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A question about questions

How teachers' questioning makes it possible to learn about the students' ways of understanding the content taught in mathematics and science

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Abstract

Title: A question about questions. How teachers' questioning makes it possible to learn about the students' ways of understanding the content taught in mathematics and science.

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This thesis reports results from a study that focuses on how teachers can learn about their students' learning in mathematics and science.

Current perspectives on learning are positioned in terms of the acquisition and participation metaphor. A third metaphor, the constitutive metaphor, is proposed and elaborated as an alternative for the current study. The theoretical framework draws upon and tries to further inform phenomenography and the "theory of variation".

The empirical material was generated from audio-taped classroom interactions and follow-up interviews. With the aid of concrete examples, teachers were probed on their understandings of selected parts of the interaction observed.

The results are described in terms of variant and invariant aspects of possible learning objects for the teacher. I pay particular attention to what the students possibly focus upon, and how they deal with the focused content in three different zones – the topical, the conceptual and the procedural zone.

The outcome of the study is discussed in relation to teachers' knowledge. Pedagogical content knowing is scrutinised and a complementary perspective where teachers' knowledge is viewed as constituted by different contextualisations of the subject matter is used to illustrate the interdependency of content as a discipline, content as taught in school and content as understood by students.

The result shows that teachers have, relatively speaking, small possibilities of making distinctions within the conceptual zone. In mathematics the topical zone dominates the interaction, in science the procedural. In other words, in mathematics the teachers mainly open for possibilities to learn, if their students remember facts and procedures; in science how they perform presentations and experiments. In both areas possibilities to make distinctions on qualities in how the students understand the content handled are rather small.

The most important finding of the study is: In order to make distinctions in relations to other persons' ways of understanding something, this something must be kept invariant and acts of knowing must be allowed to vary in relation to the invariant object of knowing.

Summary

A question about questions

How teachers' questioning makes it possible to learn about the students' ways of understanding the content taught in mathematics and science

Introduction

I want to start this summary by trying to answer the question: What can be discerned? The answer in general terms is: That which varies! I want to give some examples in order to support this claim.

A bird that sits on the branch of a tree can be very hard to notice. The variation in position when it moves or when it flies makes it possible to notice the bird. Furthermore, if the bird is coloured in a similar way to the surrounding trees it is very hard to see, but if it is coloured in contrast to the surroundings, say red, we seldom have problems in seeing it. It is the variation between different colours that gives meaning to one of them.

A common experiment in lower grade science classrooms on the sensation of temperature begins by students putting one hand in hot water and one in cold. After a while both hands are transferred to a bowl of water of intermediate temperature. With the "cold" hand the lukewarm water feels hot, while the "hot" hand feels it as cold. Hence what we experience is *differences* in temperature or *variation* in temperature rather than the temperature in itself. Temperature in this example is a dimension of variation with different instances of temperature.

These examples of movement, colour and temperature have a rather simple content, but this line of reasoning can be generalised to more complex phenomena:

> Gentlemen, I take it we are all in complete agreement on the decision here. (Everybody nodded.) Then I propose we postpone further discussion of this matter until our next meeting to give ourselves time to develop disagreement and perhaps gain some understanding of what the decision is all about.
>
> Sloan (1964)

Perhaps without thinking of variation, the director of Ford uses the idea of variation in order to get a grasp of an upcoming decision. Since everyone agrees on the decision, there is no way to contrast it against something else. Hence it is not possible to understand the meaning of the decision. He then tries to bring about variation in how the decision should be regarded in order to understand the meaning of the actual proposition. He invites different points of view. In this thesis I will term this kind of instance as *open up* opportunities for different ways of seeing, experiencing or understanding.

We see, experience or understand with the totality of all our experiences. Vladimir Nabakov has expressed this idea in the following way:

> [A] lily is more real to a naturalist than it is to an ordinary person. But it is still more real to a botanist. And yet another stage of reality is reached with that botanist who is a specialist in lilies. You can get nearer and nearer, so to speak, to reality; but you never get near enough because reality is an infinite succession of steps, levels of perception, false bottoms, and hence unquenchable, unattainable.
>
> Nabokov (1962)

In this study a lily is equally real to all of us since we live in the same ontological world, but Nabokov, in my interpretation, points to the importance of our previous experiences and how they influence how we understand what we encounter. In a similar manner teachers understand different things when observing students handling content in the classroom. Eisner has written about the importance of content knowledge in classroom observation:

> Surprisingly the quality of content being taught is frequently neglected in classroom observation. The reason I think, is that those who observe are often not specialists in the subject matter being taught and focus therefore on what the teacher and students do.
>
> Eisner (1991, p. 178)

Likewise, when teachers use their classroom observation to try to make inferences about what their students know, the distinctions they are able to make are related to their knowing both about the topic taught and about the students' ways of understanding the topic. Shulman (1986) among many others has described teachers' knowledge. I will come back to this tradition when I discuss the results of the study.

Marton (1994) has described both teaching and research on teacher thinking and their teaching in terms of "the erosion of content". By this he refers to the lack of focus on how content is handled in our classrooms. This thesis is about the learning of content, that is, teachers learning about how the students handle content in the classroom. The theoretical framework draws upon, and tries to further inform, the research approach of phenomenography (Marton, 1981; Marton & Booth, 1997). More precisely it is related to the currently named "theory of variation" (Marton & Pang, 1999; Runesson, 1999, Rovio-Johansson, 1999). In this theory, teachers and students collectively constitute a space of learning in the classroom. The teacher is internally related to the content taught and the students are internally related to the content learned. The students as well as the teachers have possibilities to experience the content handled.

In order to support students' learning in our classrooms in the best ways possible, teachers need to have a firm grasp on what the students know and how they understand. In a pilot study, teachers reported in interviews that their main source of information on their students knowledge comes from being with them in the classroom, (see also Johansson & Emanuelsson, 1997). The main study was designed to describe teachers' possibilities to learn about the students' learning in the naturalistic classroom.

Perspectives on knowing

Sfard (1998) describes contemporary perspectives on learning in terms of two metaphors, the acquisition and the participation metaphor. I want to propose a third, the constitutive metaphor. Lave (1996) suggests that all theories of learning at a minimum should address three aspects of learning, telos – the direction of learning, mechanism and subject-world relation. Combining Sfard and Lave while adding a third aspect, the agent of learning, gives the following table.

Table 1. Metaphors on learning

	Acquisition (Constructivism)	Constitution (Theory of variation /Phenomenography)	Participation (Socio-cultural perspectives.)
Subject – world relation	varying, unclear	non-dualist explicitly	varying, unclear
Telos, the direction of learning	internal consistency, resolution of contradictions, viability	increased differentiation of wholes, inclusivity	peripheral to central partici-pant, use of artefacts and discourse
Mechanism	cognitive conflict adaptation assimilation accommodation reflective abstraction	variation, discernment, simultaneity	participation pre-interpreta-tion mediation
Agent who or what, drives learning	the individual, the inner strives for equilibrium, the outer constrains	relations to other and to the world, interplay between "inner" and "outer"	the collective, discourse, culture, the "outer" precedes the "inner"

The proposed third metaphor tries to transcend the theoretical gap between the other two. The theoretical framework of this study can be related both to the tradition of learning as a social and cultural practice (cf. Vygotskij, 1978; Bruner, 1990; Wertsch, 1998; Säljö, 2000) and to the tradition emphasising the individual learner trying to make sense of the world (cf. Ausubel, 1968; Furth, 1969; Piaget, 1982; von Glasersfeld, 1995). However, penomenography can also be related to Gestalt psychology (cf. Wertheimer, 1945; Gurwitch, 1964) but involves a more precise focus upon how the learner sees, experiences or understands what is learned (Marton & Booth, 1997; Bowden & Marton 1998).

Aims

The aim of the study was to describe variation in the ways in which teachers' questions in the classroom open up possibilities for them to see, understand or experience their students' ways of understanding in the subject matter areas of mathematics and science. A subordinated aim was to describe differences and similarities, as regards to these possibilities, between these two content areas.

In classroom studies it is common to focus upon how teachers *form* their practice, that is how and what they teach. Instead I focus upon how they possibly are *formed* by their practice. Hence I view the classroom as a place for learning not only for the students, but for the teachers also.

Teachers' questions in the classroom

Black and William (1998) give an overview of studies on formative assessment by teachers in their classrooms. A total of 681 studies were reviewed and 250 selected for reading in full. Their most important findings in relation to this study are that it is qualities in the interaction between teachers and students that are most important in relation to the students' learning and that all assessments are grounded in an explicit or implicit idea of learning. Moreover, they also stress the importance of focusing not only on assessments per se, but also on how results are fed back to the students. Current practice is described as encouraging a surface approach to learning and seems to be focused on whether or not students remember isolated facts. Value and rank order are over-emphasised on behalf of learning. Norm-referenced approaches are used more than criterion-referenced.

Garnett and Tobin (1989) describe two high school chemistry teachers. On the surface the teachers seemed very different. One used a didactic approach to teaching while the other had a more progressive way of teaching. However, both of the teachers were very skilled at posing questions that made the students' understandings visible in the classroom. They conclude that it was the deep pedagogical content knowledge of both teachers that enabled them to focus upon qualities in the students' understanding rather than superficial characteristics of their teaching.

Few studies of classroom questioning have analysed teachers' learning. Davies describes teachers' *listening* in the classroom. He reports results from a longitudinal case study on teachers' ways of listening to the students while teaching mathematics. Three qualitatively different types of listening, *evaluative*, *interpretative* and hermeneutic *listening* are described (Davies, 1997). I have interpreted Davies's results in terms of two different dimensions. The first dimension is concerned with whether the students' contribution to the interaction alters the course of the teaching. This happens only in the last type. The other dimension is on whether the questioning aims at getting an anticipated answer or if it opens up for the students' ways of understanding. In the first type, "evaluative listening", the teachers opens up for anticipated answers only. Both dimensions are present simultaneously only in the last type.

The empirical study

The study can be related to Stake's naturalistic and theory generating instrumental case study approach (Stake, 1994). I analyse the possible meanings of what students know from a point of departure in classroom interaction, as seen from the perspective of what can potentially be experienced by teachers in the classroom. I describe variation within and between the different cases.

Data are generated from observations of eight teachers' classrooms and follow-up interviews. The teachers were selected on the basis of a pilot study of 24 teachers. The sampling of teachers and classrooms was made in order to maximise variation in types of questions posed, and at the same time minimise the number of school subjects. All selected teachers were non-specialists as regards to subject matter. All thaught students in the lower grades (ages 7 to 13) of the compulsory school in Sweden. Each classroom was observed two to four times in mathematics and science, respectively. Classroom observations and interviews were audiotaped and transcribed verbatim.

First- vs. second-order perspective

In studies with a "classical" phenomenographic interest, interviewees are typically probed on their conceptions, ways of understanding, seeing or experiencing some aspect of the world around them. This is normally done by analysing talk in an interview. Transcribed speech is then analysed in terms of what conceptions it possibly represents, reflects or is resonant with. In the current work transcribed speech is also analysed. Both interviews and observations were regarded as situations where people act verbally, and I interpreted these acts in terms of how the teachers' experienced their students' ways of understanding aspects of the content handled in the classroom interaction. Hence I see no fundamental difference between data from interviews and data from observations as regards to the analytical attitude I adopt. In both cases acts of speech are analysed in a second-order perspective from the teachers point of view. The second-order perspective was in this study a matter of analytical attitude.

I do not describe the result in definite terms of how the teachers experienced their students' knowledge. Instead, I describe it in terms of *possible* ways of understanding. Hence I interpret speech and other acts in the classrooms in terms of affordances and constrains for the teachers' learning rather than what they actually learned.

Question episodes

Typically teachers initiate classroom interaction by posing a question or making a statement to the students. This can be done in either a written or spoken format. By so doing they open up opportunities for the students to participate in classroom interaction. The initial question together with the related interaction that follows is called *a question episode* below. In a question episode the topic discussed or otherwise handled is invariant. This definition is not unambiguous, but in practice it has worked well as far as getting manageable units of text to analyse is concerned.

Phases of the study

A first explorative phase has been followed by a confirmative phase. In the first phase data from each teacher was analysed separately. Question episodes were delimited and analysed with respect to potential meanings for the teacher. In order to describe these meanings a model of description was developed in an abductive process (Alvesson & Sköldberg, 1994; Prawat, 1999). The model has been used to describe teachers' possibilities to learn both in the analytical case studies and in the summarised results.

Results

The results are presented in three sections. These sections follow a different sequence than the temporal one sketched above. The first section contains the tentative and abducted theoretical results, followed by examples from some of the eight teachers' classrooms. In the last section the result is summarised and the two content areas are compared.

Theoretical result

Acts and objects of knowing

In the classroom, teachers and students do a lot of things: they talk, make calculations, conduct experiments and make presentations, and so on. A common belif is that both the teacher and the students understand in certain ways,what they talk about, what they experiment with, and what they make presentations about. The meaning of the content handled in the classroom is formed both by what teachers and students orient themselves towards and how this orientation takes place. Eisner has written on the relation between what the teaching is about and how it is performed.

> The processes through which ideas are grasped and understood, which themselves are influenced by the conditions of teaching, give meaning to the content learned. Hence *how* something is taught and *what* is taught are, from an experiential perspective, part of the same whole. One can attend to both the separable and the experiential, inseparable, aspects of content. (1991, p. 178, italics in original)

Translated to teachers' possibilities to learn about the children's learning, this means that the meaning making of teachers is done in relation to both what the students are handling (students' object of learning) and how this handling takes place (students' acts of learning). The aspects are intertwined in the constitution of meaning for the teacher. This composite object has, by the same reasoning, a relation to how the teacher directs herself towards the students' knowing. These relations are described in the figure below:

Researcher	object	act	(researchers' object, act of knowing)
Teacher	object	act	(teachers' object, act of knowing)
Students	object	act	(students' object, act of knowing)

Figure 1. Architecture of the object of learning

This architecture of the object of learning is a development of the notion of "minds in contact", in the sense that both teacher and students orient themselves toward the same content. This is possible since teachers, according to the non-dualistic assumption of phenomenography, are considered internally related to the content taught, and students in the same way are considered internally related to the same content (Alexandersson, 1994, Marton, 1994), this time though as learners. The object aspect is considered invariant over the three levels, but the act aspect is allowed to vary. This means that the potential meanings that can be associated with the different agents in the levels also can vary, since meanings are constituted in relation to both objects and acts of knowing.

Different zones in the classroom interaction

The objects that are handled in the interaction can be of qualitatively different types. I describe this in terms of the objects being related to different zones in the interaction. The choice of *zone* as an aspect of the interaction is related to Vygotskij's idea of the *zone of proximal development* (1962). It is viewed as a mutual constitution of the students and the teacher in relation to the content. It is not looked upon as a trait of the student, the teacher or the content. I have termed the zones: topical, conceptual and procedural zone.

Objects in the topical zone are facts and procedures, such as solution strategies, which have to be remembered. They can be mathematical facts such as number facts or the application of an algorithm. Science examples are names of organisms, what they eat or how different morphological structures are named. Typically students' knowing is categorised in terms of right or wrong by the teacher in the topical zone. In the conceptual zone, the interaction is about ways of understanding rather than remembering. For example, why a mathematical algorithm functions in a certain situation and not in another, or why a certain pattern of dots can be described with a specific mathematical expression. In science the interaction can be about reasons for the function of different organisms or ecosystems. Distinctions are made in terms of qualitatively different understandings rather than whether the understanding is legitimate. The interaction in the procedural zone is about the form of the students' work rather than the content. The content possible for the teacher to discern is the form of the students' presentations such as oral presentations, posters or booklets. In this type of interaction the teachers make distinctions such as clear – not so clear presentation, neat – not as neat poster.

There can be shifts between different zones within the same question episode. Questions within one zone can point to another. A shift from the topical to the conceptual zone is termed *vertical* since such a question deepens the topic discussed and points to more general or "bigger ideas" within the topic. A question that points to different contexts, sometimes a context outside the common school context, is termed a *horizontal* question.

Case studies

In this section I present examples from some of the eight teachers in the study. Selected episodes are cases of situations where teachers have possibilities to make distinctions about their students' knowing.

Agneta's science presentations

The students have worked some weeks with individual presentations of their "research" on aquatic organisms. Today they will make their presentations in front of the class. Each student is supposed to make a short oral presentation of his or her written work on the octopus, lugworms, and the whale shark. Agneta initiates the activity by repeating what she wants them to do.

> Agneta: When you have told the others about your animal you must ask two questions /.../ You know ... we have two ears [and only one mouth]. We learn very much by listening /.../ What do we demand of those who talk? /.../ What demands does it put on those who present ... M?
>
> M: You must talk loud and clear.
>
> Agneta: Otherwise we cannot hear. I will stand further back so I can hear if you talk loud and clear. And then when the question comes you write short a answer. It doesn't matter if you spell wrong or something. Just write a short answer.

The explicit demands stated are all about the form of the presentation. Nothing is said about the content. The examples below are transcripts from three different students.

> P: The big whale shark: The whale shark reaches 15-20 m. It's whitish spotted, and has six fins. Amongst other places it lives outside the shores of Mexico in warm waters. It eats small squid and small fish such as anchovies, sardines and plankton.
>
> Agneta:Hmm, can you show the drawing? [student shows a drawing] ... Can you read your questions now?
>
> P: How big can it grow? /.../

Agneta: Now, I want you to read once more. Tell us about the whale shark. Read everything again; last time was too quick.

P: [Repetition, word by word, identical with the above]

Agneta: Now, can you take the next question about the whale shark!

P: Where does it live?

The next student makes a presentation about the octopus.

P: The octopus: The octopus belongs to the same group of animals as snails and clams. ... The octopus can change its colour rapidly in order to fit into the environment. ...

Agneta: Once more! Here are many facts, so one must be on one's toes.

P: [repetition word by word followed by the question:]

P: What group does the octopus belong to?

Agneta: What other animals belong to the group that the octopus belongs to? ... If you don't know the answer just mark with a line.

P: Can the octopus change its colour?

Agneta: Did you all hear? Write: Question four! Can the octopus change its colour? You can almost answer yes or no, it's fairly simple!

The next presentation is on lugworms:

P: The lugworm: The enemy of the lugworm is man. It eats sand. In the sand there is plankton which it eats. It's thick on the front side and thin on the back side. It lives in shallow water in the sand.

Agneta: Take it once more. Listen carefully. The questions come soon.

P: [repeats]

Agneta: Now you may take your questions.

P: Where does it live? /.../ What enemy does it have?

In the presentations above we have met three different ways for animals to lessen the risk of being eaten by other animals, or three different strategies to avoid predators. "Being big" (whale sharks), "changing colour" (octopuses) and "burrowing" (lugworms). The activity opens up this variation and the students have formulated questions that address these adaptations. However, the three adaptations are only mentioned as singular instances, as biological facts, without any overarching theory or idea. Agneta seems to view each presentation by itself, and she doesn't open up opportunities for interaction about similarities and differences among the different presentations. It would have been possible to ask follow-up questions that thematise the adaptations and try to get the students perspectives on why sharks grow big, why octopi change colour and why the lugworm burrows itself into the sand. In an extension these questions can address the functioning of animals in an evolutionary perspective. The students direct themselves towards the presentation in itself. The presentation becomes an invariant object in the episode, and Agneta opens up opportunities for different acts of presenting within the episode. Some presentations are rather long and are confident, "loud and clear". Others are short and hard to hear. This is an example of interaction in the procedural zone. The episode has a potential to be extended to the conceptual zone by asking the proposed vertical "why-type" questions.

Doris, the multiplication twelve

Doris sits together with a few students in one corner of the classroom. She has prepared several sets of twelve pieces of paper. The episode starts with Doris asking the question: "What multiplications become twelve?" The students answer her question by ordering the pieces of paper in rectangular patterns on a big piece of paper on the floor, and by writing the related mathematical notation beside the laid-out pattern. After the patterns $2 \cdot 6$, $3 \cdot 4$ and $1 \cdot 12$ one student proposes $6 \cdot 2$ and another student protests and says that they already laid out that pattern. After a short discussion where different opinions are given on whether $1 \cdot 12$ is the same as $12 \cdot 1$ they agree that the notations give the same result, but they mean different things, or represent different situations. When all possible integer solutions to the question are laid out as patterns, Agneta asks for reasons why a pattern can be described with a multiplication. She does not get an answer to her question, but during the silence after the question one boy says that he has found another multiplication resulting in twelve. He is allowed to lay out the pattern [9, 3]. This pattern is different from the others already laid-out and another student says that it "maybe it isn't multiplication". After agreement the interaction turns to the question of conditions on a pattern for representing a multiplication. The proposition, "It must be even numbers", is now suggested by the students. Doris doesn't seem to understand what they mean and she asks for clarification. Here she opens up toward three different meanings of the expression "even numbers". Even numbers are numbers that can be divided by two [cf. $3/2 = 1.5$], even numbers are evenly divided in twos

[4/2 = 2], and even numbers mean rows of "equal numbers of pieces of paper". In this manner, an answer which is mathematically incorrect [9 · 3=12] becomes the key to solve a much more complicated problem about multiplication. It is Doris's attitude towards the students' knowledge where she focuses on what they mean and not what they explicitly say that opens up opportunities for a variation in acts of knowing in relation to the same object of knowing in the conceptual zone. Simultaneously, the interaction takes place in both the topical (number facts) and the procedural zone (the students' presentations of and argumentation for their solutions). This is the only empirical example where all zones are present in the same question episode.

Boel's division

The third empirical example is also from mathematics. In the excerpt below Boel publicly asks a student how she had calculated the division 4282/2 and got the answer 2141.

> Boel: How did you think when you got it? The answer? Tell us how you did!
>
> P: [inaudible, but fieldnotes indicate 4282/2; 4 – 2 = 2, 2 – 1 = 1; 8 – 4 = 4; and 2 –1 = 1; answer: 2141]
>
> Boel: Uhm, you say take away! From the four you take away two... and get two left, is that what you mean? (yes) You say four take away two. Two left you say (yes). Two take away two, does one get one left then? ... ... Do you all say take away? (nooo) How... What do you say P?
>
> P: ...
>
> Boel: You divide right off... You take one number at a time?
>
> P: Yes.
>
> Boel: So you do. You don't think four, take away two, you think four...
>
> P: Half of four is two.

The teacher's question on how the student thinks opens fup the opportunity or a description of division as finding the number subtracted equal to the result of the subtraction. 4/2 is understood as 4 – 2 = 2, 2/2 as 2 – 1 = 1, 8/2 as 8 – 4 = 4, a type of repeated subtraction. The teacher fails to recognise this and instead offers her way of understanding to the student. The variation that the student offers is closed or ignored, and the "correct one" is offered instead. The student's way of understanding division includes the teacher's and even goes one step further. The student not only expresses the understanding that it is possible to take one number at a time, she also offers a way of thinking regarding the details of stepwise divisions, but the teacher does not thematise the details. In the interview we discuss this sequence and another related one:

Jonas: One of the first examples you took here was 82 divided by two, and you asked them how they were thinking ... Why are you interested in how they think?

Boel: I wanted to know if they took all [numbers] at once or if they did some sort of division. If they for example took 80 first, and half of it and the units by themselves.

Jonas: Why do you want to know that?

Boel: It's interesting to ask what they think, because you can give the others tips on how they can think.

Jonas: In this case it was one student that answered 41 and you asked how he thought, and the student answered, "take minus". And you asked why he took minus and he said that he had it in his head "82 minus 41". And you asked "why 41?" and he said "It doesn't work with 42."

Boel: [Laughs] I stopped there and didn't say anything more...

Jonas: Yes, you took the next example, which was 96 divided by 3... What does a sequence like this say to you?

Boel: Yes, he at least got to 41, but how he did it, to take minus... It was like Anna said [the student quoted above]. She said "take away". I didn't follow how she was thinking. What does it actually say to me? That he doesn't know what he is doing, perhaps.

Jonas: But he gave the right answer?

Boel: Yes you can think in different ways. It may have been just a lucky coincidence that the answer was correct. I don't know. Now I don't remember who it was.

In the task 82/2 I propose that the student immediately understood the answer to be 41 and checked its correctness by performing 82 – 41 = 41. Both of these students try to contribute meaning by relating division and subtraction, but the teacher fails to recognise what they mean. The question "Tell us how you think" becomes rhetorical rather than genuine. The minds of the teacher and the students do not come in contact. In this episode objects belong to the topical zone. Distinctions are being made in terms of mathematically right or wrong answers or in terms of the legitimate or illegitimate way of doing calculations.

Boel's fish

This example is also from Boel's teaching, but this time from science. After five short presentations where groups of students have summarised what they know about fish, six students are brought to the whiteboard and instructed to draw one fish each. Then they have a whole-class discussion. They discuss differences and similarities among the drawings. Until now the content of the lesson has been descriptive and focused on characteristic traits of fish. After one student mentioned that the fishes are similar in shape, the teacher described the fish as "bobbin-shaped" and asks:

> Boel: Is this a good shape, to live in the water, do you think? (Yeees) Why is it?

In order to answer the question one has to relate variation in shape to friction against the water, and hence to differences in possibilities to survive in an aquatic environment. Later in the same lesson during a dissection a question of a similar type is posed when discussing the colour of the fish being dissected.

> Boel: Hear me, it [the houting: lat: Coregonus] is brighter beneath and darker on top, does it matter? Why is it bright beneath and dark on top?

Here also one needs to relate several dimensions to each other. The variation in colour needs to be discerned at the same time as the potential variation in light seen from below and above. These two variations in colour need to be related to the potential risk to the fish of being noticed by different predators, those that may come from above and those that come from below. One student answers the question by saying: "It's not so easy to spot". The answer is recognised by the teacher with a nod, but it isn't thematised further. The following section is also from the dissection phase.

> Boel: What do they [the scales] have as a task then? ... Is to look good? (no!) Yes, fish can be a little bit vain. Can they have a more important task?... H?

H: Heat.

Boel: Maybe, to keep themselves warm. As a little quilted jacket that they take on. Yes, perhaps! I wonder what temperature fish have? What do you say K-E?

K-E: It's like our skin.

Boel: What's the task of the skin then?

K-E: Protection.

From her comment "I wonder what temperature fish have?" I infer that she knows that fish change their body temperature in accordance with temperature in the environment. Despite this she doesn't close this variation. It's left open and the student is not corrected. This area takes a lot to understand. Besides knowing that different types of organisms have differences in thermoregulation, you need to know that fish are not warm-blooded and hence do not have to keep their temperature up. So then you can infer that they do not need insulation. Later when discussing the slime on the scales, the students propose many functions of the slime. For example, to taste bad to other fishes, to be smooth against the water, to make friction less and as a protective coat. At no point does the teacher correct the students' ideas. In the interview she is explicit about not judging the students in terms of right and wrong. By critiquing the students she claims:

Boel: Then you take away the joy of thinking further for them. It's not for certain that they are wrong in what they say. And also, you can't be sure exactly what they mean when they speak. It's hard for them to formulate in words, many times, at these ages. Most of what they said was positive, and they thought it was fun and interesting.

Descriptive properties of fish are related to complex issues regarding the functioning of organisms and ecosystems. The questions open up opportunities for the students to express their experiences related to fish. The teacher opens up possible relations between the students' experiences and potential biological theory from areas such as ecology and evolution, and a rich environment is constituted for the teacher's learning. The questions are open in nature, and the teacher does not comment on the correctness of the students. The reason is said to be to encourage the students to think further and to be able to see their perspectives on the content. The content is viewed as problematic; it can be hard to say what is right and what is wrong since you cannot know for sure what the students mean. Learning is a collective enterprise; you learn from each other by contrasting different views with each other. The correctness is hard to judge since the same meaning can be expressed in different ways. The students may very well have correct ideas but have problems formulating them in words. The questions address fundamental biological issues, which can be related to "big ideas" within science such as evolution, ecology and physiology. The example "fish" is used to address these big ideas, at least potentially. The fish in itself seems not to be the central issue here. Teacher questions point beyond the fish and are of the vertical type. This is also confirmed in the interview. This is an example in which the students' object of knowing is invariant, and acts of knowing are varying in both the conceptual and the procedural zone.

Cecilia's "bingo"

This is a very short episode. When the math lesson is coming to an end Cecilia asks the students how far they have come in the mathbook. She then begins to read out loud from the answer book.

> Cecilia: 1066, a: 40,... b: 20,... c: 40,... now we are on 66.... 1067, a / .../ I'll stop there! So ... 10.00 we meet again. We take a break until 10.00.

She reads answers to about ten tasks. The melody in her voice is similar to the melody of the crier in a bingo hall. This rather odd sequence is typical for Cecilia in the respect that the students do not have to show Cecilia what they have done or how they have done it. In this sequence, objects are taken for granted and acts of knowing do not vary in any zone. In the interview Cecilia talks about her own knowing in terms of insecurity in relation to the content of mathematics and science. She talks about making efforts in order to keep the students from presenting their lack of knowledge to her. This is consistent with her way of handling the students' questions to her. Often, especially when they are complicated, she doesn't answer; instead, she refers to a later stage or a lesson to come. On one occasion when a boy asks why there is water in the exhaust gas of a car when gasoline doesn't contain water, she cannot answer and instead refers to me, the observer. She seems to make efforts not to show her lack of knowledge to the students in a way similar to those she uses to keep the students from showing their knowledge to her.

Hanna's examination

The last empirical example I describe is a science example. The class has been working with the human body for a couple of weeks, and today Hanna is going to test their knowledge individually. The students stand in a queue in front of her, and with the aid of a picture one boy explains how sound travels through the ear. The first time he doesn't pass, so he must tell her again:

Hanna: Where is the ear?

A: ... the outer ear is here.

Hanna: And the sound comes ... ?

A: ... in through the auditory meatus, goes, so it vibrates, what's it called, eardrum...

Hanna: What do you say about the eardrum. Please be quiet back there, I can't hear what he says.

A: It vibrates.

Hanna: Yes, it vibrates and what happens then?

A: Ehhh, it pushes, it pushes, what's it called...?

Hanna: It puts ...

A: ...

Hanna: These vibrations they propagate to ...

A: The hammer, the anvil (Yes...) hits the cochlea (Yes), is transformed to..., to... ehhhh.

Hanna: Yes you can do it! We have talked about what happens. What did we do before when I did this [Hanna waves with her thumb as if she is ringing on a bicycle bell]?

A: Yes, it was signals!

Hanna: It is transformed to signals in the cochlea, that's right!

A: ...

Hanna: These signals, what happens?

A: They go to the brain.

Hanna: What is this [points to the auditory nerve] the part that sends the signals to the brain, they are sent by the ...?

A: ...

Hanna: What did you say...?

A: That what's in the cochlea.

Hanna: No, it connects to the cochlea, but what's it called… that place…?

A: Ehhh …. auditory.…

Hanna: Yes, the auditory nerve goes up… Ok, now I only want to know what this is called [points].

A: It's called the auditory tube.

Hanna: Yes! Please give me a pen and your paper. You have passed!

In this excerpt of an episode, it is Hanna who fills the interaction with content. The type of interaction is the same in the other "examinations" Hanna does with the other students. She is piloting the student by giving clues to what he is supposed to say. The student seems to mention words without them having meaning to him. For him it is just empty words to be mentioned in a certain order. The student's invariant object is "saying certain words in the right order" and the acts vary in terms of correct word or not and the correct order or not. Hence, Hanna can make distinctions in the topical zone only.

Results summarised

Since the object aspect and the act aspect of knowing must be kept together, acts of knowing must vary in relation to the same object of knowing in order for the teacher to make distinctions in a zone. Using this idea allows the following description of the summarised results:

1 Students' object *topic* is invariant. Variation possible in three zones:
- Acts varying in relation to the topic
- Acts varying in relation to concepts
- Acts varying in relation to skills

2 Students' object *concept* is invariant. Variation possible in two zones:
- Acts varying in relation to concepts
- Acts varying in relation to skills

3 Students' object *skill* is invariant. Variation possible in one zone:
- Acts varying in relation to skills

4 Students' object taken for granted

– Acts not varying in any zone

This result is abducted from data and consistent with the theoretical result of the study. However, there are several variants of each of the above main types of interaction patterns. The outcome is theoretically based but not always empirically consistent. Instead, variants can be found within the above types. The excerpts in the previous section are placed within this extended structure after each variant.

Variants within the overarching structure

Type 1 **(four variants)**

1a Students' object *topic* is invariant. Variation in three zones:

- Acts varying in relation to the topic
- Acts varying in relation to concepts
- Acts varying in relation to skills

[Doris's multiplication]

1b Students' object *topic* is invariant. Variation in two zones:

- Acts varying in relation to the topic
- Acts varying in relation to concepts

1c Students' object *topic* is invariant. Variation in two zones:

- Acts varying in relation to the topic
- Acts varying in relation to skills

1d Students' object *topic* is invariant.
Variation in one zone:

- Acts varying in relation to the topic

[Boel's division; Hanna's examination]

Type 2 (two variants)

2a Students' object *concept* is invariant. Variation in two zones:

- Acts varying in relation to concepts
- Acts varying in relation to skills

[Boel's fish]

2b Students' object *concept* is invariant. Variation in one zone:

- Acts varying in relation to concepts

Type 3 (one variant)

3 Students' object *skill* is invariant. Variation in one zone:

- Acts varying in relation to skills

[Agneta's aquatic presentations]

Type 4 (one variant)

4 Students' object taken for granted. Acts not varying in any zone

[Cecilia's "bingo"]

The total structure can be ordered according to the scheme below:

$$\begin{array}{ccccccc} \mathbf{1a} & \supset & \mathbf{1b} & - & \mathbf{1c} & \supset & \mathbf{1d} \\ \cup & & \cup & & & & \\ \mathbf{2a} & \supset & \mathbf{2b} & & & & \\ \cup & & & & & & \\ \mathbf{3} & & & & & & \\ \cup & & & & & & \\ \mathbf{4} & & & & & & \end{array}$$

The ordering is not complete. Some of the patterns, such as 1b – 1c and 3 – 1b, are not ordered with respect to each other. To order these, one needs to apply outer criteria (Uljens, 1989) since the ordering between the above examples isn't of a hierarchical type.

Differences between mathematics and science

- With one exception, all patterns of variation where distinctions are possible in the conceptual zone come from mathematics teaching. (The exception is from Boels's teaching about fish.) [4 math, 1 science]
- Patterns of variation where distinctions within the topical zone come with one exception from mathematics teaching. [8, 1]
- Patterns of variation where distinctions within the procedural zone come with one exception from science teaching. [1, 5]
- Patterns of variation where possibilities of making distinctions in every zone are lacking come with one exception from science teaching. (The exception is Cecilia's ”bingo”, where she reads the correct answers from the mathematics textbook.) [1, 3].

The teachers seem to have good possibilities to learn about the students' ways of doing presentations and ways of doing experiments in science. They have poorer possibilities to determine what science facts the students remember and relatively few possibilities to determine the students' ways of understanding science phenomena. When the same teachers teach the same class, in mathematics the balance between the zones shifts. In mathematics, the dominant possibilities are to make distinctions in terms of right or wrong answers or right or wrong choice of calculation method. The possibilities to experience how the students understand the mathematics handled are fewer. The teachers also have limited possibilities to determine the students' knowing when it comes to presenting a mathematical solution, or making mathematical arguments for a point of view, line of reasoning, or a solution method.

This description of differences is true for all except two of the teachers in the present study. Boel was the only teacher who in science opens up for the students' acts in the conceptual zone, and Filippa open up possibilities for acts in the procedural zone in both subject matter areas. To summarise, five of seven teachers in the study followed this typical pattern. *

* Hanna is excluded from this comparison since she does not teach mathematics.

Discussion

Pedagogical content knowledge

Pedagogical content knowledge, PCK (Shulman, 1986), is criticised by McEwan & Bull (1991) for being grounded in an unclear epistemology . Cochran et.al. (1993) reformulated PCK based on a radical constructivist epistemology.

> From a constructivist perspective, the teacher's understanding of these two aspects [students' knowing and the context of learning] provides the basis for teaching because learning is created by the students, not the teacher, with the student's understandings and the learning setting forming the context for that learning. Moreover it is the student who decides whether or not the understanding constructed in the classroom is viable. (p. 267)

Hence Cochran et al prefer the term *pedagogical content knowing*. I share the conclusion that Shulman's concept of teachers' knowledge lacks a clear epistemology, but I am uncertain whether the grounding in radical constructivism makes it any clearer. In the radical constructivist formulation, teachers' pedagogical content knowledge is a construction related to the teacher, and the content of the students' knowing is a construction related to the students. (V)on Glaserfeld (1995) claims that the ontology of others in unclear within the radical formulation of constructivism since other persons and their knowing is a mere construction by the subject. This lack of epistemological and ontological contact between teachers and students seems problematic. I will therefore propose an alternative to pedagogical content knowing.

Content knowledge in a phenomenographic perspective

When Marton (1987,1989) discussed how research on learning and teaching can contribute to the knowledge base of teachers, he used the concept "a pedagogy of content". The difference between PCK and Marton's concept is on how content is viewed or rather from which position it is viewed. Shulman is occupied with the content as taught, the offered content. Marton is concerned with the content as handled and understood by students. This distinction is of the same category as the distinction between first- and second-order perspectives.

Marton claims that the two fields of research, "teachers' PCK", and "research on students' understanding within certain content areas" are rarely connected. He claims to point to a bridge between the two areas, the students' ways of being aware of a specific content and the teachers' ways of being aware of the same content (Marton, 1994). However such a bridge puts demands on the theoretical perspective to be used. The perspective must be able to handle both the students' ways of understanding a specific content and the teachers' ways of understanding the same content simultaneously. Furthermore, the perspective needs to be able to handle how these understandings are related to each other

Feynmans lectures (Feynman et al., 1963) are often referred to as a prototype of what pedagogical content knowledge is (Treagust, 1997). In my reading, his lectures lack a perspective on content as students' ways of understanding the content. His focus is instead on transforming the content in a manner which is consistent with physics and at the same time is on the correct level in relation to what can be expected by the audience. He demonstrates a deep and broad understanding of physics and an ability to relate physics to everyday phenomena, but his teaching does not seem to address what is known about the students' ways of understanding the phenomena dealt with.

Lybeck's work (1981) demonstrates what teachers content knowing can look like in a phenomenographic perspective. In this case though, it is the researcher that combines the pedagogical and content aspect of learning. When he analyses teaching and learning, Lybeck takes his departure in students' ways of understanding density as described in an outcome space. He interprets transcriptions from an experimental teaching sequence in terms of how the students' understanding of density evolves dynamically during teaching. The main aim of the study is to describe how students' understanding of the content becomes the content taught (p. 137). Other examples can be found in Strömdahl (1996), Tullberg, (1997) and Baille et al. (2001).

A possible difference between Shulman's (1986) PCK and a "pedagogy of content" is Shulman's separation of content from pedagogy. PCK is described as "that special amalgam" (s. 8) which is typical for teachers, but I claim that it's meaningful to view all contents as having pedagogical dimensions. All specific contents are understood contents that are communicated and negotiated between humans. It is not only the teacher who needs to recognise the pedagogical dimension even though this dimension may be more important for teachers since their mission requires that the students develop certain ways of understanding specific contents. The pedagogical aspects of contents ought to be a figure to a larger extent for teachers than for others.

A phenomenographic correspondence to PCK is, as a consequence of the non-dualist ontological assumption, placed as a relational entity between teachers and students, and it is related to the specific content in question. This knowing is not a psychological entity of the teacher; instead, it is constituted in the interaction with content and students.

To summarise, pedagogical content knowledge and the ability to see the pedagogical dimension of content are different but related perspectives of teacher knowledge. In a "PCK-perspective" there are different types of contents (cf. disciplinary content, pedagogical content, and content as understood by teachers and students). In a phenomenographic perspective there is one type of content with different aspects or different contextualisations. In respect to the teachers' work in the classroom, the most important aspect is the pedagogical dimension of content. Content as taught and as potentially understood. The current work can be looked upon as a way of trying to describe the pedagogical dimension of contents as I have tried to describe the possibilities teachers open up for themselves in order to achieve contact with the students' ways of understanding.

In the relatively few episodes in which the interactions took place in the conceptual zone, the teachers in this study seemed familiar and confident with the content and with students' ways of understanding the content. In the interviews, several teachers reported problems with both of these aspects of knowledge. For instance, Hanna talked about her own confusion in regard to the names of different parts in the ear. She said that she was uncertain about both choice of subject matter to teach and assess, and she reported insecurity with respect to her own knowledge in the chosen areas. Also Boel seemed to have problems in handling the students' ways of relating subtraction and division. In the interview Agneta talked about her main aim in science teaching as helping students to relate different phenomena in their world and to see the beauty in nature by understanding the interconnectedness between different organisms. When the students present their research on animals she doesn't relate what they say to an idea of, as she says, "how everthing fits together". This is perhaps because she does not discern the proposed object in the conceptual zone herself. These examples and more indications from the study give some evidence for the suggested relations to the teachers' own knowing.

Finally, I want to claim that it is worthwhile studying questions in classroom interaction. I believe questions are downgraded in relation to answers both in research and in teaching.

> The cutting edge of knowledge is not in the known but in the unknown, not in knowing but in questioning. Facts, concepts, generalizations and theories are dull instruments unless they are honed to a sharp edge by persistent inquiry about the unknown.
>
> Thompson (1969, s 467)

The citation is directed towards college graduates and their possibilities to ask questions during their education. Thompson suggests that students within their training are drilled in answering questions rather than formulating them. Further clues to the reasons for the results of this study can perhaps be found in teachers' possibilities to ask question within pre- and in-service education, especially questions that aims at open up toward other people's ways of understanding.

Final conclusions

Based on the study described I claim the following:

1. If teachers are to make distinctions about their students knowledge in any of the zones, then there must be an invariant object of knowledge for the students and the teacher must allow for a variation of acts of knowing related to that object. That is, objects and acts of knowing must be kept together in the classroom interaction.
2. The keeping together is not only a matter of questions in themselves. It has to do with how the teacher follows up the answers, how she takes care of what the students say, and in what ways she asks further questions. Question episodes must be considered as wholes if we are to analyse teachers' possibilities to learn in the classroom.
3. Interaction within episodes can relatively rarely be related to the conceptual zone. In mathematics the main emphasis lies in the topical zone. In science most of the interaction is within the procedural zone.

The results can be summarised even further. The most important finding of the study is: In order to make distinctions in other peoples' ways of understanding something, this something must be kept invariant and acts of knowing must be allowed to vary in relation to the invariant object of knowing.