m) = qn \quad (\text{II.14d})$$

$$c = y - \delta k - b_d(qn). \quad (\text{II.14e})$$

Due to the assumption that households are fully insured, development saving is non-stochastic. Combining (II.14a) and (II.14b) gives a steady state no-arbitrage condition:

$$\pi = \frac{\rho + q}{\rho + \delta} \frac{r}{z'(m)}. \quad (\text{II.15})$$

The interpretation is straightforward. If the shut down intensity,  $q$ , is high relative to the depreciation rate,  $\delta$ , the profit rate must be higher to provide households with incentive to save in development.  $z'(m)$  is the marginal "development productivity". The marginal cost of saving in development is inversely related to marginal productivity, and a high relative marginal development cost naturally makes households demand a higher pay off, i.e. a higher profit rate, for saving in development instead of real capital.

### Aggregation

Capital saving may differ among different households, but it is linear in capital wealth. Aggregating over every household's capital saving, given by (II.14c), implies:

$$\bar{s} = \delta \bar{k}. \quad (\text{II.16})$$

The model is most easily solved by assuming that  $\sigma$  is in the interval  $(0, 1)$ . If  $\sigma \in (0, 1)$  the probability to succeed in developing a new variations is at least partly, but not only, dependent on the development efforts in other developing firms.

Replacing  $\pi$  by use of (II.7c) and replacing  $z'(m)$  by use of the cost function in (II.9a), condition (II.14b) defines a unique optimum for development saving:

$$m(\bar{c}, \bar{n}, r, w_h, \bar{z}) = \bar{z} \left[ \frac{\sigma \beta (1 - \tau) \bar{c}}{(\rho + q) \bar{n}} \right]^{\frac{1}{1-\sigma}} \left[ a_h \left[ \frac{\gamma}{r} \right]^\gamma \left[ \frac{1 - \gamma}{w_h} \right]^{1-\gamma} \right]^{\frac{\sigma}{1-\sigma}}. \quad (\text{II.17})$$

The most important property of household development saving is that it depends only on aggregate, non-household specific, quantities. Therefore aggregate development saving is distributed uniformly over the population, and household and average development saving is identical.<sup>2</sup>

Given household saving in development firms, every household's steady state wealth in shares can be computed by use of relation (II.14d). Efficient development firms minimize costs. Using the inverse of the cost function in (II.9a), and noting that  $b_d = m$ ,  $n$  is solved for as:

$$n = \frac{\bar{z}}{q} \left[ \frac{a_h \sigma \beta (1 - \tau) \bar{c}}{(\rho + q) \bar{n}} \left[ \frac{\gamma}{r} \right]^\gamma \left[ \frac{1 - \gamma}{w_h} \right]^{1-\gamma} \right]^{\frac{\sigma}{1-\sigma}}. \quad (\text{II.18})$$

It is immediately clear that households' share holdings,  $n$ , only depend on aggregate quantities and therefore, every household has the same amount of wealth in shares. This is of course a logical consequence of the previous result that every household's development saving is equalized. Using that  $\bar{n} = n$  and solving for  $\bar{n}$  yields:

$$\bar{n}(\bar{c}, r, w_h, \bar{z}) = \left[ \frac{\bar{z}}{q} \right]^{1-\sigma} \left[ \frac{a_h \sigma \beta (1 - \tau) \bar{c}}{\rho + q} \left[ \frac{\gamma}{r} \right]^\gamma \left[ \frac{1 - \gamma}{w_h} \right]^{1-\gamma} \right]^\sigma. \quad (\text{II.19})$$

Inserting the average households' share holdings,  $\bar{n}$ , given by (II.19), into the expression for household *and* average development saving, given by (II.17), reduces average development

---

<sup>2</sup>With decreasing individual returns to development investment,  $\sigma < 1$ , efficiency requires uniform investments. This result parallels the inequality growth result that a necessary condition for inequality to affect growth, via human capital investments, is that capital markets are imperfect Aghion and Howitt (1998); Aghion et al. (1999). Capital on the other hand is not subject to individual decreasing returns, and the distribution of capital does not affect efficiency.

saving significantly:

$$\bar{m}(\bar{c}) = \frac{\sigma q \beta (1 - \tau) \bar{c}}{\rho + q}. \quad (\text{II.20})$$

The market clearing conditions, i.e (II.10a), (II.10b) and (II.10c), provides the basic relations necessary to solve for  $\bar{c}$ ,  $\bar{k}$  and  $w_h$ . The remaining endogenous variables can be solved or eliminated. The steady state interest rate  $r$  is pinned down by (II.14a), and the wage rate for low-skill workers is then given by the capital market clearing condition in (II.10d). Average capital saving,  $\bar{s}$ , is eliminated by (II.16), and average development saving,  $\bar{m}$ , is eliminated by (II.20). The resulting system of three equations is:

$$(1 - \alpha) [(1 - \beta)\bar{c} + \delta\bar{k}] - (1 - \phi)w_l = 0 \quad (\text{II.21a})$$

$$(\rho + q)\phi w_h - (1 - \gamma)q\sigma\beta(1 - \tau)\bar{c} = 0 \quad (\text{II.21b})$$

$$[\alpha(\rho + q)(1 - \beta) + \gamma q\sigma\beta(1 - \tau)]\bar{c} + (\rho + q)(\alpha\delta - r)\bar{k} = 0. \quad (\text{II.21c})$$

Solving this system is straightforward. To simplify the notation, let  $\Delta$  be defined as:

$$\Delta \equiv r\rho(1 - \beta)(\rho + q) + q\gamma\sigma\beta\delta(1 - \tau) > 0. \quad (\text{II.22})$$

The solution to the system is

$$r = \frac{\rho + \delta}{1 - \tau} \quad (\text{II.23a})$$

$$w_l = (1 - \alpha)a_l^{\frac{1}{1-\alpha}} \left[ \frac{\alpha}{r} \right]^{\frac{\alpha}{1-\alpha}} \quad (\text{II.23b})$$

$$w_h = \frac{\sigma q \beta (1 - \gamma)(r - \alpha\delta)}{\Delta} \frac{1 - \phi}{\phi} \frac{w_l}{1 - \alpha} \quad (\text{II.23c})$$

$$k = \frac{\alpha\rho(1 - \beta) + q[\alpha(1 - \beta) + \gamma\sigma\beta(1 - \tau)]}{\Delta} \frac{1 - \phi}{1 - \alpha} w_l \quad (\text{II.23d})$$

$$\omega = \frac{q\sigma\beta(1 - \gamma)(1 - \tau)(r - \alpha\delta)}{(1 - \alpha)\Delta} \frac{1 - \phi}{\phi}, \quad (\text{II.23e})$$

where  $\omega = w_h/w_l$  denotes the relative wage of high-skill workers, compared to low-skill workers. The skill-premium is defined as  $\ln \omega$ .

### Comparative Statics

The main concern of this paper is the return to skill. Since the steady state equilibrium conditions provide analytically traceable expressions for all endogenous variables, the skill premium is easily investigated. To investigate what determines the skill premium,  $\ln \omega$  is differentiated with respect to the key parameters of the model.

**Skill Composition** The standard increased factor supply–decreased factor return logic holds for both kinds of labor, as seen by the negative derivative of  $\ln \omega$  with respect to the fraction of high-skill households, i.e.  $\phi$ :

$$\frac{d \ln \omega}{d \phi} = \frac{-1}{\phi(1-\phi)} < 0. \quad (\text{II.24a})$$

Hence, increasing the relative supply of high-skill households decreases the skill premium.

**Preference for Variety** The impact of preference for variety on the skill premium is described by the derivative of  $\ln \omega$  with respect to  $\beta$ . After some algebra:

$$\frac{d \ln \omega}{d \beta} = \frac{(\rho + \delta)(\rho + q)}{\beta(1-\tau)\Delta} > 0. \quad (\text{II.24b})$$

Increasing  $\beta$  makes households more inclined to spread out consumption more evenly over all variations given any fixed set of prices, implying greater market power for the producer of any variation. On the one hand, it follows from (II.8b) that greater preferences for variety decreases the per firm demand for low-skill labor as the supply of each firm decreases.

On the other hand, it is clear from (II.7c) that stronger preference for variety increases the value of a firm producing a variation, which in turn increases the incentives to develop new variations. Naturally greater incentives to develop new variations translates into increasing demand for high-skill workers; see (II.14d). Therefore, in the short run, before the number production firms adjusts, increasing the preference for variety increases the skill premium.

In the long run the number of production firms and development firms,  $\bar{n}$  and  $\bar{m}$ , changes, thereby altering the demand for high-skill and low-skill labor. As seen by the comparative statics and reasoning above, it is clear that in the short run as well as the long run increased preference for variety increases the skill premium.

**Taxation** Increasing the tax rate on income from capital and owner shares, i.e. increasing  $\tau$ , has an ambiguous effect on the skill premium.

$$\frac{d \ln \omega}{d \tau} = \frac{\alpha \delta}{\rho + \delta [1 - \alpha(1 - \tau)]} + \frac{2q\gamma\sigma\delta\beta}{\Delta} - \frac{1}{1 - \tau}. \quad (\text{II.24c})$$

The sign is ambiguous and the effect is non-linear. It is easy to see, inspecting (II.22), that  $\Delta$  is bounded and strictly positive as  $\tau \rightarrow 1$ . This implies that for large distortions the derivative is infinitely negative. It follows that if  $\tau$  is sufficiently close to unity, improving the financial market increases the skill premium. Hence improving *sufficiently* distorted financial markets increase the skill premium.

It is a bit surprising that the result is ambiguous. The high-skill labor market clearing condition in (II.10b) implies that the wage rate for high-skill workers is proportional to the average saving in development firms. Saving in development firms is in turn proportional to one minus the tax rate, i.e.  $1 - \tau$ , as seen by the steady state expression for  $\bar{m}$  stated in (II.20). However, by the same expression, it is clear that average development saving is proportional to average consumption spending,  $\bar{c}$ . Increasing the tax rate on financial income increases average consumption, and, the effect on development saving is therefore ambiguous, as is the effect on the wage rate of high-skill workers.

The wage rate of low-skill workers clearly decreases as the tax rate increases. Increasing the tax rate increases the steady state interest rate, and that lowers the wage rate of low-skill workers, as seen by (II.23b). This is an equilibrium result. Capital is the numeraire good and the wage rate of low-skill workers falls out, clearing the market for new capital. As its price is fixed, the wage rate of low-skill workers must adjust to clear the market.

Clearly it is difficult to predict a priori, whether financial market distortions increase or decrease the skill premium. However, excluding capital from the model yields unambiguous results. Letting  $\alpha \rightarrow 0$  and  $\gamma \rightarrow 0$  renders capital redundant in the development and production processes. The derivative reduces to:

$$\left. \frac{d \ln \omega}{d \tau} \right|_{\substack{\alpha \rightarrow 0 \\ \gamma \rightarrow 0}} = \frac{-1}{1 - \tau} < 0. \quad (\text{II.24d})$$

**Shut Down Intensity** The expected lifetime of a variation of the consumption good is  $1/q$ . Decreasing the expected lifetime of variations of the consumption good, i.e. increasing  $q$ , in-

creases the skill premium:

$$\frac{d \ln \omega}{dq} = \frac{\rho(1-\beta)(\rho+\delta)}{q(1-\tau)\Delta} > 0. \quad (\text{II.24e})$$

On the one hand, decreasing the expected life of variations of the consumption good decreases incentives to develop new variations in the short run as lifetime profits decrease. On the other hand, in the long run shorter life spans decrease the number of variations available, and thereby increase the profit rate of each producer, increasing the incentives to save in development firms.

In steady state, increasing the shut down intensity,  $q$ , implies increasing average development saving over average consumption, see (II.20). This shift in favor of development saving increases the demand for high-skill workers, which increases the steady state wage rate. Shorter product cycles therefore raise the skill premium.

**Development Externalities** If development generates strong externalities, i.e.  $\sigma$  closer to zero, the skill premium is smaller:

$$\frac{d \ln \omega}{d\sigma} = \frac{(1-\beta)(\rho+\delta)(\rho+\delta+q)}{\sigma\Delta} > 0. \quad (\text{II.24f})$$

A smaller  $\sigma$  decreases every households marginal benefit from development saving, since the households are uncoordinated and fail to internalize their positive external effect on every other household. A lower marginal benefit decreases the incentives to save by financing development firms, thereby decreasing the demand for high-skill workers. In the end, the wage rate of high-skill workers must decrease to maintain full employment.

## 5 Conclusions

The model presented in this paper puts forward the idea that if high-skill workers are mainly used in *developing* goods and low-skill workers mainly are used in *producing* existing goods, various market imperfections can alter the skill premium.

All actions by agents in the model are based on rational maximization of lifetime utility and profits in a general equilibrium setting. However to simplify, only steady state results are considered. Therefore all results pertain to the long run.



The model assumes perfectly competitive labor markets, and thereby departs from the existing branch of literature investigating the effect of labor market imperfections on the wage rate. Capital market distortions are also introduced. The paper's main results are:

- Greater preference for variety in consumption increases the skill premium.
- Shorter product cycles increase the skill premium.
- Financial market distortions, such as taxes, changes the skill premium.

In the short run, preference for variety translates into market power for firms, increasing profits but reducing supply. Reduced supply reduces the demand for production workers, i.e. low-skill workers. Higher profits stimulates development of new variations of consumption good, thereby increasing the demand for development workers, i.e. high-skill workers. However since the supply of low-skill and high-skill workers is fixed, the decreased demand for low-skill workers translates into a lower wage rate, and the increased demand for high-skill workers translates into a higher wage rate for high-skill workers.

Shorter product cycles, all else equal, reduces the profitability of developing new variations. In the long run however, the number of variations decrease, but the income share spent on development relative consumption increases, thereby increasing the skill premium. The model thereby points out shorter product cycles to be a potential explanation for the increasing dispersion in wages during the last 30 years.

The result for taxation on non-labor income, is ambiguous. In a model without capital, a non-labor tax decreases the skill premium. With capital, a necessary condition for a non-labor tax to decrease the skill premium is that the initial tax is sufficiently high. This is a weak prediction, but nevertheless the model hints that there is a connection between financial institutions and the skill premium, pointing towards financial liberalization as a possible explanation of the changes in the skill premium during the 1980s.

The model presented in this can easily be extended on the household side in order to model, for example, distorted financial markets and different household characteristics, such as risk aversion or different degrees of precautionary saving. The model, therefore, is rich in future research prospects.

## Appendix

### II.A Record of Notation

In general, any variable with an overbar represents an average, usually with respect to households or households supplying either high-skill or low-skill labor.  $i$  is used to index a specific firm producing a variation of the consumption good while  $j$  is used to index a specific development firm. A complete list of the symbols used in the paper is presented in Table II.1.

### II.B Demand for Variations

The aim of this section is to derive the demand function for each variation of the consumption good, given a certain degree of preference for variety. Preference for variety is modeled as in Dixit and Stiglitz (1977). By consuming  $\bar{n}L$  different variations of the consumption good, household utility is:

$$\sum_{i=1}^{\bar{n}L} \left[ x_i^{1-\beta} \right]^{\frac{1}{1-\beta}}.$$

For  $\beta \in (0, 1)$ , consuming an extra unit of any of the variation decreases the marginal utility of yet an extra unit of the same variation, and therefore consumers prefer to increase consumption of all variations. Only if prices differ, will a single households consume different quantities of the different variations.

Given a fixed consumption budget  $c$ , a utility maximizing household must act as if solving the optimization problem:

$$\begin{aligned} v(\vec{x}) = \max_{\vec{x}} & \sum_{i=1}^{\bar{n}L} \left[ x_i^{1-\beta} \right]^{\frac{1}{1-\beta}} \\ s.t. & \sum_{i=1}^{\bar{n}L} p_i x_i = c. \end{aligned}$$

Let  $\mu$  denote the Lagrangian multiplier due to the budget constraint. The first order conditions

Table II.1: List of Symbols

Symbol	Range	Interpretation
$\alpha$	$[0, 1)$	Capital's "weight" in production (Cobb-Douglas).
$a_h$	$R_+$	High-skill workers' productivity.
$a_l$	$R_+$	Low-skill workers' productivity.
$\beta$	$[0, 1)$	Preference for variety.
$b_c(\cdot)$	$R$	Cost function for production firms.
$b_d(\cdot)$	$R$	Cost function for development firms.
$c$	$R_+$	An arbitrary household's consumption spending.
$C$	$R_+$	Aggregate consumption spending.
$\delta$	$R$	Depreciation rate of capital.
$\Delta$	$R$	Abbreviation, defined in (II.22).
$\gamma$	$[0, 1)$	Capital's "weight" in development (Cobb-Douglas).
$h_d$	$R_+$	High-skill labor used by an arbitrary development firm.
$k$	$R_+$	An arbitrary household's capital holding.
$k_d$	$R_+$	Capital used by an arbitrary development firm.
$K_k$	$R_+$	Aggregate capital used producing capital.
$L$	$R_+$	Number, i.e. the measure, of households.
$L_k$	$R_+$	Aggregate low-skill labor used producing capital.
$m$	$R_+$	An arbitrary household's investment in development.
$n$	$R_+$	An arbitrary household's holding of development shares.
$\phi$	$(0, 1)$	Fraction of households supplying high-skill labor.
$\pi_c$	$R_+$	An arbitrary consumption firm's profit rate.
$p$	$R_+$	Price of an arbitrary variation.
$\hat{p}$	$R_+$	Auxiliary price index.
$q$	$R$	Poisson intensity at which variations become obsolete.
$\rho$	$R$	Households' discount rate.
$r$	$R$	Interest rate.
$\sigma$	$(0, 1)$	$1 - \sigma$ : Free riding possibilities in developing.
$s$	$R$	An arbitrary household's savings.
$\tau$	$[0, 1]$	Tax rate on non-labor income.
$u(\cdot)$	$R$	Instantaneous utility function.
$v(\cdot)$	$R$	Auxiliary utility function (w.r.t. variations).
$V(\cdot)$	$R$	Lifetime utility.
$\omega$	$R_+$	The skill premium, $w_h/w_l$ .
$w_h$	$R_+$	Wage rate for households supplying high-skill labor.
$w_l$	$R_+$	Wage rate for households supplying low-skill labor.
$x$	$R_+$	Quantity of an arbitrary variation.
$y$	$R_+$	An arbitrary household's income rate.
$z$	$R_+$	Development intensity of an arbitrary development firm.

are:

$$x_i^{-\beta} \left[ \sum_{k=1}^{\bar{n}L} x_k^{1-\beta} \right]^{\frac{\beta}{1-\beta}} + \mu p_i = 0 \quad \forall i \quad (\text{II.25a})$$

$$\sum_{k=1}^{\bar{n}L} p_k x_k - c = 0. \quad (\text{II.25b})$$

The relative demand of any two variations is easily derived by division of equation (II.25a) for two distinct  $i$ 's. Using the resulting expression to eliminate  $x_k$  in (II.25b) and solving for  $x_i$ , results in a demand function for any  $x_i$  ( $i$  is arbitrary):

$$x_i = \frac{c}{p_i^{1/\beta} \hat{p}}$$

$$\hat{p} = \sum_{k=1}^{\bar{n}L} p_k^{\frac{\beta-1}{\beta}}.$$

Substituting back into the utility function and simplifying gives:

$$v(\vec{x}) = c \hat{p}^{\frac{\beta}{1-\beta}}.$$

The last three relations verify expressions (II.3a) – (II.3c).

## II.C Supply of Variations

Consider the producer of the  $i$ th variation of the consumption good. The problem of the firm manager is to maximize the instantaneous profit rate. In doing so the manger must obey the first order conditions of the optimization problem:

$$\pi_i = \max_{x_i, p_i} x_i p_i - b_c(x_i) \quad (\text{II.26a})$$

$$s.t. \quad x_i = \frac{C}{p_i^{1/\beta} \hat{p}} \quad (\text{II.26b})$$

$$b_c(x_i) = \hat{b}_c x_i. \quad (\text{II.26c})$$

Naturally managers must choose a price quantity pair on the demand curve, given by (II.26b). The demand curve was derived in Appendix II.B. The only difference is that spending by a single household,  $c$ , has been replaced by aggregate spending,  $C$ , obtained by horizontal summation over all households.

By (II.26c) managers are constrained by the constant return to scale technology defined by the second constrain in problem (II.6). For a simpler exposition  $\hat{b}_c$  is used to denote marginal production cost, accordingly defined as:

$$\hat{b}_c = \left[ \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} + \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \right] \frac{r^\alpha w_l^{1-\alpha}}{a_l}. \quad (\text{II.27})$$

As in Dixit and Stiglitz (1977), it is assumed that managers overlook the impact of changing their own price on the index  $\hat{p}$ , defined by (II.3c). This is only perfectly rational if there is an infinite number of competing firms. Solving the problem above by inserting the cost function into the objective function, and using  $\mu$  to denote the Lagrangian multiplier due to the demand constraint, the first order conditions are:

$$p_i + \hat{b}_c + \mu = 0 \quad (\text{II.28a})$$

$$x_i + \mu \frac{1}{\beta} \frac{C}{\hat{p}} p_i^{-\frac{1-\beta}{\beta}} = 0 \quad (\text{II.28b})$$

$$x_i - \frac{C}{p_i^{1/\beta} \hat{p}} = 0. \quad (\text{II.28c})$$

The output price of the  $i$ th firm is easily solved for by direct insertion of (II.28a) and (II.28c) into (II.28b) to eliminate  $x_i$  and  $\mu$ . The result is a pricing rule with a constant mark up over marginal cost:

$$p_i = \frac{\hat{b}_c}{1-\beta}. \quad (\text{II.29a})$$

Clearly if there is no preference for variety, i.e.  $\beta = 0$ , the market becomes perfectly competitive and price equals marginal cost. In order to determine the quantity supplied by each producer,  $x_i$ ,  $p_i^{1/\beta} \hat{p}$  must be computed. As is clear from the pricing rule in (II.29a), all producers set the same price, since they all have the same technology. The price index, defined in (II.3c), reduces

to:

$$\hat{p} = \bar{n}L \left[ \frac{\hat{b}_c}{1-\beta} \right]^{\frac{\beta-1}{\beta}}. \quad (\text{II.29b})$$

Given the price index and the demand curve, (II.28c), the supply of any firm, indexed by  $i$ , is:

$$x_i = \frac{(1-\beta)C}{\bar{n}\hat{b}_c L}. \quad (\text{II.29c})$$

Inserting  $p_i$ ,  $\hat{p}$  and  $x_i$  into the profit function defined in (II.26a) and simplifying gives:

$$\pi_i = \frac{\beta C}{\bar{n}L}. \quad (\text{II.29d})$$

After replacing  $C$  with  $\bar{c}L$  and  $\hat{b}_c$  by (II.27), relations (II.29a) – (II.29d) verify (II.7a) – (II.7c).

## Bibliography

- 1 Aghion, P. and Howitt, P. (1998). *Endogenous Growth Theory*, Cambridge, Mass. London: MIT Press.
- 2 Aghion, P., Caroli, E. and García-Peñalosa, C. (1999). Inequality and economic growth: The perspectives of new growth theories, *Journal of Economic Literature* XXXVII: 1615–1660.
- 3 Blanchflower, D. G., Oswald, A. J. and Sanfey, P. (1996). Wages, profits, and rent-sharing, *The Quarterly Journal of Economics* 111(1): 227–251.
- 4 Burda, M. C. and Dluhosch, B. (2002). Cost competition, fragmentation, and globalization, *Review of International Economics* 10(3): 424–441.
- 5 Dinopoulos, E. and Segerström, P. (1999). A Schumpeterian model of protection and relative wages, *The American Economic Review* 89(3): 450–472.
- 6 Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity, *The Economic Journal* 67(3): 297–308.
- 7 Ekholm, K. and Midelfart, K. H. (2005). Relative wages and trade-induced changes in technology, *European Economic Review* 49: 1637–1663.

- 8 Förster, M. F. and Pellizzari, M. (2000). Trends and driving factors in income distribution and poverty in the OECD area, *Labour Market and Social Policy - Occasional Papers 42*, OECD.
- 9 Glazer, A. and Ranjan, P. (2001). Preference heterogeneity, wage inequality, and trade, *Journal of International Economics* 60: 455–469.
- 10 Gottschalk, P. (1997). Inequality, income growth, and mobility: The basic facts, *Journal of Economic Perspectives* 11(2): 21–40.
- 11 Juhn, C., Murphy, K. M. and Brooks, P. (1993). Wage inequality and the rise in returns to skill, *Journal of Political Economy* 101(3): 410–442.
- 12 Mendez, R. (2002). Creative destruction and the rise of inequality, *Journal of Economic Growth* 7: 259–281.
- 13 Nickell, S. (1999). Product markets and labour markets, *Labor Economics* 6: 1–20.
- 14 Nickell, S., Vainiomaki, S. and Wadhvani, S. (1994). Wages and product market power, *Economica* 61: 457–473.
- 15 Ranjan, P. (2001). Productivity growth and wage inequality: The role of the product cycle and international trade, *Working Paper*, University of California, Irvine.

# Essay III

## Firm Fragmentation and the Skill Premium

### 1 Introduction

Following the recognition of the massive increase in wage inequality in the U.S. in the 1980–1990 period, economists’ slumbering interest in distributional questions was awakened. Several theories have been proposed to understand the changes. The most common revolve around skill-biased technological change (Berman et al. 1998), increased competition from low wage countries (Wood 1995), and institutional changes (Fortin and Lemieux 1997). One purpose of this paper is to augment those explanations by investigating the effect of domestic outsourcing and domestic sub-contracting on the skill premium.

The massive changes in the U.S. wage distribution during the 1970–1990 period are well documented. Wage inequality in U.S. increased rapidly during the 1980–1990 period due to increases in most of the different components of overall wage inequality. The skill premium, or returns to education, increased, returns to experience increased and residual wage inequality, or inequality among individuals with similar characteristics, also increased (Gottschalk 1997; Juhn et al. 1993).

Gottschalk points out that “... the increases in the college premium are being driven more by the decline in real earnings of high school graduates than by the increase in earnings of college workers” (Gottschalk 1997, p. 30). Any full explanation of the changes in the skill premium in the U.S. is therefore obligated to present a plausible case for an absolute *decrease* in earnings of workers with relatively low education.



The rapid increase in U.S. wage inequality during the 1980–1990 period is unmatched by any European country. Gottschalk and Smeeding (1997) summarize the changes in Europe. While the U.K. stands out in the European family by experiencing large increases in earnings inequality during the 1980–1990 period, the European experience is in general mixed. Most, but not all, countries experienced some increases in earnings inequality. For Sweden the results differ depending on choices of periods and measurement, but several studies describe increased inequality (Gottschalk and Smeeding 1997; Gustafsson and Palmer 1997; Gottschalk and Smeeding 2000; Gustafsson and Palmer 2001).

## 1.1 Contribution

The contribution of this paper is twofold. On the one hand it presents a novel framework for combining the standard marginal analysis, i.e. competitive wages, with rent sharing theories where workers bargain over wages. On the other hand it hypothesizes that changes in the skill premium can be explained by *domestic* disintegration of production which prohibits workers with different skill levels to negotiate with each other over wage rates. In addition, the model investigates what factors cause outsourcing and contracting out. An important property of the framework is that firms operate under uncertainty. This uncertainty causes firm owners to occasionally threaten to shut down or relocate production. Employees are therefore occasionally subject to the risk of unemployment.

Workers can influence firm owners not to shut down the firm by renegotiating wages, i.e. agreeing on lower wages to avoid unemployment. This assumption introduces wage bargaining in the model. As opposed to many other labor market models, workers do not bargain over profits but rather to avoid unemployment, i.e. workers bargain over losses.

Firm owners always have incentives to threaten to shutdown the firm in order to lower wages and thereby increase profits. However, rational workers only consider *credible threats*. If a firm owner credibly threatens to shut down the firm, workers agree on lowering wages precisely such that firm owners are indifferent between shutting down the firm or continuing production. Credible shut down threats put workers in a bargaining situation. Workers do not primarily bargain with firm representatives since the total reduction of the wage bill necessary for firm owners not to shut down the firm is known to all parties. Instead, workers with different characteristics must agree on the distribution of wage reductions.

The model developed in this paper focuses on two types of workers: high-skill and low-skill. Whether two types of workers, in general, should form a single union that bargains with the firm

representative or bargain separately is discussed in Horn and Wolinsky (1988). Their results indicate that high-skill and low-skill workers should form a single union if they are substitutes. The model in this paper is set such that high-skill and low-skill workers bargain over a fixed surplus. That is, the maximum total surplus that can be extracted by all workers together does not depend on whether high-skill and low-skill workers form a single union or not. Therefore it is reasonable to assume that high-skill and low-skill workers form two separate unions. To see why, consider first the case where high-skill and low-skill workers form an alliance. In this case the distribution of the surplus between high-skill and low-skill workers is determined by the political mechanisms within the single union. A median voter outcome would dictate the minority group its outside option. The minority group would then always leave and form a separate union.

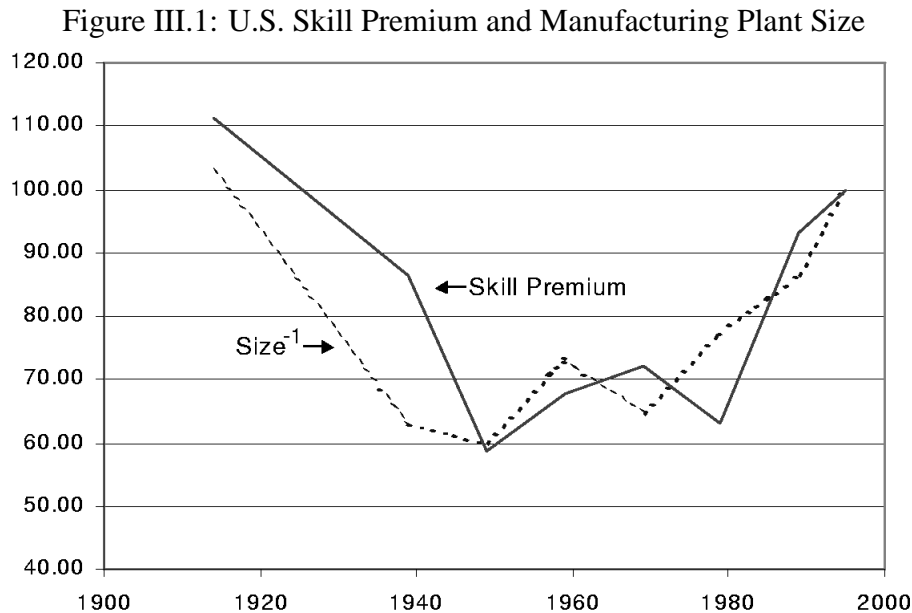
Given this basic setting, the model investigates how labor demand and wages are affected by firms' option to default on labor contracts, but also how increased utilization of external provision of labor by firms affects wage rates and the skill premium. The reliance of external provision can be categorized into two broad categories: *outsourcing* and *contracting out*. In both cases the final goods producer hands over the employment and more or less of the employer responsibilities to a third party. In the outsourcing case the final goods producer can be fully detached from the third party employee, while in the contracting out case, the final goods producer provides capital, like office space, machines or software tools, to the third party employee. Henceforth the term *fragmentation* will be used instead of outsourcing and contracting out.

In a less fragmented economy more firms employ a mix of high-skill and low-skill workers. Low-skill workers benefit from bargains relative to high-skill workers if firm owners threaten to shut down the firm. Therefore, shut down threats tend to *decrease* the skill premium in a *less* fragmented economy.

## 1.2 Some Supporting Data

The graph in Figure III.1 plots the inverse of plant size against the skill premium during the 20th century in the U.S. The correlation is striking:

- 1900–1940: Plant size increased and the skill premium decreased.
- 1940–1980: Plant size and the skill premium were relative stable.
- 1980–2000: Plant size decreased and the skill the premium increased.



The evolution of U.S. plant size during the 20th century is highly correlated with the evolution of the skill premium. Both series are indexed relative to 1995. Source: Mitchell (2005)

Needless to say, Figure III.1 does not prove that fragmentation increases the skill premium. First, plant size and firm size are related but not identical. Second, firm size and firm homogeneity, with respect to employees, are different concepts. However, it seems plausible that in an economy with smaller firms, there is a larger number of homogenous firms. This is also confirmed by Kremer and Maskin (1996) who present evidence of a trend where high-skill and low-skill workers are sorted into separate firms.

Recognizing these caveats, the figure hints that fragmentation can be important for explaining changes in the skill premium.

### 1.3 Related Literature

In the discussion of the impact of unions on wage inequality, Freeman and Medoff (1984) argue that unions favor wage equality because unions prefer single rate wage policies to individual wage policies. Freeman and Medoff put forth a few arguments: First, because of political mechanisms within the union, unions favor the majority of workers, thereby favoring redistributive contracts. This result follows, for example, by applying the median voter theorem. Second, Freeman and Medoff argue that unions tend to equalize wages due to ideological reasons favoring worker solidarity and organizational unity. This argument parallels the brief discussion in

Abraham and Taylor (1996) concerning the possibility that within larger and more heterogeneous firms, equity motives play an important role in the wage determination process.

Besides favoring single rates across its members, unions tend to decrease wage inequality by favoring single rates across firms and industries. None of those arguments are applicable for this paper since high-skill and low-skill workers are members in *separate* unions, whereby the political mechanisms within unions are sidestepped, since all members are identical. Further, every worker behaves in a neo-classical way; that is, every worker acts as if maximizing his or her utility without any egalitarian considerations. Finally, unions are firm specific and do not synchronize policies across firms or industries.

Borjas and Ramey (1995) relate to this paper by discussing the importance of the distribution of rents for the wage distribution. They claim that the industries that are hurt the most by import competition from less developed countries are manufacturing firms earning rents. These firms, according to Borjas and Ramey, employ relatively many low-skill workers. Tougher competition decreases both rents and low-skill employment in manufacturing firms. Hence, the low-skill workers are hurt “twice” from increased import competition.

The analysis in Kremer and Maskin (1996) shows that if the variation in skill levels is sufficiently low, it is efficient to match low- and high-skill workers in production. But with sufficiently large variation in the distribution of skills, efficiency requires that low-skill workers match with low-skill workers, and high-skill workers match with high-skill workers, causing a segregation of firms with respect to skill. With segregation by skill, the skill premium increases since the two production tasks are complementary.

Mitchell (2005) proposes that high-skill workers are superior to low-skill workers in being able to perform a wider variety of tasks. In the first part of the 20th century, mass production led to larger plants and a higher degree of specialization. The demand for high-skill workers diminished as every worker was required to perform a smaller number of tasks. As a result, the skill premium decreased during the first half of the century. During the last part of the 20th century new production technology decreased the cost-efficient plant size and increased the demand for workers who are able to perform a wider variety of tasks, thereby increasing the demand for high-skill workers. The increased demand for high-skill workers during the second half of the century increased the skill premium.

Caroli and Van Reenen (2001) use British and French micro data to investigate the impact of organizational change on the demand for high-skill and low skill-labor. Their definition of organizational change states not only that employees must perform more tasks but also includes flatter organizational hierarchies, implying that employees face more responsibility and

have to work more independently. This supposedly benefits high-skill workers. Caroli and Van Reenen's analysis indicates that there is a complementarity between organizational change and skill.

Acemoglu et al. (2001) focus on the distribution of rents as they build a model where high-skill and low-skill workers bargain over rents. However, they do not model vertical disintegration as a choice of firm owners but instead focus on skill biased technological change, increasing high-skill workers' gains from switching to specialized firms, thereby undermining the possibility for low-skill workers to specify redistributive wage contracts.

Harrison and Bluestone (1988) connect U.S. firms' increased use of contingent workers, i.e. temporary employed and third party workers, to the deterioration of low-skill workers' wages. Contingent workers are in general paid lower wages and receive less insurance benefits (Kalleberg et al. 1984).

The analysis in this paper can be seen as extending the analysis of Sap (1993), who integrates unionized workers into two groups – men and women. Standard bargaining theory is applied, highlighting that bargaining strength and outside options determine the wage differentials between men and women. This analysis puts Sap's analysis into a broader context and replaces the gender distinction with a skill distinction.

Thesmar and Thoenig (2004) hypothesize that increased fragmentation can be linked to financial liberalization. Financial liberalization diversifies shareholder portfolios, thereby reducing the cost of risk, *ceteris paribus*. Shareholders demand more risky assets, relative to the expected returns, and firms respond by relying more heavily on external provision of intermediate goods.

In Burda and Dluhosch (2002) firm's choice of fragmentation is endogenous. By disintegrating the production chain, demand for communication and coordination services, produced solely by high-skill workers, increases but the variable marginal production cost decreases. Burda and Dluhosch show that in the long run, if the growth rate of high-skill workers exceeds the growth rate of low skill workers, fragmentation increases, and the skill premium increases.

## 1.4 Outline

In Section 2 the basic properties of the model are presented. The section describes the endowments, and parts of the institutional setting. Section 3 presents the fundamental setup and some general results. Section 4 analyzes firms in more detail and derives the necessary expressions to analyze the impact of fragmentation on the skill premium. Section 5 discusses the possi-

Table III.1: Subscripts

Subscript	Indicates
$i$	In-house Firm
$f$	Fragmented Firm
$s$	Specialized Firm
$h$	High-Skill
$l$	Low-Skill

ble steady state equilibria and verifies the hypotheses of the paper. Section 6 summarizes the findings. Appendix III.A contains a list of symbols used, Appendix III.B and Appendix III.C complement Section 4 and Section 5 with some mathematical derivations.

## 2 Model

This section describes the fundamental parts of the model. Table III.1 depicts the general logic for subscripts used to categorize different variables. Indices over a continuum are written in parentheses. All symbols are listed in Table III.3 in Appendix III.A. Random variables are marked by  $\tilde{\cdot}$ ,  $\hat{\cdot}$ , or  $\check{\cdot}$ , depending on the information available. An upper case symbol is used for stochastic variables while lower case symbols are used to denote a particular realization of the corresponding random variables. Upper case letters are also used to denote aggregate quantities while lower case letters are also used to denote micro quantities. Symbols marked by  $\cdot^*$  are derived from an optimization problem.

### 2.1 General Setting

Consider an economy with a single consumption good, the  $Y$  good. There is a continuum of firms selling a distinct variation of the  $Y$  good. The  $Y$  good is assembled using two other goods: the  $X$  good and the  $Z$  good. The  $X$  good is produced using high-skill labor, and the  $Z$  good is produced using low-skill labor. Firms that employ workers and produce both the  $X$  and  $Z$  goods (which are necessary to assemble the  $Y$  good) are labeled *in-house* firms. Firms that do not hire any labor but purchase the  $X$  and  $Z$  goods, which are necessary to assemble the  $Y$  good, from *specialized* firms, are called *fragmented* firms. Naturally, asserting that fragmented firms hire no labor, is a crude characterization of firms relying more heavily on outside contractors.

### In-house Firms

There is a continuum of *in-house* firms with range  $K_i$ . Every in-house firm *produces* and sells a distinct variation of the consumption good. In-house firms produce both intermediate goods, i.e. both the  $X$  good and the  $Z$  good, necessary to assemble the  $Y$  good. To denote the quantity of the  $Y$  good sold by the  $k$ th in-house firm, the notation  $y_i(k)$  is used. The corresponding price is denoted  $\tilde{P}_i(k)$ .

### Fragmented and Specialized Firms

There is a continuum of firms with range  $K_f$  that only *assemble* and sell the  $Y$  good. Those firms are labeled *fragmented* firms. Fragmented firms purchase intermediate goods, the  $X$  and  $Z$  goods, necessary to assemble the  $Y$  good. Firms producing either an  $X$  or a  $Z$  good, but not both, are called *specialized* firms. To denote the  $k$ th fragmented firm's output of the  $Y$  good,  $y_f(k)$  is used, and its price is denoted  $\tilde{P}_f(k)$ .

## 2.2 Consumers' Preferences

The representative consumer does not care whether or not the consumption good is sold by an in-house or a fragmented firm, hence from the consumer point of view there is a continuum of variations of the  $Y$  good with range  $K_i + K_f$ . Due to a preference for variety, consumers are biased towards spreading their consumption across all the different variations of the  $Y$  good, thereby providing producers with some market power. Let  $C$  denote the amount the representative consumer spends on the  $Y$  good. The representative consumer behaves as if maximizing

$$\left[ \int_0^{K_i} \tilde{d}_i(k) y_i(k)^{1-\beta} dk + \int_0^{K_f} \tilde{d}_f(k) y_f(k)^{1-\beta} dk \right]^{\frac{1}{1-\beta}}$$

subject to the budget constraint

$$C = \int_0^{K_i} \tilde{p}_i(k) y_i(k) dk + \int_0^{K_f} \tilde{p}_f(k) y_f(k) dk.$$

The first integral in the objective function sums the utility derived by consuming different variations of the  $Y$  good, sold by *in-house* firms. The second integral sums the utility derived by consuming different variations of the  $Y$  good, sold by *fragmented* firms. Demand uncertainty is modeled using the stochastic demand variables  $\tilde{D}_i$  and  $\tilde{D}_f$ , and the consumer preference for

a variation of the good depends on the realizations of those demand variables,  $\tilde{d}_i(k)$  and  $\tilde{d}_f(k)$ . Consumer preference for variety is parameterized by  $\beta \in [0, 1)$ . If  $\beta$  equals zero, consumers only purchase the cheapest variation of the  $Y$  good, given that the realizations of the demand variables are equal.

The first integral in the budget constraint sums the representative consumer's expenditures on all the  $K_i$  different variations of the  $Y$  good sold by *in-house* firms. The second integral sums the representative consumer's expenditure on the  $K_f$  different variations of the  $Y$  good sold by *fragmented* firms.

Demand uncertainty is modeled using the stochastic variables  $\tilde{D}_i$  and  $\tilde{D}_f$ , with appropriate indices. That is, the demand for every variation of the  $Y$  good is stochastic. Demand shocks are deviations from expected demand. The stochastic demand variables are uniformly distributed with an expected value of 1 and range  $2\Delta$ . The cumulative density function is therefore:

$$F_i(d) = F_f(d) = \frac{d - (1 - \Delta)}{2\Delta} \quad 0 < \Delta \leq 1. \quad (\text{III.1})$$

Solving for the inverse demand functions yields:

$$\forall k \in [0, K_i] \quad : \quad \tilde{p}_i(k) = \left[ \frac{C}{\bar{p}} \right]^\beta \frac{\tilde{d}_i(k)}{y_i(k)^\beta} \quad (\text{III.2a})$$

$$\forall k \in [0, K_f] \quad : \quad \tilde{p}_f(k) = \left[ \frac{C}{\bar{p}} \right]^\beta \frac{\tilde{d}_f(k)}{y_f(k)^\beta} \quad (\text{III.2b})$$

$$\left[ \frac{C}{\bar{p}} \right]^\beta = \frac{C}{K_i \times \overline{\tilde{d}_i y_i^{1-\beta}} + K_f \times \overline{\tilde{d}_f y_f^{1-\beta}}}. \quad (\text{III.2c})$$

Relations (III.2a) and (III.2b) together with (III.2c) provide the inverse demand function for every firm in the model. The  $\bar{p}$  variable is a price index. Since there is a continuum of firms, the price index is unaffected by each firm's price and quantity choice and is therefore taken as given by each firm. The averages in the expression for  $\bar{p}$  are taken over the continuum of in-house firms and the continuum of fragmented firms.

Notice that a demand shock by some percentage increases revenues,  $\tilde{p}_i y_i$  or  $\tilde{p}_f y_f$ , by the same percentage, independent of the production levels,  $y_i$  or  $y_f$ . This in turn implies that even though the revenue function is concave with respect to the production level, the revenue function is linear with respect to the demand shock. Hence a mean preserving spread in demand changes neither the expected profit rate nor the size of the firm, if the firm owner is risk neutral and must



commit to an employment choice prior to the realization of the demand shock.

### 2.3 Firms' Technology

The production of the  $Y$  good requires two intermediate goods: the  $X$  good and the  $Z$  good. The production of the  $X$  ( $Z$ ) good requires high-skill (low-skill) labor. The production functions for the  $X$  and  $Z$  goods are:

$$x = h \quad z = l. \quad (\text{III.3a})$$

That is, one unit of high-skill labor,  $h$ , produces one unit of the  $X$  good and one unit of low-skill labor produces one unit of the  $Z$  good.

The production of the  $Y$  good is described by the Cobb-Douglas production function in the  $X$  and  $Z$  goods as:

$$y = x^\alpha z^{1-\alpha}. \quad (\text{III.3b})$$

### 2.4 Institutional Setting

The following paragraphs define the different institutional settings for in-house and fragmented firms. Both firms are of course subject to the same institutional constraints, but the different ways of organizing production implies some differences.

**In-house Firms** In-house firms and their employees are limited by institutional constraints. In-house firms post skill specific job vacancies, either high-skill or low-skill. It is assumed that firms find it easy to fill vacancies with workers with appropriate skills, while workers find it costly or time consuming to find employment. However, once contracted, in-house firms cannot, for whatever reason, replace or dismiss workers, during the contract period, unless workers threaten to strike in order to increase their wages. Employers and employees agree on one period contracts. Hence, firms cannot decrease production levels by changing employment during the period.

During the contract period, firms are subject to a demand shock. Firm owners cannot change employment or lower wages during the contract period without incurring a prohibitive cost, but firm owners always have the option to shut down the firm instantly and thereby avoid paying

wages for the remainder of the contract period. Naturally, workers suffer if the firm is shut down, since unemployed workers cannot find work instantly.

To simplify the analysis, the following assumptions are made. First, since it is easy for firms to recruit employees, workers and firm owners agree on competitive wage rates, i.e. standard wage rates determined by marginal productivity.

Second, a demand shock is not realized at any arbitrary point in time during the contract period, but immediately after signing wage contracts. The assumption magnifies the effect of demand uncertainty, but does not alter qualitative results.

Third, firm owners are not allowed to increase production and thereby employment during the period, even if the realization of the demand shock is favorable. This assumption is made only to simplify the analysis but can be rationalized by assuming that new workers need some training before becoming productive. Note that firm owners still benefit from favorable demand shocks as the price of their good increases.

Fourth, high- and low-skill workers do not bargain over employment in order to save the firm. This assumption is not unreasonable since workers find unemployment costly.

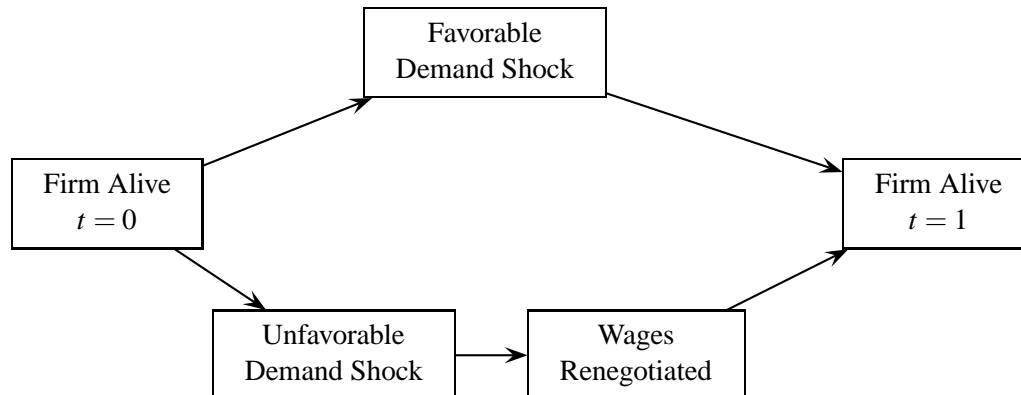
Figure III.2 illustrates the sequence of events for an in-house firm during a single period. First, firms employ workers and agree on the competitive wage rates. Second, the demand shock is realized. If the demand shock is favorable, the firm produces as planned but if the demand shock is unfavorable, high-skill and low-skill workers renegotiate wages, and the firm again produces as planned. This sequence of events is repeated every period.

Notice that a demand shock is considered to be favorable if the firm owner does not threaten to shut down the firm. The probability that a firm owner does not threaten to shut down the firm is denoted  $Q_i$ .  $Q_i$  is endogenous and derived from the behavior of rational firm owners, maximizing the discounted profit stream.

**Fragmented and Specialized Firms** Each specialized firm sells its good, either the  $X$  good or the  $Z$  good, to a continuum of fragmented firms. To make the analysis as simple as possible, it is assumed that fragmented firms can purchase the  $X$  and  $Z$  good *after* the realization of the demand variable. This is reasonable only if the  $X$  and  $Z$  goods are homogeneous, i.e. identical across fragmented firms, and transport costs are negligible.

While this assumption is questionable it simplifies the analysis because it is possible to apply the mean value theorem for the demand shocks faced by fragmented firms. That is, the demand shocks of the different fragmented firms even out and each specialized firm faces a certain demand. Since the demands for the  $X$  and  $Z$  goods are certain, employees of specialized

Figure III.2: Sequence of Events for In-house Firms



If the demand shock is favorable, the firm produces as planned. If the realization of the demand variable is unfavorable, the in-house firm's employees re-negotiate lower wages and the firm produces as planned. The sequence of events is repeated in the next period.

firms never face shutdown threats and never renegotiate wage rates.

It is however worth pointing out that it is not the lack of shutdown threats for specialized firms that drives the results derived ahead. Because specialized firms employ either high-skill or low-skill workers but not both, renegotiating wages in specialized firms would not have any redistributive effect.

### 3 Intermediate Results

The following sections present some general results governing the decisions of firm owners and workers. Due to the generality of the discussion some terms, such as profit rates or investment costs, are not formally defined or properly subscripted. Formal definitions and proper subscripts follow in later sections where the results, derived in this section, are applied.

#### 3.1 In-house Firms

Starting a firm requires a capital investment of  $I_i$ . The depreciation rate of capital, whether used or not, is  $\delta$ . The demand for the firm's product is uncertain, due to the market demand shock. It is assumed that firm owners observe the realization of demand shocks after hiring employees. The owner of a firm can shut down the firm in order to avoid paying wages, knowing that variable costs will exceed revenues. The possibility for firm owners to terminate operations

gives the model a foundation for wage bargains which is a central feature of the setup and necessary to derive the results.

Owners of in-house firms and workers employed by in-house firms face two possible scenarios in every period. Either the firm owner threatens to shut down the firm and workers renegotiate new wages, or the firm owner does not threaten to shut down the firm and workers are paid the wage agreed upon at the beginning of the period. Uncertainty arises because the demand for the in-house firm's good is uncertain. This uncertainty carries over to profit and wage rates, to price, as well as to revenues.

There exists an endogenous firm-specific threshold,  $\underline{d}_i$ , such that if the realization of the stochastic demand variable,  $\tilde{D}_i$ , is greater than or equal to this threshold,  $\tilde{d}_i \geq \underline{d}_i$ , then the firm *does not* threaten to shut down the firm. Note that the  $\tilde{\cdot}$  notation is used for  $\tilde{d}_i$  since it is a realization of a stochastic variable. The threshold  $\underline{d}_i$  on the other hand is non-stochastic and therefore not marked by  $\tilde{\cdot}$ .

If the realization of the demand variable is less than this threshold,  $\tilde{d}_i < \underline{d}_i$ , then the firm owner *does* threaten to shut down the firm. In the latter case, workers renegotiate wages to motivate the firm owner to continue operations and not shut down the firm.

Demand uncertainty is described by the firm-specific stochastic variable  $\tilde{D}_i$ , which is uniformly distributed with mean 1 and range  $2\Delta$ . The cumulative density function for  $\tilde{D}_i$  is denoted by  $F(\cdot)$  and is given by (III.1). Given the threshold  $\underline{d}_i$ , the probability that the firm owner does not threaten to shut down is  $1 - F(\underline{d}_i)$ . From here on this probability is denoted by:

$$Q_i \equiv \text{Prob}(\tilde{d}_i \geq \underline{d}_i) = 1 - F(\underline{d}_i). \quad (\text{III.4})$$

It follows immediately that the probability that the firm owner does threaten to shut down the firm is  $1 - Q_i = F(\underline{d}_i)$ .

Due to the stochastic demand, the profit rate,  $\tilde{\Pi}_i$ , the wage rates,  $\tilde{W}_i$ , the price of the good,  $\tilde{P}_i$ , and firm revenue,  $\tilde{R}_i$ , are also stochastic. In the derivation that follows it is often convenient to rewrite expectations conditionally. For example consider the expected value of the wage rate,  $E\{W_i\}$ :

$$E\{W_i\} = Q_i E\{W_i | \tilde{d}_i \geq \underline{d}_i\} + (1 - Q_i) E\{W_i | \tilde{d}_i < \underline{d}_i\}.$$

Here,  $E\{W_i | \tilde{d}_i \geq \underline{d}_i\}$  is the expected wage rate given that the firm owner does not threaten to shut down (that is, it is known that the realization of the demand variable,  $\tilde{d}_i$ , is greater than  $\underline{d}_i$ ) and  $Q_i$  is the probability that the firm owner does not threaten to shut down the firm; see

(III.4).  $E \{W_i | \tilde{d}_i < \underline{d}_i\}$  is the expected wage rate given that the firm owner threatens to shut down the firm and workers renegotiate wages. This happens only if  $\tilde{d}_i < \underline{d}_i$ , which occurs with probability  $1 - Q_i$ ; again see (III.4).

Because it becomes cumbersome to write the conditional expectation operator everywhere, the following notations are used.  $\tilde{W}_i$  denotes the wage rate, which is stochastic,  $\hat{W}_i$  denotes the wage rate given that the firm owner does not threaten to shut down the firm, and  $\check{W}_i$  denotes the wage rate given that the firm owner does threaten to shut down the firm. Given these definitions, the expected wage rate can be written as:

$$E \{W_i\} = Q_i E \{\hat{W}_i\} + (1 - Q_i) E \{\check{W}_i\}. \quad (\text{III.5})$$

The wage rate was used as an example above, but the same notational convention is used for the profit rate,  $\tilde{\Pi}_i$ , firm revenue,  $\tilde{R}_i$ , the price of the good,  $\tilde{P}_i$ , and the demand variable,  $\tilde{D}_i$ . To summarize,  $\hat{\cdot}$  and  $\check{\cdot}$  are used to distinguish the scenarios where the firm owner does not and does threaten to shut down the firm, respectively. In terms of information sets,  $\hat{\cdot}$  is used if the only information given is that the firm owner does not threaten to shut down the firm and  $\check{\cdot}$  is used if the only information given is that the firm owner does threaten to shut down the firm. Of course this notation is only meaningful before the realization of the stochastic demand variable is known. Once it is known, there is no uncertainty about profit rates, wage rates, or firm revenues and given the notation convention stated earlier, lower case letters are used.

It is possible to simplify the analysis further by noting the following. First, if the firm owner shuts down the firm, the profit rate is simply the replacement cost of capital. If the firm owner threatens to shut down the firm, workers will renegotiate wages such that the firm owner is indifferent about shutting down the firm and keeping it alive. Therefore, the profit rate given that the firm owner threatens to shut down the firm is simply  $-\delta I_i$ , i.e. the replacement cost for capital.

Second, given that the firm owner does not threaten to shut down the firm, wage rates are not renegotiated and thereby not affected by the realization of the demand shock. Therefore the wage rate given that the firm owner does not threaten to shut down the firm is simply  $w$ . Note that  $w$  is determined at the start of the period and hence it is not stochastic.

The profit rate with respect to time is a stochastic variable, denoted by  $\tilde{\Pi}_i$ . Given the realization of the the demand shock,  $\tilde{d}_i$ , the owner of the firm can either shut down the firm, with a *non-stochastic* profit rate  $-\delta I_i$ , or keep the firm alive, with the given profit rate  $\hat{\pi}_i$ .

Let  $\tilde{v}_i$  denote the value of a firm, *after* the outcome of the demand shock is realized, let  $\rho$

denote the discount rate, and let  $\tilde{V}'_i$  denote the value of the firm in the next period.  $\tilde{v}_i$  satisfies:

$$\tilde{v}_i = \max \left\{ \hat{\pi}_i + \frac{E \{ \tilde{V}'_i | \tilde{d}_i > \underline{d}_i \}}{1 + \rho}, -\delta I_i + \frac{I_i}{1 + \rho} \right\}. \quad (\text{III.6})$$

The first member of the set is the value of the firm given that the owner, knowing the realization of the demand shock, decides to keep the firm alive. The second member of the set is the value of shutting down the firm.

If the owner decides to keep the firm alive, the instantaneous profit received is  $\hat{\pi}_i$  plus the discounted continuation value. The continuation value is  $E \{ \tilde{V}'_i | \tilde{d}_i > \underline{d}_i \}$ , which is the expectation operator, conditioned on the information that the firm was not shut down. However, it is assumed that demand shocks are serially uncorrelated. Therefore, it is possible to replace  $E \{ \tilde{V}'_i | \tilde{d}_i > \underline{d}_i \}$  by  $E \{ V_i \}$ , i.e. the unconditional expectation operator.

If the owner decides to shut down the firm, he or she earns profits  $-\delta I_i$  before selling the capital, worth  $I_i$ , at the end of the period.

The continuation value, keeping the firm alive, is the expectation of the next period value,  $V'_i$ . The expectation operator is necessary since the demand in the next period is unknown in the current period. However, in equilibrium, the expected value of owning a firm must equal its investment costs. Therefore:

$$E \{ V_i \} = I_i. \quad (\text{III.7})$$

Hence, in the steady state equilibrium,  $E \{ \hat{V}_i \} = E \{ V_i \}$  in (III.6) can be replaced by  $I_i$ . Simplifying this implies:

$$\tilde{v} = \max \{ \hat{\pi}_i, -\delta I_i \} + \frac{I_i}{1 + \rho}. \quad (\text{III.8})$$

Naturally, the owner threatens to shut down the firm only if:

$$\hat{\pi}_i < -\delta I_i. \quad (\text{III.9})$$

The profit rate if the firm owner does not threaten to shut down,  $\hat{\Pi}_i$ , is written in lower case letters since the firm owner makes the decision of whether or not to threaten to shut down the firm when the realization of the random variable is known, so that the profit rate is  $\hat{\pi}_i$ .

$Q_i$  denotes the probability that the firm owner *does not* threaten to shut down the firm. The

*expected* value of owning a firm in terms of conditional expectations can be found by rewriting (III.6):

$$E \{V_i\} = Q_i \left[ E \{ \hat{\Pi}_i \} + \frac{E \{ \tilde{V}'_i | \tilde{d}_i > \underline{d}_i \}}{1 + \rho} \right] + (1 - Q_i) \left[ -\delta I_i + \frac{I_i}{1 + \rho} \right]. \quad (\text{III.10})$$

That is,  $Q_i$  is the probability that the firm owner does not threaten to shut down and is defined endogenously from the condition in (III.9).  $E \{ \hat{\Pi}_i \}$  is the expectation given that it is known that the firm owner did not threaten to shut down the firm. That is, the realized profit rate is greater than the profit rate if the firm owner threatens to shut down, i.e.  $\hat{\pi}_i \geq -\delta I_i$ .

Simplifying  $E \{V_i\}$  using the steady state equilibrium condition,  $E \{ \tilde{V}'_i | \tilde{d}_i > \underline{d}_i \} = E \{V_i\} = I_i$  implies:

$$E \{V_i\} = \frac{1 + \rho}{\rho} [Q_i E \{ \hat{\Pi}_i \} - (1 - Q_i) \delta I_i] = I_i. \quad (\text{III.11})$$

Notice that in a world without uncertainty and continuous time, this condition reduces to  $\pi/\rho = I_i$ . Remember that this condition stems from the steady state equilibrium condition  $E \{V_i\} = I_i$ , and holds due to free entry. New firms enter or leave at a rate such that the value of starting a new firm is always zero. Solving for  $E \{ \hat{\Pi}_i \}$  gives:

$$E \{ \hat{\Pi}_i \} = \frac{\rho + (1 - Q_i)(1 + \rho)\delta}{(1 + \rho)Q_i} I_i. \quad (\text{III.12})$$

## 3.2 Workers

The economy is populated by  $H_i + H_s$  high-skill workers and  $L_i + L_s$  low-skill workers. The  $H_i$  high-skill workers are employed by in-house firms, and the  $H_s$  high-skill workers are employed by firms specialized in producing intermediate goods necessary to assemble the  $Y$  good.  $L_i$  and  $L_s$  are interpreted analogously.

### In-house Workers

While the losses of firm owners are limited by the depreciation of capital and foregone interest payments, workers are left without any wage payments if the firm is shut down. By assuming that unemployment benefits are paid only if workers are unemployed at the beginning of the period, workers and firm owners always reach an agreement in order to save the firm. Therefore

workers always accept lower wage rates in order to assure that the owner does not shut down the firm.

The firm owner accepts *losses* less than the capital replacement cost  $\delta I_i$ ; see (III.9). Profits are defined including capital replacement costs; that is, revenues minus the wage bill minus capital replacement costs:  $\tilde{\Pi}_i = \tilde{R}_i - wb_i - \delta I_i$ . The owner therefore shuts down the firm if  $\tilde{r} < wb$ , i.e. if the revenue realization is insufficient to cover variable costs. Again, lower case letters are used in the condition, since firm owners base their decision on the realization of revenues and the non-stochastic wage bill.

To save the firm, workers must agree on wage rates such that wage costs are covered by revenues. Utility maximizing employees naturally agree on wage rates such that the owner is indifferent about shutting down the firm and keeping it alive. That is, workers renegotiate wages such that wage costs equal revenues,  $wb = \tilde{r}$ . Therefore, the firm is never shut down.

The wage rate paid to workers depends on whether the firm owner is inclined to shut down the firm or not. Expected wages of workers satisfy:

$$E \{W_i\} = Q_i w + (1 - Q_i) E \{\check{W}_i\}. \quad (\text{III.13})$$

The expected wage rate for workers is simply the sum of the expected value if the firm owner does not threaten to shut down and the expected value if the firm owner threatens to shut down, weighted by the appropriate probabilities. From (III.9) it is clear that the firm owner threatens to shut down if and only if  $\hat{\pi}_i < -\delta I_i$ , which occurs with probability  $1 - Q_i$ .

The wage rate paid if the firm owner does not threaten to shut down is non-stochastic and the expected value, conditioned on the information that the firm owner does not threaten to shut down is simply  $w$ .

Given only the information that the firm owner threatens to shut down the firm, i.e. that  $\hat{\pi}_i < -\delta I_i$ , there is a range of possible realizations for the demand variable satisfying this condition. Each such realization implies a different wage rate if the workers of the firm agree on lowering their wage rates. Therefore the wage rate, if the firm owner threatens to shut down, is stochastic and the expectation must be conditioned on the information that  $\hat{\pi}_i < -\delta I_i$ , hence the use of  $E \{\check{W}_i\}$ .

### Bargaining Positions

In the Nash solution to the bargaining problem, the difference between the parties' outside options is the major determinant of the outcome. In order to determine the outside option of high-



skill workers and low-skill workers, every worker's lifetime utility, employed and unemployed, must be derived.

There is frictional unemployment, implying that unemployed workers cannot find employment instantaneously. An unemployed worker receives unemployment benefits. The unemployment benefit is a fraction,  $u_b$ , of the worker's average, i.e. expected, wage. Therefore the unemployment benefit is  $u_b E \{W_i\}$ . Note that an unemployed low-skill worker receives a fraction of the average wage of low-skill workers, while a high-skill worker receives a fraction of the average wage of high-skill workers. In both cases this fraction is  $u_b$ . Let  $E \{J_i\}$  denote the expected discounted lifetime utility of a currently employed worker, and let  $E \{U_i\}$  denote the expected discounted lifetime utility of a currently unemployed worker. In steady state,  $E \{J_i\}$  and  $E \{U_i\}$  satisfy:

$$E \{J_i\} = E \{W_i\} + \frac{E \{J_i\}}{1 + \rho} \quad (\text{III.14a})$$

$$E \{U_i\} = u_b E \{W_i\} + \frac{\theta E \{J_i\} + (1 - \theta) E \{U_i\}}{1 + \rho}. \quad (\text{III.14b})$$

Employed workers are paid the stochastic wage rate  $\tilde{W}_i$  and the expected continuation value is  $E \{J_i\}$ . Note that firms are never shut down due to adverse demand shocks, since high-skill and low-skill workers always reach an agreement on lower wages. The unemployed worker receives unemployment benefits equal to  $u_b E \{W_i\}$ , becomes employed in the next period with probability  $\theta$ , and stays unemployed with probability  $1 - \theta$ . The  $\theta$  coefficient parameterizes the matching quality in the labor market. Solving for  $E \{J_i\}$  and  $E \{U_i\}$  implies:

$$E \{J_i\} = \frac{1 + \rho}{\rho} E \{W_i\} \quad (\text{III.15a})$$

$$E \{U_i\} = \frac{1 + \rho}{\rho} \frac{\rho u_b + \theta}{\rho + \theta} E \{W_i\}. \quad (\text{III.15b})$$

This specification implies a logic inconsistency. If workers do not face any risk of becoming unemployed, in the long run the economy must converge to full employment. The common solution to this problem is to add an exogenous shock such that the firm is shut down with some exogenous probability. The analysis in this paper can easily be extended in that direction without changing any of the results. However, to minimize the notation this is not done, and this inconsistency is overlooked.

### In-house Bargaining

If an in-house firm is about to be shut down, high-skill and low-skill workers negotiate new wage rates via union representatives in order to motivate the firm owner not to shut down. Let  $\check{r}_i$  denote the revenues to be distributed among high-skill and low-skill workers. The lower case notation is used since negotiations are done ex post the realization of the demand variable and the revenue of the firm is known to all parties. The outcome is described by the Nash solution for the bargaining problem where  $\gamma$  denotes the bargaining power of high-skill workers, and  $1 - \gamma$  the bargaining power of low-skill workers. The share of revenues captured by high-skill workers,  $\psi^*$ , is the share of revenues which maximizes the Nash product:

$$\psi^* = \underset{\psi}{\operatorname{argmax}} \quad \gamma \log \left[ \frac{\psi \check{r}_i}{h_i} + \frac{E \{J'_{ih}\} - E \{U'_{ih}\}}{1 + \rho} \right] + \\ (1 - \gamma) \log \left[ \frac{(1 - \psi) \check{r}_i}{l_i} + \frac{E \{J'_{il}\} - E \{U'_{il}\}}{1 + \rho} \right]. \quad (\text{III.16})$$

The expected lifetime utility can be decomposed into an instantaneous pay-off and a continuation value. The instantaneous pay-off for high-skill workers, if the parties reach an agreement, is  $\psi$  times the revenues of the firm,  $\check{r}_i$ , divided by the number of high-skill time units employed,  $h_i$ . The continuation value is the discounted lifetime utility being employed.

If the parties cannot reach an agreement, the firm is shut down and the high-skill worker becomes unemployed. His or her continuation value and expected discounted lifetime utility is in this case  $E \{U'_{ih}\} / (1 + \rho)$ , which is the threat point of high-skill workers. This specification is a consequence of the assumption that unemployed workers do not receive any unemployment benefits the period they become unemployed.

Given that  $\psi$  denotes the share of revenues captured by high-skill workers, the share of revenues captured by low-skill workers is  $1 - \psi$ . The interpretation of the second term, i.e. the bargaining position of low-skill workers, is analogous. Solving this problem for  $\psi^*$ :

$$\psi^* = \gamma + \gamma l_i \frac{E \{J'_{il}\} - E \{U'_{il}\}}{(1 + \rho) \check{r}_i} - (1 - \gamma) h_i \frac{E \{J'_{ih}\} - E \{U'_{ih}\}}{(1 + \rho) \check{r}_i}. \quad (\text{III.17})$$

In steady state,  $E \{J'_i\} = E \{J_i\}$  and  $E \{U'_i\} = E \{U_i\}$ . Replacing  $E \{J'_{ih}\} - E \{U'_{ih}\}$  and  $E \{J'_{il}\} - E \{U'_{il}\}$  using (III.15a) – (III.15b) simplifies the steady state bargaining outcome such

that:

$$\psi^* = \gamma + \gamma(1 - u_b)l_i \frac{E\{W_{il}\}}{(\rho + \theta_{il})\check{r}_i} - (1 - \gamma)(1 - u_b)h_i \frac{E\{W_{ih}\}}{(\rho + \theta_{ih})\check{r}_i}. \quad (\text{III.18})$$

## 4 Firms Revisited

The following sections derive the optimal management of firms, or how to maximize the rate of profit given the firm owners decision of whether or not to produce. Hence, derivations in the following section pin down the flows generated by firms, such as profit, wage, and employment rates. This is in contrast to the problem of the firm owners, such as whether or not to keep the firm alive or when to invest, which was analyzed in previous sections. It is assumed that there is no conflict in the objectives of owners and managers, so those words can be used interchangeably.

The first sub-section analyzes in-house firms, while the second sub-section analyzes fragmented and specialized firms. In-house firms must commit to an employment choice ex ante the realization of the demand shock, while fragmented firms purchase intermediate goods ex post the realization of the demand shock.

Quantities referring to in-house firms are subscripted by an  $i$  and quantities referring to fragmented firms are subscripted by a  $f$ . Quantities derived from an optimization problem are superscripted by  $*$ . As before,  $\hat{\cdot}$  and  $\check{\cdot}$  are used to distinguish scenarios where firm owners do not threaten to shut down the firm and where firm owners do threaten to shut down the firm, respectively.

### 4.1 In-house Firms

The choices of in-house firm owners involve shutting down the firm or keeping it alive. The firm owner must commit to an employment choice prior to deciding whether or not to threaten to shut down the firm. This is a reasonable assumption if demand changes frequently, relative to the turnover rate of workers.

Before deriving the optimal choices of firm owners, remember that the profit rate if the firm owner shuts down the firm is the non-stochastic capital replacement cost, equal to  $-\delta I_i$ . Given that the firm owner does not threaten to shut down the firm, the wage rate for high-skill workers is non-stochastic and equals  $w_{ih}$ , while the wage rate for low skill workers, which is also non-stochastic, is  $w_{il}$ .

In-house firms produce the  $X$  and  $Z$  goods by hiring high-skill and low-skill workers. Augmenting the production functions in (III.3a) by an  $i$  subscript for in-house firms gives

$$x_i = h_i \quad z_i = l_i \quad y_i = x_i^\alpha z_i^{1-\alpha} \quad (\text{III.19})$$

where  $h_i$  is the firm's total use of high-skill labor and  $l_i$  is the firm's total use of low-skill labor. The firm owner maximizes the expected profit rate:

$$E \{ \Pi_i \} = Q_i E \{ \hat{\Pi}_i \} y_i - (1 - Q_i) \delta I_i.$$

The first term captures the expected profit rate, if the firm owner does not threaten to shut down the firm. The second term captures the non-stochastic profit rate, if the firm owner threatens to shut down the firm. The profit rate if the firm owner does not threaten to shut down the firm is

$$\hat{\Pi}_i = \hat{P}_i y_i - w_{ih} h_i - w_{il} l_i - \delta I_i$$

where the inverse demand function, given by (III.2a), restricts the owners feasible choices of  $y_i$ . The price of the consumption good is written in upper case since it, via (III.2a), is stochastic. Note that  $y_i$  is certain since the firm owner can control the number of workers to employ and thereby the output of the firm, hence also  $h_i$  and  $l_i$  are non-stochastic. Even though the wage rates are stochastic, the wage rates conditional on the firm owner not threatening to shut down, are not. Hence  $w_{ih}$  and  $w_{il}$  are used.

The problem for the firm owner is complicated by the fact that the probability that the firm owner will not find it optimal to threaten to shut down the firm,  $Q_i$ , depends on the choice of employment,  $h_i$  and  $l_i$ . That is, a rational firm owner must take into account the impact of his or her employment choice today, on the probability that he or she will threaten to shut down the firm during the period.

There exists a minimal realization of  $\tilde{D}_i$ , the stochastic demand variable, such that the firm owner is willing to keep producing. This threshold value, denoted  $\underline{d}_i$ , is defined by relation (III.9) as:

$$\hat{\pi}_i \left| \begin{array}{l} \\ \underline{d}_i = \underline{d}_i \end{array} \right. = -\delta I_i . \quad (\text{III.20})$$

The probability that the firm owner is not inclined to threaten to shut down the firm, given  $\underline{d}_i$ ,

is simply  $Q_i = 1 - F_i(\underline{d}_i)$ . The cumulative density function,  $F_i(\cdot)$ , is in turn given by (III.1).

### Employment and Firm Size

The problem solved by the firm owner in order to determine employment of high-skill and low-skill workers becomes:

$$\begin{aligned} \max_{\underline{d}_i, h_i, l_i} \quad & [1 - F(\underline{d}_i)] [E \{\hat{P}_i\} y_i - w_{ih}h_i - w_{il}l_i] - \delta I_i \\ \text{s.t.} \quad & \text{(III.2a), (III.19), (III.20).} \end{aligned}$$

The wage rates for high-skill and low-skill labor, taken as given by the firm, are denoted  $w_{ih}$  and  $w_{il}$ , respectively. These wage rates are called the competitive wage rates and are paid to workers, only if the firm owner decides to keep the firm alive. Due to capital depreciation, the firm owner must pay  $\delta I_i$  to replace depreciated capital.

The solution to this problem is derived in Appendix III.B and the unique maximizing choice of  $(\underline{d}_i, h_i, l_i)$  is:

$$\underline{d}_i^* = \begin{cases} \frac{(1-\beta)(1+\Delta)}{1+\beta} & \beta \leq \Delta \\ 1 - \Delta & \beta > \Delta \end{cases} \quad \text{(III.21a)}$$

$$h_i^{*\beta} = (1 - \beta)E \{\hat{D}_i^*\} \left[ \frac{C}{\bar{P}} \right]^\beta \left[ \frac{\alpha}{w_{ih}} \right]^{1-(1-\alpha)(1-\beta)} \left[ \frac{1-\alpha}{w_{il}} \right]^{(1-\alpha)(1-\beta)} \quad \text{(III.21b)}$$

$$l_i^{*\beta} = (1 - \beta)E \{\hat{D}_i^*\} \left[ \frac{C}{\bar{P}} \right]^\beta \left[ \frac{\alpha}{w_{ih}} \right]^{\alpha(1-\beta)} \left[ \frac{1-\alpha}{w_{il}} \right]^{1-\alpha(1-\beta)}. \quad \text{(III.21c)}$$

As is shown in Appendix III.B there are two solutions. If the variation in demand is small compared to the degree of market power,  $\Delta < \beta$ , firm owners never find it optimal to exercise the option to shut down the firm. In this case  $\underline{d}_i^* = 1 - \Delta$ ,  $Q_i^* = 1$  and  $E \{\hat{D}_i^*\} = E \{D_i\} = 1$ .

The more interesting case, where firm owners occasionally exercise their right to shut down the firm, applies in the opposite case, when the variation in demand is large compared to the degree of market power, , i.e.  $\beta \leq \Delta$ . In this case the firm owner threatens to shut down the firm if the realization of  $\tilde{D}_i$  is less than  $\underline{d}_i^* = (1 - \beta)(1 + \Delta)/(1 + \beta)$  and  $E \{\hat{D}_i^*\} > E \{\tilde{D}_i\}$ .

This of course implies that market power in the product market shelters workers from variation in wages, i.e. risk, and can be welfare improving if insurance markets are absent and workers are risk averse. Given the shutdown threshold,  $\underline{d}_i^*$ , it is possible to compute the differ-

ent conditional expectations of the demand variable:

$$E\{\hat{D}_i^*\} = E\{\hat{D}_i\} \left| \begin{array}{l} \underline{d}_i^* \end{array} \right. = \begin{cases} \frac{1+\Delta}{1+\beta} & \beta \leq \Delta \\ 1 & \beta > \Delta \end{cases} \quad (\text{III.22a})$$

$$E\{\check{D}_i^*\} = E\{\check{D}_i\} \left| \begin{array}{l} \underline{d}_i^* \end{array} \right. = \begin{cases} \frac{1-\beta\Delta}{1+\beta} & \beta \leq \Delta \\ 1-\Delta & \beta > \Delta \end{cases} . \quad (\text{III.22b})$$

A firm owner threatens to shut down the firm with probability  $1 - Q_i$ , i.e. only if  $\tilde{d}_i < \underline{d}_i^*$ . The probability that the firm owner does not threaten to shut down the firm is therefore  $1 - F(\underline{d}_i^*)$ . From the definition of the cumulative density function in (III.1), and the solution to the profit maximization problem in (III.21a), it follows that:

$$Q_i^* = Q_i \left| \begin{array}{l} \underline{d}_i^* \end{array} \right. = \begin{cases} \frac{1+\Delta}{\Delta} \frac{\beta}{1+\beta} & \beta \leq \Delta \\ 1 & \beta > \Delta \end{cases} . \quad (\text{III.23})$$

This implies that as demand uncertainty increases ( $\Delta$  closer to unity), the probability that firm owners pay the competitive wage rate decreases. Hence, greater demand uncertainty tends to increase the competitive wage rate but also to decrease the probability that the worker receives the competitive wage rate. The neatest property of  $Q_i^*$  is that it is independent of endogenous variables.  $Q_i^*$  only depends on two parameters: the variation in demand,  $\Delta$ , and the preference for variety,  $\beta$ .

It is difficult to predict the effect of greater market power, i.e. more preference for variety, on the size of the firm since  $\beta$  is present in the exponents in the expressions for  $h_i$  and  $l_i$ . However, an interesting result concerning the effect of shutdown threats and firm size is easily obtained:

**Proposition III.1 (Firm Size and Demand Uncertainty)** *If firm owners have the option to shut down the firm in order to avoid variable costs, i.e. paying the wage bill, greater demand uncertainty ( $\Delta$  greater) implies larger firms.*

**Proof** The result that firm size increases with demand uncertainty is easily verified by noting that the derivative of  $h_i$  and  $l_i$  with respect to  $\Delta$  is greater than zero. ■

It might appear surprising that firm size increases with uncertainty. However, as noted in Section 2.2, the revenue function is linear with respect to the demand shock. This in turn implies

that the risk neutral firm owner, without the option to shut down the firm, is not affected by a mean preserving spread in demand. Hence, if the variation in demand increases, this firm owner does not change the employment level, and the expected profit rate stays unchanged.

However, given the option to shut down the firm, the expected profit rate must increase, or at least not decrease. This follows because the firm owner can choose to ignore the option to shut down the firm. However, as is shown above, the firm owner does indeed occasionally utilize the option to shut down the firm, if  $\beta \leq \Delta$ . By hiring more workers and threatening to shut down the firm in case of sufficiently low demand, the firm owner can increase the expected profit rate. If  $\beta > \Delta$  the firm owner never threatens to shut down the firm.

$H_i$  and  $L_i$  denote aggregate employment of high-skill and low-skill workers by in-house firms. Because  $h_i^*$  and  $l_i^*$  are identical across in-house firms, no variable on the right hand side is firm specific. The competitive wage rates are easily obtained by integrating labor demand over the range of in-house firms and solving for  $w_{ih}$  and  $w_{il}$ :

$$w_{ih} = \alpha(1 - \beta)E\{\hat{D}_i^*\} \frac{K_i}{H_i} \left[\frac{C}{\bar{p}}\right]^\beta \left[\frac{H_i^\alpha L_i^{1-\alpha}}{K_i}\right]^{1-\beta} \quad (\text{III.24a})$$

$$w_{il} = (1 - \alpha)(1 - \beta)E\{\hat{D}_i^*\} \frac{K_i}{L_i} \left[\frac{C}{\bar{p}}\right]^\beta \left[\frac{H_i^\alpha L_i^{1-\alpha}}{K_i}\right]^{1-\beta}. \quad (\text{III.24b})$$

These wage rates are called competitive since they are derived from the demand of profit maximizing firms, taking the wage rate as given. However, they are not identical to wage rates on perfectly competitive markets since firms do not take the price of their output as given. The relative competitive wage rates reduce to the standard Cobb-Douglas case where the relative wage is determined by relative employment and the elasticity of substitution between high-skill and low-skill labor.

The wage rates are easily interpreted. Given the Dixit and Stiglitz preferences, workers are paid a share of revenues equal to  $1 - \beta$ , while firm owners receive the remaining share,  $\beta$ . Due to the Cobb-Douglas production function, high skill workers, as a group, receive a fraction equal to  $\alpha$  while low-skill workers, as a group, receive the remaining part, as will be clear below. Hence, competitive wage rates increase one-to-one with expected productivity. This in turn implies that the competitive wage rates increase with greater demand uncertainty, i.e.  $\Delta$  closer to unity. The interpretation is straightforward with greater variation in demand, the threshold for not threatening to shut down the firm is higher; see (III.21a). Therefore the expected competitive wage rate is higher. It is of course important to remember that changing demand uncertainty,

$\Delta$ , also changes the probability that the firm owner does not threaten to shut down the firm and pays the workers the competitive wage rates.

### Entry and Exit

The expected profit rate  $E\{\hat{\Pi}_i\}$  can be reduced using the competitive wage rate expressions, (III.24a) and (III.24b), together with the symmetric employment conditions,  $h_i = H_i/K_i$  and  $l_i = L_i/K_i$ :

$$E\{\hat{\Pi}_i\} = \beta E\{\hat{D}_i^*\} \left[\frac{C}{\bar{p}}\right]^\beta \left[\frac{H_i^\alpha L_i^{1-\alpha}}{K_i}\right]^{1-\beta} - \delta I_i.$$

New firms enter or existing firms leave the market unless the value of owning an in-house firm equals the initial investment cost. This occurs unless (III.12) is satisfied. The steady state equilibrium number of in-house firms is be:

$$K_i = \left[ \beta Q_i^* E\{\hat{D}_i^*\} \frac{1+\rho}{[\rho + \delta(1+\rho)] I_i} \right]^{\frac{1}{1-\beta}} \left[\frac{C}{\bar{p}}\right]^{\frac{\beta}{1-\beta}} H_i^\alpha L_i^{1-\alpha}. \quad (\text{III.26})$$

This relation provides a necessary condition for determining the number, i.e. range, of in-house firms in the steady state equilibrium. Because capital can be resold if the firm is shut down, the only real cost of starting a firm is the capital depreciation,  $\delta > 0$ , and the inter-temporal cost of giving up  $I_i$  while the firm is operating. The latter cost hinges on  $\rho > 0$ . Without depreciation and without impatience,  $\rho = \delta = 0$ , the cost of starting a firm is zero, and the steady state equilibrium number of firms must equal infinity, i.e.  $K_i \rightarrow \infty$ .

The following results are easily verified and most of them are intuitive:

- Higher investment costs decrease the number of firms.
- A higher rate of depreciation decreases the number of firms.
- More impatient investors decreases the number of firms.
- More demand uncertainty decreases the number of firms.

The first three results are intuitive while. The last result is an equilibrium result. There is a fixed number of workers and more demand uncertainty increases the firm size, hence in equilibrium the number of firms must decrease. The effect of greater market power,  $\beta$  greater, is again ambiguous since  $\beta$  appears in the exponents in (III.26).



### Wages

If the firm owner does not threaten to shut down the firm, the competitive wage rates, denoted  $w_{ih}$  and  $w_{il}$ , are paid to high-skill and low-skill workers. These wage rates are non-stochastic but depend on the variation in demand,  $\Delta$ , and the preference for variety, i.e. the degree of market power,  $\beta$ . If the firm owner credibly threatens to shut down the firm, high-skill and low-skill workers negotiate new wage rates. The negotiated wage rate for high-skill workers is stochastic and denoted by  $\check{W}_{ih}$  and the negotiated wage rate for low-skill workers, also stochastic, is denoted by  $\check{W}_{il}$ .

The expected wage rate of high-skill workers,  $E \{ \check{W}_{ih} \}$ , is the probability that the firm owner does not threaten to shut down the firm,  $Q_i^*$ , times the non-stochastic competitive wage rate  $w_{ih}$ , plus the probability that the firm owner threatens to shut down the firm,  $1 - Q_i^*$ , times the conditional expectation of the stochastic negotiated wage rate,  $E \{ \check{W}_{ih} \}$ . The expected wage rate for low-skill workers,  $E \{ \check{W}_{il} \}$ , is analogous. Therefore:

$$E \{ \check{W}_{ih} \} = Q_i^* w_{ih} + (1 - Q_i^*) E \{ \check{W}_{ih} \} \quad (\text{III.27a})$$

$$E \{ \check{W}_{il} \} = Q_i^* w_{il} + (1 - Q_i^*) E \{ \check{W}_{il} \}. \quad (\text{III.27b})$$

The firm's revenue is stochastic and denoted  $\check{R}_i$ . Because all in-house firms are ex-ante identical, they employ the same amount of high-skill and low-skill labor units, namely  $H_i/K_i$  and  $L_i/K_i$  respectively, implying that revenues for an in-house firm reduce to:

$$\check{R}_i = \check{D}_i \left[ \frac{C}{\bar{p}} \right]^\beta \left[ \frac{H_i^\alpha L_i^{1-\alpha}}{K_i} \right]^{1-\beta}. \quad (\text{III.28})$$

The negotiated wage rates for high-skill and low-skill workers equal  $\check{W}_{ih} = \psi^* \check{R}_i K_i / H_i$  and  $\check{W}_{il} = (1 - \psi^*) \check{R}_i K_i / L_i$ , respectively. The share of revenues captured by high-skill workers,  $\psi^*$ , is given by the Nash solution to the bargaining problem in (III.16). By comparing firm revenue, see (III.28), and the competitive wage rates, see (III.24a) and (III.24b), the competitive wage rates can be rewritten in terms of firm revenue. Hence:

$$\check{W}_{ih} = \psi^* \check{R}_i \frac{K_i}{H_i} \quad w_{ih} = \alpha(1 - \beta) \frac{K_i}{H_i} E \{ \hat{R}_i \} \quad (\text{III.29a})$$

$$\check{W}_{il} = (1 - \psi^*) \check{R}_i \frac{K_i}{L_i} \quad w_{il} = (1 - \alpha)(1 - \beta) \frac{K_i}{L_i} E \{ \hat{R}_i \}. \quad (\text{III.29b})$$

The expected wage rates for in-house high-skill and low-skill workers can in turn be written:

$$\begin{aligned} E\{W_{ih}\} &= [\alpha(1-\beta)Q_i^*E\{\hat{R}_i\} + (1-Q_i^*)E\{\psi^*\check{R}_i\}] \frac{K_i}{H_i} \\ E\{W_{il}\} &= [(1-\alpha)(1-\beta)Q_i^*E\{\hat{R}_i\} + (1-Q_i^*)E\{(1-\psi^*)\check{R}_i\}] \frac{K_i}{L_i}. \end{aligned}$$

It is now possible to solve for  $E\{\psi^*\check{R}_i\}$  and  $E\{(1-\psi^*)\check{R}_i\}$  by using the Nash solution for the bargaining problem; see (III.18). To do so, replace  $E\{W_{ih}\}$  and  $E\{W_{il}\}$  in (III.18) using the expected wage expressions above. After some cumbersome algebra

$$\begin{aligned} E\{\psi^*\check{R}_i\} &= \frac{\gamma\bar{\rho}_{ih}[\bar{\rho}_{il} + (1-Q_i^*)(1-u_b)]E\{\check{R}_i\}}{\bar{\rho}_{ih}\bar{\rho}_{il} + (1-Q_i^*)(1-u_b)[\gamma\bar{\rho}_{ih} + (1-\gamma)\bar{\rho}_{il}]} \\ &\quad - \frac{Q_i^*(1-u_b)(1-\beta)[\alpha(1-\gamma)\bar{\rho}_{il} - \gamma(1-\alpha)\bar{\rho}_{ih}]E\{\hat{R}_i\}}{\bar{\rho}_{ih}\bar{\rho}_{il} + (1-Q_i^*)(1-u_b)[\gamma\bar{\rho}_{ih} + (1-\gamma)\bar{\rho}_{il}]} \end{aligned} \quad (\text{III.31a})$$

$$\begin{aligned} E\{(1-\psi^*)\check{R}_i\} &= \frac{(1-\gamma)\bar{\rho}_{il}[\bar{\rho}_{ih} + (1-Q_i^*)(1-u_b)]E\{\check{R}_i\}}{\bar{\rho}_{ih}\bar{\rho}_{il} + (1-Q_i^*)(1-u_b)[\gamma\bar{\rho}_{ih} + (1-\gamma)\bar{\rho}_{il}]} \\ &\quad + \frac{Q_i^*(1-u_b)(1-\beta)[\alpha(1-\gamma)\bar{\rho}_{il} - \gamma(1-\alpha)\bar{\rho}_{ih}]E\{\hat{R}_i\}}{\bar{\rho}_{ih}\bar{\rho}_{il} + (1-Q_i^*)(1-u_b)[\gamma\bar{\rho}_{ih} + (1-\gamma)\bar{\rho}_{il}]} \end{aligned} \quad (\text{III.31b})$$

where

$$\bar{\rho}_{ih} \equiv \rho + \theta_{ih} \quad \bar{\rho}_{il} \equiv \rho + \theta_{il}.$$

The expressions above are quite messy. Most interestingly, equal bargaining power  $\gamma = 1/2$ , similar discount rates and labor market conditions for high-skill and low-skill workers, i.e.  $\bar{\rho}_{ih} = \bar{\rho}_{il}$  and  $\alpha > 1/2$ , imply that  $w_{ih} > w_{il}$ , and  $E\{(1-\psi^*)\check{R}_i\} > E\{\psi^*\check{R}_i\}$ .

With probability  $1 - Q_i^*$ , high- and low-skill workers are paid the negotiated wage rates,  $\check{W}_{ih}$  and  $\check{W}_{il}$ . These are easily rewritten in terms of the competitive wage rates,  $w_{ih}$  and  $w_{il}$ , by use of (III.29a) and (III.29b). The expected negotiated high-skill wage rates are:

$$\begin{aligned} E\{\check{W}_{ih}\} &= \frac{1}{(1-\beta)} \frac{E\{\psi^*\check{R}_i\}}{\alpha E\{\hat{R}_i\}} w_{ih} \\ E\{\check{W}_{il}\} &= \frac{1}{(1-\beta)} \frac{E\{(1-\psi^*)\check{R}_i\}}{(1-\alpha)E\{\hat{R}_i\}} w_{il}. \end{aligned}$$

Given the expected wage rates in both scenarios, the expected wage rates satisfy:

$$E \{W_{ih}\} = \left[ Q_i^* + \frac{1 - Q_i^*}{(1 - \beta)} \frac{E \{\psi^* \check{R}_i\}}{\alpha E \{\hat{R}_i\}} \right] w_{ih} \quad (\text{III.32a})$$

$$E \{W_{il}\} = \left[ Q_i^* + \frac{1 - Q_i^*}{(1 - \beta)} \frac{E \{(1 - \psi^*) \check{R}_i\}}{(1 - \alpha) E \{\hat{R}_i\}} \right] w_{il}. \quad (\text{III.32b})$$

The relative competitive wage rate reduces to the standard Cobb-Douglas relative wage. That is, from (III.24a) and (III.24b) it follows that:

$$\frac{w_{ih}}{w_{il}} = \frac{\alpha}{1 - \alpha} \frac{L_i}{H_i}$$

Using (III.32a) and (III.32b) it is straightforward to show that:

$$\frac{E \{W_{ih}\}}{E \{W_{il}\}} \leq \frac{w_{ih}}{w_{il}} \iff \frac{E \{\psi^* \check{R}_i\}}{E \{(1 - \psi^*) \check{R}_i\}} \leq \frac{\alpha}{1 - \alpha}. \quad (\text{III.33})$$

If this condition is fulfilled, then the possibility for owners to shut down the firm increases the relative wage of low-skill workers compared to high-skill workers. By imposing symmetry conditions, i.e.  $\bar{p}_{ih} = \bar{p}_{il}$  and  $\gamma = 1/2$ , in (III.31a) and (III.31b) this is clearly true for  $\alpha > 1/2$ , while not true for  $\alpha < 1/2$ . This implies that shutdown threats moderate wage differences across skills since relative wages are determined by bargains if the firm owner threatens to shut down the firm.

In the simplest case with a 100 percent replacement rate in case of unemployment, i.e.  $u_b = 1$ ,  $\psi^*/(1 - \psi^*)$  reduces to  $\gamma/(1 - \gamma)$ . A lower replacement ratio ( $u_b$  closer to zero) decreases the outside option of high-skill and low-skill workers. This benefits low-skill workers relative to high-skill workers, because the surplus to bargain over, which is divided equally if  $\gamma$  equals 1/2, increases. While it is difficult to verify this claim algebraically, numerical examples (see below) support this intuitive conjecture.

This completes the description of in-house firms. Taking employment, i.e.  $H_i$  and  $L_i$ , as given, all endogenous in-house firm variables are pinned down, either explicitly or implicitly.

## 4.2 Fragmented and Specialized Firms

The following section analyzes fragmented and specialized firms. Fragmented firms subcontract production of intermediate goods to specialized firms. There are two types of specialized firms:

producers of the  $X$  good, i.e. the high-skill intermediate good, and producers of the  $Z$  good, i.e. the low-skill intermediate good.

The markets for intermediate goods are not analyzed in detail. Due to either transaction costs or non-competitive markets, the markup over marginal cost is  $m_x$  for the  $X$  good and  $m_z$  for the  $Z$  good. Since the  $X$  good is produced by a constant returns to scale technology in high-skill labor only, the price of the  $X$  good is simply the markup times the wage rate for high-skill labor, i.e.  $m_x w_{sh}$ . Analogously, the price of the  $Z$  good is  $m_z w_{sl}$ , where  $w_{sl}$  is the wage rate for low-skill workers and  $m_z$  is the markup factor over marginal cost.

Specialized firms, i.e producers of the  $X$  or  $Z$  good, supply the intermediate good to a continuum of fragmented firms. Therefore, by the mean value theorem, the aggregate demand faced by a producer of the  $X$  or  $Z$  good is certain. It is assumed that any  $X$  or any  $Z$  good can be sold to any fragmented firm which implies that fragmented firms can choose the quantity of intermediate goods to use ex post the realization of the demand shock. This assumption can be rationalized if transport time and transport costs are negligible, so that specialized firms are indifferent about which fragmented firm purchases their products.

### Fragmented Firms

The owner of a fragmented firm maximizes profit by solving:

$$\tilde{\pi}_f = \tilde{p}_f y_f \left| \begin{array}{l} - m_x w_{sh} h_s - m_z w_{sl} l_s - \delta I_f. \\ \tilde{D}_f = \tilde{d}_f \end{array} \right.$$

The first term is total revenue, given that the realization of the stochastic demand variable is  $d_f$ . The next two terms are the costs of purchasing the high-skill intermediate good ( $X$ ) and the low-skill intermediate good ( $Z$ ). The last term is the cost of replacing depreciated capital. The quantity purchased of the  $X$  good is denoted  $h_s$ . Since the production technology for the  $X$  good maps one unit of high-skill labor into one unit of the  $X$  good,  $h_s$  also denotes high-skill labor requirements. The  $l_s$  symbol is interpreted analogously.

This specification implies that specialized firms supply intermediate goods on demand, that isale ex post the realization of the demand variables. This is reasonable since every specialized firm supplies intermediate goods to a large number, i.e. a continuum, of fragmented firms. Since there is no aggregate demand uncertainty, the idiosyncratic demand shocks observed by fragmented firms sum to zero and specialized firms face a certain demand.

To solve the problem of the owner of a fragmented firm, replace  $\tilde{p}_f y_f$  using the inverse demand function in (III.2b), then replace  $y_f$  using the production function in (III.3b), and finally replace  $x_f$  and  $z_f$  using the production functions in (III.3a). The problem for the firm owner is to maximize:

$$\tilde{\pi}_f = \tilde{d}_f \left[ \frac{C}{\bar{P}} \right]^\beta h_s^{\alpha(1-\beta)} l_s^{(1-\alpha)(1-\beta)} - m_x w_{sh} h_s - m_z w_{sl} l_s - \delta I_f.$$

Be careful to notice that  $h_s$  and  $l_s$  do not denote employment of high-skill and low-skill workers, instead they denote the quantity purchased of the intermediate goods (the  $X$  and  $Z$  goods) necessary to assemble the  $Y$  good. Solving this problem is straightforward. The first order conditions are:

$$\alpha(1-\beta) \tilde{d}_f \left[ \frac{C}{\bar{P}} \right]^\beta h_s^{\alpha(1-\beta)-1} l_s^{(1-\alpha)(1-\beta)} = m_x w_{sh} \quad (\text{III.34a})$$

$$(1-\alpha)(1-\beta) \tilde{d}_f \left[ \frac{C}{\bar{P}} \right]^\beta h_s^{\alpha(1-\beta)} l_s^{(1-\alpha)(1-\beta)-1} = m_z w_{sl}. \quad (\text{III.34b})$$

Solving this system for  $h_s$  and  $l_s$ , or the demand for the  $X$  and  $Z$  good, is straightforward:

$$h_s = \left\{ (1-\beta) \tilde{d}_f \left[ \frac{C}{\bar{P}} \right]^\beta \left[ \frac{\alpha}{m_x w_{sh}} \right]^{1-(1-\alpha)(1-\beta)} \left[ \frac{1-\alpha}{m_z w_{sl}} \right]^{(1-\alpha)(1-\beta)} \right\}^{1/\beta} \quad (\text{III.35})$$

$$l_s = \left\{ (1-\beta) \tilde{d}_f \left[ \frac{C}{\bar{P}} \right]^\beta \left[ \frac{\alpha}{m_x w_{sh}} \right]^{\alpha(1-\beta)} \left[ \frac{1-\alpha}{m_z w_{sl}} \right]^{1-\alpha(1-\beta)} \right\}^{1/\beta}. \quad (\text{III.36})$$

The total supply of high-skill labor employed by firms producing the  $X$  ( $Z$ ) good is  $H_s$  ( $L_s$ ), and since the  $X$  ( $Z$ ) technology maps one unit of high-skill (low-skill) labor into one unit of the  $X$  ( $Z$ ) good,  $H_s$  ( $L_s$ ) is also the aggregate supply of the  $X$  ( $Z$ ) good. The reduced first order conditions above give the demand for the  $X$  and  $Z$  goods by a single fragmented firm. Aggregate demand is easily obtained by integrating over the continuum of fragmented firms. Clearing the market for high-skill and low-skill labor implies that the equilibrium wage rates must satisfy:

$$w_{sh} = \frac{\alpha(1-\beta)}{m_x} \left[ \frac{C}{\bar{P}} \right]^\beta \left[ E \left\{ \tilde{D}_f^{1/\beta} \right\} \right]^\beta \frac{K_f}{H_s} \left[ \frac{H_s^\alpha L_s^{1-\alpha}}{K_f} \right]^{1-\beta} \quad (\text{III.37})$$

$$w_{sl} = \frac{(1-\alpha)(1-\beta)}{m_z} \left[ \frac{C}{\bar{P}} \right]^\beta \left[ E \left\{ \tilde{D}_f^{1/\beta} \right\} \right]^\beta \frac{K_f}{L_s} \left[ \frac{H_s^\alpha L_s^{1-\alpha}}{K_f} \right]^{1-\beta}. \quad (\text{III.38})$$

### Entry and Exit

To find the expected profit rate it is necessary to find each fragmented firm's relative use of the  $X$  and  $Z$  good, meaning it is necessary to find  $h_s/H_s$  and  $l_s/L_s$ . The  $h_s$  and  $l_s$  differ among fragmented firms since different firms experience different demand shocks. Aggregating using (III.35) and (III.36), it is easy to see that:

$$\frac{h_s}{H_s} = \frac{\tilde{d}_f^{1/\beta}}{E \left\{ \tilde{D}_f^{1/\beta} \right\}} \frac{1}{K_f} \quad \frac{l_s}{L_s} = \frac{\tilde{d}_f^{1/\beta}}{E \left\{ \tilde{D}_f^{1/\beta} \right\}} \frac{1}{K_f}. \quad (\text{III.39})$$

First inserting the first order conditions in (III.34a) and (III.34b) into the objective function, (III.34), then replacing  $h_s$  and  $l_s$  using the relative use of the  $X$  and  $Z$  good above, the profit rate of a fragmented firm becomes:

$$\tilde{\pi}_f = \beta \frac{\tilde{d}_f^{1/\beta}}{\left[ E \left\{ \tilde{D}_f^{1/\beta} \right\} \right]^{1-\beta}} \left[ \frac{C}{\bar{p}} \right]^\beta \left[ \frac{H_s^\alpha L_s^{1-\alpha}}{K_f} \right]^{1-\beta} - \delta I_f. \quad (\text{III.40})$$

The initial cost for starting a fragmented firm is  $I_f$ . Unless the value of owning a fragmented firm equals the initial investment cost, new firms are started or existing firms are shut down. In the steady state equilibrium, (III.12) must be satisfied. Using (III.12), letting  $Q_i = 1$ ,  $I = I_f$ , and  $E \left\{ \tilde{\Pi}_i \right\} = E \left\{ \tilde{\Pi}_f \right\}$  provides the equation necessary to solve for  $K_f$ :

$$K_f = \left[ \frac{\beta}{I_f} \frac{1 + \rho}{\rho + (1 + \rho)\delta} \right]^{\frac{1}{1-\beta}} \left[ \frac{CE \left\{ \tilde{D}_f^{1/\beta} \right\}}{\bar{p}} \right]^{\frac{\beta}{1-\beta}} H_s^\alpha L_s^{1-\alpha}.$$

It is interesting to compare this relation with the corresponding expression for in-house firms in (III.26). Treating  $\bar{p}$  as given and looking at either a pure in-house equilibrium,  $H_i = H$  and  $L_i = H$ , or a pure fragmented equilibrium,  $H_s = H$  and  $L_s = L$ , the difference in the range of variations of the consumption good is determined by the difference in the expectation of the demand shock,  $\tilde{D}$ . If  $\left[ E \left\{ \tilde{D}_f^{1/\beta} \right\} \right]^\beta \geq Q_i^* E \left\{ \tilde{D}_i^* \right\}$  the range of variations in the fragmented equilibrium exceeds the range of variations in the in-house equilibrium.

### 4.3 Firms Summary

The only endogenous variables left to determine are the employment variables, i.e.  $H_i$ ,  $L_i$ ,  $H_s$ , and  $L_s$ . To solve for the employment variables, some long run steady state equilibrium conditions are necessary.

## 5 Equilibrium results

This section discusses some of the results that are possible to derive from the model. Results based on closed form solutions are complemented by figures. First the model is used to analyze what factors affect a potential firm owner's choice to start an in-house or a fragmented firm. Next the effect from shutdown threats on the skill premium is discussed. Finally the results are illustrated in a numerical example where the skill premium is graphed for various parameter values.

### 5.1 Steady State Equilibria

To solve for the steady state equilibrium it is necessary to determine  $H_i$ ,  $L_i$ ,  $H_s$  and  $L_s$ . Appendix III.C shows how to determine the type of equilibrium that will exist, using the steady state equilibrium conditions:

$$E \{ \tilde{W}_{ih} \} = w_{sh} \quad E \{ \tilde{W}_{il} \} = w_{sl}. \quad (\text{III.41})$$

The equilibrium conditions, which simply state that expected wage rates must be equal in in-house and specialized firms, are a bit simplified. While the proper conditions should be written in terms of lifetime utilities, simplifying them does not alter the results qualitatively, but rather simplifies the exposition.

From Appendix III.C it follows that the economy will be in a fragmented equilibrium only if  $\Delta I_{if}$  is positive where  $\Delta I_{if}$  is defined as:

$$\Delta I_{if} = \left[ \frac{w_s}{Q_i^* w_i} \right]^{1/\beta} \left[ \frac{1}{\left( \frac{m_s G_{ih}}{Q_i^*} \right)^\alpha \left( \frac{m_s G_{il}}{Q_i^*} \right)^{1-\alpha}} \right]^{\frac{1-\beta}{\beta}} I_i - I_f > 0. \quad (\text{III.42})$$

If this condition is violated, the economy will be in an in-house equilibrium. The interpretation

is quite simple. If the investment cost for fragmented firms is large relative to in-house firms, an in-house equilibrium is more likely, and vice versa. This follows from the relation between  $I_f$  and  $I_i$  in the expression for  $\Delta I_{if}$ .

The  $m_x$  and  $m_z$  parameters determine the markup factor over marginal cost for specialized firms producing intermediate goods, i.e. producers of the  $X$  and  $Z$  goods. The parameters can be interpreted as the degree of competitiveness in the market for the  $X$  and  $Z$  good. A larger  $m$  implies less competitive markets which in turn implies higher prices of intermediates goods. An intuitive conjecture is that a higher degree of market power for specialized firms should decrease the likelihood of a fragmented equilibrium, which is also verified since increasing either of the  $m$ 's decreases the first term on left hand side.

To investigate the impact of a mean preserving spread in demand, it is assumed that all workers are identical. This assumption sidesteps any distributional consideration and simplifies the expressions. To see this, note that if  $\alpha = \gamma = 1/2$  and  $\bar{p}_{ih} = \bar{p}_{il}$ , it follows that  $G_{ih} = G_{il}$  and  $G_{ih}^\alpha G_{il}^{1-\alpha} / Q_i^*$  is independent of  $\Delta$ . The only remaining term depending on the variation in demand, i.e. depending on  $\Delta$ , is:

$$\frac{w_s}{Q_i^* w_i} = \frac{\left[ E \left\{ \tilde{D}_f^{1/\beta} \right\} \right]^\beta}{Q_i^* E \left\{ \hat{D}_i^* \right\}}. \quad (\text{III.43})$$

To see how the right hand side follows from the left hand side, see the definitions in Appendix III.C. To analyze the numerator, note that since

$$\frac{\partial E \left\{ \tilde{D}_f^{1/\beta} \right\}}{\partial \Delta} = \frac{(1 + \Delta/\beta)(1 - \Delta)^{1/\beta} - (1 - \Delta/\beta)(1 + \Delta)^{1/\beta}}{2(1 + 1/\beta)\Delta^2}, \quad (\text{III.44})$$

it follows that:

$$\frac{\partial \left[ E \left\{ \tilde{D}_f^{1/\beta} \right\} \right]^\beta}{\partial \Delta} > 0.$$

This relation tells that the value of a fragmented firm increases due to a mean preserving spread in demand. The rationale for this result follows from the demand function derived in Section 2.2. If a firm owner must commit to an employment choice before the demand shock is revealed, a mean preserving spread in demand does not affect the value of the firm. However, an owner of a fragmented firm has the option to choose the production level ex post the demand realization, and this extra option must increase, or at least not decrease, the expected profit and thereby the



Table III.2: Default Parameter Values

$$\begin{array}{llll} \delta = 1/10 & \rho = 1 & I_i = I_f = 1 \\ \theta_{ih} = \theta_{il} = 1/2 & C = 1 & H = L = 1 \end{array}$$

value of the firm.

Turning to the denominator, when  $\beta > \Delta$  the denominator is simply 1. However, expanding  $Q_i^* E \{ \hat{D}_i^* \}$  assuming that  $\beta \leq \Delta$  and using (III.23) and (III.22a) gives:

$$Q_i^* E \{ \hat{D}_i^* \} \left| \begin{array}{l} \\ \beta < \Delta \end{array} \right. = \frac{\beta}{\Delta} \left[ \frac{1+\Delta}{1+\beta} \right]^2 . \quad (\text{III.45})$$

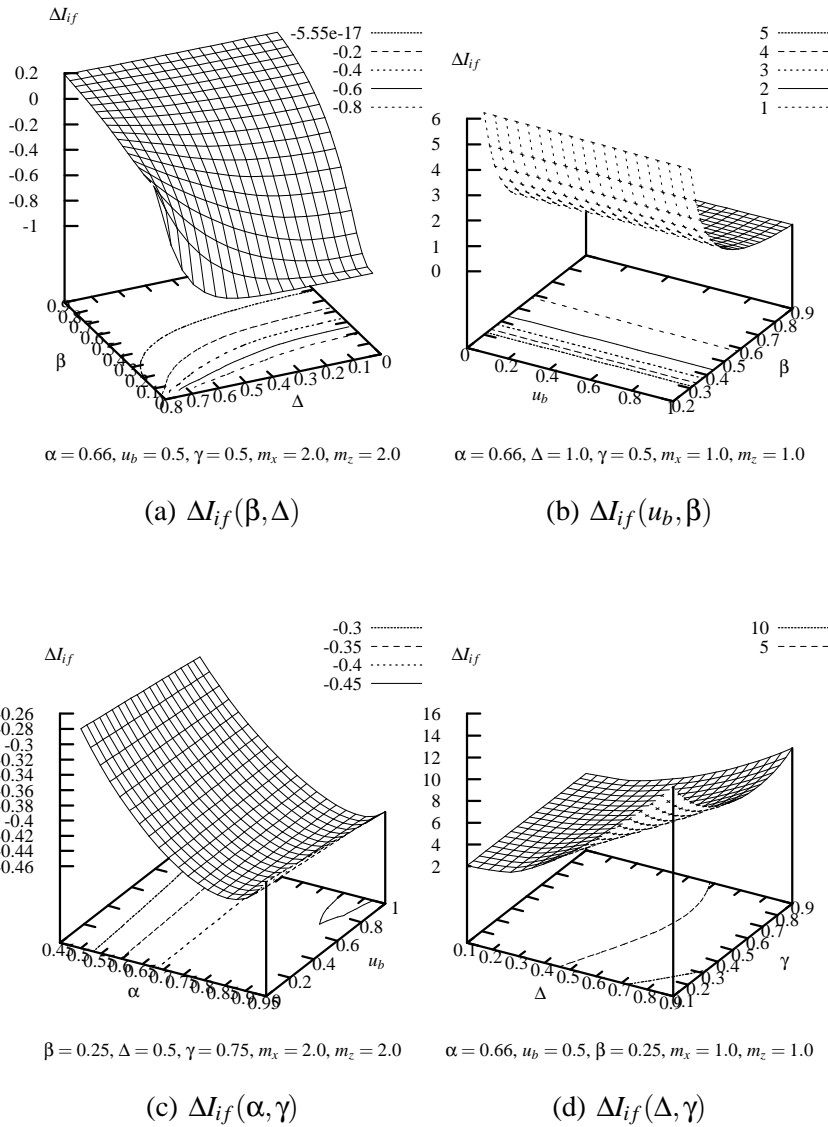
The derivative of this expression with respect to  $\Delta$  is clearly negative. The analysis of the numerator and the denominator implies that if workers are identical, a greater variation in demand implies that a fragmented equilibrium is more likely. This implies that the value derived from the option to purchase intermediate goods ex post the realization of the demand shock increases as the variation in demand increases, relative to the value of having the option to shut down the firm.

In Figure III.3 relation (III.42) is graphed. If  $\Delta I_{if}$  is greater than zero the equilibrium is a fragmented equilibrium; otherwise it is an in-house equilibrium. Figure III.3(a) verifies that increasing the variation in demand pushes the economy towards a fragmented equilibrium.

Figures III.3(a) and III.3(b) indicate that  $\beta$  has an ambiguous effect on the type of equilibrium. Note that in Figure III.3(a) the potential fragmented equilibrium is heavily distorted by non-competitive markets for intermediate goods; the markup over marginal cost is 100%. If  $\Delta$  is small, increasing  $\beta$  pushes the economy towards a fragmented equilibrium. If  $\Delta$  is close to unity, increasing  $\beta$  has a less clear effect.

A greater  $\beta$  decreases the frequency of shutdown threats, bringing the in-house equilibrium closer to a competitive equilibrium where firm owners maximize expected profits and cannot dismiss workers ex post the realization of the demand shock. A larger  $\beta$  also decreases the effect of the demand shock on revenues, thereby decreasing the value of choosing employment ex post the realization of the demand shock, an option only available for owners of fragmented firms. This implies that increasing  $\beta$  brings (III.43), i.e. the first bracketed term in (III.42), closer to unity from above.

Figure III.3: Equilibrium Type



For  $\Delta I_{if} < 0$  only in-house firms operate in the steady state equilibrium; otherwise only fragmented firms do. See Table III.2 for default parameter values.

The second effect from increasing  $\beta$  is that  $\frac{G_{jh}}{Q_i}$  and  $\frac{G_{il}}{Q_i}$  approach unity from above. Therefore, the term within the second brackets in (III.42) approaches  $m_x^{-\alpha} m_z^{\alpha-1}$  from below as  $\beta$  increases. However, due to the  $(1 - \beta)/\beta$  exponent the entire second term in (III.42) approaches unity from below as  $\beta$  increases. This tends to push the economy towards a fragmented equilibrium. Taken together, the effect of  $\beta$  on the type of equilibrium is ambiguous.

An interesting hypothesis is that the more distorted the relative prices in the in-house scenario, the more likely that a fragmented equilibrium arises. A test for falsifying this hypothesis can be carried out by varying the unemployment replacement ratio,  $u_b$ . Increasing  $u_b$  decreases the surplus to bargain over and thereby brings the expected relative wages closer to the relative marginal revenue product of each factor.

Also, bear in mind that for  $\alpha = \gamma$ , relative wages are not distorted by bargaining between high-skill and low-skill workers, and that in the fragmented equilibrium the relative wages of high-skill and low-skill workers are not distorted. Figure III.3(c) graphs  $\Delta I_{if}$  varying  $\alpha$  and  $u_b$  given that the bargaining power is  $\gamma = 3/4$ .

If the hypothesis that more distorted relative prices in the in-house scenario push the economy towards a fragmented equilibrium is correct, then  $\Delta I_{if}$  should be minimized at  $\alpha = \gamma$ , since this minimizes the distortion in relative wages. Further, increasing  $u_b$  should decrease  $\Delta I_{if}$ , because the surplus to bargain over decreases. The situation depicted in Figure III.3(c) indeed shows that  $\Delta I_{if}$  is minimized at  $\alpha = \gamma$  and that increasing  $u_b$  decreases  $\Delta I_{if}$ ; hence, the hypothesis cannot be falsified by those tests.

A second test of the same hypothesis is found in III.3(d) where  $\Delta I_f$  is graphed for combinations of  $\Delta$  and  $\gamma$ . Besides once again verifying that as  $\Delta$  increases so does the likelihood of a fragmented equilibrium, note that, as before, at  $\gamma = \alpha = 2/3$  the likelihood of an in-house equilibrium is maximized. Further, the greater the  $\Delta$  relative to  $\beta$ , the more distorted the relative prices and the less the likelihood of an in-house equilibrium.

It is tempting to fall back on the analysis in Acemoglu et al. (2001), where the degree of redistributive contracts that can be specified by low-skill workers is limited by the wage rate paid by firms hiring only high-skill workers. In this setting there are no outside firms hiring only high-skill workers, and even though a high degree of redistribution from high-skill and low-skill workers increases the value of starting a specialized firm producing the high-skill intermediate good, it decreases the value of starting a firm producing the low-skill intermediate good. It is therefore not straightforward to see why more distorted relative wages of high-skill and low-skill workers tend to decrease the likelihood of an in-house equilibrium.

## 5.2 The Bargaining Effect

In Appendix III.C, it is shown that a steady state equilibrium will not exist with both in-house firms and fragmented firms. Therefore this section presents results by comparing the two possible steady state equilibria: one in which all firms are in-house firms, and one in which all firms

are fragmented or specialized firms. Let  $H$  denote total employment of high-skill workers,  $H \equiv H_i + H_s$ , and let  $L$  denote total employment of low-skill workers,  $L \equiv L_i + L_s$ .

**The Fragmented Skill Premium** If there are no in-house firms, it follows that every employed high-skill worker is employed by a specialized firm;  $H_s = H$  and  $H_i = 0$ . Similarly, every employed low-skill worker is employed by a specialized firm;  $L_s = L$  and  $L_i = 0$ .

The skill premium,  $\omega_s = \frac{E\{W_{sh}\}}{E\{W_{sl}\}} = \frac{w_{sh}}{w_{sl}}$ , is easily computed using (III.37) and (III.38):

$$\omega_s = \frac{\alpha}{1-\alpha} \times \frac{L}{H} \times \frac{m_z}{m_x}. \quad (\text{III.46})$$

This is the standard Cobb-Douglas skill premium, augmented by a market competitiveness term. The first term corresponds to the standard Cobb-Douglas weights in the production technology. The second term is the standard Cobb-Douglas relative supply term. The last term corrects for differences in the markup over marginal cost for firms employing high-skill and low-skill workers.

**The In-house Skill Premium** If there are no specialized firms, it follows that every employed high-skill worker is employed by an in-house firm;  $H_i = H$  and  $H_s = 0$ . Similarly, every employed low-skill worker is employed by an in-house firm;  $L_i = L$  and  $L_s = 0$ .

The skill premium,  $\omega_i = \frac{E\{W_{ih}\}}{E\{W_{il}\}}$ , is easily computed using (III.32a), (III.32b), (III.24a) and (III.24b):

$$\omega_i = \frac{\alpha}{1-\alpha} \times \frac{L}{H} \times \frac{Q_i^* + \frac{1-Q_i^*}{1-\beta} \frac{E\{\psi\check{R}_i\}}{E\{\alpha\check{R}_i\}}}{Q_i^* + \frac{1-Q_i^*}{1-\beta} \frac{E\{(1-\psi)\check{R}_i\}}{E\{(1-\alpha)\check{R}_i\}}}. \quad (\text{III.47})$$

The three different terms are easily interpreted. The first term corresponds to the standard Cobb-Douglas weights in the production technology. The second term is the standard Cobb-Douglas relative supply term. The third term is the novel term. Due to shutdown threats, wage bargaining is introduced into the model.

The bargaining term,  $G_i$ , augments the standard Cobb-Douglas skill premium and is defined

as

$$G_i = \frac{Q_i^* + \frac{1-Q_i^*}{1-\beta} \frac{E\{\psi\check{R}_i\}}{E\{\alpha\check{R}_i\}}}{Q_i^* + \frac{1-Q_i^*}{1-\beta} \frac{E\{(1-\psi)\check{R}_i\}}{E\{(1-\alpha)\check{R}_i\}}} \left( = \frac{G_{ih}}{G_{il}} \right), \quad (\text{III.48})$$

where the expected revenue shares obtained by high-skill and low-skill workers are defined by (III.31a) and (III.31b). The expression in parentheses reconciles the bargaining term with the definitions in Appendix III.C.

**Proposition III.1 (The Skill Premium with Shutdown Threats)** *The possibility for firm owners to shut down the firm if revenues are low creates a bargaining situation where low-skill workers in general increase their relative wage rate, compared to high-skill workers.*

**Proof** First note that in general there is no reason to expect the market for high-skill intermediate goods to be more or less competitive than the market for low-skill intermediate goods. Therefore, in general  $m_x = m_z$ . Further, a necessary and sufficient condition for high-skill workers to have a higher competitive, given the same supply of labor, wage is that  $\alpha > 1/2 > 1 - \alpha$ .

If  $G_i < 1$ , bargaining under shutdown threats in general decreases the skill premium. From a simple inspection of (III.48), it is clear that a necessary and sufficient condition is that:

$$\frac{\alpha}{1-\alpha} > \frac{E\{\psi\check{R}_i\}}{E\{(1-\psi)\check{R}_i\}}$$

By inspecting (III.31a) and (III.31b), it is immediately clear that with a replacement rate equal to unity,  $u_b = 1$ , this condition reduces to:

$$\frac{\alpha}{1-\alpha} > \frac{\gamma}{1-\gamma}.$$

While high-skill workers might be in a superior bargaining position, i.e. have a better outside option, there is no reason to assume that high-skill workers have a greater bargaining power, i.e.  $\gamma > 1/2$ . Therefore, bargaining under shutdown threats decreases the skill premium with full unemployment coverage.

In the case of a less than 100 percent replacement rate,  $u_b < 1$ , it is still possible to prove the proposition under the assumption that  $\bar{p}_{ih} = \bar{p}_{il}$  and  $\gamma = 1/2$ . If so, the denominators in (III.31a) and (III.32a) are identical and the first term in the numerators are also identical, while

the sign of the second term differs. With  $\alpha > 1/2$ , it is easy to see that  $E\{\psi\check{R}_i\}$  is less than  $E\{(1-\psi)\check{R}_i\}$ , which proves the proposition. ■

The *bargaining power* parameter,  $\gamma$ , could be interpreted as capturing differences in the bargaining skills of high-skill and low-skill union representatives. It seems far fetched to assume any systematic differences in the bargaining skills across the parties. Hence, it seems reasonable to assume  $\gamma = 1/2$ . Given the vague definition of high-skill and low-skill, it appears to be difficult to assert anything specific about the relation between the markup factor in markets for specialized goods. The assumption that  $m_x$  equals  $m_z$ , therefore seems reasonable.

The assumption that  $\bar{\rho}_{ih}$  equals  $\bar{\rho}_{il}$  is more problematic. It seems reasonable that high-skill workers find new employment more easily than low-skill workers, implying that  $\bar{\rho}_{ih}$  is greater than  $\bar{\rho}_{il}$  (recall that  $\bar{\rho} = \rho + \theta$ ), which intuitively should increase the relative wage rate for high-skill workers. To verify this intuitive claim, maintain the assumption that  $\gamma$  equals  $1/2$ . The ratio  $E\{\psi\check{R}_i\}/E\{(1-\psi)\check{R}_i\}$  is computable given the expressions in (III.31a) and (III.31b). In order to make this ratio a bit simpler, let the unemployment replacement ratio be zero,  $u_b = 0$ . The ratio then becomes:

$$\begin{aligned} & \frac{E\{\psi\check{R}_i\}}{E\{(1-\psi)\check{R}_i\}} \\ & = \\ & \frac{[\bar{\rho}_{ih}\bar{\rho}_{il} + (1-Q_i^*)\bar{\rho}_{ih}]E\{\check{R}_i\} - Q_i^*(1-\beta)[\alpha\bar{\rho}_{il} - (1-\alpha)\bar{\rho}_{ih}]E\{\hat{R}_i\}}{[\bar{\rho}_{ih}\bar{\rho}_{il} + (1-Q_i^*)\bar{\rho}_{il}]E\{\check{R}_i\} + Q_i^*(1-\beta)[\alpha\bar{\rho}_{il} - (1-\alpha)\bar{\rho}_{ih}]E\{\hat{R}_i\}}. \end{aligned}$$

Now consider a change increasing  $\theta_h$  and decreasing  $\theta_l$  such that the product  $\bar{\rho}_{ih}\bar{\rho}_{il}$  remains constant. This is clearly beneficial for high-skill workers relative to low-skill workers because the matching quality on the labor market for high-skill workers increases while the matching quality on the labor market for low-skill workers decreases. Intuitively, this should improve the bargaining position of high-skill workers relative to low-skill workers and thereby increase the bargained relative wage for high-skill workers; that is,  $G_i$  should increase. Inspecting the ratio above, this is clearly the case, because the numerator increases while the denominator decreases. This of course raises some concerns about the importance of the result in Proposition III.1. It should however be noted that if workers have a high discount rate, the importance of the matching quality in the labor market is low, since  $\bar{\rho}$  equals  $\rho + \theta$  and  $\theta$  never appears outside this sum.

The bargaining term,  $G_i$ , is plotted in Figure III.4 for various parameter settings. A  $G_i$  less than unity implies a lower skill premium relative to the standard competitive economy and the fragmented equilibrium. The lower the  $G_i$ , the lower the skill premium. From Figure III.4(a)

it is evident that wage bargaining only decreases inequality if  $\gamma < \alpha > 1/2$ , meaning only if the superior productivity of high-skill workers is not matched by an at least equally superior bargaining power.

By inspecting Figure III.4(b), it is clear that a higher  $\beta$  decreases the moderating effect from shutdown threats. The moderating effect on the skill premium from shutdown threats on the skill premium decays as  $\beta$  approaches  $\Delta$  from below. The reason is that a higher  $\beta$  implies that shutdown threats occur less frequently, greater  $Q_i^*$ , and workers are more frequently paid their marginal revenue product. Figure III.4(b) also verifies that increasing the unemployment benefit ratio,  $u_b$ , decreases the moderating effect of wage bargaining.

Figure III.4(c) verifies that shutdown threats are redistributive as long as  $\alpha > \gamma$ , i.e. as long as the superior marginal productivity of high-skill workers is not matched by an equally superior bargaining power of high-skill workers relative to low skill workers. It is clear from the figure that high-skill workers benefit from shutdown threats relative to low-skill workers only if  $\gamma > \alpha$ .

Figure III.4(d) summarizes the results neatly. As long as demand shocks are small ( $\Delta < \beta = 1/4$ ), shutdown threats do not affect the skill premium. With a more uncertain demand ( $\Delta \gg \beta$ ), shutdown threats decrease the skill premium more dramatically, unless high-skill worker bargaining power is sufficiently superior ( $\gamma > \alpha$ ). In the latter case, demand uncertainty and shutdown threats magnify the skill premium, but this is unlikely unless there is some explicit reason as to why the bargaining power of high-skill workers should be greater than the bargaining power of low-skill workers.

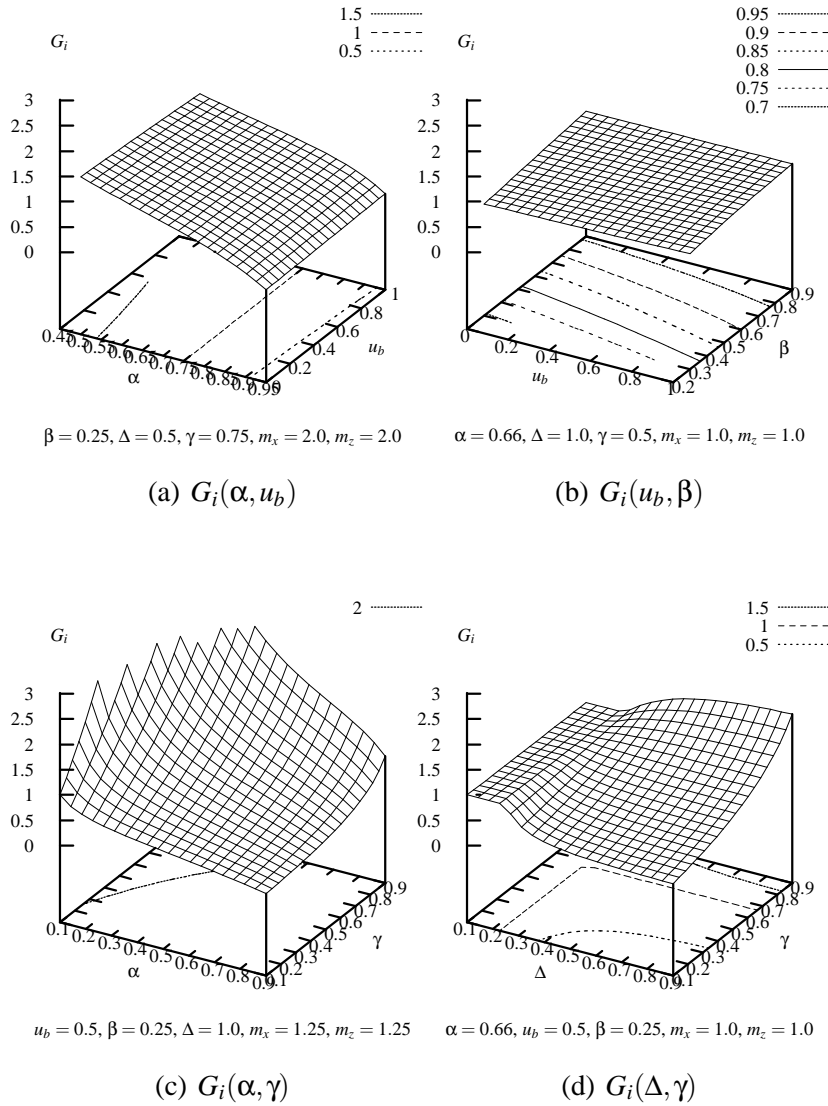
From the results above it is clear that fragmentation in general increases the skill premium since low-skill workers never bargain over wages with high-skill workers. This hurts low-skill workers, relative to high-skill workers. In a fragmented economy low-skill workers can no longer compensate their inferior productivity via a relatively stronger bargaining position.

### 5.3 The Skill Premium

To exemplify the full results of the model, consider a scenario where the markets for intermediate goods are not perfectly competitive, such that  $m_x = 2 > 1$  and  $m_z = 2 > 1$ . Figures III.5(a) – III.5(d) depict the model's prediction of the skill premium for different combinations of demand uncertainty,  $\Delta$ , and degrees of market power,  $\beta$ .

Figure III.5(a) illustrates the skill premium in a hypothetical fragmented equilibrium. In a fragmented equilibrium the skill premium is simply  $\alpha/(1 - \alpha) = 2$ . Changing the variation in demand,  $\Delta$ , or the degree of market power,  $\beta$ , does not alter the skill premium as both high-skill

Figure III.4: Bargaining Effect



If  $G_i < 1$ , the skill premium is lower in the in-house equilibrium, while for  $G_i > 1$ , the skill premium is higher in the in-house equilibrium. See Table III.2 for default parameter values.

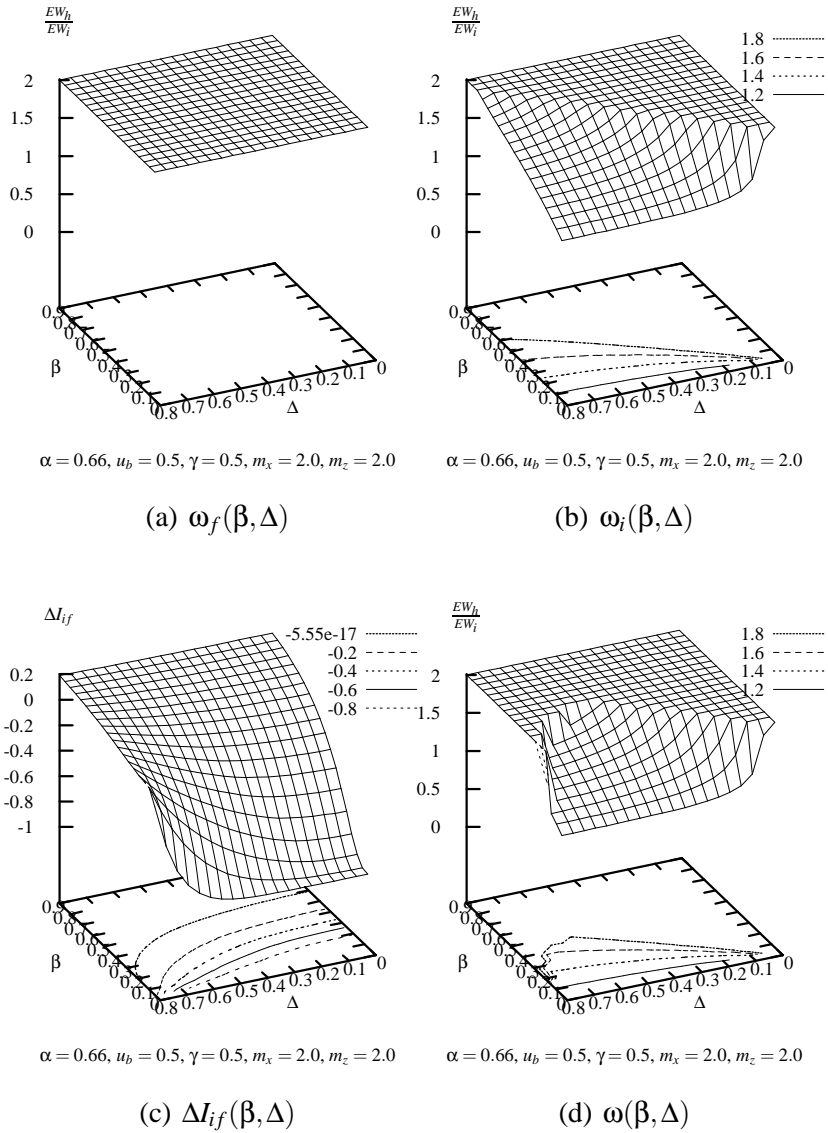
and low-skill worker wages are proportional to the marginal revenue product.

Figure III.5(b) depicts the skill premium in a hypothetical in-house equilibrium. In this scenario the skill premium decreases as  $\beta/\Delta$  increases. The reason is that as  $\beta/\Delta$  increases, shutdown threats and thereby renegotiations of wages become less frequent.

Note that as long as  $\beta > \Delta$ , firm owners never exercise their right to shut down the firm and



Figure III.5: Skill Premium



Figures III.5(a) and III.5(b) illustrate the skill premium in a fragmented equilibrium and an in-house equilibrium, respectively. The type of equilibrium is determined by  $\Delta I_{if}$  illustrated in Figure III.5(c). Figure III.5(d) illustrates the skill premium, taking the type of equilibrium into account. See Table III.2 for default parameter values.

the skill premium is identical to the skill premium in the hypothetical fragmented equilibrium. However, as the  $\Delta/\beta$  ratio increases, the skill premium decreases. At most the skill premium is about 37 percent lower than in the hypothetical fragmented equilibrium.

Figure III.5(c) maps each combination of  $\beta$  and  $\Delta$  with either an in-house or a fragmented

equilibrium. Everywhere in the graph where  $\Delta I_{if} < 0$ , the economy is characterized by an in-house equilibrium, and otherwise by a fragmented equilibrium. As can be seen in III.5(c), an in-house equilibrium is most likely for low values of  $\Delta$  compared to  $\beta$ .

Combining III.5(a) – III.5(c) yields III.5(d), which plots the skill premium taking into account the type of equilibrium. As can be seen in III.5(d), the skill premium is clearly non-linear in  $\beta$  as well as in  $\Delta$ . Taking into account the type of equilibrium hampers the potential for redistribution, since some equilibria where the in-house equilibrium redistributes, i.e. where  $\beta < \Delta$ , are discarded.

## 6 Conclusions

Relating the squeeze of wages for low-skill workers to outsourcing or linking wages to profits via bargains is nothing new. However, this paper spins those stories by considering domestic outsourcing or domestic contracting out and linking wages to shutdown threats.

**Shutdown Threats** Firm owners always have the option to default, i.e. to shut down the firm and sell its assets. If demand is lower than some endogenous threshold, firm owners can minimize losses by shutting down. In this case the losses for firm owners are limited to the depreciation of initial capital investments, since labor can be disposed without cost. Workers on the other hand always have the option to leave the firm and become unemployed, but the expected lifetime utility of being unemployed is inferior to the expected lifetime utility of being employed.

The firm owner's option to default on labor contracts leads to a bargaining situation. Firm owners threaten to shut down the firm if the realized profit rate is sufficiently low. Workers are reluctant to become unemployed and therefore negotiate new wage contracts to motivate the firm owner to keep the firm alive. Renegotiated wage rates are determined by the bargaining positions of high-skill versus low-skill workers. In the simple setting of the model, marginal productivity only matters indirectly via the outside options of high-skill and low-skill workers.

The superior marginal productivity of high-skill workers relative to low-skill workers is not matched by an equally superior bargaining power of high-skill workers relative to low-skill workers. Therefore low-skill workers benefit from bargaining relative to high-skill workers.

Another interesting consequence is the firm size effect. If owners have the option to shut down the firm, they are no longer inclined to face the full cost of low demand realizations. This asymmetry motivates firm owners to increase the size of the firm in response to greater demand

uncertainty. Workers are paid higher wages if the firm owner does not threaten to shut down the firm, but shutdown threats become more frequent.

**Fragmentation** Looking at the fragmentation process (i.e. outsourcing or contracting out) and taking into account wage bargains over losses, it is easy to see that fragmentation is likely to increase the skill premium.

In an economy with a low degree of fragmentation, each firm carries many tasks and requires a wide range of workers with different skills and skill levels. In the presence of shutdown threats, low-skill workers can increase their wage rate relative to high-skill workers via bargaining.

In an economy with a high degree of fragmentation, each firm carries out a much smaller set of tasks and hires a more homogenous group of workers. As more firms employ only high-skill *or* low-skill workers, the possibility for low-skill workers to compensate for low marginal productivity by bargaining with high-skill workers vanishes, and the skill premium increases.

The analysis shows that fragmentation is more likely to occur with greater variation in demand and as the market power of firms selling the consumption good declines, i.e. if consumer preferences for variety decreases. As would be expected intuitively, when markets for intermediate goods become less competitive, the likelihood of fragmentation declines. The quantitative analysis in the paper also suggests that the more distorted the relative wages of high-skill and low-skill workers (i.e. the more the relative wage rates of high-skill and low-skill workers deviate from their relative marginal productivity), the more likely that a fragmented equilibrium arises. However, this remains a conjecture that could neither be proven nor falsified.

## Appendix

### III.A Record of Notation

Table III.1 depicts the general logic for subscripts used to categorize different variables. Indices over a continuum are written in parentheses. All symbols are listed in Table III.3. Random variables are marked as  $\tilde{\cdot}$ ,  $\hat{\cdot}$ , or  $\check{\cdot}$ , depending on the information available. An upper case symbol is used for the stochastic variable while a lower case symbol is used to denote a particular realization of the corresponding random variable. Upper case letters are also used to denote aggregate quantities, while lower case letters are also used to denote micro quantities. Symbols marked by a superscripted  $*$  are derived from an optimization problem.

Table III.3: List of Symbols

Symbol	Meaning
$\alpha$	Cobb-Douglas exponent.
$\beta$	Representative agent's preference for variety.
$G$	Bargaining terms.
$C$	Consumption spending.
$\delta$	Depreciation rate.
$\Delta$	Parameterizes the variation in demand.
$\Delta I_{if}$	Adjusted investment cost difference.
$D$	Stochastic demand parameter.
$F(\cdot)$	Cumulative Density Function.
$\gamma$	High-skill workers' relative bargaining strength.
$H$	High-skill employment.
$I$	Investment cost.
$J$	Lifetime utility, employed.
$K$	Range of firms.
$L$	Low-skill employment.
$m$	Markup over marginal cost.
$N$	Range of specialized firms.
$\omega$	Relative wage (skill premium).
$\Pi$	Profit rate
$P$	Price of the consumption good.
$\bar{P}$	Price index.
$\Psi$	Bargaining outcome, high-skill workers' share.
$Q$	Probability of shutdown threat.
$\bar{\rho}$	$\rho + \theta$ .
$R$	Firm revenue.
$\underline{d}$	Shutdown threat threshold.
$\theta$	Labor market matching quality.
$u_b$	Unemployment benefit, fraction of expected income.
$U$	Lifetime utility, unemployed.
$V$	Value of a firm, post investment.
$W$	Wage rate.
$wb$	Wage bill.
$x$	Quantity of the $X$ good.
$y$	Quantity of the $Y$ good.
$z$	Quantity of the $Z$ good.

### III.B Profit Maximization

The shutdown condition in (III.20) is greatly simplified by using the inverse demand function in (III.2a) together with the production functions in (III.19) and (III.3b):

$$\left[\frac{C}{\bar{p}}\right]^\beta h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)} \underline{d}_i - w_{ih}h_i - w_{il}l_i = 0.$$

The objective for the in-house firm owner is therefore to maximize the expected profit rate  $E\{\Pi\}$ :

$$[1 - F(\underline{d}_i)] \left\{ \left[\frac{C}{\bar{p}}\right]^\beta h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)} E\{\hat{D}_i\} - w_{ih}h_i - w_{il}l_i \right\} - \delta I_i, \quad (\text{III.49a})$$

subject to the shutdown condition (solved for  $\underline{d}_i$ ):

$$\underline{d}_i = \left[\frac{\bar{p}}{C}\right]^\beta \frac{w_{ih}h_i + w_{il}l_i}{h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)}}. \quad (\text{III.49b})$$

The impact of  $\underline{d}_i$  on the objective function is twofold. On the one hand, a higher  $\underline{d}_i$  increases the probability, via  $F(\underline{d}_i)$ , that the firm owner will threaten shut down. On the other hand, increasing  $\underline{d}_i$  increases the expected productivity of workers, via  $E\{\hat{D}_i\}$ , and thereby increases expected profits if the firm owner does not threaten to shut down the firm. The constraint guarantees that the firm manager is loyal to the firm owner by assuring that the value of the firm is maximized.

#### B.1 The Unconstrained Solution

To solve the problem: First expand  $1 - F(\underline{d}_i)$  using the definition in (III.1), then expand  $E\{\hat{D}_i\}$  using the conditional expectation formula such that  $E\{\hat{D}_i(\underline{d}_i)\} = 1/2 \times (1 + \Delta + \underline{d}_i)$ , and finally replace  $\underline{d}_i$  in the objective function, using the constraint. The objective function simplifies to

$$(1 + \Delta)^2 \Theta^{1-\beta} - 2 \left[\frac{\bar{p}}{C}\right]^\beta (1 + \Delta) \Phi + \left[\frac{\bar{p}}{C}\right]^{2\beta} \Phi^2 \Theta^{\beta-1},$$

where the auxiliary variables  $\Theta$  and  $\Phi$  are defined as:

$$\Theta = h_i^\alpha l_i^{1-\alpha} \quad \Phi = w_{ih}h_i + w_{il}l_i.$$

The first order conditions with respect to  $h_i$  and  $l_i$  simplify to:

$$\begin{aligned} \alpha(1-\beta)(1+\Delta)^2\Theta^{1-\beta} - \alpha(1-\beta) \left[\frac{\bar{p}}{C}\right]^{2\beta} \Phi^2\Theta^{\beta-1} \\ = \\ 2 \left[ \left[\frac{\bar{p}}{C}\right]^\beta (1+\Delta) - \left[\frac{\bar{p}}{C}\right]^{2\beta} \Phi\Theta^{\beta-1} \right] w_{ih}h_i \end{aligned} \quad (\text{III.50a})$$

$$\begin{aligned} (1-\alpha)(1-\beta)(1+\Delta)^2\Theta^{1-\beta} - (1-\alpha)(1-\beta) \left[\frac{\bar{p}}{C}\right]^{2\beta} \Phi^2\Theta^{\beta-1} \\ = \\ 2 \left[ \left[\frac{\bar{p}}{C}\right]^\beta (1+\Delta) - \left[\frac{\bar{p}}{C}\right]^{2\beta} \Phi\Theta^{\beta-1} \right] w_{il}l_i. \end{aligned} \quad (\text{III.50b})$$

Dividing the first order conditions results in the familiar Cobb-Douglas mix of factors:

$$\frac{\alpha}{1-\alpha} = \frac{w_{ih} h_i}{w_{il} l_i}.$$

This implies that firms minimize costs, whichever quantity the firm plan to produce, an intuitive result. The auxiliary variables simplify as:

$$\Theta = \left[ \frac{1-\alpha}{\alpha} \frac{w_{ih}}{w_{il}} \right]^{1-\alpha} h_i \quad \Phi = \frac{w_{ih}h_i}{\alpha}. \quad (\text{III.51})$$

Eliminating  $h_i$  in the first order condition with respect to  $h_i$  using the  $\Phi$  expression:

$$\left[ \frac{\Theta^{1-\beta}}{\Phi} \right]^2 - \frac{2 \left[\frac{\bar{p}}{C}\right]^\beta}{(1-\beta)(1+\Delta)} \frac{\Theta^{1-\beta}}{\Phi} = \frac{(1+\beta) \left[\frac{\bar{p}}{C}\right]^\beta}{(1-\beta)^2(1+\Delta)^2}.$$

Solving the second order equation in  $\Theta^{1-\beta}/\Phi$  gives two solutions:

$$\frac{\Theta^{1-\beta}}{\Phi} = \frac{1 \pm \beta}{1+\Delta} \left[\frac{\bar{p}}{C}\right]^\beta. \quad (\text{III.52})$$

Solving for  $h_i$ ,  $l_i$  and  $\underline{d}_i$  is trivial given the intermediate results above:

$$h_i^\beta = \left[ \frac{\alpha}{w_{ih}} \right]^{1-(1-\alpha)(1-\beta)} \left[ \frac{1-\alpha}{w_{ih}} \right]^{(1-\alpha)(1-\beta)} \frac{\Phi}{\Theta^{1-\beta}}$$

$$l_i^\beta = \left[ \frac{\alpha}{w_{ih}} \right]^{\alpha(1-\beta)} \left[ \frac{1-\alpha}{w_{ih}} \right]^{1-\alpha(1-\beta)} \frac{\Phi}{\Theta^{1-\beta}}$$

$$\underline{d}_i = \frac{(1-\beta)(1+\Delta)}{1+\beta}.$$

To verify that  $\underline{d}_i = (1-\beta)(1+\Delta)/(1+\beta)$  is a local maximum, note that the first order conditions imply that both  $\Theta$  and  $\Phi$  are linear in  $h_i$ . Therefore, the objective function and the constraint defining  $\underline{d}_i$  can be written as

$$E\{\Pi\} = a_1 h_i^{1-\beta} - a_2 h_i + a_3 h_i^{1+\beta} - \delta I_i$$

$$\underline{d}_i = a_4 h_i^\beta,$$

with  $a_i > 0$ .

Clearly, for  $h_i = 0$  the objective function is  $-\delta I_i$ , while for sufficiently small choices of  $h > 0$ , the objective function is greater than  $-\delta I_i$ . However, the solution implying that  $\underline{d}_i = 1 + \Delta$  implies that  $1 - F(\underline{d}_i) = 0$ . Again the objective function equals  $-\delta I_i$ .

Taken together, this implies that at  $h_i = 0$ , the objective function equals  $-\delta I_i$ , but increases as  $h_i$  increases. Eventually  $h_i$  equals  $h_1^*$  which implies that  $\underline{d}_i = (1+\Delta)(1-\beta)/(1+\beta) < 1 + \Delta$ . Increasing  $h_i$  further decreases the value of the objective function until  $h_i = h_2^*$  which implies that  $\underline{d}_i = 1 + \Delta$ , where again the objective function equals  $-\delta I_i$ , since  $1 - F(1 + \Delta) = 0$ .

Therefore the solution with

$$\underline{d}_i = \frac{(1-\beta)(1+\Delta)}{1+\beta}$$

$$h_i^\beta = (1-\beta) \frac{1+\Delta}{1+\beta} \left[ \frac{C}{\bar{p}} \right]^\beta \left[ \frac{\alpha}{w_{ih}} \right]^{1-(1-\alpha)(1-\beta)} \left[ \frac{1-\alpha}{w_{il}} \right]^{(1-\alpha)(1-\beta)}$$

$$l_i^\beta = (1-\beta) \frac{1+\Delta}{1+\beta} \left[ \frac{C}{\bar{p}} \right]^\beta \left[ \frac{\alpha}{w_{ih}} \right]^{\alpha(1-\beta)} \left[ \frac{1-\alpha}{w_{il}} \right]^{1-\alpha(1-\beta)},$$

is indeed a maximum.

## B.2 The Constrained Solution

Before accepting the solution derived above it is necessary to check that the cut-off value  $\underline{d}_i$  is within the range of the realizations of the stochastic demand variable,  $\tilde{D}_i$ . That is, it is necessary

to check that  $1 - \Delta < \underline{d}_i < 1 + \Delta$ , which in turn is equivalent to checking that:

$$0 \leq F(\underline{d}_i) \leq 1. \quad (\text{III.53})$$

Given the definition of  $F(d)$  in (III.1) and maximizing choice of  $\underline{d}_i$  this implies checking that:

$$0 \leq \frac{(1 - \beta)(1 + \Delta)/(1 + \beta) - (1 - \Delta)}{2\Delta} \leq 1. \quad (\text{III.54})$$

Note that this function is decreasing in  $\beta$ , for  $0 \leq \beta \leq 1$  ( $\Delta \leq 1$ ). For  $\beta = 0$ ,  $F(\underline{d}_i) = 1$ , implying that the maximizing choice of  $\underline{d}_i$  never violates the condition  $F(\underline{d}_i) \leq 1$ .

However, increasing  $\beta$  starting at  $\beta = 0$  violates  $F(\cdot) \geq 0$  at  $\beta > \Delta$ .  $F(\cdot)$  is nothing but  $1 - Q_i$ . So for  $\beta > \Delta$ , the solution to an unconstrained problem dictates the firm owner to shut down the firm with a negative probability. This is of course the same as to say that the firm owner keeps the firm running with a probability greater than one.

The proper way to handle this problem would have been to solve the optimization problem adding the constraints  $0 \leq F(\underline{d}_i) \leq 1$ . The result would, given the discussion above, be that for  $\beta \leq \Delta$  the unconstrained solution derived above would apply, while for  $\beta > \Delta$ , the first constraint would bind and the solution to the problem would be argmax of:

$$\begin{aligned} [1 - F(\underline{d}_i)] & \left\{ \left[ \frac{C}{\bar{p}} \right]^\beta h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)} E \{ \hat{D}_i \} - w_{ih} h_i - w_{il} l \right\} - \delta I_i \\ \text{s.t.} \quad & F(\underline{d}_i) = 0. \end{aligned}$$

Consequently, the first bracketed term equals unity while  $F(\underline{d}_i) = 0$  implies that  $\underline{d}_i = 1 - \Delta$ , which in turn implies that  $E \{ \hat{D}_i \}$  equals  $E \{ \tilde{D}_i \}$ . Therefore the solution to a firm owner's problem when  $\beta > \Delta$  is:

$$\max_{h_i, l_i} \left[ \frac{C}{\bar{p}} \right]^\beta h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)} E \{ \tilde{D}_i \} - w_{ih} h_i - w_{il} l - \delta I_i.$$

The solution to this problem is simpler. Straightforward use of the first order conditions for  $h_i$  and  $l_i$  implies:

$$\underline{d}_i = 1 - \Delta$$



$$\begin{aligned}
h_i^\beta &= (1-\beta) \left[ \frac{C}{\bar{P}} \right]^\beta \left[ \frac{\alpha}{w_{ih}} \right]^{1-(1-\alpha)(1-\beta)} \left[ \frac{1-\alpha}{w_{ih}} \right]^{(1-\alpha)(1-\beta)} \\
l_i^\beta &= (1-\beta) \left[ \frac{C}{\bar{P}} \right]^\beta \left[ \frac{\alpha}{w_{ih}} \right]^{\alpha(1-\beta)} \left[ \frac{1-\alpha}{w_{ih}} \right]^{1-\alpha(1-\beta)}.
\end{aligned}$$

### B.3 The Complete Solution

By combining the constrained and unconstrained solutions, the complete solution can be stated as:

$$\begin{aligned}
\underline{d}_i^* &= \begin{cases} \frac{(1-\beta)(1+\Delta)}{1+\beta} & \beta \leq \Delta \\ 1-\Delta & \beta > \Delta \end{cases} \\
h_i^\beta &= (1-\beta) E\{\hat{D}_i\} \left| \begin{array}{l} \left[ \frac{C}{\bar{P}} \right]^\beta \left[ \frac{\alpha}{w_{ih}} \right]^{1-(1-\alpha)(1-\beta)} \left[ \frac{1-\alpha}{w_{ih}} \right]^{(1-\alpha)(1-\beta)} \\ \underline{d}_i^* \end{array} \right. \\
l_i^\beta &= (1-\beta) E\{\hat{D}_i\} \left| \begin{array}{l} \left[ \frac{C}{\bar{P}} \right]^\beta \left[ \frac{\alpha}{w_{ih}} \right]^{\alpha(1-\beta)} \left[ \frac{1-\alpha}{w_{ih}} \right]^{1-\alpha(1-\beta)} \\ \underline{d}_i^* \end{array} \right.
\end{aligned}$$

Note that for  $\beta < \Delta$ , firm owners choose the threshold level  $\underline{d}_i = 1 - \Delta$  which is at the lowest realization of the demand variable. In this case, the firm owner is never inclined to shut down the firm ex post the realization of the demand variable, i.e.  $Q_i = 1$ . Further, expected demand  $E\{\hat{D}_i\}$  evaluated at  $\underline{d}_i = 1 - \Delta$  is simply  $E\{\tilde{D}_i\}$ . Therefore, for low values of  $\beta$  the firm owner never threatens to shut down the firm and the standard competitive results hold.

## III.C Equilibrium Conditions

To solve for the steady state equilibrium it is necessary to determine  $H_i$ ,  $L_i$ ,  $H_s$  and  $L_s$ . To do this it is assumed that in the steady state equilibrium:

$$E\{\tilde{W}_{ih}\} = w_{sh} \quad E\{\tilde{W}_{il}\} = w_{sl}. \quad (\text{III.55})$$

The expected wage rates are rewritten as

$$E \{ \tilde{W}_{ih} \} = \alpha w_i G_{ih} \frac{K_i}{H_i} \left[ \frac{H_i^\alpha L_i^{1-\alpha}}{K_i} \right]^{1-\beta} \quad E \{ \tilde{W}_{il} \} = (1-\alpha) w_i G_{il} \frac{K_i}{L_i} \left[ \frac{H_i^\alpha L_i^{1-\alpha}}{K_i} \right]^{1-\beta} \quad (\text{III.56a})$$

$$w_{sh} = \alpha \frac{w_s K_f}{m_x H_s} \left[ \frac{H_s^\alpha L_s^{1-\alpha}}{K_f} \right]^{1-\beta} \quad w_{sl} = (1-\alpha) \frac{w_s K_f}{m_z L_s} \left[ \frac{H_s^\alpha L_s^{1-\alpha}}{K_f} \right]^{1-\beta},$$

where

$$w_i = (1-\beta) E \{ \hat{D}_i^* \} \left[ \frac{C}{\bar{p}} \right]^\beta \quad (\text{III.57a})$$

$$w_s = (1-\beta) \left[ E \{ \tilde{D}_f^{1/\beta} \} \right]^\beta \left[ \frac{C}{\bar{p}} \right]^\beta \quad (\text{III.57b})$$

$$G_{ih} = Q_i^* + \frac{1-Q_i^*}{1-\beta} \frac{E \{ \psi \check{R}_i \}}{E \{ \alpha \hat{R}_i \}} \quad (\text{III.57c})$$

$$G_{il} = Q_i^* + \frac{1-Q_i^*}{1-\beta} \frac{E \{ (1-\psi) \check{R}_i \}}{E \{ (1-\alpha) \hat{R}_i \}}. \quad (\text{III.57d})$$

Solving the system in (III.55) for  $H_s$  and  $L_s$  implies:

$$H_s = \left[ \frac{w_s}{w_i} \frac{1}{(m_x G_{ih})^{1-(1-\alpha)(1-\beta)} (m_z G_{il})^{(1-\alpha)(1-\beta)}} \right]^{1/\beta} \frac{K_f}{K_i} H_i \quad (\text{III.57e})$$

$$L_s = \left[ \frac{w_s}{w_i} \frac{1}{(m_x G_{ih})^{\alpha(1-\beta)} (m_z G_{il})^{1-\alpha(1-\beta)}} \right]^{1/\beta} \frac{K_f}{K_i} L_i. \quad (\text{III.57f})$$

These expressions show that if the joint marginal revenue product for workers is higher in specialized firms than in in-house firms, then

$$w_s > w_i (m_x G_{ih})^{\alpha(1-\beta)} (m_z G_{il})^{(1-\alpha)(1-\beta)},$$

meaning that more workers are employed by specialized firms and vice versa.

In the steady state equilibrium, the value of owning a firm must equal the start-up cost. This condition is met for in-house firms only if (III.12) is satisfied. The expected profit rates for in-house firms not threatening to shut down are given by (III.25a), which rewritten using the

definitions above gives:

$$E \{ \hat{\Pi}_i \} = \frac{\beta w_i}{1 - \beta} \left[ \frac{H_i^\alpha L_i^{1-\alpha}}{K_i} \right]^{1-\beta} - \delta I_i.$$

Since the value of owning a firm must equal the start-up cost, (III.12) must be satisfied. Solving for  $K_i$ :

$$K_i = \left[ \frac{\beta}{1 - \beta} \frac{1 + \rho}{\delta + \rho + \delta \rho} \frac{Q_i^* w_i}{I_i} \right]^{\frac{1}{1-\beta}} H_i^\alpha L_i^{1-\alpha}. \quad (\text{III.58})$$

In order to investigate if in-house firms and fragmented firms can co-exist in the steady state equilibrium,  $H_s$  and  $L_s$  in the expected profit expression for fragmented firms, see (III.40) are first eliminated using (III.57e) and (III.57f). In the resulting expression,  $K_i$  is eliminated using (III.58). The resulting expected profit rate for fragmented firms is:

$$E \{ \tilde{\Pi}_f \} = \frac{\rho + (1 + \rho)\delta}{1 + \rho} \left[ \frac{w_s}{w_i} \right]^{1/\beta} \left[ \frac{1}{(m_x G_{ih})^\alpha (m_z G_{il})^{1-\alpha}} \right]^{\frac{1-\beta}{\beta}} \frac{I_i}{Q_i^*} - \delta I_f.$$

As for in-house firms, in the steady state equilibrium the value of owning a fragmented firm must equal the start-up cost. That is, (III.12) must hold with  $E \{ \hat{\Pi}_i \}$  replaced by  $E \{ \tilde{\Pi}_f \}$ ,  $I_i$  replaced  $I_f$ , and  $Q_i$  replaced by 1, implying:

$$\left[ \frac{w_s}{w_i} \right]^{1/\beta} \left[ \frac{1}{(m_x G_{ih})^\alpha (m_z G_{il})^{1-\alpha}} \right]^{\frac{1-\beta}{\beta}} \frac{I_i}{Q_i^*} - I_f = 0. \quad (\text{III.59})$$

This relation depends only on parameters and exogenous variables, which in turn implies that a mixed equilibrium can occur only for a specific set of parameter values, with measure zero. That is, in the steady state equilibrium there exist only in-house firms or only fragmented firms, but not both.

If (III.59) is greater than zero, it is more profitable to start a fragmented firm than an in-house firm, while if (III.59) is less than zero then it is more profitable to start an in-house firm. As it turns out, the main determinant for which type of equilibrium to occur is the relation between fixed investment costs and marginal productivity. A higher investment cost for starting a fragmented firm,  $I_f > I_i$ , must be compensated by high marginal productivity,  $w_s > w_i (m_x G_{ih})^{\alpha(1-\beta)} (m_z G_{il})^{(1-\alpha)(1-\beta)}$ , corrected for the additional cost due to markup over

marginal cost for goods purchased on the market.

It is not surprising that a mixed equilibrium will not exist, since there is no mechanism generating an interior equilibrium. Consumers do not care whether goods are produced by in-house or fragmented firms and in equilibrium the most cost efficient production method is used, taking into account that the expected wage rates paid by in-house and specialized firms cannot deviate.

## Bibliography

- 1 Abraham, K. G. and Taylor, S. K. (1996). Firm's use of outside contractors: Theory and evidence, *Journal of Labor Economics* 14(3): 394–424.
- 2 Acemoglu, D., Aghion, P. and Violante, G. L. (2001). Deunionization, technical change and inequality, *Carneige-Rochester Conference Series of Public Policy* 55(1): 229–264.
- 3 Berman, E., Bound, J. and Machin, S. (1998). Implications of skill-biased technological change: International evidence, *The Quarterly Journal of Economics* pp. 1245–1279.
- 4 Borjas, G. J. and Ramey, V. A. (1995). Foreign competition, market power, and wage inequality, *The Quarterly Journal of Economics* pp. 1075–1110.
- 5 Burda, M. C. and Dluhosch, B. (2002). Cost competition, fragmentation, and globalization, *Review of International Economics* 10(3): 424–441.
- 6 Caroli, E. and Van Reenen, J. (2001). Skill-biased organizational change? Evidence from a panel of British and French establishments, *The Quarterly Journal of Economics* pp. 1449–1492.
- 7 Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity, *The Economic Journal* 67(3): 297–308.
- 8 Fortin, M. N. and Lemieux, T. (1997). Institutional change and rising wage inequality: Is there a linkage?, *Journal of Economic Perspectives* 11(2): 75–96.
- 9 Freeman, R. B. and Medoff, J. L. (1984). *What Do Unions Do?*, Basic Books.
- 10 Gottschalk, P. (1997). Inequality, income growth, and mobility: The basic facts, *Journal of Economic Perspectives* 11(2): 21–40.

- 11 Gottschalk, P. and Smeeding, T. M. (1997). Cross-national comparisons of earnings and income inequality, *Journal of Economic Literature* XXXV: 633–687.
- 12 Gottschalk, P. and Smeeding, T. M. (2000). Empirical evidence on the income inequality in industrialized countries, in A. Atkinson and F. Bourguignon (eds), *Handbook of Income Distribution*, first edn, Vol. 1, Elsevier Science, chapter 5.
- 13 Gustafsson, B. and Palmer, E. (1997). Changes in Swedish inequality, in P. Gottschalk, B. Gustafsson and E. Palmer (eds), *Changing Patterns in the Distribution of Economic Welfare: An Economic Perspective*, Cambridge University Press, pp. 293–325.
- 14 Gustafsson, B. and Palmer, E. (2001). 90-talets inkomstklyftor – hur och varför de ökade, *Ekonomisk Debatt* 29(7): 487–498.
- 15 Harrison, B. and Bluestone, B. (1988). *The Great U-Turn: Corporate Restructuring and the Polarization of America*, Basic Books Inc. New York.
- 16 Horn, H. and Wolinsky, A. (1988). Worker substitution and patterns of unionisation, *The Economic Journal* 98(391): 484–497.
- 17 Juhn, C., Murphy, K. M. and Brooks, P. (1993). Wage inequality and the rise in returns to skill, *Journal of Political Economy* 101(3): 410–442.
- 18 Kalleberg, A. F., Reskin, B. F. and Hudson, K. (1984). Bad jobs in America: Standard and non-standard employment relations and job quality in the United States, *American Sociological Review* 65: 256–278.
- 19 Kremer, M. and Maskin, E. (1996). Wage inequality and segregation by skill, *Working Paper 5718*, NBER. *Forthcoming in Quarterly Journal of Economics*.
- 20 Mitchell, M. F. (2005). Specialization and the skill premium in the 20th century, *International Economic Review* 46(3): 935–955.
- 21 Sap, J. (1993). A game-theoretic model of gender differences in union wage bargaining, *Labour Economics* 1: 25–48.
- 22 Thesmar, D. and Thoenig, M. (2004). Financial liberalization, firm organization and the labor market, CSEF Seminar, Spring Term 2004. <http://www.dise.unisa.it/PDF/thesmarthoenig.pdf>.

- 23 Wood, A. (1995). How trade hurt unskilled, *Journal of Economic Perspectives* 9(3): 57–80.

