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After Work – Investing for Retirement

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After Work—Investing for Retirement

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Abstract

The first three papers are the result of work on various aspects of pension savings. The framework for analysis and common to all papers; is a life-cycle model of a borrowing-constrained individual's consumption- and portfolio-choices in the presence of uncertain labour-income. The income-process, taxes and pension systems are also realistically calibrated.

The first paper investigates some welfare effects of forced saving through a mandatory pension scheme. Pension benefits stem from both a defined-benefit and a notionally defined-contribution part, the latter indexed to stochastic aggregate labour-income. It is shown that, early in life, individuals attribute little value to their pension savings. Furthermore, for individuals in mid-life, the welfare-loss associated with the dependency between pension-returns and labour-income growth is estimated to 1.2% of annual consumption.

The second paper investigates the diversification-demand of an individual faced with the alternative, through an individual account in a mandatory-pension scheme, of exchanging aggregate labour-income risk for equity-exposure. It is shown that, depending on age and exchange-premium, individuals will be either buyers or sellers of such swaps, and that inter-generational risk-sharing can therefore be achieved.

The third paper explores the recent transition from defined benefit to defined contribution for white-collar workers in Sweden. The main result is that individuals with the characteristic of a low expected pre-retirement income relative to average income and high variance in earnings are winners (men with university degree in the private sector), and that those with the opposite characteristic (women with university degree in the public sector) would be losers.

The final paper is an application of the Cox-Ingersoll-Ross model to the term-structure of Swedish treasuries. As with other studies, it was found that, when estimated from cross-section, the parameters are quite unstable. This instability stands in stark contrast to the relatively stable term-structure and implicit volatilities in traded options. The parameter instability is partially resolved by adding more information from options data.

Introduction

Apart from the paper on the Cox-Ingersoll-Ross model, which was one of five papers in my *Licentiate* thesis[1]; this thesis is the result of our work on various pension issues. My interest in this topic began when I was responsible for Asset and Liability Management at Skandia; and later, at the Second National Pension Fund when asset management benchmarks were to be formulated. For such (and similar) organisations the objective function is not easily identifiable, as the assets belong to a very large and diverse group of savers. However, some of the savers are constrained. The most important constraint is that human capital cannot be mortgaged; which results in too little equity exposure in early life. Intuitively, the conclusion was that the assets should be managed according to the objective of the "constrained" savers, typically the young; since the unconstrained savers can easily compensate for any sub-optimal pension allocations by reallocating within their private savings.

Regulators have also stressed the importance of individual situations—“*what is a good and reasonable advice in one case...can be reckless or erroneous in another”*[2]. Individual financial advice has so far been restricted to the wealthy few, but the trend towards large defined contribution systems, creates a new demand for cost-effective advisory techniques, for almost all individuals. The work by William Sharpe in creating Financial Engines as an advisory service to individuals is a major step in this direction. Our contribution supports the view, that a large welfare loss is associated with the idea of—one-size-fits-all.

Merton[3] also have an individual portfolio choice in mind, when arguing that what we should expect from advisors is to ”*help the customer design a financial plan to determine his optimal life cycle needs and then find the products necessary to implement that integrated plan in a cost-efficient manner*”, rather than letting the individuals make “*decisions that they had not had to make in the past, are not trained to make in the present, and are unlikely to execute efficiently in the future*”; but the responsibility and execution rest with the financial advisor.

The inspiration behind these papers comes largely from Campbell[4] others, who demonstrated decision rules for individuals in terms of portfolio choice. Other studies on *ex post* individual portfolio choice have been inconclusive, since they typically only covered a subset of total assets—pension assets are often neglected from the analysis. Pension systems even within a single country are often so different that little, if anything can be said about whether individuals behave "optimally" or not,

[1] Carlsson E., 2002, *Papers on Econometric Models*, Licentiate Thesis in Economics, School of Business, Economics and Law, Göteborg University.

[2] Regeringen, 2003, Regeringens proposition 2002/03:133, *Lagen om finansiell rådgivning till konsumenter*"—Government white paper for an act regulating financial advice to individuals".

[3] Merton R., 1999, Finance Theory and Future Trends: The Shift to Integration, *Risk Magazine*, July.

[4] In this introduction most of the references are left out, since they will appear in the articles.

if pension assets are excluded.

The effect of wage-indexation is analysed in the first paper, where it is found that wage-indexation of pension benefits increases the exposure of the young to the largest asset—their human capital. The young therefore attribute very little value to their pension savings. In the same paper it is also shown that volatility of wage-indexation benefits causes less welfare-loss than correlation between wage-indexation and equities.

The importance of both borrowing constraints and the correlation of risks are well documented in several papers by Shiller and Campbell. The problem to be solved is the risk-sharing between individuals with primarily human capital (young) and those with real capital (retired), as well as risk-sharing between countries. Shiller[5] also underlines the importance of individual portfolio choice, when he states that—"*the redesign (of) social security ought to be the time when we carefully consider the fundamental inter-generational risk-management problem and define choices in individual accounts".*

In the second paper, we therefore invent a risk-sharing method in the mandatory pension system, which would allow individuals to exchange wage- and equity-exposure within the mandatory pension system. It is found that different generations would voluntarily take opposing positions, and that inter-generational risk sharing is therefore achievable.

The effects from participating in different pension systems, is analysed in the third paper. This paper investigates the effects of moving from a defined benefit to a defined contribution pension system for individuals with differences in both income profiles and income uncertainty. Not only are winners and losers from such a change of pension system identified, but we also show that individual portfolio choices display a large variation, solely depending on the individual situation.

Acknowledgements and ...

Little did I know, that this long and winding road upon which I embarked some 25 years ago, should have so many detours. The twenty years I have spent in professional finance, coincided with both a complete deregulation of markets as well as a convergence of pricing between markets. During such a transition there were several instances when only theory can be the foundation of effective analogies to appraise opportunities in value and risk. While being based on theory, such evaluations have a friend in a healthy respect for history and detail. It has therefore been very rewarding to work in an environment where hypotheses in finance can be tested daily with both an abundance and frequency of data. In this environment, the idea of profit maximising agents makes more sense than elsewhere; and data is generated without the common problems of aggregation. The hypotheses

[5] Shiller R.J., 2003, Social Security and Individual Accounts as Elements of Overall Risk-Sharing, *American Economic Review* 93, 343-347.

that survive the test of trading, get the status of Theories and become the effective sort engines that can facilitate future pattern recognition. However, the verdict that “nothing is more practical than theory” has an *obiter dictum* that “only artists are allowed to fall in love with their models”. What often seems to be an opportunity—turns out to be a fight with the Devil in the detail, and then frets away in a pale hue. I am therefore very thankful to both my *Alma mater* and previous employers in exposing me to theory and practice.

Coming up to the door that ends the winding road, far more friends than can be mentioned has added α to this random tour. However, nothing is more important than having a good mentor as a superior, I therefore have to mention: Ove Rydin, at both Swedbank and Skandia during very turbulent times, he is the definition of integrity and a man of valour; Lars Lundquist, at Swedbank, Carnegie and SEB, who is also a disciple of the Rydin School of Management, Lars is the living proof that enthusiasm, intelligence and integrity can be combined; Mikael Nachemson, at Öhman, in addition to all the previously mentioned attributes, he also has a radar that senses and finds flaws in reasoning and opportunities before anyone else.

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Privately, I am blessed with six lovely kids—my pride; Agnes, Emil, Hugo, August, Edvard and Astrid, (Nanne, Memi, Guggo, Votte, Dedde and Lilla Syster) who keeps me young at heart and makes me weep at joy. The caos of: College applications, Driving lessons, Military ordeals, Finance with an untutored mind, Rock singing, Narnia with sharks, and the smiles of recognition; is the stuff that life should be made of. My little sister—Nanna, who takes care of the big family; and my childhood friends—*Les Trois Mousquetaires*—Anders, Peter and Magnus (how enlightened the world would be if our discussions were broadcasted), who during more than 40 years patiently have listen to all my trials and tribulations; . I love you all.

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Finally, Jeanette—this beautiful lady that's walking around with me, who with happiness and joy instills the seeds of love in our family—let me laud the lips that was meant for mine.

The Dark Side of Wage Indexed Pensions

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This paper investigates some welfare effects of forced saving through a mandatory pension scheme. The framework for the analysis is a life-cycle model of a borrowing constrained individual's consumption and portfolio choice in the presence of uncertain labour income and realistically calibrated tax and pension systems. Pension benefits stem from both a defined benefit and a notionally defined contribution part, the latter being indexed to stochastic aggregate labour income. We show that agents attribute little value to their pension savings in early life. Furthermore, we estimate the welfare loss for individuals in mid-life associated with the dependency between pension returns and labour income growth to 1.2% of annual consumption.

Key Words: Life-cycle, portfolio choice, pensions.

JEL classification: D91, G11, G23

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1. INTRODUCTION

Mandatory pension schemes is at the very centre of academic and political debates (*cf.* CSSS (2001), Shiller (2003)). Mainly, because of the changes necessary in order to compensate for—too large benefits relative to the contributions of older generations; but also because of how different designs of unfunded pension systems (*eg.* Campbell (2005) and Holzmann & Palmer (2006)) will create substantial welfare consequences. Both the contribution profile across age and indexation of pension system liabilities are crucial parts of the design, since it will define the asset that the individual is forced to hold. Intuitively, large contributions at an early age will exacerbate the negative effect of forced savings for young individuals facing a positive income-profile, but postponing contributions too much will make the pays-as-you-go system insolvent. Wage-indexation of pension assets, is an important tool to enable the retireés to participate in the general growth of the resources in society. However, as we will argue in this paper, there are negative consequences associated with such an indexation as it increases the wage exposure during working life.

Sweden was the first country to introduce a Notional Defined Contribution (NDC) system, where contributions are credited to an individual notional account with a return set to aggregate labour income growth. This reform has attracted a lot of interest as a potential blueprint for other countries (*cf.* Schieber & Shoven (1996), Diamond (2002)). To analyse the effects of such a system for the individual, we use a life-cyclel model.

Life-cycle models have generated a lot of interest as a tool for explaining the accumulation and distribution of wealth as well as portfolio choice over the life-cycle. For agents with uncertain income and liquidity constraints, savings serve several purposes, *eg.* precautionary, retirement and bequest. The importance of each of these motives varies over the life-cycle and will consequently affect both the consumption and the allocation between assets. Over the life-cycle, retirement savings will dominate in absolute size and are to a large extent accumulated in mandatory pension schemes. The introduction of such a scheme into a life-cycle model will motivate an optimising agent to respond by adjusting her consumption and portfolio composition over time. Furthermore, the design of the mandatory pension scheme will have important welfare effects.

This paper has its origin in the literature that highlights uncertainty and market incompleteness as important factors in explaining individual choice and welfare. The first papers on this subject came from the consumption literature on buffer-stock saving. The life-cycle / permanent income hypothesis of Modigliani and Brumberg (1954) and Friedman (1957) implies that there should be no correlation between consumption and predictable income change, since agents would borrow against future incomes as a mean to equalise consumption over life. However, data shows a positive correlation between the two (*cf.* Flavin (1981), Hall and Mishkin (1982), Zeldes (1989)).

Deaton (1991), Carroll (1997) and Gourinchas and Parker (2002) created life-cycle models with uncertain wage income and where human capital could not be used as collateral for borrowing and where saving was invested in a risk-free asset. These models could explain the positive correlation between consumption and predictable income change as a rational response to uninsurable income risk in the presence of borrowing constraints[1].

Cocco *et al.* (2005) and others have extended these models by allowing the agents to allocate between risk-free and risky assets. In order to analyse the effects of different retirement savings systems, Campbell, Cocco, Gomes & Maenhout (2001) (henceforth CCGM) augmented the Cocco *et al.* (2005) model by including a mandatory pension scheme. The authors also showed that a lower pension contribution makes younger generations increase their welfare by postponing private savings to a time when their labour incomes are higher.

In this paper we want to analyse some welfare effects of forced saving through mandatory pensions schemes. Our model is set in partial equilibrium, whereas *eg.* Heaton and Lucas (2004) investigate equilibrium effects of alternative pension systems. We restrict our analysis to the problem of an individual who disregards any societal consequences of her choice. The individual welfare effect from forced saving can originate from (at least) four sources. Firstly, it may increase pension savings above the unrestricted level, especially early in life when savings are driven primarily by precautionary motives. Secondly, wealth in retirement accounts cannot be used to accommodate negative income shocks and will therefore incentivise the individual to make additional savings as a precaution. Thirdly,

[1] Deaton (1991) and Gourinchas and Parker (2002) impose borrowing constraints, while Carroll (1997) sets up a model where the agents choose never to borrow.

the risk and return characteristics of the "pension asset" may differ from the optimal choice and, finally, it redistributes income from early to later in life, when different tax rates may apply. In order to analyse these effects, we have chosen a model similar to the life-cycle model of CCGM.

We model individuals rather than households. Our *rationale* is that: female labour participation and divorce rates are high, which together with an age difference between male and spouse can obscure the expected earnings profile if estimated on family data[2] and consequently the "optimal" behaviour in terms of choices will be erroneous; pension contributions and benefits are often based on individual rather than family incomes; taxes are usually progressive and primarily dependent on the individual instead of family incomes.

We have chosen Sweden as a benchmark for the calibration of our model, due to the relative simplicity and transparency of both the tax[3] and pension systems. Since the Swedish pension reform in 1999, pension contributions have been credited to an individual notional account with a return set to aggregate labour income growth. Furthermore, both tax and pension systems are solely dependent on individual rather than on a mixture of individual and family incomes. Finally, the availability of high quality register based data also alleviates some of the quality problems associated with survey data. While calibrated on Swedish data and rules for taxes and pensions, there are several similarities to systems in other countries, *eg.* the US Social Security retirement system. In both Sweden and the US, contributions and benefits are dependent on gross individual income and most importantly, benefits are indexed by average wage growth.

Our model extends the CCGM model by including a realistically calibrated tax and pension system. The main contribution of this paper is that we can attribute a value to mandatory pension savings and analyse the welfare effects of pension returns linked to stochastic labour income growth. Our findings show that young individuals save primarily due to precautionary motives and will therefore attribute little value to savings in retirement accounts. Furthermore, there is a loss of welfare associated with uncertain pension returns indexed by labour income growth. This loss stems primarily from the dependency between labour income and pension returns, rather than from the volatility of pension

[2] When estimating on family data, the educational status, age and retirement date is typically defined by the head of household only.

[3] Most people can file their declaration of their income tax by sending an SMS or e-mail.

returns.

The paper is organised as follows. Section 2 describes the model and the solution algorithm, while Section 3 demonstrates how the model is calibrated. Results are presented in Section 4. Finally, we end with some concluding remarks in Section 5.

2. THE MODEL

2.1. Individual preferences

The individual (rather than the household) maximises the expected utility over a finite life-cycle, which is divided into pre- and post-retirement. Each individual starts her "optimization life" at the age of 20 or 23 (the latter for those with a university degree) τ_0, retires at 65 and dies at a maximum age of 100 T. Individuals have constant relative risk aversion preferences on a single non-durable consumption good.

Individual preferences at time m are defined as

$$\frac{C_m^{1-\gamma}}{1-\gamma} + E_m \sum_{\tau=m+1}^{T} \delta^{\tau-m} \left(\prod_{j=m}^{\tau-2} p_j \right) \left\{ p_{\tau-1} \frac{C_\tau^{1-\gamma}}{1-\gamma} + b(1-p_{\tau-1}) \frac{D_\tau^{1-\gamma}}{1-\gamma} \right\}, \tag{1}$$

where C_τ represent consumption at age τ, γ is the coefficient of relative risk aversion, p_τ is the one year age contingent survival probability, δ is the discount factor, b is the bequest parameter and D_τ is the bequest amount.

2.2. Labour income

The labour income process follows Carroll and Samwick (1997) with the exception that it is based on an individual rather than a household. Individuals were divided into six mutually exclusive groups with respect to sex and education. While in the labour force, the individual experiences idiosyncratic as well as common shocks to gross income. During the pre-retirement period the log labour real income $l_{ik\tau}$ for an individual i belonging to group k is exogenous and given as[4]

$$l_{ik\tau} = f_k(\tau, \mathbf{Z}_{ik\tau}) + v_{ik\tau} + \epsilon_{ik\tau}, \tag{2}$$

where f_k is a function of the individual characteristics[5] $\mathbf{Z}_{ik\tau}$ as well as an average national labour productivity growth μ^l, $\epsilon_{ik\tau}$ is an idiosyncratic temporary shock distributed as $N(0, \sigma_{\varepsilon_k})$ and $v_{ik\tau}$ is a random walk

$$v_{ik\tau} = v_{ik\tau-1} + u_{ik\tau}. \tag{3}$$

The innovation, $u_{ik\tau}$, is divided into a group aggregate $\xi_{k\tau} \sim N(0, \sigma_{\xi_k})$ and an individual uncorrelated component $\omega_{ik\tau} \sim N(0, \sigma_{\omega_k})$ as below

$$u_{ik\tau} = \xi_{k\tau} + \omega_{ik\tau}. \tag{4}$$

2.3. Mandatory savings and retirement benefits

The Swedish mandatory pension system is divided into a notionally defined contribution part, *NDC*, and a funded defined contribution part, *FDC*. Contributions are paid by the employer and are set to 18.5% of gross income, 16% is added to the *NDC* account[6] and 2.5% to the *FDC* (*cf.* RFV (2002)). In our portfolio choice model, each part of the pension system unfortunately requires a separate state variable, adding to the *curse of dimensionality*, *cf.* Bellman (1961). We therefore disregard the smaller *FDC* part of this system.

Contributions to the national pension plan are capped above an income of 300 KSEK[7]. The return on the national pension plan R^l_τ is set to the national labour income growth[8]

[4] Throughout this paper, we discriminate between the future time periods, τ, which belongs to the optimization problem, and the historic time, t, which is used for estimation.

[5] *i.e.* age, martial status, family size, number and age of children.

[6] In the US, benefits are funded through a Social Security tax of 12.4% of the employee´s income up to an amount of $90,000.

[7] In the following, KSEK - thousands of Swedish Kronor will be omitted. The present exchange rate is *circa* 7 SEK / USD.

[8] The National Social Insurance Board (Riksförsäkringsverket) is responsible for the actuarial estimation of liabilities and the appropriate discount rate. In reality, if the assets in terms of estimated future contributions and return from the buffer funds do not support the growth of liabilities, then the actual benefits paid out will be reduced until the assets match the liabilities and *vice versa*. In this paper we will assume that this will not happen.

An important difference between the Swedish and the US system is that there is no real appreciation of benefits after retirement in the US.

$$R^l_\tau = e^{\mu^l + \xi^A_\tau}, \tag{5}$$

$$NDC_{i\tau} = \begin{cases} R^l_{\tau-1} \cdot NDC_{i\tau-1} + 0.16 \min\left[L_{i\tau}, 300\right] & , \tau < 65 \\ R^l_{\tau-1} \cdot NDC_{i\tau-1} - PO_\tau(R^l_{\tau-1} \cdot NDC_{i\tau-1}) & , \tau \geq 65, \end{cases} \tag{6}$$

where PO_τ is the age specific annualised mortality adjusted payout function after retirement, and μ^l is the expected national labour income growth aggregated over all groups with noise, $\xi^A_\tau \sim N(0, \sigma_{\xi^A})$.

Due to the cap on contributions to the NDC plan, the employer partly compensates by paying into a negotiated plan with the individual as the beneficiary. The vast majority of such plans are at present—defined benefit plans, with benefits depending on the wage at retirement. Albeit there is some variation, most of these company defined benefit plans have a payout of 10%, 65% and 32.5% of incomes in the intervals $[0, 320)$, $[320, 850)$, and $[850, 1270)$ respectively at retirement. The return of the company plan is insured to pay a defined benefit depending on the wage at retirement and guaranteed for the remaining life with no real appreciation after retirement. In reality it depends on the wage during the five years prior to retirement. However, modelling this rule correctly would have necessitated additional state variables. We therefore approximate this by only including the permanent income changes until retirement,

$$L^P_{i64} = e^{f_k(\tau, \mathbf{Z}_{ik64}) + v_{ik64}}. \tag{7}$$

Payout from this plan during retirement will be denoted $DBPO_{i\tau}$. Its dynamics are given by

$$\begin{gathered} DBPO_{i\tau} = 0.1 \min\left[L^P_{i64}; 320\right] + \\ 0.65 \min\left[\max\left(L^P_{i64} - 320; 0\right); 850 - 320\right] + \\ 0.325 \min\left[\max\left(L^P_{i64} - 850; 0\right); 1270 - 850\right]. \end{gathered} \tag{8}$$

All payouts from the NDC pension plan are forfeited in the event of death and for simplicity, we assume the same for the defined benefit plan.

2.4. Taxes

Wage and retirement income $L_{i\tau}$ can be defined as

$$L_{i\tau} = \begin{cases} e^{l_{i\tau}} & ,\tau < 65 \\ PO_\tau(R^l_{\tau-1} \cdot NDC_{i\tau-1}) + DBPO_{i\tau} & ,\tau \geq 65. \end{cases} \tag{9}$$

According to the present[9] Swedish tax rules, labour income and pension benefit are taxed at the same rate, and separate from capital income. To calculate net income $L^n_{i\tau}$, we first deduct a general allowance of 10, then a municipal tax of 30%, a government tax of 20% on all income above 300 and finally an additional government tax of 5% on income above 450. Net income is bounded below by the social welfare minimum benefit at 60, which also applies to retirees in the form of a government guaranteed pension.

$$\begin{aligned} L^n_{i\tau} = \max[&L_{i\tau} - 0.3\max(L_{i\tau} - 10; 0) - \\ &0.2\max(L_{i\tau} - 300; 0) - 0.05\max(L_{i\tau} - 450; 0); 60]. \end{aligned} \tag{10}$$

All the threshold values that create kinks in tax rates and benefits[10] are appreciated by the expected national labour income growth μ^l, except the social welfare minimum benefit which is constant in real terms.

2.5. Assets

There exist one risky and one risk-free asset with after tax real simple returns equal to R^s_τ and R^f respectively. Excess return is defined as

$$R^s_\tau - R^f = \mu^s + \eta_\tau, \tag{11}$$

[9] We use the tax rules for incomes earned in 2003.

[10] This is the same as in the US since the "bend points" when calculating the primary insurance amount (PIA) are adjusted by average earnings growth.

and correlated with the group aggregate innovation in permanent labour income, ξ_k which allows for a group specific sensitivity to the risky asset,

$$\begin{bmatrix} \boldsymbol{\xi} \\ \eta \end{bmatrix} \sim N\left(\begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma} & \boldsymbol{\sigma}_{\xi\eta} \\ \boldsymbol{\sigma}'_{\xi\eta} & \sigma^2_\eta \end{bmatrix}\right). \tag{12}$$

2.6. Private savings and consumption

Each individual starts her "optimization life" with initial wealth set to F. In pre-retirement years the individual receives a wage, and in subsequent years the individual will receive retirement benefits. The individual has two control variables: the proportion of cash on hand to consume, θ_τ, and what proportion of savings, α_τ, to allocate to the risky asset. The cash on hand, or disposable wealth, is therefore,

$$X_{i\tau} = \begin{cases} [R^f + \alpha_{i\tau-1}(\mu^s + \eta_\tau)](1-\theta_{i\tau-1})X_{i\tau-1} + L^n_{i\tau} & ,\tau > \tau_0 \\ F_i + L^n_{i\tau} & ,\tau = \tau_0 \end{cases} \tag{13}$$

and

$$C_{i\tau} = \theta_{i\tau} X_{i\tau}. \tag{14}$$

Finally, we impose both borrowing and short sales constraints, *i.e.*,

$$0 \le \theta_{i\tau} \le 1, \tag{15}$$

$$0 \le \alpha_{i\tau} \le 1. \tag{16}$$

2.7. Optimization

The individual's problem can now be characterised as having four state variables (τ, v, X and NDC) and two choice variables (θ and α) as well as four stochastic variables (ϵ, u, ξ^A and η). The value function of the investor's intertemporal consumption and investment problem can be written as

$$V_\tau\left(\Gamma_\tau\right) = \max_{\theta_\tau,\, \alpha_\tau} \left\{ \frac{C_\tau^{1-\gamma}}{1-\gamma} + \delta E_\tau \left[p_\tau V_{\tau+1}\left(\Gamma_{\tau+1}\right) + \left(1-p_\tau\right) b \frac{D_{\tau+1}^{1-\gamma}}{1-\gamma} \right] \right\}$$
$$\Gamma_\tau = \left\{X_\tau, v_\tau, NDC_\tau\right\}. \tag{17}$$

The solution to this maximisation problem gives us the state dependent policy rules,

$$\theta_\tau = \theta_{k\tau}(\Gamma_\tau), \tag{18}$$

$$\alpha_\tau = \alpha_{k\tau}(\Gamma_\tau). \tag{19}$$

The problem is solved by backward recursion from the final year - 100. Since there is no analytical solution to this constrained optimization problem, we solve the problem numerically using standard methods. A description of the procedure is found in Appendix A

3. CALIBRATION OF PARAMETERS

3.1. Estimation of labour income process

This section describes the estimation of the labour income process. Appendix B gives a more detailed description of the methodology used.

We estimate the parameters in the labour income process by using LINDA data for the years 1992 to 2002. LINDA covers 3.35% of the Swedish population (more then 300,000 individuals plus their family members) and is described in Edin and Fredriksson (2000). The definition of income includes, in addition to wages, all taxable social benefits, primarily compensation for unemployment, sickness and early retirement. Data was divided into six non-intersecting groups, defined by educational status[11] and sex. The predictable component of labour income was estimated separately for each group and the regressors include dummy variables for marital status and age as well as the number of children in four separate age intervals. Parameter estimates are presented in Table B.1.

We then estimate a polynomial of degree three on the age dummies and the averages of the characteristics to obtain the deterministic component of labour income $\exp\left\{f_k(\tau, \overline{\mathbf{Z}}_{k\tau})\right\}$, *cf.* Table B.2. Two

[11] The three educational groups are: individuals without a gymnasium (high school) degree, individuals with gymnasium degree but no university degree, and finally, individuals with an university degree.

findings are notable. First, individuals with a university degree experience significantly faster income growth in mid-life[12]. This result matches the stylised facts from the US *(cf.* Cocco *et al.* (2005) Gourinchas and Parker (2002) Hubbard *et al.* (1995)). Secondly, within each educational group, men have higher incomes in all stages of the life-cycle than their female counterparts.

The variances of the permanent and transitory components, σ_u^2 and σ_ε^2, of shocks to labour income as specified in Equation (2) were estimated using the methodology of Carroll and Samwick (1997). The results are presented in Table B.3 along with the results by Carroll and Samwick (1997) who based their study on household gross income, and Cocco *et al.* (2005) who used household net income.

Our results show a strikingly lower variation in the transitory component when compared to both of these studies. This was surprising, since we expected a diversification effect within the family and that the lower variation in net *vs* gross income, would reduce the residual variation.

One possible explanation could be that measurement errors are treated in the same way as transitory income shocks and thereby increase the estimated variance. Comparing register based and survey data, Duncan and Hill (1985) and Bound *et al.* (1994) demonstrate that survey data, such as the PSID, can give rise to measurement errors, which may have a large effect on estimated variances.

Our lower estimates can therefore partially be explained by the data in LINDA being based on filed tax reports which are more precise. Gourinchas and Parker (2002), p. 81, state that: "*a reasonable guess might be that roughly a third of the variance of measured income growth is due to mismeasurement and that most of this is transitory*".

3.2. Individual parameters

We use a standard set of assumptions with respect to the individual parameters for the reference case. First, we set the coefficient of relative risk aversion γ to 5 and the discount factor δ to 0.98. The survival probabilities p are sex dependent and taken from the Swedish life insurers (*cf.* Figure 3.1) when underwriting new policies, *i.e.* it is forward looking[13]. The bequest parameter b is set to 1. The importance of the risk aversion parameter γ will be elaborated on when we do the sensitivity

[12] This is probably partly due to a selection bias, since we would expect those with university aptitude to perform better even without a degree, *cf.* Hausmann and Taylor (1981).

[13] The difference from todays realised mortality table is that both sexes are expected to live longer and that the difference in longevity between men and women is increasing.

analysis in Section 4.4.

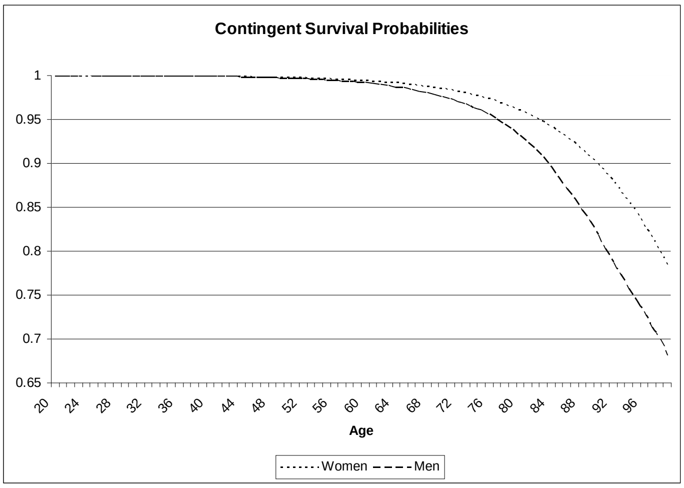

FIG. 3.1 Age contingent survival probabilities

3.3. Assets and correlations

In the optimization, we set the risk-free after tax rate $R^f - 1$ to 1.5%, which fits with the present gross return of less than 2% for long-dated index-linked bonds. The mean after tax equity premium μ^s is set to 3%, which is low when compared with historical average, but corresponds well with forward-looking estimates (*cf.* Claus and Thomas (2001); Fama and French (2002), among others). Volatility σ_η was set to 20% for the risky asset.

Next, we follow the procedure of Cocco *et al.* (2005) to estimate the correlation $\varrho_{\xi_k \eta}$ between group specific permanent labour income shocks $\xi_{k\tau}$ and lagged equity returns $\eta_{\tau-1}$. In Table B.4, we present the estimated correlation using the returns of the Swedish equity index OMX and the 12-month Swedish Treasury Bill as proxies for equity returns and the risk-free rates, respectively. Due to the uncertainty in the equity premium, we analyse the sensitivity in Section 4.4 of our results with respect to an increase in this parameter.

It can be noted that university educated women and men define the range for the correlation, with women having the lowest. One possible explanation could be that women with a university degree are to a larger degree publicly employed. We also set the growth in average labour income μ^l to 1.8%, which is the estimate used by the National Social Insurance Board. Finally, the initial wealth F is set to 47, corresponding to the mean wealth for individuals between the ages of 20 and 23.

4. RESULTS

To study the behaviour of an individual belonging to a specific group, we now use the policy functions in Equations (18) (consumption share) and (19) (risky share) that describe the optimal state dependent behaviour. The contour plots in Figures 4.1 and 4.2 show the policy functions for risky weight and consumption share respectively, with the state variables age and NDC held constant.

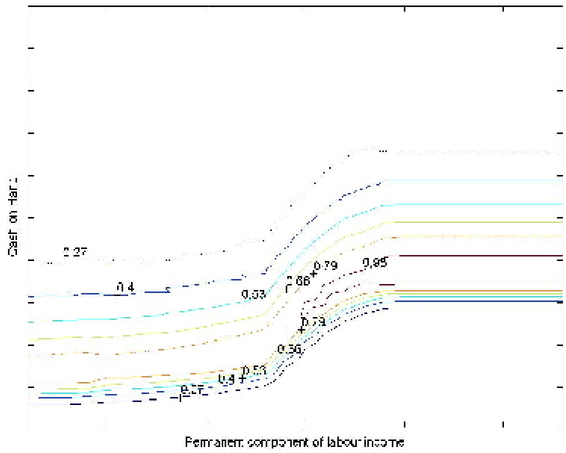

FIG. 4.1 Proportion of savings allocated to the risky asset α at the age of 64 with NDC held constant for men with a high school degree.

At retirement, the permanent component of labour income shock ν decides the defined benefit for the remainder of the life. For large values of cash on hand, the optimal policy in risky weight coincides with the complete market solution[14]. When the ratio of cash on hand to the implicit pension assets (NDC and present-value of future Defined Benefits) decreases, both the consumption and risky share will increase. As the ratio decreases even further, the dominating savings motive is bequest and therefore the risky share is reduced back to the complete market solution. The curvature in policy functions is caused by the defined benefit payout being more sensitive to changes in permanent income (*cf.* Equation (8)). Since defined benefit resembles a risk-free asset, the agent will compensate by increasing both the risky and consumption share.

[14]Where risky share is $\alpha = \frac{\mu^s}{\gamma\sigma_\eta^2}$.

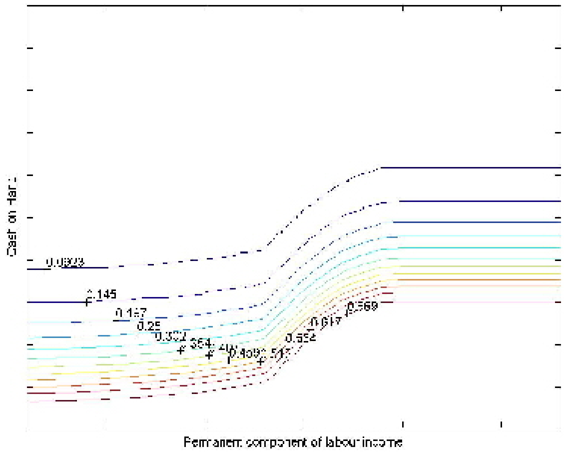

FIG. 4.2 Proportion of cash on hand consumed θ at the age of 64 with NDC held constant for men with a high school degree.

To study the potential outcomes of the model, we simulate the behaviour of an individual (one from each group) by generating 30,000 random trajectories through time. These simulated distributions cannot be directely compared with the actual distributions of today, since the latter are conditional on one realisation for several individuals. In addition, the present population lived under very different conditions in terms of productivity level, pension systems, *etc.* compared to the present and future that we model.

The top and bottom pictures in Figure 4.3 show the simulated individual frequency distribution across age for the risky weight and consumption share, respectively. We note that the short sale constraint is effective for most trajectories in mid-life. Outside this period, there is a wide range of optimal choices dependent on state variables other than age.

4.1. Reference case

In this section, we present the cross-section averages from the simulation. In Figures 4.4 and 4.5 we plot the average of consumption, consumption share, retirement wealth in the NDC account, after tax income, portfolio allocation and cash on hand for an individual from each group.

We note that the individual tries to smooth consumption over life, which can be seen if we compare the wage and consumption profiles. However, an increasing and uncertain labour income together

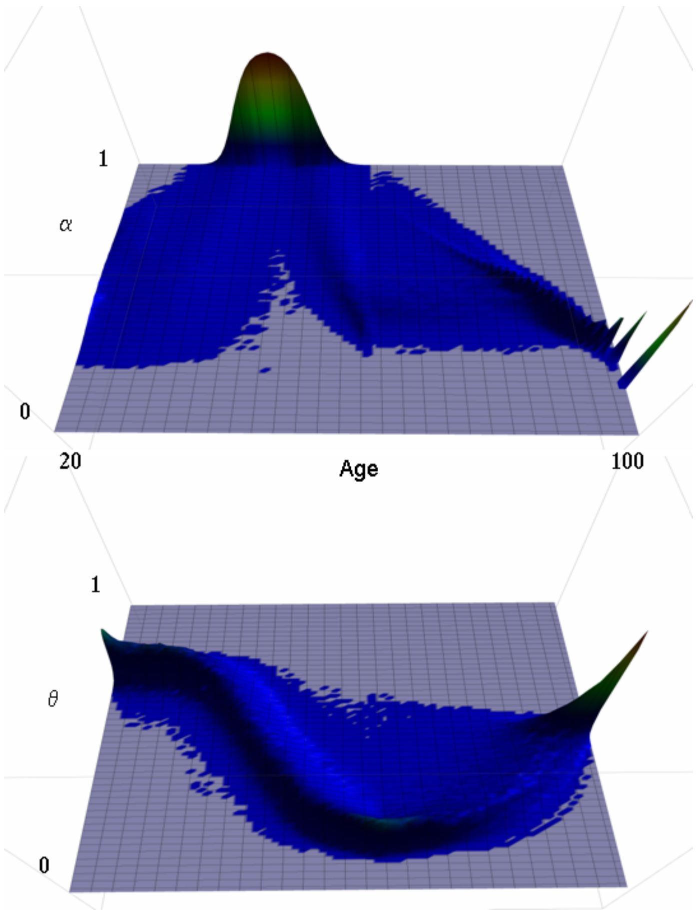

FIG. 4.3 Simulated frequency distributions for the choice variables across ages for men with a high school degree. The upper picture is the proportion of savings allocated to the risky asset α. The lower picture is the proportion of cash on hand consumed θ.

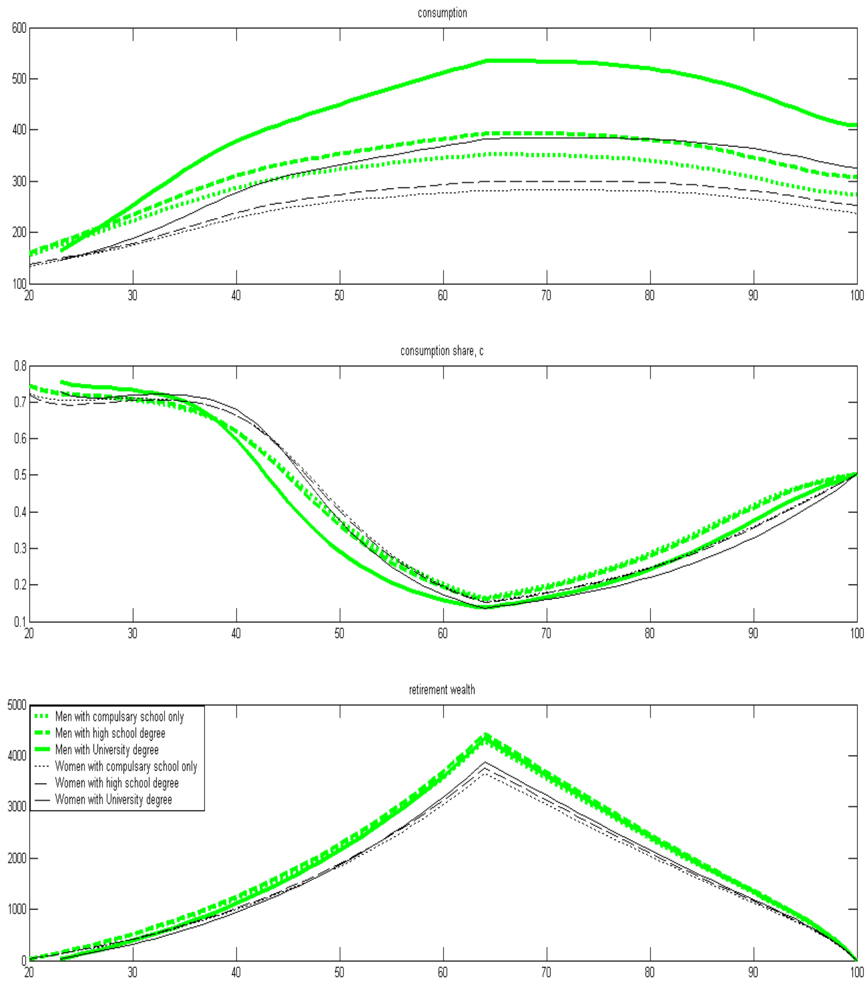

FIG. 4.4 Reference case. Age dependent averages from the simulated distributions for each group.

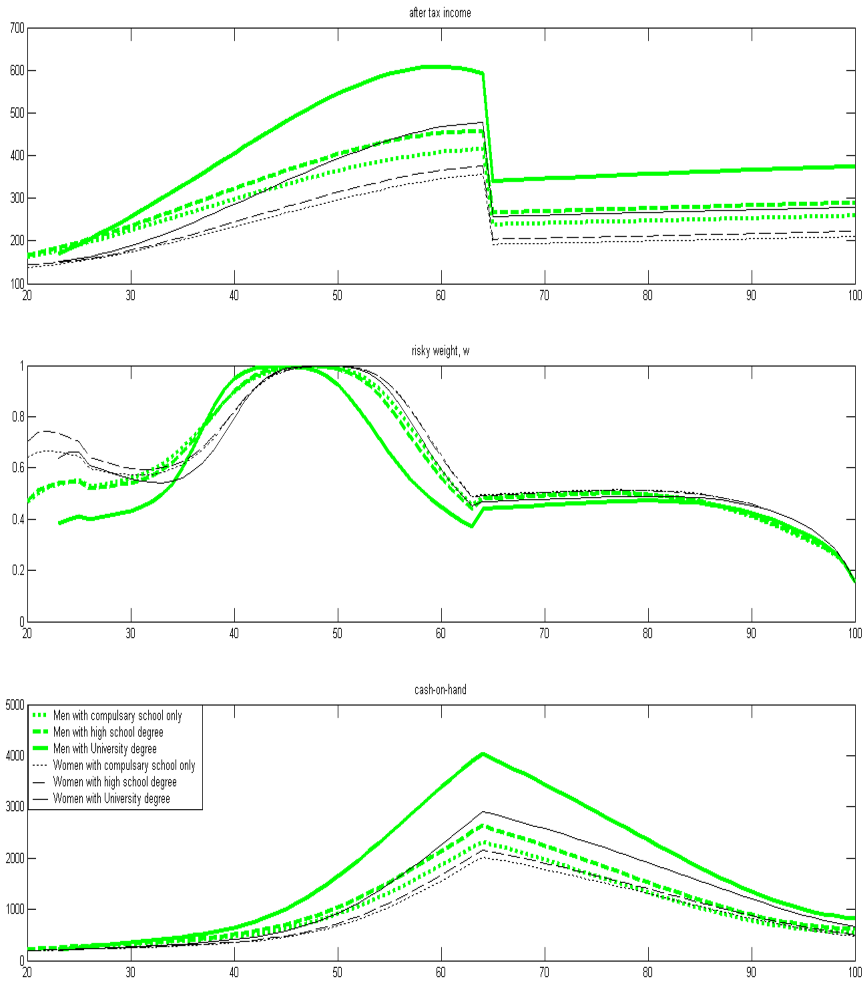

FIG. 4.5 Reference case. Age dependent averages from the simulated distributions for each group.

with the borrowing constraint create a hump-shaped consumption profile, as in *eg.* Gourinchas and Parker (2002). The decline in consumption during retirement is primarily due to mortality risk, which gives rise to a more flat consumption profile for women than for men. It should also be noted that consumption increases with after tax income and peaks close to retirement.

The peak in wages occurs later than in empirical cross-section data since the wage profile is forward looking, *i.e.* it includes the expected increase in wages from the average productivity growth.

The cap on the contribution amount has the effect that there are only small differences in NDC wealth at retirement (*cf.* Figure 4.4), even though wages differ markedly between groups. However, the company sponsored defined benefit plan compensates high income earners for the cap, making the retention rates almost equal across the groups (*cf.* Table 4.1). This result corresponds well with projections from dynamic micro-simulations (*cf.* Flood (2003)).

TABLE 4.1
Retention rates

Median retention rates as a percentage of after tax income at retirement in relation to previous year			
	Total retention rate	Defined Benefit	NDC
Men			
Compulsory school	57	13	44
High school	55	13	42
University	58	29	29
Women			
Compulsory school	54	13	41
High school	55	13	42
University	53	16	37

In order to alleviate the drop in income at retirement, the individual also saves voluntarily to even out consumption over the life-cycle. As can be seen in the consumption share average (Figure 4.4), retirement saving does not start before mid-life. Prior to this, savings are driven by precautionary and bequest motives. Consequentially, the cash on hand during early life is largely invested in the risk-free asset. When private savings for retirement increases during mid-life, wealth is allocated to the risky asset, since the implicit assets in mandatory pension schemes and future wages are closer substitutes to the risk-free asset.

As cash on hand becomes relatively larger in comparison to the implicit assets, the investor compensates by reducing the risky weight. Although the implicit assets are less risky, they are not risk-free. The risk is most pronounced for university-educated men. This group has both a larger relative exposure to the defined benefit asset and a higher correlation between their wages and the stock market.

They will therefore choose a lower risky allocation from mid-life until retirement, when the defined benefit asset becomes risk-free.

The profile for cash on hand (*cf.* Figure 4.5) has the charateristic life-cycle shape that we find in the data[15] (*cf.* Figure 4.6). We also see the effect of the gender difference in longevity on consumption and savings behaviour. During retirement women will decrease their cash on hand at a lower rate than men will, since they expect to distribute their savings for consumption over a longer period.

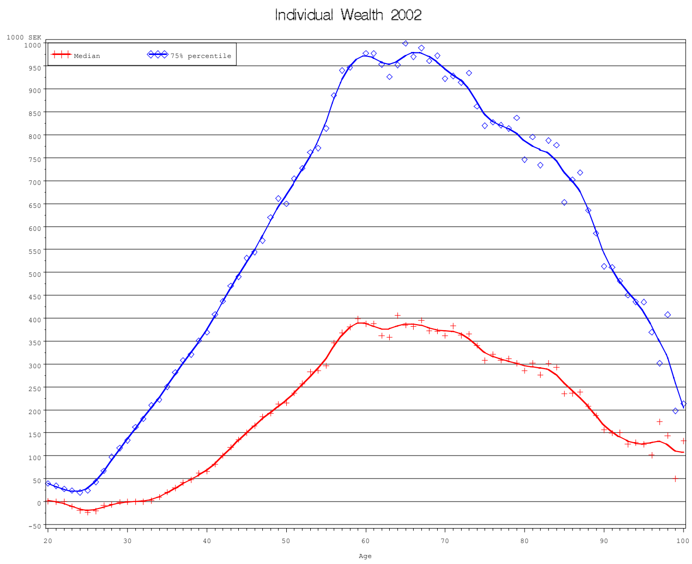

FIG. 4.6 Individual net-wealth. Cross-section LINDA data for 2002 based on 499,241 individuals.

4.2. Valuing the NDC account

Forced saving in early life, when the wage profile is increasing, raises the question of the value of the NDC asset. In Equation (20), we express the value of the NDC asset[16] in terms of cash on hand X, by calculating the expected value of the ratio of the respective derivatives of Equation (17),

$$E_{\tau_0}\left[\frac{\partial V_\tau(\Gamma_\tau)}{\partial NDC}\Big/\frac{\partial V_\tau(\Gamma_\tau)}{\partial X}\right]. \tag{20}$$

We note in Figure 4.7 that the marginal value that an individual attributes to the NDC account

[15] The two profiles are not directely comparable since the simulated profile relates to future wealth whereas the actual data is a cross-section from year 2002. In our simulations, there is a peak at age 65 since we assume a fixed retirement date. In reality, there is a lot of variation in retirement age, making the peak less pronounced. After the age of 80 the number of survivors decreases rapidly in our sample and expected wealth is potentially biased upwards, due to survival probabilities being correlated with wealth (*cf.* Modigliani and Jappelli (1998)).

[16] For comparative purposes, since bequest can only come from cash on hand X, we change the model in two ways. First, we allocate the wealth in the NDC account to bequest in case of death, but taxed with the median tax rate of 30%. Second, we remove the inheritance gains in the NDC system.

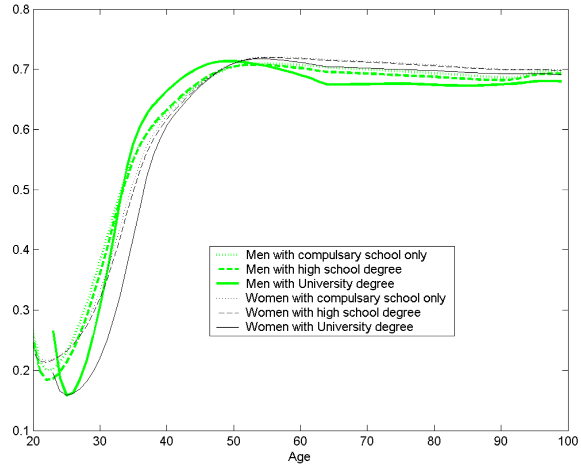

FIG. 4.7 Expected Ratio of Marginal Utilities of NDC and cash on hand. These ratios converge to *cirka* 70% since most retireés will only pay the municipal tax of 30%.

is low in early life. This result is primarily caused by forced retirement saving at a time in life when consumption is preferred. Moreover, since NDC wealth cannot serve the precautionary motive, the agent will make additional savings as a precaution. As the wealth in the NDC account becomes larger in comparison to private wealth (*cf.* Figure 4.8), the marginal value of the NDC asset will decrease even further, since the NDC asset primarily fulfils a small bequest motive. The increase in marginal value until age 40 stems from the retirement savings motive becoming stronger[17].

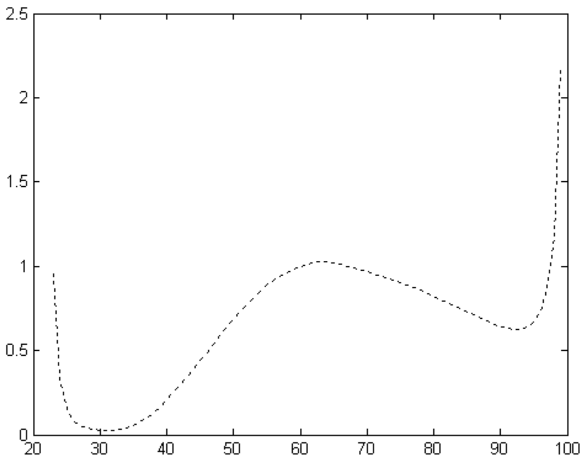

FIG. 4.8 Ratio of private to NDC wealth for university educated men

[17]The ratio has a peak at age 50 for men with an university degree, as their income is often taxed at a higher rate, whereas the bequest of NDC wealth is only taxed at 30 %. As the probability of bequest prior to retirement decreases with age, the ratio falls.

The NDC asset has characteristics similar to a combination[18] of risk-free and risky assets; and the relative value converges to the median after tax value of 70% (*cf.* Equation (10)). As pension benefits are generally lower than wages, only a few trajectories will be in the higher tax brackets during retirement, which is why only the highest income group will experience a slightly smaller value due to higher taxation.

4.3. Risk in the NDC account

This model has five "assets": risk-free, risky, defined benefit, future wages and the NDC asset, of which the last three are non-tradeable. In this section we analyse the diversification properties of the NDC asset. As described in Section 3, the NDC return has a volatility of about 2% and is correlated with both group permanent income shocks and the return of the risky asset. Albeit that the volatility is rather low, the risk in the NDC asset has a major impact on individual utility since this is on average the largest asset at retirement.

The economic importance of the risk in the NDC system can be analysed by computing the utility gains associated with two alternative regimes. The first regime makes the NDC return risk-free, while the second regime makes the return independent of both group permanent income shocks and the return of the risky asset. In both cases, the expected simple return[19] of the NDC account is held constant and equal to that of the reference case.

The utility gains are presented as consumption and bequest equivalent units CBE. A one percent change in CBE represents the same percentage change in consumption and bequest in all possible states for the remainder of life. Equation (17) is solved with the new covariance matrix and the corresponding policy responses, $\theta_{k\tau}(\Gamma_\tau)$ and $\alpha_{k\tau}(\Gamma_\tau)$, are derived. We then compute the value function $V_\tau^a\left(\overline{\Gamma}_\tau^r\right)$ for each age, where $\overline{\Gamma}_\tau^r$ is set equal to the average of each state variable from the reference case. Superindices a and r refer to the alternative and reference case, respectively. The gain in CBE is then defined as

$$G_\tau = \left[\frac{V_\tau^r\left(\overline{\Gamma}_\tau^r\right)}{V_\tau^a\left(\overline{\Gamma}_\tau^r\right)}\right]^{\frac{1}{1-\gamma}} - 1. \tag{21}$$

In Figure 4.9 we have plotted G_τ from making the NDC account independent or risk-free. The groups shown are men with a university degree and women with compulsory school only. These are our extreme groups with respect to earnings, but still depicting a similar pattern.

In the previous section, we showed that the value that a young individual attributes to the NDC

[18] The same Sharpe ratio, the NDC asset is approximately equal to a 10% investment into the risky and the remainder into the risk-free asset.

[19] μ_l is increased by $\frac{1}{2}\sigma^2_{\xi^A}$ in the risk-free regime.

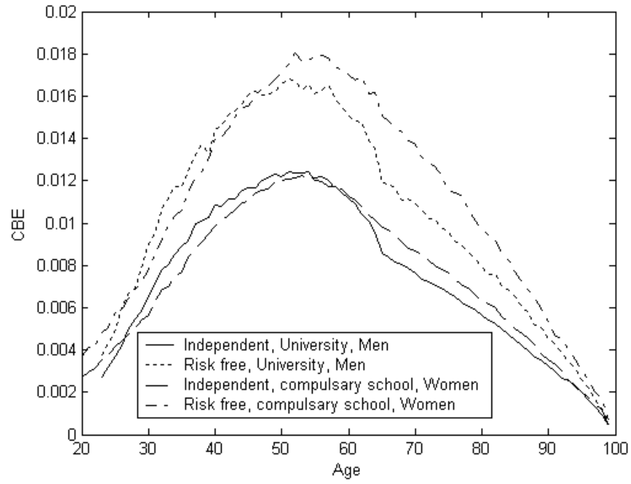

FIG. 4.9 Gain in consumption and bequest equivalent units CBE for the remainder of the life, due to making the NDC account independent or risk free for men with a university degree and women with compulsary school only.

account is very small, since consumption is preferred and savings are primarily driven by precautionary motives. The risk characteristics will therefore be of little importance for a young individual when determining expected lifetime utility.

As the expected ratio of marginal utilities increases (*cf.* Figure 4.7), the sensitivity to changes in risk characteristics increases as well. As one approaches the retirement date, the part of the implicit assets that originates from future wages decreases, making G_τ decline. At retirement, the defined benefit asset will become risk-free, resulting in a more pronounced drop in G_τ for the high-income group. After retirement, the G_τ continues to decrease, as the ratio of private to NDC wealth increases (*cf.* Figure 4.8).

At the peak, the gain in consumption and bequest equivalent units from having a risk-free NDC asset is considerable and ranges from 1.6% to 1.8%. Interestingly, about two-thirds of the gains originate from the elimination of the dependency between NDC asset returns and both group permanent income shocks and risky asset returns. During working life, the NDC asset return is correlated with future wages and defined benefits, as well as with risky asset returns. When in retirement, only the latter correlation remains, but the proportional gain ($\frac{2}{3}$) still holds regardless of age[20]. The existence of a risky asset, correlated with NDC wealth, underlines the risk aspects in NDC during retirement.

[20] Therefore, to the extent that NDC schemes are supported by buffer funds (as in Sweden) to even out differences between contributions and benefits, such funds should actively manage assets with the aim to minimise the correlation with wages. Idiosyncratic risks in such schemes, *eg.* demographic risks, are of less relevance. This is in sharp contrast to how some of these funds interpret their risks. When describing its optimization problem, one fund stated that: "*The optimization of risk-return was done relative to the minimum risk portfolio...This was considered to be the portfolio showing the smallest tracking error relative to the income index*" (Wassum (2002)).

This would have been overlooked if private wealth only could be invested in a risk-free asset.

4.4. Robustness and sensitivity

It is important to note that our parameterisation is conservative with respect to the effects on the valuation of the NDC account analysed in Section 4.2: including the FDC contribution of 2.5% would increase the forced saving; the NDC return is high and the equity premium is low, making these assets approximately equal with respect to Sharpe ratios; and finally the impatience parameter is high which alleviates the negative consequences of forced saving. In order to test to what extent our results are influenced by the parameterisation, we perform two sensitivity tests[21]. Firstly, we decrease the coefficient of relative risk aversion - γ from 5 to 2. This parameter has the property of controling both the elasticity of intertemporal substitution and the risk aversion. Our results, especially in Section 4.2, are driven by the unevenly distributed consumption across ages and are therefore likely to be affected by a change in the elasticity of intertemporal substitution. Furthermore, γ determines the risk aversion and hence the analysis of the risk in the NDC account in Section 4.3.

Secondly, we increase the risk premium to 4%, since our reference case with 3% is low both in comparison with the empirical average and with other studies (*eg.* Cocco (2005); Cocco *et al.* (2005); Gomes and Michaelides (2005); Yao and Zhang (2005)) where 4% was assumed.

Changing the coefficient of relative risk aversion γ from 5 to 2 has a dual effect on the value attributed to the NDC asset. On the one hand, the increase in elasticity of intertemporal substitution raises the value of the NDC asset when the individuals are young and liquidity constrained, since it makes agents more willing to substitute consumption over time. On the other hand, a lower γ induces a higher demand for risky assets and the agent will become constrained by the short sales constraint on the risk-free asset. The NDC asset is a closer substitute for the risk-free asset than for the risky asset and hence an undesirable investment for an agent with relatively low risk aversion.

Early in life the two effects offset each other, making the value attributed to the NDC account approximately equal to that in the reference case (*cf.* Figure 4.11). Later in life as consumption reaches the lifetime average, the effect of a higher elasticity of intertemporal substitution diminishes, resulting in a substantially lower value attributed to the NDC asset than in the reference case. During the last ten years of life, the importance of the non-negativity constraint on the risk-free asset decreases, making the marginal value of the NDC asset equal to the after tax value.

Changing the risk premium, depreciates the relative value of the NDC asset to cash on hand. This

[21] We only present the results for the group Men with a High School Degree, since this group is the most numerous and their wage profile is closest to the average of all groups.

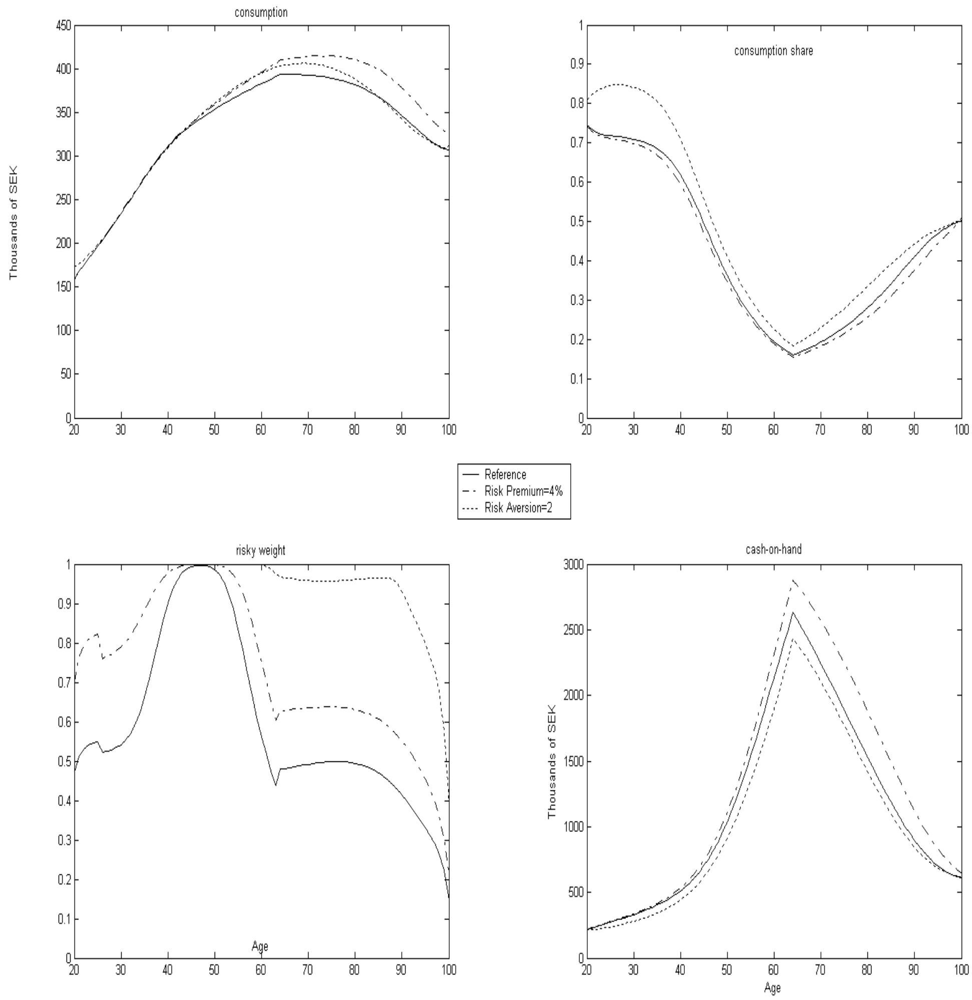

FIG. 4.10 Simulated profiles for men with a high school degree. Effects from changing the risk premium and relative risk aversion.

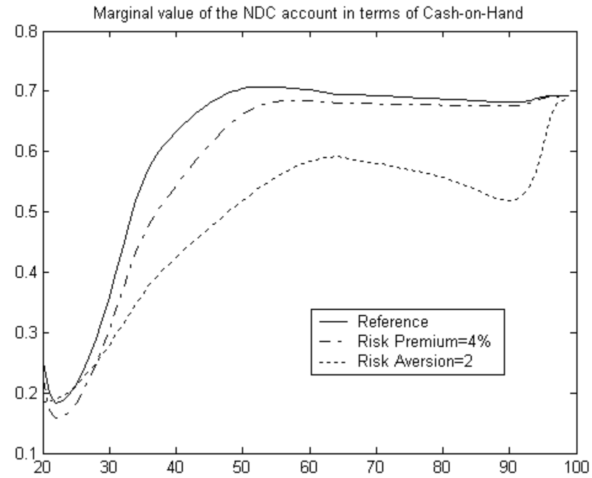

FIG. 4.11 Effects on the marginal value of the NDC from changing the risk premium and risk aversion for men with a high school degree.

effect is most pronounced for a young individual, who is forced to hold this asset for a longer period.

In Figures 4.12 and 4.13, we demonstrate the sensitivity of our results in Section 4.3, with respect to the same changes in risk premium and risk aversion.

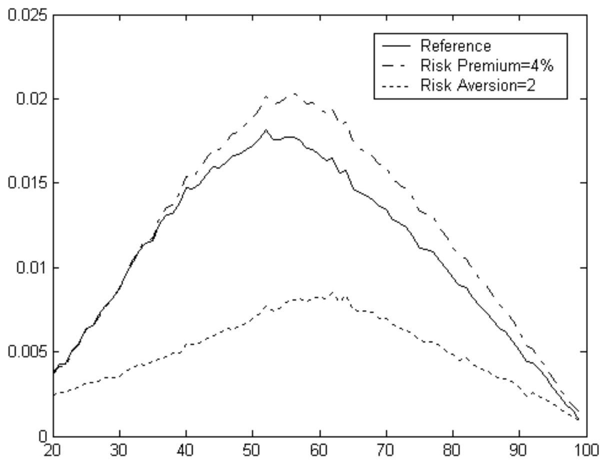

FIG. 4.12 Gain in consumption and bequest equivalent units CBE for the remainder of the life, due to making the NDC account risk free for men with a high school degree.

Lowering the individual risk aversion will reduce the gains from making the NDC asset return independent (*cf.* Figure 4.13) or risk-free (*cf.* Figure 4.12). A higher risk premium relative to the reference case will increase the gains, because the individual will now have a larger proportion of cash on hand in the risky asset. However, the proportion of gains $(\frac{2}{3})$ that stems from independency

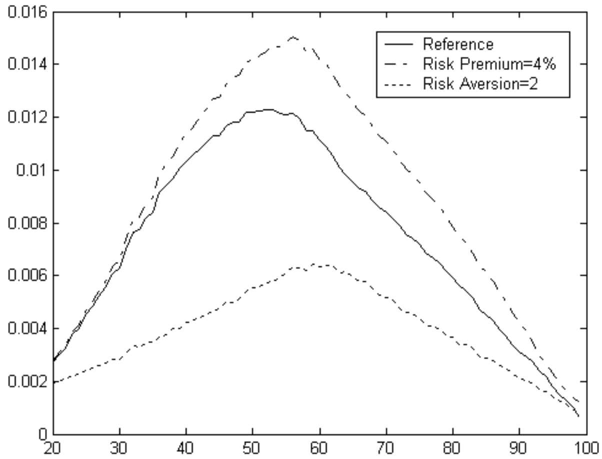

FIG. 4.13 Gain in consumption and bequest equivalent units CBE for the remainder of the life, due to making the NDC account independent for men with a high school degree.

relative to zero risk will be approximately intact.

5. CONCLUSION AND COMMENTS

This paper contributes to the understanding of the risk characteristics and welfare effects of NDC pension systems. We present a life-cycle model of a borrowing constrained individual's consumption and portfolio choice in the presence of uncertain labour income and realistically calibrated tax and pension systems. The pension scheme consists of both a defined benefit and a notionally defined contribution part, the latter being indexed to stochastic aggregate labour income growth.

We investigate the utility effects from forced retirement savings in an asset with returns determined by labour income growth. Firstly, our findings show that individuals attribute little value to their pension savings in early life. We expect this result to hold qualitatively for any pension scheme with a flat contribution rate across ages and therefore to be independent of the country specific calibration. Welfare gains can consequently be achieved by postponing the contribution from early to mid working life, while keeping the duration of the NDC account unchanged. Such a change could also lead to an increase in supply and demand for young labour and thereby encourage an earlier entry into the labour market.

Secondly, we find that it is the dependency between labour income growth and NDC returns rather than the volatility in NDC returns that is the most important source of welfare loss. The reason is that it makes the returns of the NDC asset correlate with individual future wages and defined benefits, as well as with risky asset returns. This effect is essential when analysing the consequences of the proposed change (*cf.* CSSS (2001)) in the US Social Security Benefit indexation that will make retention rates negatively correlated with aggregate wages (*cf.* Munnell and Soto (2005)). The positive welfare effects of this negative correlation will therefore partly compensate for the lower expected retention rate.

Another implication is that to the extent that NDC schemes are supported by buffer funds (as in Sweden) to even out differences between contributions and benefits, such funds should actively manage their assets with the aim to minimise the correlation with wages. Idiosyncratic risks in such schemes, *eg.* demographic risks, are of less relevance. Societal gains could therefore be achieved by reducing dependency between the NDC asset and aggregate income. This topic has been discussed earlier in *eg.* Shiller (2003).

The introduction of a risky asset into a model with these risk characteristics demonstrates the limited inter-generational risk sharing that can be achieved through an NDC asset dependent on average wage growth.

APPENDIX A: OPTIMIZATION

Since no analytical solution to the Equation (17) exists, we solved it numerically by backward recursion. The continuous state variables (cash on hand X, retirement savings account NDC and permanent component of shocks to labour income υ) and choice variables (consumption weight θ and risky weight α) were discretised. The grid of points in X and NDC was distributed exponentially, whereas a uniformly spaced grid was used for υ, θ and α. The introduction of tax and pension systems that are not proportional to wages prohibits us from normalising the problem, *i.e.* we cannot reduce the dimension of the state space, as was done in *eg.* CCGM and Gomes and Michaelides (2005).

Due to zero survival probability in the final year, the value function is simplified since cash on hand is partly consumed and the remainder is bequeathed. Policies and the value of the Bellman equation were therefore easily determined for each combination of state variables. We approximated the value and policy functions for intermediate points with a third degree polynomial B-spline, *cf.* de Boor (1978). The value function in the terminal period was then used to compute the value function in the previous period, where the expectation in Equation (17) was evaluated using a Gaussian-Hermite quadrature approximation (*cf.* Golub and Welsch (1969)). Close to any of the kinks in payout or tax functions, we refined the approximation by using a higher number of nodes. For every combination of states, the optimum was found by a grid search in each choice dimension θ and α. This procedure was then iterated backwardly until the initial year.

APPENDIX B: ESTIMATION

We estimated the parameters in the labour income process by using LINDA data for the years 1992 to 2002. LINDA—a register-based longitudinal data set—consists of a large panel of individuals, which is representative for the population from 1960 and onwards. The data set covers 3.35% of the Swedish population (more then 300,000 individuals plus their household members) and is described in Edin and Fredriksson (2000).

The income data is based on filed tax reports. Our definition of income includes, in addition to wages, all taxable social benefits, primarily compensation for unemployment, sickness and early retirement. We assumed that all individuals with an income less than 100 were voluntarily unemployed, and they were therefore excluded from the estimation. The data was divided into six non-intersecting groups, defined by educational status and sex. The three educational groups were individuals without a gymnasium (high school) degree, individuals with a gymnasium degree but no university degree, and finally individuals with a university degree. Individuals with missing values for educational status (approximately 1.2 percent of the sample) were deleted.

The predictable component of labour income was estimated separately for each group using a balanced panel random effects model with an $AR(1)$ disturbance term,

$$\begin{aligned} l_{ikt} &= \beta_{k0} + \mathbf{Z}_{ikt}\boldsymbol{\beta}_k + \vartheta_{ki} + e_{ikt} \quad &, i = 1, \ldots N_k \\ e_{ikt} &= \rho e_{ikt-1} + \varkappa_{ikt} \quad &, t = 1, \ldots, T, \end{aligned} \tag{22}$$

where

$$\vartheta_{ik} \sim NIID(0, \sigma^2_{\vartheta_k}) \tag{23}$$

and

$$\varkappa_{ikt} \sim NIID(0, \sigma^2_{\varkappa_k}), \tag{24}$$

and where l_{ikt} is the logarithm of income adjusted for overall income growth in the economy. $\mathbf{Z}_{ikt}$ includes dummy variables for marital status and age as well as number of children in four separate age intervals. The estimated coefficients are presented in Table $B.1$.

TABLE B.1
Labour Income Process: Coefficients from Regression

	AR(1) Random effects Regression								
Log real income		#Children at age			Status	AR	Std. in	Std. in	R^2
2004 KSEK	0 β_0	1-2 β_1	3-5 β_2	6-17 β_3	Single=1 β_4	ρ	fixed σ_ϑ	overall σ_e	within
Men									
Compulsory school	-.00228	-.00389	-.00056	-.00393	-.00778	.510	.212	.128	.029
High school	.00075	-.00624	-.00364	-.00187	-.01099	.510	.249	.129	.066
University	.006	-.00772	-.0028	.00012	-.00956	.512	.347	.159	.153
Women									
Compulsory school	.04562	-.1064	-.04637	-.01248	.02118	.557	.194	.102	.045
High school	.03433	-.12407	-.0568	-.02393	.02816	.525	.200	.113	.135
University	.03138	-.13274	-.06368	-.02142	.02971	.515	.253	.134	.202

After adjusting the age dummies for expected future labour productivity growth, we estimated a polynomial of degree three on the age dummies and the averages of the characteristics to obtain the forward-looking deterministic income profile, $\exp\left\{f_k(\tau, \overline{\mathbf{Z}}_{k\tau})\right\}$. Table $B.2$ shows the estimated parameters of the income polynomial for each group.

TABLE B.2
Coefficients in the age polynomial of the forward-looking income profile

	Income profile, 2004 KSEK, (AGE-18)			
	Constant	Age	Age^2	Age^3
	a_0	a_1	a_2	a_3
Men				
Compulsory school	211.7	5.960	.2514	-.0048
High school	221.7	4.730	.4363	-.00779
University	190.4	4.283	1.051	-.0188
Women				
Compulsory school	189.5	.758	.3318	-.00473
High school	200.9	-1.108	.4566	-.00645
University	222.1	-6.304	.9188	-.01277

The variances of the permanent and transitory components, σ^2_u and σ^2_ε, of shocks to labour income as specified in Equation (2), were estimated using the methodology of Carroll and Samwick (1997). Define individual residuals from Equation (22), as $l^*_{ikt} = l_{ikt} - f_k(\tau, \mathbf{Z}_{ik\tau})$. The d-year variance is therefore

$$E(l^*_{ikt+d} - l^*_{ikt})^2 = d\sigma^2_{u_k} + 2\sigma^2_{\epsilon_k}. \tag{25}$$

The two parameters in Equation (25) were estimated using OLS on the differences - d and a constant term. We allow for a serial correlation in the transitory term of order $MA(2)$ by only including differences $d > 2$. Furthermore, we exclude the maximum difference $d = 10$ since there is only one observation. This gives us 35 observations for each individual. The results are presented in Table B.3, along with the results by Carroll and Samwick (1997) and Cocco *et al.* (2005).

TABLE B.3
Variance Decomposition

Description	Men	Women	C & S	CGM
Variance of transitory shocks ($\sigma^2_{\epsilon_k}$)				
Compulsory school	.00867	.00623	.0658	.1056
High school (gymnasium)	.00981	.00741	.0431	.0738
University degree	.01208	.01000	.0385	.0584
Variance of permanent shocks ($\sigma^2_{u_k}$)				
Compulsory school	.00462	.00403	.0214	.0105
High school (gymnasium)	.00564	.00460	.0277	.0106
University degree	.00958	.00634	.0146	.0169
Sensitivity to equity returns (β_k)				
Compulsory school	.0291	.0279		.0956
High school (gymnasium)	.0284	.0265		.0627
University degree	.0347	.0205		.0733
Correlation with equity returns ($\rho_{\xi_k\eta}$)				
Compulsory school	.5167	.4767		.3280
High school (gymnasium)	.5136	.4436		.3709
University degree	.5393	.2899		.5155

C & S refers to Carroll and Samwick (1997) and CGM to Cocco *et al.* (2005).

We then follow the procedure of Cocco *et al.* (2005) to estimate the correlation between labour income shocks and stock returns. Using Equation (2), the first difference in l^*_{ikt} can be written as

$$\triangle\, l^*_{ikt} = \xi_{kt} + \omega_{ikt} + \triangle\, \epsilon_{ikt}. \tag{26}$$

Taking the average over individuals in each group gives us the group aggregate component:

$$\overline{\triangle\, l^*_{kt}} = \xi_{kt}. \tag{27}$$

Finally, we estimate the correlations $\varrho_{\xi_k\eta}$ by running the following OLS regression:

$$\overline{\triangle\, l^*_{kt}} = \beta_k(R^s_{t-1} - R^f_{t-1} - \mu_s) + \phi_t. \tag{28}$$

In Table B.4 we present the result from Equation (28), using the return of the Swedish equity index OMX as a proxy for equity returns and the 12-month Swedish Treasury Bill as the risk-free rate.

TABLE B.4
Variance decomposition and estimated return covariances

Group	Number of individuals	Estimated variance of the permanent component, $\sigma^2_{u_k}$	Estimated variance of the transitory component, $\sigma^2_{\varepsilon_k}$	Std. of the permanent aggregate component, σ_{ξ_k}	Correlations with Swedish equity returns, $\varrho_{\xi_k\eta}$
Full sample	55 532			.01989	.482
Men	31540				
Compulsory school	6878	.00462 (.000137)	.00867 (.000379)	.02008	.517
High school (gymnasium)	14978	.00564 (.000112)	.00981 (.000313)	.02020	.514
University degree	9684	.00958 (.000226)	.01208 (.000625)	.02187	.539
Women	23992				
Compulsory school	3485	.00403 (.000140)	.00623 (.000386)	.02014	.477
High school (gymnasium)	11119	.00460 (.000085)	.00741 (.000235)	.01943	.444
University degree	9388	.00634 (.000126)	.01000 (.000348)	.02269	.290

Standard errors in parentheses

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The Bright Side of Shiller-Swaps: A Solution to Inter-generational Risk-sharing

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This paper investigates the diversification-demand of an agent faced with the alternative to swap aggregate labour-income risk for equity-exposure, through her individual account in a mandatory-pension scheme. The framework for the analysis is a life-cycle model of a borrowing-constrained individual's consumption- and portfolio-choices in the presence of uncertain labour-income and realistically calibrated tax- and pension-systems. Pension benefits stem from both defined benefit and notionally defined contributions parts, the latter indexed to stochastic aggregate labour-income. We show that depending on age and swap premium, agents will be either buyers or sellers of such a swap, and that inter-generational risk-sharing can therefore be achieved.

Key Words: Life-cycle, portfolio choice, pensions, Shiller-swap.

JEL classification: D91, G11, G23

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1. INTRODUCTION

Politicians and academics (*cf.* Commission to Strengthen Social Security (2001), Shiller (2003a)) have recently put focus on mandatory pension schemes. Although, this debate has arisen largely due to the legacy cost for future generations; it has also demonstrated the need for management of individual and inter-generational risks, associated with mandatory pensions. New and more cost effective techniques for administrating individual accounts and recent important insights into the risk management of large societal risks (*cf.* Shiller (2003a)) has created new opportunities to address the inter-generational risk-sharing problem. In this paper we propose, a technique for using individual accounts within mandatory pension schemes to increase societal risk-sharing.

Shiller (2003b) argues that: *"the time when we redesign social security ought to be the time when we carefully consider the fundamental intergenerational risk-management problem and define choices in individual accounts".* The inter-generational risk-management problem, can simplistically be thought of as: how to transfer the risks and the benefits between two groups of agents—young and old; where the first group disproportionately have human capital and benefits from labour productivity; whereas the other is primarily a beneficiary of owning real capital and receives return from securities, either directly through private savings or indirectly *via* mandatory pensions. Albeit, individual accounts are important as a vehicle for creating the appropriate incentives, by connecting contributions with benefits; individual accounts, are also motivated by the need to tailor exposure and diversify risks that originate from individual differences; in age, wage uncertainty, implicit insurances, preferences, and assets that the individual may have.

When reforming their pension systems, some countries have introduced a notional—or non-funded—defined contribution system (NDC), with individual accounts, and return indexed to wage growth. This indexation, has the benefit to the retireés of giving them a share of future labour productivity, while distributing the volatility in wages to the entire society. The problem with this type of indexation is that it exacerbates the wage-related risk for younger generations who are already exposed to too much of this risk in their human capital.

Campbell (2005) discusses a risk-sharing system, where contributions are negatively correlated with capital returns, which effectively creates a swap-contract between owners of real and human capital. Campbell and Nosbusch (2006) found that a social security system that optimally shares risks across generations exposes future generations to a share of the risk in physical capital returns. Shiller have in numerous articles (*cf.* Shiller (1993), Shiller (2003a)) advocated the introduction of a swap, that pays the domestic aggregate labour income growth in exchange for risk-free or a global labour income growth.

In this paper we propose an NDC system, which allow the individual some freedom of choice in the allocation of the mandatory savings, while keeping the system for contributions intact. We do this by introducing a swap-contract; where the individual, can choose to enter into a swap, that swaps—aggregate labour income growth in exchange for equity return (henceforth—a Shiller-swap), and thereby addressing the inter-generational risk-management problem. Net positions within the NDC system could be zero, leaving assets and liabilities in the system unchanged. Allowing the individual to enter into specific positions of the Shiller-swap, would assuage the consequences of forcing all individuals into one-size-fits-all, in terms of risk and return characteristics of some of their pension assets. To demonstrate the properties of our proposal, we use a model similar to the life-cycle model of Campbell *et al*(2001), (henceforth—CCGM) and Carlsson and Erlandzon (2005).

In the complete market solution; an individual would equalise consumption over life, while keeping the residual savings in assets (including human capital) optimally diversified. However, restricting the individual from capitalising future income has the consequence that; young individuals will attribute little value to their mandatory pension-savings, since they face a positive earnings profile, (*cf.* CCGM, Carlsson and Erlandzon (2005)). Furthermore, young adults will also have a disproportionate exposure to human capital; which is the *rationale* for our proposal—to use the mandatory savings account in order to create a more balanced portfolio.

Not surprisingly, the largest welfare gains would be achieved by allowing the individual to freely determine, when to save and how to allocate the savings. However, we cannot expect "the market", to solve this problem. The fact that most societies require a system with mandatory savings for pensions among consenting adults, and that such systems have existed in many countries for more than a century; is maybe a tribute to the insights of past politicians into individual behaviour—modern societies will not permit people to consume too much of their income, and then allow the same individuals to rely on society to care for their pension. The individual preferences and perceived insurances that made it necessary for society to introduce the restriction of mandatory savings, will most likely, also reduce the likelihood of these individuals making spontaneously good choices among various investment alternatives. Government has therefore a rôle to play as an administrator of mandatory pensions, but the design of such systems, should minimize the distortions.

We use the Swedish NDC system as a benchmark against which to measure the potential of our proposal. Sweden was the first country to introduce an NDC system, where contributions are credited to an individual notional account with a return set to aggregate labour income growth. This reform—initiated 1999; has attracted a lot of interest as a potential blueprint for other countries (*cf.* Schieber & Shoven (1996), Diamond (2002), Holzmann & Palmer (2006)). Furthermore, the relative simplicity and transparency of both the tax and pension systems also facilitates a realistically calibrated model. While calibrated on Swedish data and rules for taxes and pensions, there are several similarities to systems in other countries, *e.g.* Italy.

This paper has its origin in the life-cycle models that highlights uncertainty and market incompleteness as important factors in explaining individual choice and welfare. Deaton (1991), Carroll (1997) and Gourinchas and Parker (2002) created life-cycle models with uncertain wages and borrowing constraints. Cocco *et al.* (2005) and others extended these models with a portfolio choice between a risk-free and a risky asset. In order to analyse the effects of different pension systems, CCGM added a mandatory pension scheme to the model. They demonstrated a positive welfare effect from a lower pension contribution; due to a postponement of savings until a time when labour incomes are higher. Carlsson and Erlandzon (2005), showed that wage-indexation of mandatory savings, exacerbates the negative welfare effects.

Our model extends the CCGM-model by including realistically calibrated tax- and pension-systems and Carlsson and Erlandzon (2005) by an additional choice; *i.e.* we allow the individual to swap some of the aggregate income exposure within the mandatory pension system for equity exposure. The main contribution of this paper is that: we can identify the age, when the individual would be a buyer or seller of a Shiller-swap and estimate the required risk premium to attract both buyers and sellers. We find that the young would be buyers—and then sellers when older—and that this pattern is invariant to individual risk-aversion. Therefore, the objective of societal risk-sharing can be achieved by allowing the individuals to take positions in Shiller-swaps.

The rest of this paper is organised as follows. Section 2 describes the model, while Section 3 demonstrates how the model is calibrated. The results are presented in Section 4. Finally, we end with some concluding remarks in Section 5.

2. THE MODEL

2.1. Individual preferences

The individual maximises the expected utility over the adult life-cycle, which is divided into pre- and post-retirement. "Optimization", starts at the age[1] of $\tau_0(20)$, retires at a fixed age $K(65)$, and dies at a maximum age of $T(100)$. We assume that the individual has constant relative risk aversion preferences, on a single non-durable consumption good, C_τ.

Individual preferences at time m are defined as

$$\frac{C_m^{1-\gamma}}{1-\gamma} + E_m \sum_{\tau=m+1}^{T} \delta^{\tau-m} \left(\prod_{j=m}^{\tau-2} p_j \right) \left\{ p_{\tau-1} \frac{C_\tau^{1-\gamma}}{1-\gamma} + b(1-p_{\tau-1}) \frac{D_\tau^{1-\gamma}}{1-\gamma} \right\}, \tag{2.1}$$

where C_τ represent consumption at age τ, γ is the coefficient of relative risk aversion, p_τ is the one year age contingent survival probability, δ is the discount factor, b is the bequest parameter and D_τ is the bequest amount.

2.2. Labour income

The labour income process follows Carroll and Samwick (1997). During working life, the individual experiences idiosyncratic as well as common shocks to gross income. The log labour real income $l_{ik\tau}$ prior to retirement, for an individual i belonging to group k is exogenous; *i.e.* the individual cannot

[1] Or at the age of 23, for those with a university degree.

change her labour supply or education to *e.g.* accomodate income shocks, and given as

$$l_{ik\tau} = f_k(\tau, \mathbf{Z}_{ik\tau}) + v_{ik\tau} + \epsilon_{ik\tau}, \tag{2.2}$$

where f_k is a function of the individual characteristics[2] $\mathbf{Z}_{ik\tau}$ as well as an average national labour productivity growth μ^l, $\epsilon_{ik\tau}$ is an idiosyncratic temporary shock distributed as $N(0, \sigma_{\varepsilon_k})$ and $v_{ik\tau}$ is a random walk

$$v_{ik\tau} = v_{ik\tau-1} + u_{ik\tau}. \tag{2.3}$$

The innovation, $u_{ik\tau}$, is divided into a group aggregate $\xi_{k\tau} \sim N(0, \sigma_{\xi_k})$, which we allow to be correlated with excess returns in the risky asset, and an individual uncorrelated component $\omega_{ik\tau} \sim N(0, \sigma_{\omega_k})$ as

$$u_{ik\tau} = \xi_{k\tau} + \omega_{ik\tau}. \tag{2.4}$$

2.3. Present mandatory savings and retirement benefits

Mandatory pension-savings and retirement benefits are part of an NDC-system in which 16% of gross pre-retirement income[3] is contributed by the employer and accounted for in individual accounts. Contributions are capped above an income of 300 KSEK[4]. The return on the accounts—R^l_τ is set equal to the national labour income growth, (*cf.* RFV (2002)).

$$R^l_\tau = e^{\mu^l + \xi^A_\tau}, \tag{2.5}$$

$$NDC_{i\tau} = \begin{cases} R^l_{\tau-1} \cdot NDC_{i\tau-1} + 0.16 \min\left[L_{i\tau}, 300\right] & , \tau < 65 \\ R^l_{\tau-1} \cdot NDC_{i\tau-1} - PO_\tau(R^l_{\tau-1} \cdot NDC_{i\tau-1}) & , \tau \geq 65, \end{cases} \tag{2.6}$$

where PO_τ is the age specific annualised mortality-adjusted payout-function after retirement, and $\xi^A_\tau \sim N(-\frac{1}{2}\sigma^2_{\xi^A}, \sigma_{\xi^A})$.

[2] *i.e.* age, martial status, family size, number and age of children.

[3] An additional contribution of 2.5% can be managed by the individual in a funded account. For simplicity, we disregard this account.

[4] In the following, KSEK—thousands of Swedish Kronor will be omitted. The present exchange rate is *circa* 7 SEK / USD.

Most individuals also have a negotiated supplementary defined benefit plan, where the employer partly compensates for the cap on NDC contributions. The benefits are constant in real terms, guaranteed for the remainder of life, and depend on the wage at retirement[5]. These company defined benefit plans have a payout of 10%, 65% and 32.5% of incomes in the intervals $[0, 320)$, $[320, 850)$, and $[850, 1270)$ respectively at retirement.

Payout from this plan during retirement will be denoted $DBPO_{i\tau}$, and its dynamics is

$$\begin{aligned} DBPO_{i\tau} &= 0.1 \min\left[L^P_{i64}; 320\right] + \\ 0.65 \min &\left[\max\left(L^P_{i64} - 320; 0\right); 850 - 320\right] + \\ 0.325 \min &\left[\max\left(L^P_{i64} - 850; 0\right); 1270 - 850\right], \end{aligned} \tag{2.7}$$

with

$$L^P_{i64} = e^{f_k(\tau, \mathbf{Z}_{ik64}) + v_{ik64}}. \tag{2.8}$$

All payouts from the NDC pension plan are forfeited in the event of death and for simplicity, we assume the same for the defined benefit plan.

2.4. Taxes

Wage and retirement income $L_{i\tau}$ can be defined as

$$L_{i\tau} = \begin{cases} e^{l_{i\tau}} & , \tau < 65 \\ PO_\tau(R^l_{\tau-1} \cdot NDC_{i\tau-1}) + DBPO_{i\tau} & , \tau \geq 65. \end{cases} \tag{2.9}$$

According to current Swedish tax rules, labour income and pension benefit are taxed at the same rate, and separate from capital income[6]. To calculate net income—$L^n_{i\tau}$, first, we deduct a general allowance of 10; second, a municipal tax of 30%; third, a government tax of 20% on all income above 300; and finally, an additional government tax of 5% on income above 450. Net income is bounded

[5] In reality it depends on the wage during the five years prior to retirement. However, modelling this rule correctly would have necessitated additional state variables. We therefore approximate this by only including the permanent income changes until retirement.

[6] We use the tax rules for incomes earned in 2003.

below at 60 by the social welfare minimum-benefit, which also applies to retirees in the form of a government-guaranteed minimum pension.

$$
\begin{aligned}
L_{i\tau}^{n} = \max[L_{i\tau} - 0.3\max\left(L_{i\tau} - 10; 0\right) - \\
0.2\max\left(L_{i\tau} - 300; 0\right) - 0.05\max(L_{i\tau} - 450; 0); 60].
\end{aligned}
\tag{2.10}
$$

All the threshold-values that create kinks in tax-rates and benefits[7] are indexed to the expected national labour income growth—μ^{l}, except the social welfare minimum benefit which is kept constant in real terms.

2.5. Assets

There is one risky and one risk-free asset with after-tax real log-returns equal to r_{τ}^{e} and r^{f} respectively. Excess return is defined as

$$
r_{\tau}^{e} - r^{f} = \mu^{e} + \eta_{\tau}, \tag{2.11}
$$

where the noise—η, is correlated with the group aggregate innovation in permanent labour income ξ_{k}, which allows for a group specific sensitivity to the risky asset,

$$
\begin{bmatrix} \boldsymbol{\xi} \\ \eta \end{bmatrix} \sim N\left(\begin{bmatrix} -\frac{1}{2}\boldsymbol{\sigma}_{\xi}^{2} \\ -\frac{1}{2}\sigma_{\eta}^{2} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma} & \boldsymbol{\sigma}_{\xi\eta} \\ \boldsymbol{\sigma}_{\xi\eta}' & \sigma_{\eta}^{2} \end{bmatrix}\right). \tag{2.12}
$$

2.6. Private savings and consumption

Each individual starts her "optimization life" with initial wealth set to F. In pre-retirement years the individual receives a wage, and in subsequent years the individual will receive retirement benefits. The individual have two control variables: the proportion of cash on hand to consume—θ_{τ}, and what proportion of savings—α_{τ}, to allocate to the risky asset. The cash on hand, or disposable wealth, is therefore,

[7] This is the same as in the US since the "bend points" when calculating the primary insurance amount (PIA) are adjusted by average earnings growth.

$$X_{i\tau} = \begin{cases} e^{r^f}\left[1+\alpha_{i\tau-1}(e^{\mu^e+\eta_\tau}-1)\right][1-\theta_{i\tau-1}]X_{i\tau-1}+L^n_{i\tau} & ,\tau>\tau_0 \\ F_i + L^n_{i\tau} & ,\tau=\tau_0 \end{cases} \tag{2.13}$$

of which consumption is,

$$C_{i\tau} = \theta_{i\tau}X_{i\tau}. \tag{2.14}$$

There is also constraints for both borrowing and short-sales,

$$\begin{aligned} 0 \le \theta_{i\tau} \le 1, \\ 0 \le \alpha_{i\tau} \le 1. \end{aligned} \tag{2.15}$$

2.7. Mandatory savings with Shiller-swaps

With our proposal, the agent can exchange (buy the contract) the risk and return in their NDC account for equity exposure, through a Shiller-swap. The design of this contract is as follows: first, we define the premium—s, to the seller of the contract; in such a way, that if a Shiller-bond existed, with log-return—$(\mu^l + \xi^A_\tau + s)$, (equal to labour income growth plus premium) it would be a non-redundant asset; *i.e.* attracting an unrestricted (international) investor with power utility,

$$\begin{gathered} \mu^l + s = r^f - \frac{\sigma^2_{\xi^A}}{2} + \beta(\mu^e + \frac{\sigma^2_\eta}{2}), \\ \text{where } \beta = \frac{Cov(\eta,\xi^A)}{Var(\eta)}. \end{gathered} \tag{2.16}$$

We then create a zero-investment portfolio with zero expected payoff,

$$E\left[e^{r^f}(e^{\mu^e+\eta_\tau}+\Lambda-1)-\Lambda e^{\mu^l+\xi^A_\tau+s}\right]=0, \tag{2.17}$$

which determines the exchange multiple—Λ; the ratio by which returns are swapped in a Shiller-swap. We do this since, we are primarily interested in the demand for Shiller-swaps because of their different risk-characteristics, not due to different expected returns.

In Figure 2.1, we show the cash-flows and returns from the constituent securities of the zero-investment portfolio. The portfolio combination of securities is equivalent to the Shiller-swap.

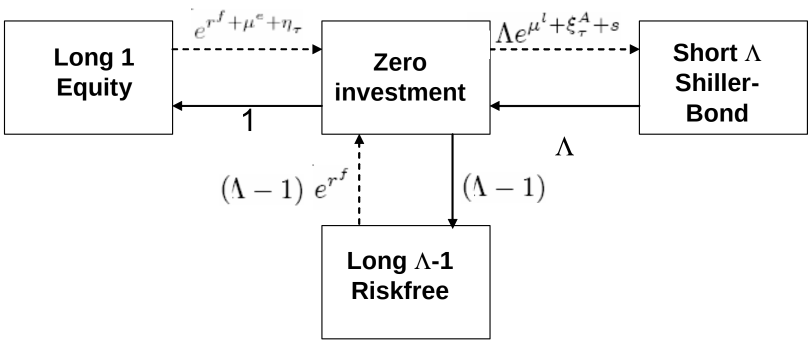

FIG. 2.1 Zero investment portfolio. The dashed lines represent the returns and the the solid lines the investments.

The individuals can now choose—λ, the proportion of their NDC account they wish to swap. Consequently the overall return on the NDC account (Equation (2.5)) changes to

$$R_\tau^l = e^{\mu^l+\xi_\tau^A} + \lambda \left[e^{r^f} \left(e^{\mu^e+\eta_\tau} + \Lambda - 1 \right) - \Lambda e^{\mu^l+\xi_\tau^A+s} \right]. \tag{2.18}$$

2.8. Optimization

The optimization problem has four state-variables (τ, v, X and NDC) and three choice-variables (θ, α and λ), as well as four stochastic variables (ϵ, u, ξ^A and η). The value function of the individual intertemporal consumption and investment problem can then be written as

$$\begin{gathered} V_\tau\left(\Gamma_\tau\right) = \max_{\theta_\tau,\, \lambda_\tau,\, \alpha_\tau} \left\{ \frac{C_\tau^{1-\gamma}}{1-\gamma} + \delta E_\tau \left[p_\tau V_{\tau+1}\left(\Gamma_{\tau+1}\right) + (1-p_\tau)\, b \frac{D_{\tau+1}^{1-\gamma}}{1-\gamma} \right] \right\}, \\ \Gamma_\tau = \left\{ X_\tau, v_\tau, NDC_\tau \right\}. \end{gathered} \tag{2.19}$$

The solution to this problem determines the state-dependent policy-rules

$$\begin{aligned} \theta_\tau &= \theta_{k\tau}(\Gamma_\tau), \\ \alpha_\tau &= \alpha_{k\tau}(\Gamma_\tau), \\ \lambda_\tau &= \lambda_{k\tau}(\Gamma_\tau). \end{aligned} \tag{2.20}$$

We solve the problem numerically by backward recursion from the final year—T, using by-now standard methods, *cf.* Cocco *et al.* (2005) and *Judd*(1998).

3. CALIBRATION OF PARAMETERS

3.1. Labour income process

We use the same parameter estimates of the labour income process as in Carlsson and Erlandzon (2005). The definition of income includes, in addition to wages, all taxable social benefits, primarily compensation for unemployment, sickness and disability retirement. Data was calculated from the LINDA data set for the years 1992 to 2002. LINDA is a register-based longitudinal data set, which consists of a large panel of individuals, and representative of the population from 1960 and onwards. The data set covers 3.35% of the Swedish population (more then 300,000 individuals plus their household members) and is described in Edin and Fredriksson (2000) and this data augmented with wealth information, has recently received attention in *cf.* Calvet *et al* (2006), Campbell (2006) and Flood (2003).

The data was divided into six non-intersecting groups defined by educational attainment and sex. The predictable component of labour income was estimated separately for each group. Table 3.1 shows the estimated parameters of the income polynomial or the most numerous group—Men with High-school degree. The results from the other groups are similar, and income in this group was closest to the national average.

TABLE 3.1
Coefficients in the age polynomial of the forward-looking income profile

Income profile, 2004 KSEK, (AGE-18)				
	Constant	Age	Age^2	Age^3
	a_0	a_1	a_2	a_3
Men with a High school degree	221.7	4.730	.4363	-.00779

Table 3.2 presents result from the variance decomposition.

3.2. Individual parameters

For the reference case we used a standard set of assumptions regarding the individual parameters. First, we set the coefficient of relative risk aversion—γ to 5, the discount factor—δ to 0.98, and the bequest parameter—b is set to 1, making consumption and bequest equally important in the final year. The survival probabilities p are sex dependent and taken from the Swedish life insurers when

TABLE 3.2
Variance decomposition

Group	Estimated variance of the permanent component, $\sigma^2_{u_k}$	Estimated variance of the transitory component, $\sigma^2_{\varepsilon_k}$	Std. of the permanent aggregate component, σ_{ξ_k}
Full sample			.01989
Men with a High school degree	.00564	.00981	.02020

underwriting new policies, *i.e.* they are forward looking.

3.3. Assets and correlations

In Table 3.3, we present the asset and correlation parameters. The mean after tax equity premium μ^e was set to 3%, which is low when compared with the historical average, but corresponds well to some forward-looking estimates (*cf.* Claus and Thomas (2001), Fama and French (2002)). We set the risk-free after-tax rate—r^f to 1.5%, consistent with the present gross return of less than 2% for long-dated index-linked bonds. Volatility σ_η was set to 17% for the risky asset. Following Cocco *et al.* (2005), we then estimated the correlations between aggregate permanent labour income shocks and lagged equity returns $\eta_{\tau-1}$.

TABLE 3.3
Asset and correlation parameters

r^f	.015
μ^e	.03
σ_η	.17
$\varrho_{\xi_k \eta}$	.51
$\varrho_{\xi^A \eta}$	.47

To keep the risk level of the NDC account[8] at approximately the same level as the current system, we restricted the Shiller-swap exposure in the NDC account to ±20% of the account. We also set the growth in average labour income μ^l to 1.8%, which is the estimate used by the Swedish National Social Insurance Board. Finally, initial wealth F was set to 47, the mean wealth for individuals between the ages of 20 and 23.

4. RESULTS

To study the potential outcomes generated by the model; we simulated individual behaviour from age 20, by generating 30,000 random trajectories through time. Subject to the stochastic experience,

[8] Standard Deviation in the value of the NDC account at retirement. The risk in a single year is of course higher.

the individual will choose a response, defined by the policy functions in Equation (2.20) (shares of consumption, risky assets and Shiller swaps), that describe the optimal state dependent behaviour.

4.1. International pricing of Shiller-swaps

As a reference, we plot the averages of the simulated trajectories for cash on hand (Figure 4.1), risky weight (Figure 4.2) and consumption (Figure 4.3) for two scenarios; with and without the existence of an internationally priced Shiller-swap.

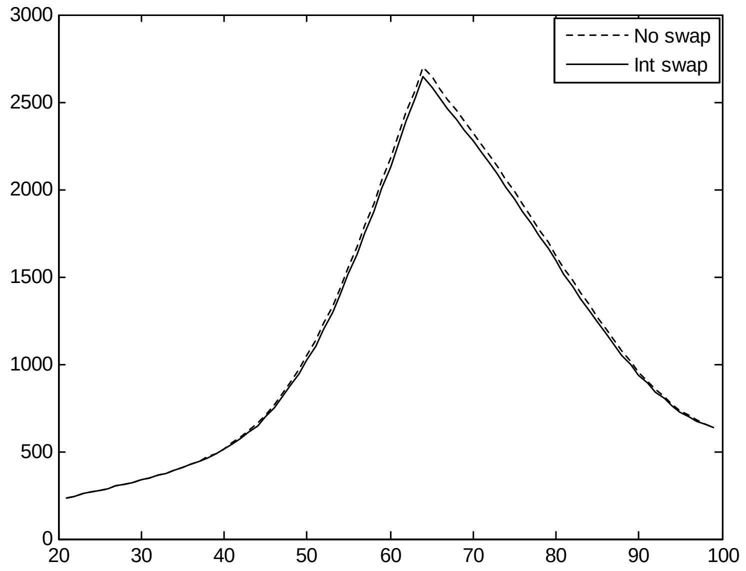

FIG. 4.1 Cash on hand—X, patterns when swap premium is set for international investors and wihout Shiller-swaps.

Since the swap was priced with zero expected return, we do not expect these profiles to be more than marginally different. However, the risk exposure of the NDC account is completely changed. In Figure 4.4, we plot the profile for the average size of the NDC account together with the hedged amount of aggregate wage risk ($-\lambda_\tau \Lambda NDC_\tau$) for an internationally priced swap and the total wage exposure within the NDC account.

Prior to mid-life the agent is constrained by our arbitrary rule of a maximum Shiller-swap share—λ_τ in the NDC account of 0.2 (*cf.* Figure 4.10).

Before retirement, the individual hedge not only the NDC account, but also the discounted expected value of both the defined benefit contract and future wages. When the defined benefit pension is fixed at retirement and future wages are zero, the agent still continues to hedge the discounted

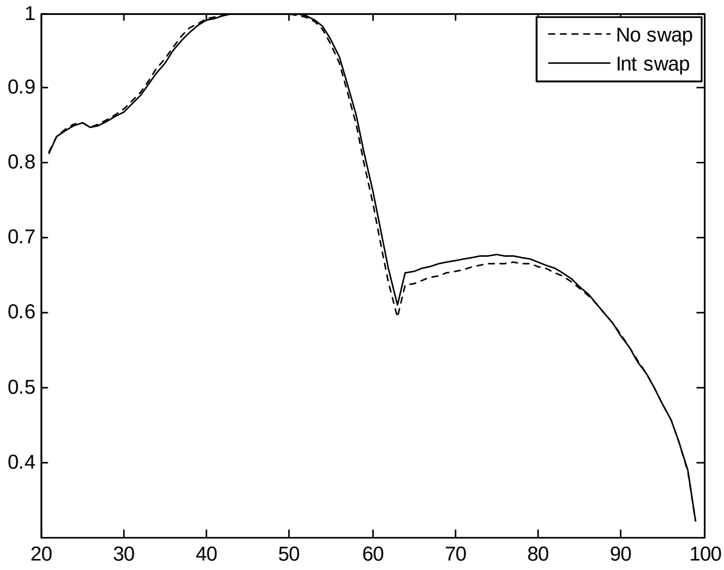

FIG. 4.2 Risky weight—α_τ when swap premium is set for international investors and without Shiller-swaps.

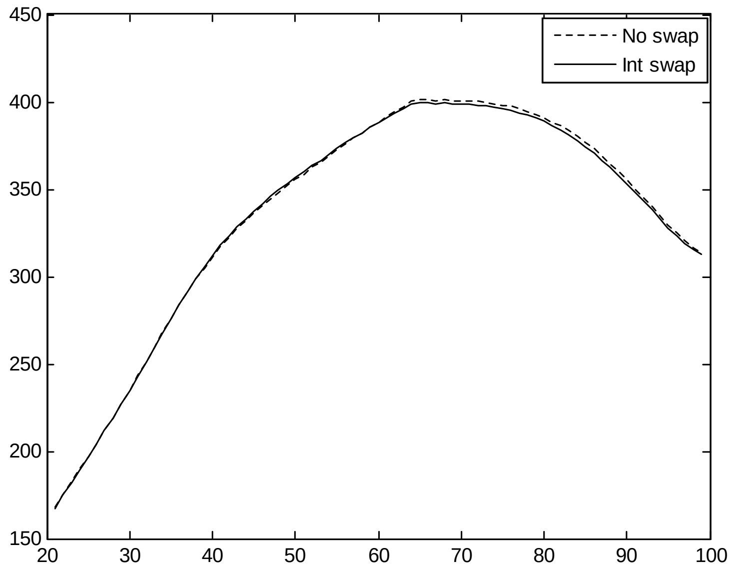

FIG. 4.3 Consumption—C, patterns when swap premium is set for international investors and without Shiller-swaps.

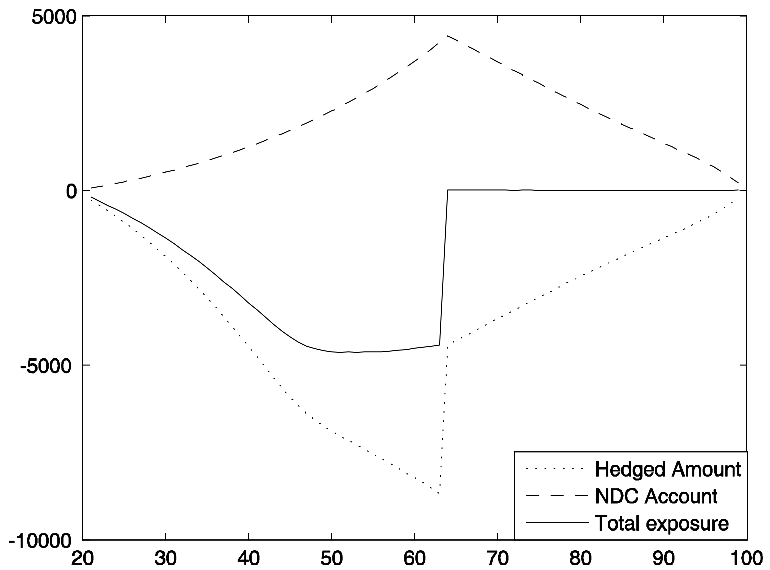

FIG. 4.4 Hedged wage exposure—$\Lambda\lambda_\tau NDC$, value of NDC account and total exposure to wage risk in the NDC account when the swap premium is set for international investors.

expected value of the NDC account.

4.2. Inter-generational pricing of Shiller-swaps

To investigate the potential demand for Shiller-swaps from domestic investors only, without resorting to an international market for Shiller-bonds, we increased the swap premium—s (Equation (2.16)) with $10, 15$ and $25bp$ (basis points) respectively, in addition to what an international investor requires. Figure 4.5 shows the simulated profiles for Shiller-swap share—λ_τ, while Figure 4.6 and Figure 4.7 demonstrates the avarage exposure to equity risk ($\lambda_\tau NDC_\tau$) and wage risk $NDC_\tau(1 - \lambda_\tau \Lambda)$, respectively.

The additional premium will encourage the agent to be both a buyer and seller of such a swap, but at different ages. Before retirement, the individual will on average be a buyer of the Shiller-swap and afterwards a seller; thereby creating a voluntary inter-generational transfer of wage-growth risk. Selling the contract implies a negative equity exposure in the NDC account, that is partly compensated for by a higher risky share—α in private savings (*cf.* Figure 4.8).

The higher risky share in cash on hand does not fully compensate for the negative equity-exposure in the NDC account, as is demonstrated in Figure 4.9, which shows total equity exposure from both private savings and the NDC account.

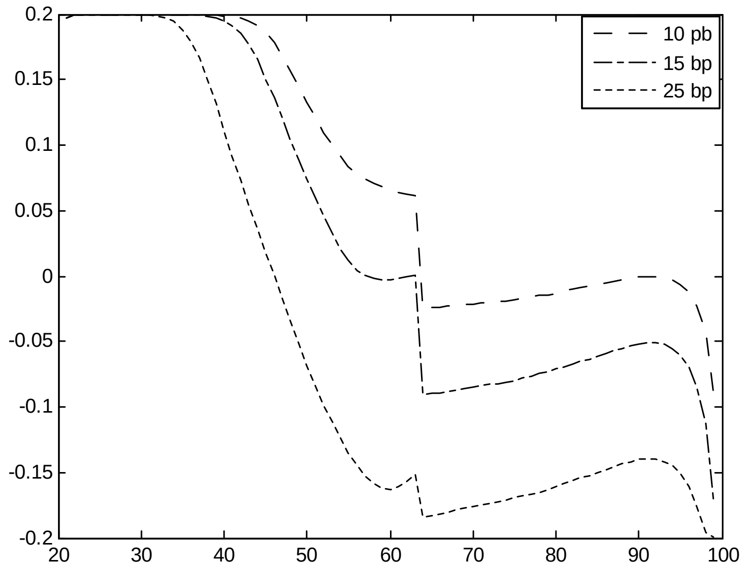

FIG. 4.5 Shiller-swap weights—λ with different swap premia in addition to the international requirement—s.

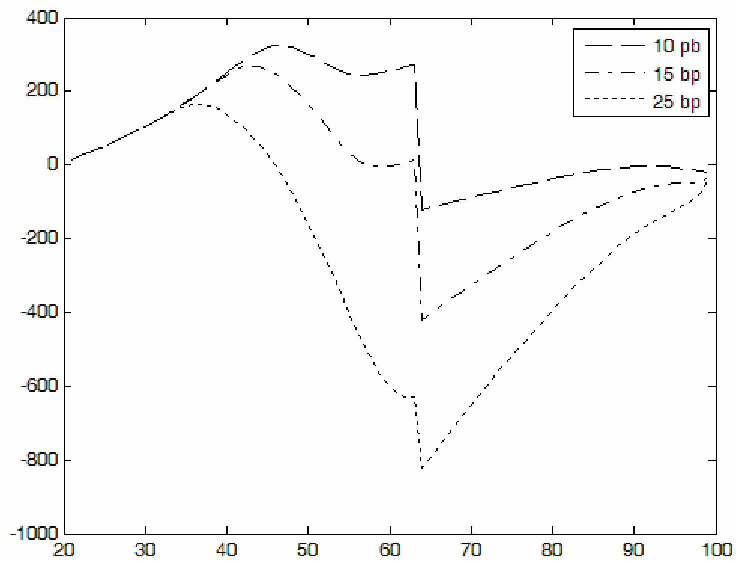

FIG. 4.6 Average equity exposure λNDC in the NDC account.

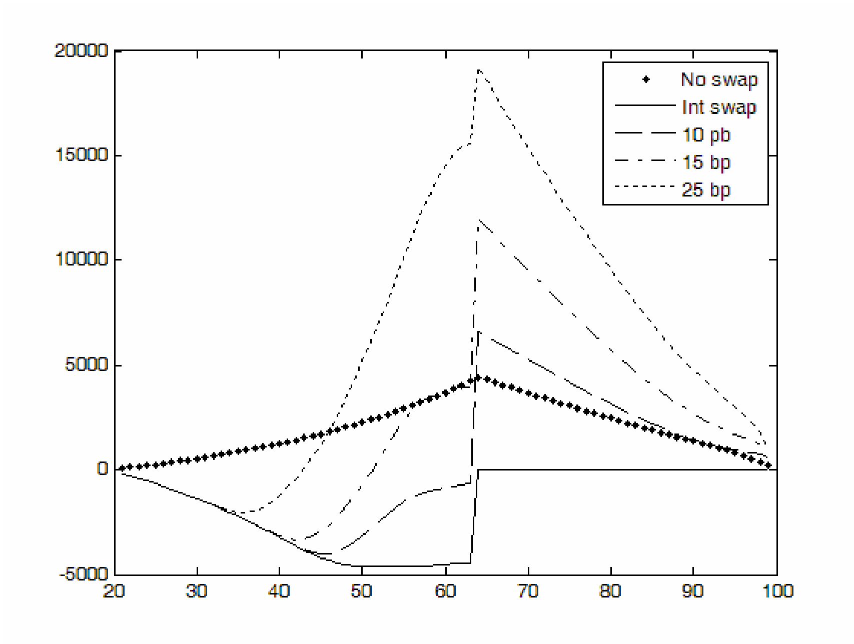

FIG. 4.7 Average exposure to wage risk—$NDC(1 - \lambda\Lambda)$ in the NDC account.

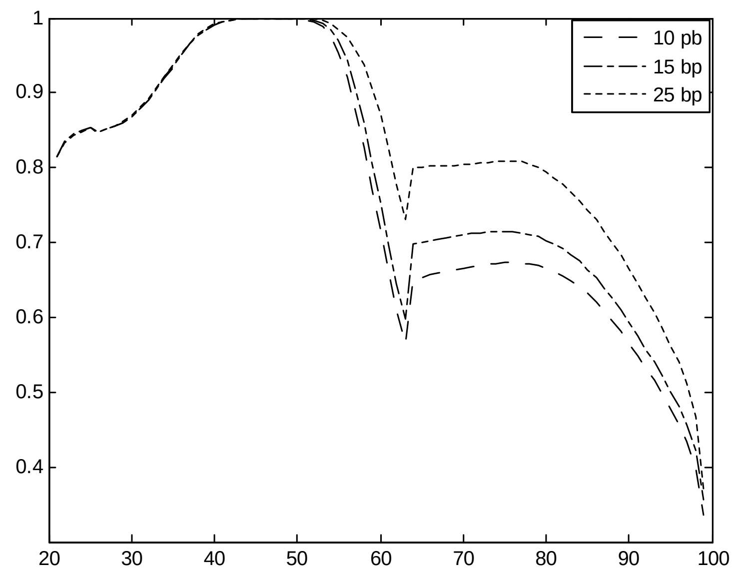

FIG. 4.8 Risky weights of cash on hand—α_τ with different Shiller-swap premia in addition to the international requirement—s.

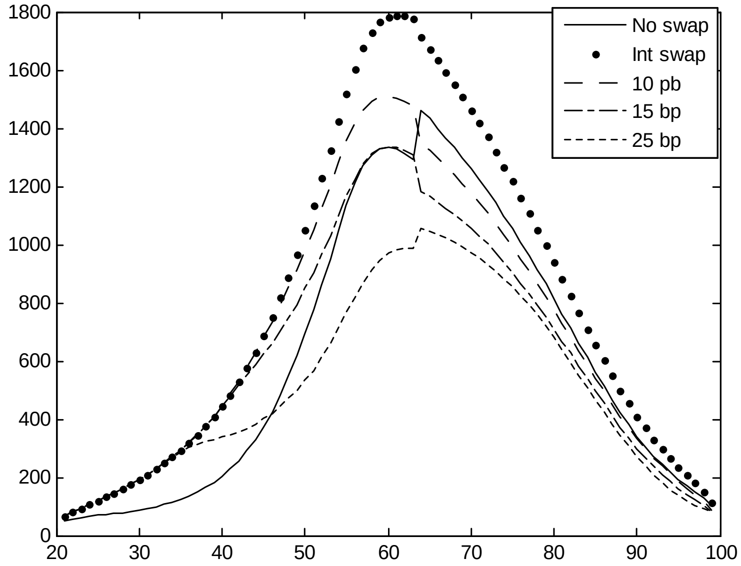

FIG. 4.9 Total exposure to risky assets from private savings and the NDC account—$\alpha X + \lambda NDC$.

In early life, the agent tries to maximise the equity exposure (subject to our constraint of 20%), irrespective of the premium. Later in life and as private savings increase, the increased swap-premium make the agent more inclined to sell the Shiller-swap.

For the single agent belonging to this group and with these preferences (risk aversion); the 15bp additional swap premium would approximately "clear" the demand and supply across the individuals age; increasing the premium further would create excess supply. We can therefore expect that—if overall demand and supply is unaffected by individual differences in risk-aversion—a Shiller-swap market could be established within the NDC framework without resorting to an international market.

Changing the relative risk aversion from 5 to 2 or to 10 has only a minor effect on preference for Shiller-swap exposure (*cf.* Figure 4.10), whereas the same changes in risk aversion have a dramatic impact on the risky weight (*cf.* Figure 4.11).

The reason is: as long as individuals are risk-averse, and with the expected risk premium—s and multiple—Λ, set so that the Shiller-swap has zero expected return (*cf.* Equation (2.17)), there will still be demand for the Shiller-swap, but for an hedging purpose; whereas the demand for risky assets are primarily motivated by higher expected returns.

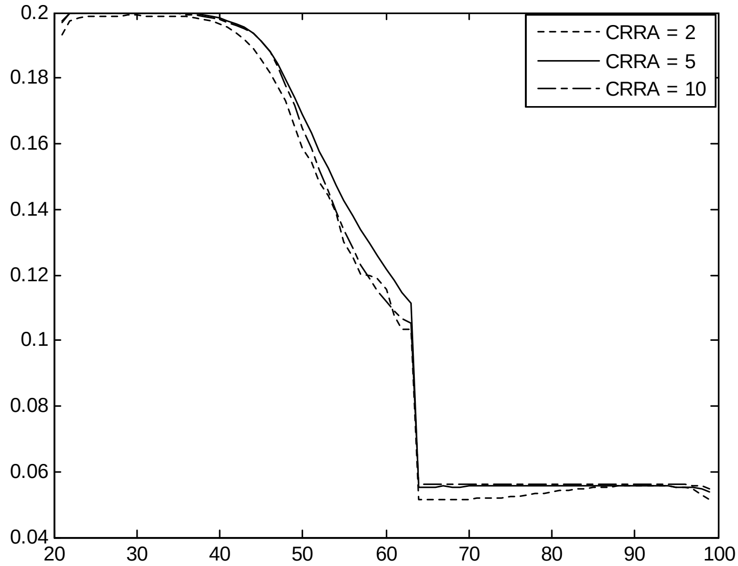

FIG. 4.10 Shiller-swap weights—λ when swap premium is set for international investors, but with different constant relative risk aversions(CRRA)—γ.

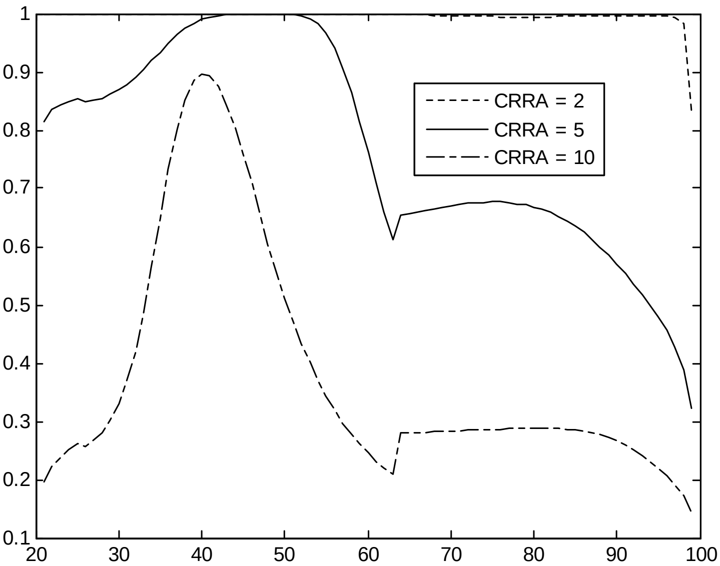

FIG. 4.11 Risky weights when swap premium—s is set for international investors, but with different constant relative risk aversions(CRRA)—γ.

5. CONCLUSION AND COMMENTS

This paper proposes the introduction of a Shiller-swap to mandatory individual pension accounts in order to allow the agents to swap wage-risk for equity-risk. We present a life-cycle model of a borrowing constrained individual's consumption- and portfolio-choice in the presence of uncertain labour income and realistically calibrated tax and pension systems. The pension scheme consists of both a defined benefit and a notionally defined contribution part, the latter being indexed to stochastic aggregate labour income growth.

Our result show that, an internationally priced swap premium, suffices to create large domestic demand for Shiller-swaps across age. However, the swap premium was derived from an international market for Shiller-bonds, which is yet to be established. We therefore show that a "minor" increase of the premium by $15bp$ will create both demand and supply for this swap, but at different ages. It would therefore be possible to create a Shiller-swap market within the pension system, thereby allowing a voluntary inter-generational sharing of wage-risk.

With the implementation of our proposal; forced saving in mandatory pension schemes can be maintained, but with substantially reduced distortions.

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A Tale of Two Systems: Winners and Losers when moving from Defined Benefit to Defined Contribution Pensions

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There is a trend among employers to prefer Defined Contribution instead of Defined Benfit pension plans, since the former transfer all risks associated with investment return, longevity, *etc* from the employer to the employeé. However, Defined Contribution plans also allow the individual to enter into positions contingent on the individual situation. This paper investigates the individual welfare consequences of different plans. We used the recent transition from defined benefit to defined contribution for white-collar workers in Sweden as the benchmark for our analysis. The framework for our analysis is a life cycle model of a borrowing-constrained individual's consumption- and portfolio choices in the presence of uncertain labour income. The main result is that individuals with the characteristic of a low expected pre-retirement income relative to average income and high variance in earnings are winners (men with university degree in the private sector), and that those with the opposite characteristic (women with university degree in the public sector) would be losers.

Key Words: Life-cycle, portfolio choice, defined contribution, defined benefit, income process.

JEL classification: D31, D91, G11, G23, H24, J31

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1. INTRODUCTION

Defined Contribution—DC pension plans[1] are now often the preferred pension system among employers. This is not very surprising since the shift from Defined Benefit—DB[2], transfers all the risks associated with investment return and longevity from the employer to the employeé. However, there are also several advantages for the individual with a DC plan: it allows the individual to enter into specific positions, which reduces the consequences of forcing all individuals into one-size-fits-all, in terms of risk and return characteristics; it facilitates portability when the agent transfers from one employer to another; and not the least it assuages the risks of lower wages in the final years of employment. Which of these systems that are beneficial to the individual is very state dependent and merits this research.

In this paper we have analysed the welfare consequences for the individual when transferring from a DB to a DC system. As a benchmark for this analysis, we have chosen the recently negotiated transfer from a DB to a DC plan between private white-collar workers union and their employers in Sweden, *cf.* Svenskt Näringsliv (2006). This analysis is even more pertinent, since this transfer will most likely be the blueprint for a similar future settlement for the public employeés.

The main result is that individuals with the characteristic of a low expected pre-retirement income relative to average income and high variance in earnings are winners (men with university degree in the private sector), and that those with the opposite characteristic (women with university degree in the public sector) are losers.

Our analysis draws heavily on the literature highlighting: life-cycle saving and consumption, Modigliani and Brumberg (1954) and Friedman (1957); and portfolio-choice, Samuelson (1969) and Merton (1971). Deaton (1991), Carroll (1997) and Gourinchas and Parker (2002) created life-cycle models with uncertain wages and borrowing constraints; which showed that market-incompleteness is important when explaining individual choice and welfare effects. Cocco *et al.* (2005) and others extended the model with portfolio-choice between a risk-free and a risky asset. Campbell *et al* (2001) added a mandatory pension scheme to the model.

[1] A DC plan accumulates a proportion of every salary as a contribution.
[2] A DB plan pays a proportion of final salary as a pension.

The introduction of non-tradeable human capital into the intertemporal life cycle model with portfolio choice and consumption, creates an asset that will influence—how much the individual saves and the optimal portfolio choice in savings. These choices depend on the expected individual dividend profile from human capital and associated uncertainties, but also on the characteristics of other assets; primarily private savings, pension savings and housing.

Labour generates two types of dividends: wages and pension contributions. In this paper we estimated the income process that should be used as the underlying for calculating the derivatives—net wages and pension contributions; keeping the "dividends" from human capital separate from other types of asset-income.

Carroll and Samwick (1997), Gourinchas and Parker (2002), Cocco *et al.* (2005), and other similar earlier studies generally treated returns from human capital as equal to earned income net of return from private savings. Such a wide definition lead to some double-counting, for those who retire early or receive pension benefits dependent on contributions during their working life. In our definition of returns from human capital, we only included income that stem from individual productivity and insurances against, *e.g.* disability, parental leave, unemployment *etc.*—not from early withdrawals from retirement savings.

We are interested in the expected income profile as the underlying for pension contributions and taxes, which influence the individuals future choices in terms of saving and portfolio allocation. It is therefore natural to model individuals rather than households[3], since pension contributions and taxes are primarily dependent on the individual instead of family incomes.

The remainder of the paper is organised as follows: Section 2 describes the model, while Section 3 describes the optimisation problem, and Section 4 the calibration of the model. Section 5 discusses the results, and the final Section 6 summarizes and draws some conclusions.

[3] Furthermore, female labour-participation and divorce rates are high, which—together with an age-difference between man and wife—could have obscured the expected-wage profile if estimated on family data. When estimating on family data, the educational status, age and retirement date is typically defined by the head of household only.

2. THE MODEL

2.1. Individual preferences

We assume that an individual maximise the expected utility over their adult life-cycle, which starts at the age of τ_0, and dies no later than at the age of T. We assume that an individual has constant relative-risk-aversion preferences for a single non-durable consumption good—C_τ.

Individual preferences at time—m are defined as

$$\frac{C_m^{1-\gamma}}{1-\gamma} + E_m \sum_{\tau=m+1}^{T} \delta^{\tau-m} \left(\prod_{j=m}^{\tau-2} p_j \right) \left\{ p_{\tau-1} \frac{C_\tau^{1-\gamma}}{1-\gamma} + b(1-p_{\tau-1}) \frac{D_\tau^{1-\gamma}}{1-\gamma} \right\}, \tag{2.1}$$

γ is the coefficient of relative risk aversion, p_τ is the one-year age-contingent survival-probability, δ is the discount factor, b is the bequest parameter and D_τ is the bequest amount.

2.2. The labour-income process

Following Carroll and Samwick (1997), we assume that the individual income process during working life—L_{it}, is exogenously given by

$$\log(L_{it}) = l_{i\tau} = f(\tau, \mathbf{Z}_{i\tau}) + v_{i\tau} + \varepsilon_{i\tau}, \quad \tau \leq K, \tag{2.2}$$

where—$f(\tau, Z_{i\tau})$ is a deterministic function of individual $i's$ age—τ, and a vector of the individual characteristics[4]—$\mathbf{Z}$, where—K is the retirement age, and—$v_{i\tau}$ is given by

$$v_{i\tau} = v_{i\tau-1} + u_{i\tau}, \tag{2.3}$$

where the permanent shock—$u_{i\tau} \sim N(0, \sigma_u^2)$ is independent from the idiosyncratic temporary shock—$\varepsilon_{i\tau} \sim N(0, \sigma_{\varepsilon_k}^2)$. The permanent shock—$u_{i\tau}$, consists of a group aggregate component—$\xi_{k\tau} \sim N(0, \sigma_{\xi_k}^2)$

[4] *i.e.* age, martial status, family size, and number and age of children.

as well as an idiosyncratic component—$\omega_{i\tau} \sim N(0, \sigma^2_{\omega_k})$,

$$u_{i\tau} = \xi_{k\tau} + \omega_{i\tau}. \tag{2.4}$$

2.3. Assets

There are two assets, one risky and one risk-free asset with after-tax real log-returns equal of r^e_τ and r^f respectively. Excess return is defined as

$$r^e_\tau - r^f = \mu^e + \eta_\tau, \tag{2.5}$$

where the noise—η is correlated with the group-aggregate innovation in permanent labour-income—ξ_k, which allows for a group specific sensitivity to the risky asset,

$$\begin{bmatrix} \boldsymbol{\xi} \\ \eta \end{bmatrix} \sim N\left(\begin{bmatrix} -\frac{1}{2}\boldsymbol{\sigma}^2_\xi \\ -\frac{1}{2}\sigma^2_\eta \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma} & \boldsymbol{\sigma}_{\xi\eta} \\ \boldsymbol{\sigma}'_{\xi\eta} & \sigma^2_\eta \end{bmatrix} \right). \tag{2.6}$$

2.4. Past and present mandatory savings and retirement benefits

In the old system[5], individuals have a defined-benefit and a defined contribution plan. The defined benefit plan has a payout of 10%, 65% and 32.5% of incomes at retirement[6] in the intervals $[0, 320)$, $[320, 850)$, and $[850, 1270)$ respectively[7].

Payout from this plan is constant in real terms, and guaranteed for the remainder of life, $PODB_{i\tau}$,

$$\begin{gathered} PODB_{i\tau} = 0.1 \min\left[L^P_{i64}; 320\right] + \\ 0.65 \min\left[\max\left(L^P_{i64} - 320; 0\right); 850 - 320\right] + \\ 0.325 \min\left[\max\left(L^P_{i64} - 850; 0\right); 1270 - 850\right]. \end{gathered} \tag{2.7}$$

The defined contribution plan has contributions at 4.5% of annual labour income up to 320. Pre-

[5] Individuals born before 1979.

[6] In reality it depends on the wage during the five years prior to retirement. However, modelling this rule correctly would have necessitated additional state variables. We therefore approximate this by only including the permanent income changes until retirement.

[7] In the following, we express all amounts in thousands of SEK. The present exchange rate is *circa* 6 SEK / USD.

retirement labour income—L^P_{i64}, is defined as,

$$L^P_{i64} = e^{f_k(\tau,\mathbf{Z}_{i64})+v_{i64}}. \tag{2.8}$$

The new system is only based defined contributions (*cf.* Svenskt Näringsliv (2006)), with contributions set to: 7% for annual incomes up to 320 and 30% for incomes above this limit. Contributions to the defined contribution plans—DC are therefore,

$$DC_{i\tau} = \begin{cases} 0.045 \min\left[L_{i\tau}; 320\right], & \text{if in old system,} \\ 0.07 \min\left[L_{i\tau}; 320\right] + 0.3 \max\left(L_{i\tau} - 320; 0\right), & \text{if in new system.} \end{cases} \tag{2.9}$$

Individuals can choose the fraction, λ of the defined contribution wealth, DCW to allocate to the risky asset,

$$DCW_{i\tau} = \begin{cases} e^{r^f}\left[1 + \lambda_{i\tau-1}(e^{\mu^e+\eta_\tau} - 1)\right] DCW_{i\tau-1} + DC_{i\tau}, & \tau < 65, \\ e^{r^f}\left[1 + \lambda_{i\tau-1}(e^{\mu^e+\eta_\tau} - 1)\right] DCW_{i\tau-1} - PODC_{i\tau}, & \tau \geq 65, \end{cases} \tag{2.10}$$

where, $PODC$, is the mortality-adjusted annuity from the defined contribution plan.

Irrespective of system, all individuals also receive social security pension benefits—SS, which depend on the individual's labour-income trajectory during working life. In Carlsson and Erlandzon (2005), we modelled this system as state dependent and from the simulated trajectories we have estimated a piece-wise linear retention-rate,

$$SS_{i\tau} = 0.4 \min\left[L^P_{i64}; 320\right] + 0.1 \min\left[\max\left(L^P_{i64} - 320; 0\right); 850 - 320\right]. \tag{2.11}$$

All payouts from these pension plans are assumed to be forfeited in the event of death.

2.5. Labour income and taxes

Wage and retirement income—L can now be defined as

$$L_{i\tau} = \begin{cases} e^{l_{i\tau}}, & \tau < 65, \\ PODC_{i\tau} + PODB_{i\tau} + SS_{i\tau}, & \tau \geq 65 \text{ if in old system,} \\ PODC_{i\tau} + SS_{i\tau}, & \tau \geq 65 \text{ if in new system,} \end{cases} \tag{2.12}$$

According to Swedish tax rules[8], labour income and pension benefits are taxed at a common rate, separate from capital income. To calculate net income—$L^n_{i\tau}$, we first deduct a general allowance of 10; then a municipal tax of 30%; then national tax of 20% on all income above 300; and finally, an additional national tax of 5% on income above 450. Net income is bounded below by the social welfare minimum-benefit and government-guaranteed minimum pension at 60. Therefore

$$\begin{aligned} L^n_{i\tau} = \max[&L_{i\tau} - 0.3\max\left(L_{i\tau} - 10; 0\right) - \\ &0.2\max\left(L_{i\tau} - 300; 0\right) - 0.05\max(L_{i\tau} - 450; 0); 60]. \end{aligned} \tag{2.13}$$

All the threshold-values that create kinks in tax-rates and benefits[9] are indexed to the expected growth in national labour income—μ^l, except the social welfare minimum benefit which is kept constant in real terms.

2.6. Private savings and consumption

An individual starts her optimisation life with initial wealth set to $\digamma$. In the following pre-retirement years they receive wages, and in subsequent years retirement benefits. The individual has two control variables: the proportion of cash on hand to consume—θ_τ, and what proportion of savings—α_τ, to allocate to the risky asset. The cash on hand—disposable wealth, is therefore,

$$X_{i\tau} = \begin{cases} e^{r^f}\left[1 + \alpha_{i\tau-1}(e^{\mu^e + \eta_\tau} - 1)\right]\left[1 - \theta_{i\tau-1}\right] X_{i\tau-1} + L^n_{i\tau}, & \tau > \tau_0, \\ \digamma_i + L^n_{i\tau}, & \tau = \tau_0, \end{cases} \tag{2.14}$$

[8] We use the tax rules for incomes earned in 2003.

[9] This is similar to the US since the "bend points" when calculating the primary insurance amounts (PIA) are adjusted by average earnings growth.

of which consumption is,

$$C_{i\tau} = \theta_{i\tau} X_{i\tau}. \tag{2.15}$$

There are also constraints on both borrowing and short-sales,

$$\begin{aligned} &0 \leq \theta_{i\tau} \leq 1, \\ &0 \leq \alpha_{i\tau} \leq 1. \end{aligned} \tag{2.16}$$

3. OPTIMISATION

To simplify the calculation[10], we introduce a decision rule that defines the asset allocation in the defined contribution account. This rule originates from Merton (1971) and states that; in complete markets—the allocation to risky assets—λ, is dependent on the relative size of investable assets to total wealth. In our model, total wealth is the sum of: present value of human capital, cash on hand and expected after-tax[11] DC wealth—DCW^{at}. The present value of human capital is the sum of: income plus defined benefits and defined contributions, net of taxes and adjusted for survival probabilities. Prior to retirement, the human capital is discounted with the complete market rate—s,

$$\begin{gathered} s = r^f - \tfrac{\sigma_\xi^2}{2} + \beta_k(\mu^e + \tfrac{\sigma_\eta^2}{2}), \\ \text{where } \beta_k = \tfrac{Cov(\eta,\xi_k)}{Var(\eta)}, \end{gathered} \tag{3.1}$$

and with the risk-free rate after retirement. Our decision rule is adjusted for the implicit equity exposure through the present value of human capital—$\beta_k \Delta$,

$$\lambda = \min\left\{ \frac{\mu^e \left[DCW^{at} + (1-\theta)X + PV(HC)\right]}{\gamma\sigma_\eta^2 \left[DCW^{at} + (1-\theta)X\right]} - \frac{\beta_k \Delta PV(HC)}{DCW^{at} + (1-\theta)X}; 1 \right\}, \tag{3.2}$$

where Δ is the change in present value of human capital from a group specific permanent income shock—ξ_k.

The individual's problem therefore has four state variables (τ, v, X and DCW) and two choice

[10] The portfolio choice in the DC-account and for private savings is highly interdependent, making a simultaneous choice very complicated numerically.

[11] The after-tax rate is set to the municipal-tax only, since this is typically the only tax that an agent pays when in retirement.

variables (θ and α) as well as four stochastic variables (ϵ, ω, ξ and η). The value function of their intertemporal consumption and investment problem can then be written as

$$\begin{gathered} V_\tau(\Gamma_\tau) = \max_{\theta_\tau,\,\alpha_\tau}\left\{\frac{C_\tau^{1-\gamma}}{1-\gamma} + \delta E_\tau\left[p_\tau V_{\tau+1}(\Gamma_{\tau+1}) + (1-p_\tau)\, b\frac{D_{\tau+1}^{1-\gamma}}{1-\gamma}\right]\right\} \\ \Gamma_\tau = \{X_\tau, v_\tau, DCW_\tau\}\,. \end{gathered} \tag{3.3}$$

The solution to this maximisation problem together with our decision-rule from (3.2) gives us the state dependent policy rules,

$$\begin{aligned} \theta_\tau &= \theta_{k\tau}(\Gamma_\tau), \\ \alpha_\tau &= \alpha_{k\tau}(\Gamma_\tau), \\ \lambda_\tau &= \lambda_{k\tau}(\Gamma_\tau). \end{aligned} \tag{3.4}$$

We solved the problem numerically by backward recursion from the final year—T, using by-now standard methods, *cf.* Judd (1998) and Cocco *et al.* (2005).

4. CALIBRATION OF PARAMETERS

4.1. Estimation of labour income process

Follwing Carroll and Samwick (1997), we modelled the log of real income as deterministic part with both permanent and temporary shocks. Their description of the income-process has been used in several life cycle models, *cf.* Campbell *et al* (2001), Cocco *et al.* (2005), Carlsson and Erlandzon (2005), Cocco (2005), Carlsson and Erlandzon (2006) and Zhou (2006). The deterministic part of Equation (2.2) was estimated (*cf.* Appendix A.1 for details) using a longitudinal panel of data—LINDA, (*cf.* Edin and Fredriksson (2000) for details), that covers the Swedish population in the age interval $[28, 64]$ for fourteen years during $1992 - 2005$, resulting in more than 1.4 million observations.

The data set augmented with wealth information, has recently received attention in *cf.* Calvet *et al* (2006), Campbell (2006) and Flood (2003). The data-set was divided into twelve non-intersecting groups, depending on sex, education and sector (private and public). Using the methodology of Carroll and Samwick (1997), we estimated the variances of the permanent σ_u^2 and transitory σ_ε^2 components of shocks to income as specified in Equation (2.2) (*cf.* Appendix A.2).

4.2. Individual parameters

We used a standard set of assumptions with respect to the individual parameters for the reference case. First, we set the coefficient of relative risk aversion—γ to 5 and the discount factor—δ to 0.98. The gender specific survival probabilities—p were taken from the Swedish life-insurers when underwriting new policies,*i.e.* they are forward looking. The bequest parameter—b was set to 1. Adult life is divided into two intervals: working life $[28, 64]$ and retirement $[65, 100]$. The importance of the risk aversion parameter—γ will be elaborated on when we report on the sensitivity analysis in Section 5.4.

4.3. Assets and correlations

In the optimisation, we set the risk-free after-tax rate—r^f to 1.5%, which is consistent with the present gross return of less than 2% for long-dated index-linked bonds. The mean after-tax equity premium—μ^e was set to 3%, which is lower than the historical average, but corresponds well with forward-looking estimates (*cf.* Claus and Thomas (2001), Fama and French (2002)). Because of uncertainty about the equity-premium, we analysed its sensitivity in Section 5.4. Volatility σ_η was set to 17% for the risky asset.

Next, we followed the procedure of Cocco *et al.* (2005) to estimate the correlation—$\varrho_{k\eta}$ between group specific permanent labour income shocks—$\xi_{k\tau}$ and lagged equity returns—$\eta_{\tau-1}$. Table A.3, shows the estimated correlation, using the returns on the Swedish equity-index—OMX and on the 12-month Swedish Treasury Bills as proxies for risky returns and the risk-free rate respectively.

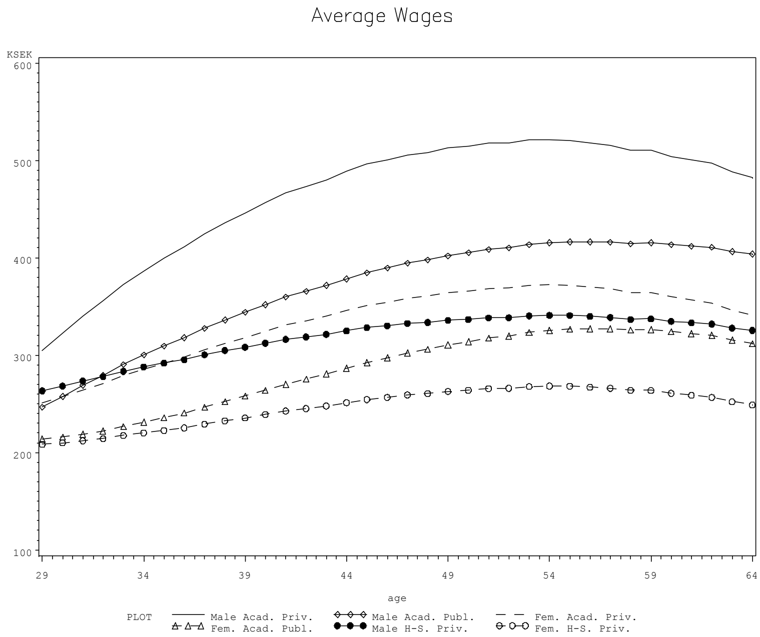
We also set the growth in average labour income—μ^l to 1.8%, which is the estimate used by the National Social Insurance Board. Finally, the initial wealth—F is set to 40, corresponding to the mean wealth for individuals at the age of 28.

5. RESULTS

5.1. Labour income process

For reference, we plotted the average of the simulated income profiles for some[12] of the groups, *cf.* Figure 5.1.

FIG. 5.1 Income profiles for different groups



Simulated real gross wages—L without productivity change.

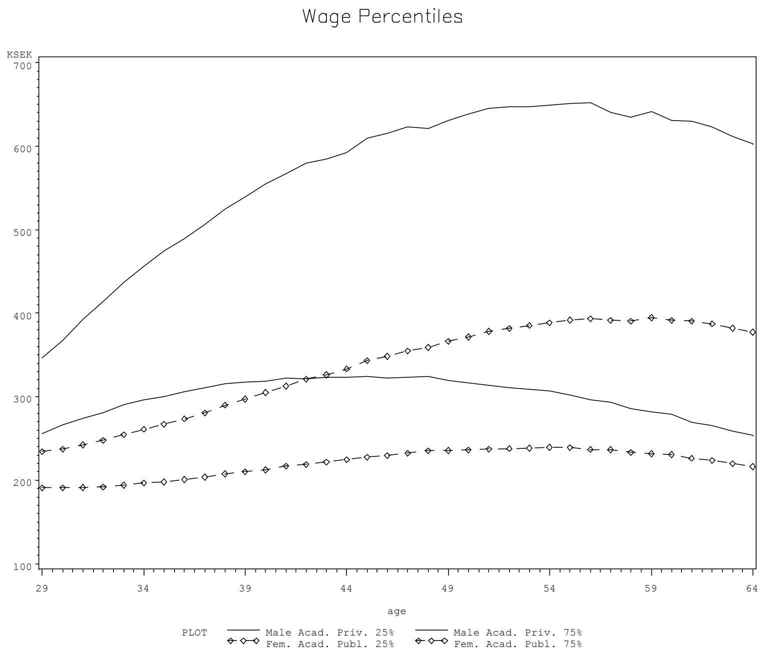
Three findings are notable: First, individuals with a university degree experienced a significantly faster income growth in mid-life than did the other groups, a result which matches stylised facts from the US, *cf.* Hubbard *et al* (1995), Gourinchas and Parker (2002) and Cocco *et al.* (2005). Secondly, at each level of education, men had higher income than did women, at all stages of the life-cycle. Thirdly, that remunerations in the private sector was typically higher than in the public sector.

Our results also show a strikingly lower permanent variance if the agent is employed by a public *vs.* a private entity, whereas the temporary variance was similar, except for those with university degrees. After controlling for private *vs.* public sector, most of the gender differences in variance, that we found in our previous study, Carlsson and Erlandzon (2005) disappeared.

[12] In order to increase readablity, we omitted the groups with similar profiles to the group with the lowest income.

We also note that the permanent shocks to income has the highest correlation with the equity market for the privately employed with an university degree, and that gender is less of an importance. Figure 5.2 shows the large effect that a higher variance in the permanent component for Men in Private sector will have on labour income variation during life, when compared to Females in Public sector, albeit both groups have a University degree.

FIG. 5.2 Income variation



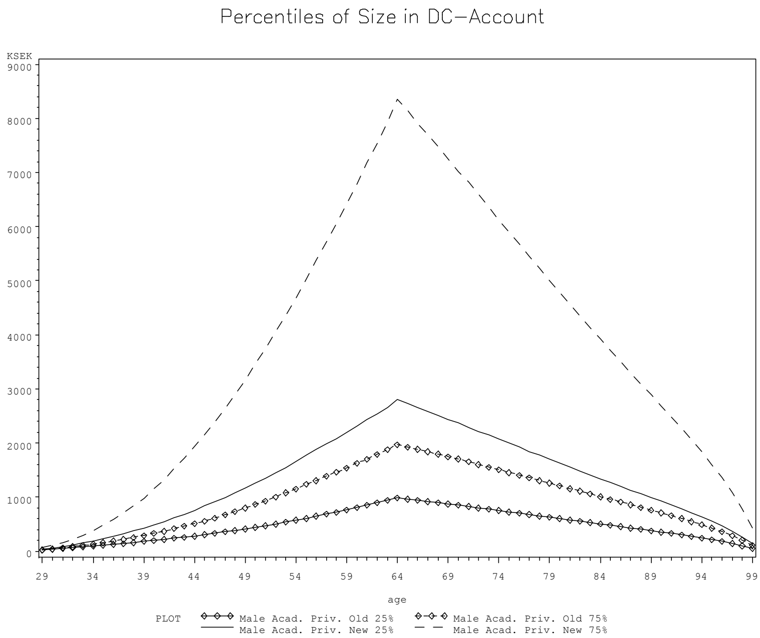
25^{th} and 75^{th} percentiles for simulated real gross wages—L without productivity for Men and Females with University degrees employed in the Private and Public sector.

5.2. Winners & Losers

We simulated the individual behaviour from age—28 until 100 with 10000 trajectories. Contingent on their random experience, individuals choose responses determined by the policy rules in Equation (3.4).

Since the change of pension system was negotiated by consenting adults—we would expect that on average the two systems would generate similar benefits. However, under the new system, individuals have a much larger responsibility for the appropriate management of the DC-account, since the outcome rests solely with the employeé. In Figure 5.3 we show (for the highest income group—Men with University degree in the private sector), the variation in size of the DC-accounts.

FIG. 5.3 DC-account variation



25^{th} and 75^{th} percentiles of the DC-account in the Old and the New pension system for Males with a University degree in the Private sector.

In order to discover to what extent this new pension system generated winners and losers, we evaluated the value function (Equation (3.3)), for the different groups in the first period; using both the old and the new pension system. For each group, we equalised the value of the value functions associated with the two pension systems; by adding an initial amount to the DC-account that was associated with the lowest value of the value function. The results for a subset of the groups are presented in Table 5.1.

Intuitively, we would expect the group with the highest expected final pre-retirement income relative to average income, to lose from the transition and *vice versa*. Another factor, is that high uncertainty in final pre-retirement income will decrease the expected utility of a defined benefit pension. Men with an university degree in the private sector has an early earnings career and a more pronounced decline in income prior to retirement. They are therefore the winners from a transition. The gain for this group is increased, as they also have a higher variance in income, which makes their

TABLE 5.1
Initial amount in Old or New DC-account necessary to equalise the value to the individual of the pension systems

Amount in KSEK		Pension System	
		Old	New
Private	Men		
	High school		19
	University	101	
	Women		
	High school		32
	University	12	
Public	Men		
	University		40
	Women		
	University		80

expected final income less certain.

The defined-contribution system recently negotiated in the private sector is a likely blueprint for a potential change of system for those in the public sector as well. Our analysis shows (*cf.* Table 5.1) that publicly employed would on average lose and that this loss is most pronounced for women. Women typically have their earnings-career later in life (*cf.* Figure 5.1), and therefore have less benefits from early contributions; and secondly that, the lower variance in earnings among publicly employed and will make the Defined Benefit pension closer to a risk-free asset.

5.3. Effects on portfolio choice

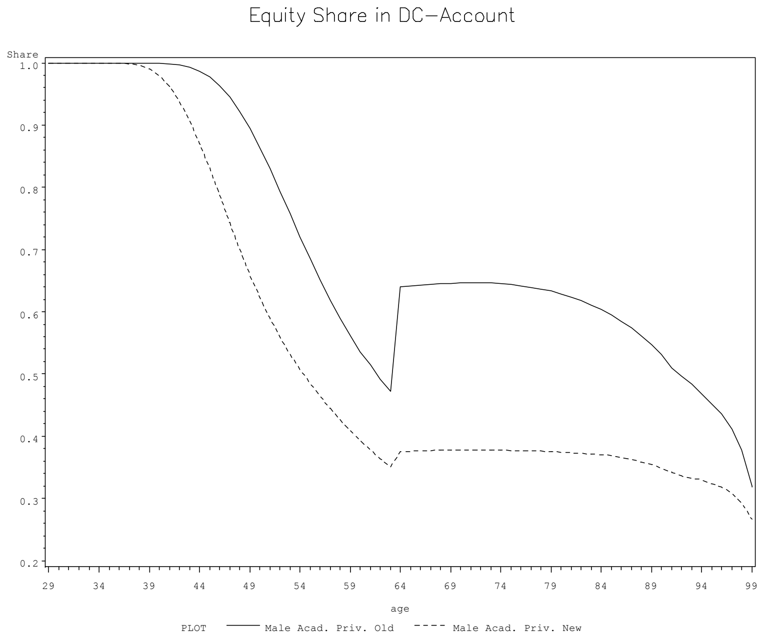
The positive labour income profile and short-sales constraints will typically make younger individuals "more" constrained, *i.e.* with an equity allocation quite different from the complete market solution.

Figure 5.4, shows the average equity proportion of the DC-account. For young individuals is cash on hand very small in comparison to the human capital and since their DC-account cannot be used for precaution or bequest, we get a maximum allocation to equities in the DC-account.

With increasing age, the combined effect of: the DC-account being a much larger proportion of total wealth in the new system and the old Defined Benefit pensions being less risky; will lead to a more conservative behaviour for an agent in the new system. After retirement, when the Defined Benefit benefits become risk-free and hence $\Delta = 0$ in Equation (3.2), we can identify a large increase in the equity exposure for an individual in the old system.

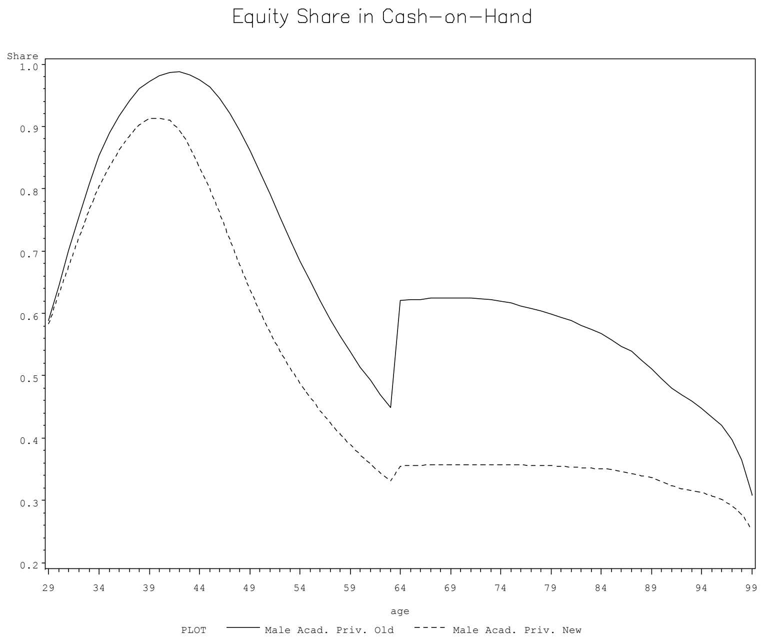
In Figure 5.5, we show the same profile, but now for risky weight in cash on hand. There is a large difference between the risky weight in cash on hand *vs.* DC-account in early life, for precautionary and bequest reasons. After retirement, with decreasing present value of human capital, there is a gradual decline in equity-exposure towards the complete-market solution.

FIG. 5.4 Simulated Average Equity Exposure in DC-Account



Simulated average equity share—λ in the DC-account for Males with University degree in the Private sector, for the old and new pension system.

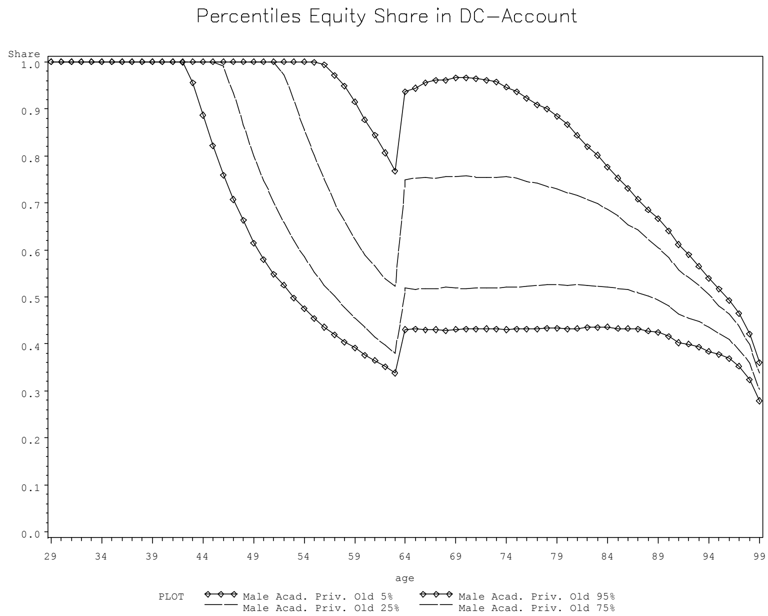
FIG. 5.5 Simulated Average Equity Exposure in Cash-on-Hand



Simulated average equity share—α in the cash on hand-X for Males with University degree in the Private sector, for the old and new pension system.

It is important to note that; the profiles reflect the simulated averages for one individual. Figure 5.6 shows some percentiles of equity exposure for an agent in the old pension system. The large variation is solely due to the accumulated effect of individual experiences. If we in addition, also could account for differences among individuals in: *e.g.* risk aversion, discounting or expected equity premia; then the variation would most likely be even larger.

FIG. 5.6 Variation in equity exposure in DC-Account

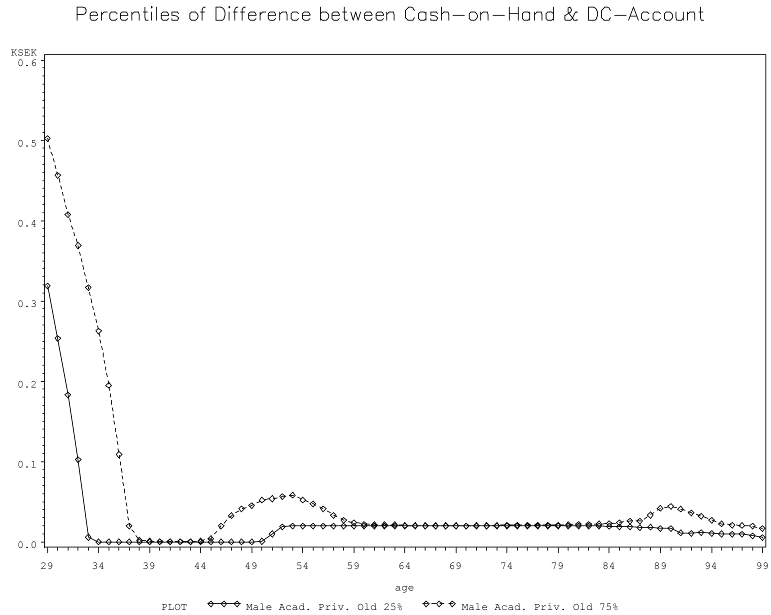


Simulated 5^{th}, 25^{th}, 75^{th} and 95^{th} percentiles percentiles for the equity share—λ in the $DC-$account for Males with University degree in the Private sector in the Old pension system.

In Equation (3.2), we created a decision-rule for the equity share in the DC-account. If this rule is too crude, we would expect individuals to compensate for any such errors in the allocation of their private savings. We therefore "tested" this rule by calculating the difference between the equity share in DC-account and in cash on hand—$(\lambda_{i\tau} - \alpha_{i\tau})$.

A priori, we would expect this difference to be small and show little variance for unconstrained individuals when the precautionary motive is weak, *e.g.*, after retirement. Early in life, however, when individuals are borrowing-constrained, we know that differences between trajectories can be large. Figure 5.7 plots this difference and the variation after retirement is not very large, which indicates that our rule seems to work.

FIG. 5.7 Variation in the difference in equity exposure between DC-Account and cash on hand



Simulated $25^{th}$and 75^{th} percentiles for the difference in equity share between $DC-$account—DCW and cash-on-hand—X for Males with University degree in the Private sector in the Old pension system—$(\lambda - \alpha)$

5.4. Sensitivity analysis

In order to analyse to what extent our results are parameter-dependent, we performed a sensitivity analysis using the group whose benefits are most affected by the change in pension systems—men with a university degree in the private sector. Table 5.2 shows the initial amounts that the DC-account must be increased with, in order to equalise the value of the two pension systems, with respect to changing risk-aversion and a higher equity-premium.

TABLE 5.2
Initial amount in the Old DC-account necessary to equalise the value of the pension systems to the individual, with respect to equity and risk premia for Men with University degree in the Private sector

	μ^e	γ	KSEK
Reference	3	5	101
Low risk aversion	3	2	179
High risk aversion	3	8	70
High equity premium	4	5	106

In all cases, it was beneficial for this group to move into the new system. The result show that changes in the equity premium—μ^e does not have a large impact, whereas the benefit to the less risk-averse was increased substantially.

6. CONCLUSIONS

This paper aims to contribute to the understanding of the welfare effects of moving from primarily a defined-benefit to a defined-contribution pension system, and the changes in optimal individual behaviour required by such a change. The setting is a borrowing-constrained individual's consumption- and portfolio-choice in the presence of uncertain labour-income, with group-dependent labour-income processes and realistically-calibrated tax- and pension-systems. We found that those employed in the private sector had higher income-variance than those in the public sector, while gender differences (after controlling for private *vs.* public employment) were small.

We have used the recently negotiated change from defined benefit to defined contribution pension systems as a benchmark for our analysis. The finding was that agents with low expected final income relative to average income and those with high income variance are set to gain from this transfer. Winners are men with an university degree within in the private sector, and losers would be women in the public sector with an university degree. The value of the different systems to the individual are dependent on the risk preferences, but will not change the preference of system.

Introducing a defined contribution system will necessitate that the individual has to manage the assets differently in private and pension savings; and that the differences in portfolio choice between agents due to individual situations are relatively large, even if we do not account for differences in terms of risk-aversion, *etc*, between individuals. One-size-fits-all kind of life-cycle funds, where the equity allocation depend on age alone, will therefore not solve the asset allocation problem for the individual.

APPENDIX A: DATA AND ESTIMATIONS

A.1. Estimation of the labour-income process

The data set was divided into twelve non-intersecting groups, depending on sex (Male, Female), education (Compulsory school-, High-school- or University-degree) and employer (Public, Private). The matrix of individual characteristics—$\mathbf{Z}$, includes variables for the number of children in different age-intervals as wells as a dummies for maritial status, age. Income was adjusted to real values by deflating with the official consumer price-index. Measured income is an aggregate including gross wages, also all social security benefits (primarily income-compensation for unemployment, disability and childcare) and pension benefits.

To avoid double-counting, we deleted all observations where income included voluntary pension benefits, *i.e.*, individuals above the age of 55 receiving pension pay-outs at their own request. Pension benefits paid prior to age 55 can be considered as insurance payouts and were therefore included. Progressive taxation will induce most agents to make these early withdrawals only if the individual has simultaneously reduced the ordinary wage income. Finally, we exclude an observation if income is less than 100.000 SEK. Individuals with income lower than this level are assumed to be voluntarily unemployed.

The following random-effects linear model was used to estimate the deterministic function for each group,

$$\begin{gathered} l_{it} = \beta_0 + \mathbf{Z}_{it}\boldsymbol{\beta} + \vartheta_i + e_{it}, \\ e_{it} = \rho e_{it-1} + \varkappa_{it}, \\ \vartheta_i \sim N(0, \sigma_\vartheta^2), \\ \varkappa_{it} \sim N(0, \sigma_\varkappa^2), \end{gathered} \tag{A.1}$$

where—$\mathbf{Z}_{it}$ are the nonstochastic regressors and $\boldsymbol{\beta}$ is the vector of regression coefficients. Estimation results are presented in Table A.1.

TABLE A.1
Labour Income Process: Coefficients from Regression

AR(1) Random effects Regression											
Log real income 2004 KSEK			#Children at age 1-2	3–5	6-17	Married=0 Single=1	AR ρ	Std. in fixed σ_ϑ	Std. in overall σ_e	R^2 within
Private	Men									
		Compulsory	-.00599	.00076	-.00284	-.02846	.5305	.2378	.2378	.185
		High school	-.00762	-.00242	.000027	-.02957	.5356	.2688	.1644	.235
		University	-.00786	.00077	.00035	-.03185	.5469	.3784	.2096	.317
	Women									
		Compulsory	-.08666	-.04532	-.02430	.03215	.5433	.2223	.1338	.271
		High school	-.12572	-.06728	-.03316	.03444	.5116	.2306	.1625	.300
		University	-.17466	-.10077	-.05480	.01108	.4645	.3194	.2085	.328
Public	Men									
		Compulsory	-.02575	-.00889	-.00838	-.04473	.4857	.2462	.1214	.213
		High school	-.01951	-.01389	-.00617	-.03341	.5076	.2642	.1296	.267
		University	-.00655	.00014	.00201	-.01905	.5394	.3190	.1449	.381
	Women									
		Compulsory	-.06417	-.03328	-.01257	.02817	.5480	.2018	.1152	.270
		High school	-.10207	-.04860	-.02063	.03488	.5274	.1699	.1224	.376
		University	-.13186	-.06781	-.02867	.03442	.5038	.2368	.1455	.443

We then calculate the deterministic component of labour income—$\exp\left\{f_k(\tau, \overline{\mathbf{Z}}_{k\tau})\right\}$, adjusted for age dummies with the averages of the characteristics. This was then used to estimate a third-degree polynomial with respect to age, *cf.* Equation (A.2) Table A.2, and Figure 5.1,

$$\exp\left\{f_k(\tau, \overline{\mathbf{Z}}_{k\tau})\right\} = \sum_{m=0}^{3} a_{km}(AGE_\tau - 18)^m. \tag{A.2}$$

A.2. Variance Decomposition

We followed Carroll and Samwick (1997) in decomposing permanent and temporary variances. By combining the error terms from Equation (2.2)—$v_{it} + \varepsilon_{it}$ with the estimated residual—e_{it} from Equation (A.1), we get:

$$\Delta e_{it}(d) = e_{it+d} - e_{it} = (v_{it+d} + \varepsilon_{it+d}) - (v_{it} + \varepsilon_{it}) = (u_{it+d} + ... + u_{it}) + (\varepsilon_{it+d} - \varepsilon_{it}) \tag{A.3}$$

and consequentially the variance is,

$$Var(\Delta e_{it}(d)) = d \cdot \sigma_u^2 + 2 \cdot \sigma_\varepsilon^2. \tag{A.4}$$

Following Carroll and Samwick (1997), we allowed for serial correlation in the transitory shock of the order $MA(2)$, and therefore excluded observations with a time distance less than 3. OLS on

TABLE A.2
Coefficients in the age polynomial of the forward-looking income profile

		Income profile, 2004 KSEK, (AGE-18)	Constant a_0	Age a_1	Age^2 a_2	Age^3 a_3
Private	Men					
		Compulsory	187.4410	3.7553	-0.0149	-0.0009
		High school	192.5214	5.4665	-0.0449	-0.0008
		University	50.6314	26.8242	-0.5615	0.0028
	Women					
		Compulsory	170.8340	0.0714	0.0986	-0.0020
		High school	200.2780	-2.5127	0.2277	-0.0037
		University	170.6430	5.3380	0.0553	-0.0027
Public	Men					
		Compulsory	150.9325	3.0467	0.0011	-0.0008
		High school	176.5778	2.5334	0.0325	-0.0013
		University	91.3953	14.9683	-0.2048	0.0000
	Women					
		Compulsory	154.1382	0.4386	0.0563	-0.0012
		High school	181.6734	-2.1792	0.1781	-0.0027
		University	217.9771	-5.5608	0.4423	-0.0065

Equation (A.4) was then used to estimate σ_u^2 and σ_ε^2.

A.3. Income correlation with the equities

We followed Cocco *et al.* (2005) in estimating the correlation between labour-income shocks and equity–returns. Using Equation (2.2), the first difference in $l_{ikt}^* = l_{ikt} - f_k(\tau, \mathbf{Z}_{ik\tau})$ can be written as

$$\triangle\, l_{ikt}^* = \xi_{kt} + \omega_{ikt} + \triangle\, \epsilon_{ikt}. \tag{A.5}$$

Taking the average over individuals in each group gives us the group-aggregate component,

$$\overline{\triangle\, l_{kt}^*} = \xi_{kt}. \tag{A.6}$$

Finally, we estimated the correlations—$\varrho_{\xi_k \eta}$, by applying OLS to,

$$\overline{\triangle\, l_{kt}^*} = \beta_k (r_{t-1}^e - r_{t-1}^f) + \phi_t. \tag{A.7}$$

Table A.3 presents the result from this regression using the real return of the Swedish equity index OMX as a proxy for equity-returns—r^e and the real return on 12-month Swedish Treasury Bill as the risk-free rate—r^f.

TABLE A.3
Variance decomposition and equity correlations

			Number of observations	Estimated variance of the permanent component $\sigma^2_{u_k}$	Estimated variance of the transitory component $\sigma^2_{\varepsilon_k}$	Std. of the permanent aggregate component σ_{ξ_k}	Correlations with Swedish equity returns $\varrho_{\xi_k \eta}$
Full sample			1 423 930			0.0211	0.40
Private	Men		585 446				
		Compulsory	140 413	0.0042	0.0152	0.0222	0.39
		High school	310 835	0.0054	0.0183	0.0229	0.40
		University	134 198	0.0098	0.0284	0.0270	0.61
	Women		290 776				
		Compulsory	67 364	0.0048	0.0104	0.0187	0.45
		High school	152 254	0.0054	0.0182	0.0173	0.45
		University	71 158	0.0079	0.0306	0.0258	0.51
Public	Men		152 243				
		Compulsory	17 039	0.0021	0.0083	0.0249	0.25
		High school	47 543	0.0029	0.0096	0.0219	0.23
		University	87 661	0.0044	0.0115	0.0216	0.24
	Women		395 465				
		Compulsory	41 921	0.0034	0.0082	0.0236	0.30
		High school	176 607	0.0030	0.0103	0.0207	0.25
		University	176 937	0.0038	0.0138	0.0233	0.22

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The Cox, Ingersoll & Ross Model on Swedish Treasuries and Options

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This paper applies the CIR-model to the term-structure of Swedish treasuries. As with other studies, it was found that parameters, when estimated from cross-section, are quite unstable. This instability stands in stark contrast to the relatively stable implicit volatilities in traded options, and could possibly stem from a co-linearity problem. The problem is partially resolved by adding more information about the diffusion process from options data.

Key Words: Term-structure, CIR, options, co-linearity.

JEL classification: C21, E43, G12

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1. INTRODUCTION

From an economist's perspective, the determinants of the term-structure of interest are of prime importance. In one sense, it contains an embodiment of almost all underlying variables' aggregate effects on market anticipations of future events. Hypotheses in the literature focus on differences between implied forward rates and expected rates; with explanatory power added, by allowing for liquidity, risk-premia or market-segmentation, *cf.* Hicks (1947) and Culbertson (1957).

The introduction of general equilibrium models, which derive asset prices from an optimal consumption plan, *cf.* Merton (1971), Lucas (1978), Cox *et al* (1985a), Breeden (1986) and Longstaff and Schwartz (1992); and the related arbitrage-free models, *cf.* Merton (1973), Vasicek (1977), Brennan and Schwartz (1979), Ho and Lee (1986) and Heath *et al* (1992); has incorporated many of the "stylised facts" about the relative pricing of securities, in a systematic way.

From a practitioner's perspective, changes in the term-structure raise numerous problems both in the valuation of fixed-income assets and in the measurement of various risk-exposures. A model that can identify erroneous prices of fixed-income assets and, even more important, evaluate risk-exposures, can therefore have a great practical value. Lack of stability in parameter estimates, either over time (*i.e.* large standard errors from cross-section estimates), or because a different subset of secuirities was used in estimation, is a type of uncertainty that the practitioner is reluctant to accept[1].

In this paper, the original one factor Cox *et al* (1985b) (henceforth CIR) model, was used as a representation of the Swedish term structure for Treasury Bills and Bond prices. Estimation of this model can be found in *eg.* Brown and Dybwig (1986), Barone *et al* (1991), and Chan *et al* (1992). In our opinion, the parameter estimates in these studies display too-large a variation, when contrasted with the relative stability in both the shape of the term-structure and the implied volatility of option-prices.

The contribution of this paper is to partly resolve this problem; by using additional information about the model from option prices. When addressing the empirical use of their model, (CIR p. 396),

[1] The analogy is with the actual usage of partial-equilibrium models; such as Black & Scholes-type formulas for valuation of options. It is well known that many of the exogenous variables and parameters in these models vary over time in a way that contradicts model-assumptions. However, for shorter time-periods, and for close strikes, the variation may not be so large as to decrease the popularity of these models among professionals.

notes that "*applications to other securities may permit richer and more powerful empirical tests than could be done with the bond market alone*".

Section 2 gives a describes the CIR model, while Section 3 discusses some earlier results, Section 4 then discusses the estimation technique, while Section 5 describes the data, and Section 6 presents the results and then finally Section 7 summarises and draws conclusions.

2. THE CIR-MODEL

The CIR-model[2] specifies the instantaneous default-free interest-rate dynamics as:

$$d\rho = \kappa(\theta - \rho)dt + \sigma\sqrt{\rho}dW. \tag{2.1}$$

The interpretation of the parameters of this model is as follows: with κ, $\theta > 0$, this is an autoregressive process with a long-term mean—θ, and where—κ determines the speed of mean reversion. The instantaneous drift—κ $(\theta - \rho)$ and variance $\sigma^2\rho$ in interest rate changes are derived as a special case from the Cox *et al* (1985a) model where ρ is the equilibrium interest rate equal to expected return from optimally invested wealth and W is proportional to random changes in production possibilities.

Subject to the usual assumptions of continuous trading and absence of transaction costs, it is possible to derive the arbitrage-free price of a default-free zero-coupon bond—B, *i.e.*, with boundary condition—$B(\rho, T, T) = 1$,

$$\begin{gathered} B(\rho,\tau) = F(\tau)e^{-\rho G(\tau)}, \\ F(\tau) = \left\{\tfrac{\alpha e^{\beta\tau}}{\beta(e^{\alpha\tau}-1)+\alpha}\right\}^{\gamma}, \\ G(\tau) = \tfrac{e^{\alpha\tau}-1}{\beta(e^{\alpha\tau}-1)+\alpha}, \\ \alpha = \sqrt{(\kappa+\lambda)^2 + 2\sigma^2}, \\ \beta = \tfrac{\kappa+\lambda+\alpha}{2}, \\ \gamma = \tfrac{2\kappa\theta}{\sigma^2}. \end{gathered} \tag{2.2}$$

[2] The description of the model here is for completeness only, and draws heavily on CIR.

From the diffusion process, we can expect that prices will be decreasing convex in both instantaneous—ρ and long-term mean interest-rate—θ and decreasing in maturity. The speed of adjustment will give an increasing concave (decreasing convex) if the interest rate is greater (less) than its long-term mean.

Increasing the risk premium *i.e.* lowering λ, will reduce prices. In a general-equilibrium context, this effect is due to the covariance between optimally invested wealth and changes in the interest rate. Decreasing λ will therefore coincide with high wealth and make the marginal utility of default free bonds lower.

The effect of σ on prices is somewhat counterintuitive; in Cox *et al* (1985a), σ reflects more uncertainty about future production / consumption and therefore increases the value to risk-averse investors of a default free bond.

Yield-to-maturity can now be calculated as:

$$R(\rho, \tau) = \frac{-\log(B)}{\tau} = \frac{\rho G(\tau) - \log[F(\tau)]}{\tau}. \tag{2.3}$$

The limit of R(ρ, τ) dependent on τ converge to,

$$R(\rho, \tau) = \begin{cases} \rho, & \tau \longrightarrow 0, \\ \gamma(\alpha - \beta), & \tau \longrightarrow \infty. \end{cases} \tag{2.4}$$

CIR derive the price of a call-option—C, on a zero-coupon bond of maturity s, with exercise-price K and expiration-date T, in period t, as

$$
\begin{aligned}
C(\rho, t, T, s, K) &= B(\rho, t, s)\chi^2(x_1; df; nc_1) - KB(\rho, t, T)\chi^2(x_2; df; nc_2),\\
x_1 &= 2r[\phi + \zeta + G(T, s)],\\
x_2 &= 2r[\phi + \zeta],\\
nc_1 &= \tfrac{2\phi^2\rho e^{\alpha(T-t)}}{\phi+\zeta+G(T,s)},\\
nc_2 &= \tfrac{2\phi^2\rho e^{\alpha(T-t)}}{\phi+\zeta},\\
df &= \tfrac{4\kappa\theta}{\sigma^2},\\
\phi &= \tfrac{2\alpha}{\sigma^2(e^{\alpha(T-t)}-1)},\\
\zeta &= \tfrac{\kappa+\lambda+\alpha}{\sigma^2},\\
r &= \tfrac{\log\left[\frac{F(T,s)}{K}\right]}{G(T,s)}.
\end{aligned}
\tag{2.5}
$$

χ^2 is non-central chi-square distribution, where the two last arguments are the degrees of freedom and the coefficient of non-centrality. The call option Equation (2.5) is another contingent claim on the process in Equation (2.1), but subject to the terminal condition:

$$C(\rho, T, T, s, K) = \max[B(\rho, T, s) - K; 0]. \tag{2.6}$$

3. PREVIOUS STUDIES

Brown and Dybwig (1986) and Barone *et al* (1991) estimated the parameters of Equation (2.2) from a cross-section of bills and bonds through time. Brown and Dybwig (1986) finds skewness in residuals for bonds trading at off par value, arguing also for a, possible "*neglected tax effect*" (p.629). Barone *et al* (1991) argues for using net-of-tax-coupons because of the introduction of withholding taxes and because households —"*who base their calculations on net yields*" (p. 90), hold the majority of bonds. It is however, not clear which tax-rate that should be used. The effective tax-rate is probably varying in time and most likely different from the official tax-rate that they used[3]. In this paper, no adjustment was made for taxes, but instead it was tested whether off-par bonds displayed a different pricing pattern[4].

[3] This is probably more of an issue in Italy (their study), where restitution of withholding taxes at the time was problematic for foreigners who otherwise would had the possibility to arbitrage any tax-differences.

[4] Schaefer (1981) used a linear programming model and notes that, U.K. gilts seemed to be effectively priced for

Both Brown and Dybwig (1986) and Barone *et al* (1991), observed heteroscedasticity in errors, potentially because of quotation limits. Errors would then be a function of maturity and correlated between coupon bonds, we therefore hoped to dispense with this problem by weighting the residuals with standard duration.

It is well known that at most a three factor (parameters) model—covering changes in the short and long-rate and in curvature—can describe the variation in yields across the term-structure very well. The lack of variation in the term-structure and the large number of parameters (four) to be estimated in Equation (2.2) hints at a problem of multi-collinearity. This is reported in the Brown and Dybwig (1986), with "high" standard errors[5] and "*month by month changes,...,*[which] *are quite dramatic*" (p. 621), with a wide range of estimates of σ, (even negative) as a result. However, they find this result consistent with the sampling error in estimates, as they cannot reject the hypothesis that the annual average of σ is an unbiased predictor of the standard deviation in short rates.

This problem was also encountered by Barone *et al* (1991), although not to the same extent. One possible reason for higher precision could be the large number of bonds they included[6]. However, including many securities may create another problem, varying degree of liquidity and non-contemperaneous prices can exacerbate the problem with high volatility in parameters. They conclude by, "*the instantaneous rate moves closely with three month bills*" and that the estimated implied volatilities "*are reasonably close to sample standard deviation, of the instantaneous rate*" (p. 94). But the other parameters, like the long-term rate, depicts an unlikely pattern. In the final year of their estimation; the long-term rate—$R(\infty)$ has a range of 6.5% and a standard deviation of 3.7%, whereas the instantaneous rate had a range of 2% and a standard deviation of 1.5%; intuitively we would expect the long term yield to be the least volatile (*cf.* Figure 7.5 for one of their graphs).

A more fundamental critique of the CIR model is that it is simply wrong. Rebonato (1996) (p. 200) states that the "*time series of the cross-sectional estimates...tend to be very far from smooth*" and that "*such erratic behaviour of the fitting parameters is a tell-tale indication of the fact that the*

an investor with only a small or zero tax. The investor base for Swedish fixed income securities was typically not the private domestic investor and a large proportion of assets are owned and traded by international institutions exempt from any taxes.

[5] Parameter estimates are not explicitly presented, but a graph of their volatilities is presented for reference in the Figure 7.5.

[6] The number of securities used for estimation, range from 7 in first year to 57 in the final year.

optimisation procedure is trying to 'twist' a wrong model into an observed data set".

The hypothesis here is instead that the high variability in parameter-estimates can be largely attributed to co-linearity. The securities used in estimation have a too little variation in maturity, yield, and cash-flows, for estimating all four parameters. To counter this, more information is needed, ideally from another claim, whose prices are generated by the same diffusion-process, but with other terminal conditions, and therefore potentially more sensitive to specific parameters. For this, we will turn to Equation (2.5) and the option values to estimate σ. The *raison d'être* behind this procedure is that options are typically very dependent on volatility, *i.e.,* even small changes in this parameter will have relatively large impact on option prices.

4. ESTIMATION

The model is derived from a rational-expectations general equilibrium, and is therefore a representation of real interest rates. However, it was assumed here that it is also an appropriate representation of nominal rates at a fixed date.

Since it is only possible to identify four parameters when estimating Equation (2.2) the risk-premium parameter—λ, was set to zero, as has been done in the other studies. Then, to account for the existence of coupons, the equations for zero-coupon bonds and options were rewritten as

$$B_{ti}B(\kappa_t,\theta_t,\rho_t,\sigma,t,s_{i0}) = \Sigma_j c_{ij}B(\kappa_t,\theta_t,\rho_t,\sigma_t,t,s_{ij}) + \epsilon_{ti}/D, \tag{4.1}$$

$$C_{tih} = \Sigma_j c_{ij}C(\kappa_t,\theta_t,\rho_t,\sigma_t,t,T_i,s_{ij},K_{tijh}) + \epsilon_{tihc},$$

$$P_{tih} = C_{tih} - \Sigma_j c_{ij}B(\kappa_t,\theta_t,\rho_t,\sigma_t,t,s_{ij}) + \Sigma_j c_{ij}B(\kappa_t,\theta_t,r_{tih},\sigma_t,T_i,s_{ij})B(\kappa_t,\theta_t,r_{tih},\sigma_t,t,T_i) + \epsilon_{tihp}, \tag{4.2}$$

where B_{ti}, is the observed price of security i on day t; c_{ij} is the coupon[7] of i:th security on the j:th coupon date, s_{ij} is the maturity date for coupon payment, D_{ti} is the standard duration measure and ϵ_{ti} the estimated residual.

[7] In this context coupon plus principal if any or zero.

Since the settlement dates on on bills and bonds are different, we discount the observed prices to business day settlement using the estimated CIR-price—$B(\kappa_t, \theta_t, \rho_t, \sigma_t, t, s_{i0})$, the estimated price of the non-existent security—0, where 0 is a label for either the bill, bond or forward settlement date.

C_{tih} and P_{tih} are the observed call and put prices on day t for strike price h and where r_{tih} and K_{tijh} are the solutions to:

$$\begin{gathered}\Sigma_j c_{ij} B(\kappa_t, \theta_t, r_{tih}, \sigma_t, T_i, s_{ij}) = \{\Sigma_j c_{ij} (1+\delta_{tih})^j\}/(1+\delta_{tih})^J, \\ B(\kappa_t, \theta_t, r_{tih}, \sigma_t, T_i, s_{ij}) = K_{tijh}.\end{gathered} \tag{4.3}$$

In our estimation, we also have to solve for the "instantaneous strike-rate"—r^z_{tih}. For each iteration—z and for each iterated parameter set $(\kappa_t,\ \theta_t,\ \rho_t,\ \sigma_t)^z$, we solve Equation (4.3), where $r^z{}_{tih}$ corresponds to the h:th strike yield-to-maturity—δ_{tih}. We then calculate the strike price for each coupon—K_{tijh}, which is then used in the estimation of call prices. Put prices are then estimated by using the call prices and Put-Call parity. It was assumed that there was no dependency between residuals either through time, across equations or between strikes[8].

Estimation was performed in two steps: first, the estimation of Equation (4.1), from which the parameters, $(\kappa_t, \theta_t, \rho_t, \sigma_t)$ were derived without any restrictions; then the set of option prices and Equation (4.2), were used to estimate σ_t, contingent on the estimates of $(\kappa_t, \theta_t, \rho_t)$. The parameters $(\kappa_t, \theta_t, \rho_t)$ were then re-estimated from Equation (4.1) conditional on σ_t. These steps were repeated, until successive changes in parameter-values, are small enough.

5. DATA AND SECURITY SPECIFICATION

The data on bond yields was collected from a Swedish investment bank, which used the data to value its portfolio of fixed income assets on each day, during 1991 and OM—the Options Exchange. Prices were then calculated using the market convention formulas.

Riksgäldskontoret (the Swedish National Debt Office), the issuer of Swedish treasuries has an agreement with the Primary Dealers (PD), which requires them to bid in auctions and to quote firm

[8] This is of course most improbable, since at least for each strike rate it seems reasonable with a negative correlation between calls and puts, due to bid / ask spreads.

bid / offer's on a subset (Benchmarks) of the outstanding notes / bonds (bonds). The main implication of this agreement has been to concentrate turnover to these benchmarks.

As long as the remaining maturity is greater than two years, Riksgäldskontoret issues benchmarks on a "tap-basis" through fortnightly auctions, and Riksbanken (the Swedish central bank) always quotes repo's equivalent to the repo-rate in these benchmarks to PD's; which minimizes the risk of a "short-squeeze" in these issues. Riksgäldskontoret also issues bills with the same frequency in two maturities, six months and one year. When a benchmark has less than one year to maturity, it can be changed at no cost into a bill with similar maturity, which minimizes the liquidity-risk of holding short-dated bonds.

In the discussion to the Brown and Dybwig (1986) paper, Ferson argues that Treasury bills of shorter maturity than two months are only used by investment banks for margin calls and are not actively traded. We therefore deleted such observations and instead included the over-night repo[9]. Each bill matures on the 3^{rd} Wednesday of each month. On each trading day, there was at least two forward contracts traded on each underlying security, with an expiration-cycle of three months. The underlying securities are notional three and six months bills at expiration and a five year bond. Under a forward contract, the seller can deliver similar (in duration) securities. Cash securities are traded OTC and options are traded and cleared through the options exchange—OM. Options have 0.2% in yield difference between strikes and a notional five-year forward as underlying.

The full data set consists of prices on one repo, ten bills, six bonds, and six forwards and (at least) sixteen options[10] on each business day, all in all 12.856 observations.

6. RESULTS

The Figure 6.1 shows the over-night-rate and its standard deviation in annualised daily-differences, for a moving three-month period. We would expect something similar from the estimation of $\sigma\sqrt{\rho}$. Cox *et al* (1985b) (p. 397) states that the comparative static's on option prices from σ are indeterminate. However, numerical experiments on the average of the estimates in this paper, have calls increasing

[9] Riksbanken has an agreement with the PDs similar to Riksgäldskontoret, requiring them to be active in the daily repo market.

[10] For liquidity reasons, it was decided to only include the closest at-the-money-forward option-values.

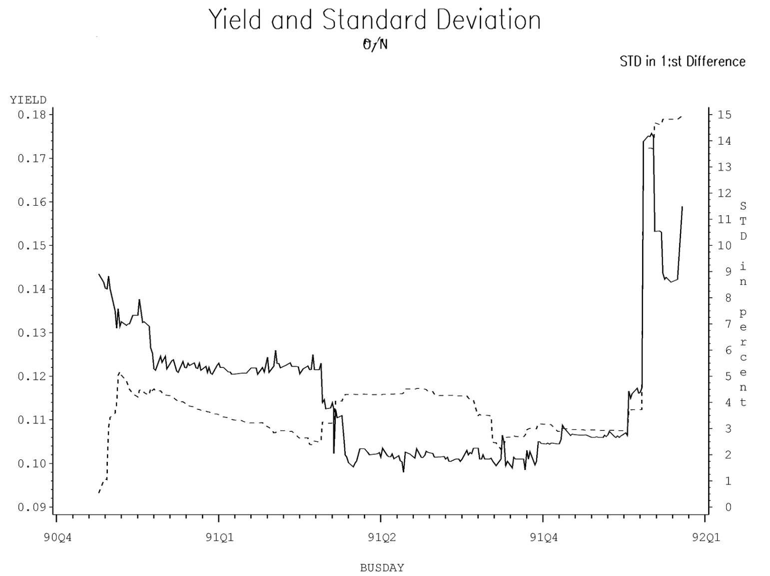

FIG. 6.1 Over-Night (O/N) repo rate (solid) and its standard deviation in 1:st difference (dashed).

in σ.

In Section 7, we show both sets of parameter estimates as graphs (*cf.* Figures 7.1 to Figure 7.4). The results from using bills and bonds only, are similar to those reported in other studies, most notably in the long-term mean—θ, being more variable than the instantaneous rate—ρ and with large variation. We also show the estimated short and long rates in Figures 7.6 and 7.6.

The instantaneous rate is determined by the short rate, whereas the long rate is set by a combination of the other parameters. For short-dated securities there was many observations and a large variation in yield, but few observations and small variation in yield for the longer-dated securities. We would therefore expect any potential problem of co-linearity to appear among the parameters, θ, κ and σ.

Comparing the two panels of parameter-estimates in Figures 7.1 to Figure 7.4; we note that the second estimation generates θ values that are more in line with *a priori* expectations. Unfortunately, the estimates of κ and σ still display variation (although to a lesser degree) that we find unlikely.

Since one of the conclusions of the other studies cited was a "neglected tax-effect", the data-set was divided into two subsets, one containing securities trading at a discount and the other those with premiums. However, an opposite effect was found from what would be expected if there was a

TABLE 6.1
Parameter estimates

	Parameter estimates from the estimation using							
	Repo, Bills and Bonds, Equation (2.2)				Repo, Bills, Bonds and Options Equations (2.2) and (2.5)			
	Mean	Std.	Min.	Max.	Mean	Std.	Min.	Max.
ρ	.1171	.0120	.0951	.1719	.1133	.0111	.0977	.1543
κ	1.136	.4950	.3960	2.578	.7064	.3641	.0180	2.035
θ	.1069	.0183	.0828	.1728	.1007	.0060	.0894	.1124
σ	.3990	.5240	0	2.646	.1167	.0580	.0286	.3320

tax-effect. the subset of securities trading at a premium was expensive relative to the model and *vice versa*, but the difference from zero being economically insignificant[11].

6.1. Caveats

The proper description of the term structure in terms of CIR-parameters could be obscured for a number of reasons of which; transactions costs, contract-construction, taxes, contemporaneously recorded prices and differences in liquidity between issues; are all potentially very important.

Firstly, we have used a model for call prices on a cash bond. In reality, options are traded with a forward as underlying. If the forward trades at fair value, then this is not a problem. However, if there is a cost (beside transactions cost, *i.e*, repo yield different from the corresponding treasury), associated with the execution of arbitraging between cash and forward, then there can be a persistent deviation from our fair value. Secondly, a forward also typically trades with a smaller effective bid / ask spread than the comparative cash-bond plus the spread in the repo; the forward can therefore deviate somewhat from its fair value before arbitrage is possible. Thirdly, a forward that trades at the limit of this bid / ask range will have a large impact for an in-the-money option with few days to expiration. These deviations, are potentially important for the parameter estimation, especially if the likelihood function is very flat. The existence of a functioning repo-market and the restriction imposed here to only include benchmark bonds, with the highest liquidity and close to continuous trading, should reduce these problems.

It was assumed that the forward price was derived from a single cash-bond price and a repo rate;

[11] The difference was about 2.5 basis points for those trading at a premium, whereas the securities trading below par value were priced at discount by about 0.3 basis point.

but underlying to the forward contract is a range of bonds, where the seller decides which bond to deliver. Since the "cheapest-to-deliver" can change from the time of execution to the time of expiration, the buyer is implicitly writing an option to the seller on a change in the slope of the yield curve. Contracts of this type will always trade at a higher yield[12] than a contract were only one bond is accepted for delivery. The size of this deviation, *i.e.* price of this option, depends on the probability of such changes in the slope of the term-structure and on the variety of bonds being eligible for delivery at expiration. This error can be of importance for this study, since the slope of the term-structure changed during our estimation period.

7. CONCLUDING REMARKS

This study attempted to explain why in empirical studies; the large number of parameters, the lack of variation in the term structure, and the parameter interdependence; can lead to unstable parameter-estimates, which seem most unlikely. Estimates from cross-sectional studies on Italian, US and Swedish treasuries, all found unstable parameter-estimates over time, as did a time-series study. Attributing this variability to normal sampling-error, because of high correlation between the instantaneous rate and observed short rates, stands in stark contrast to the relatively stable implicit volatilities in traded options. We decided to adhere to the CIR idea of "more powerful empirical test" and included option prices to our study, which greatly enhanced stability in parameters, albeit not to a "practical" standard.

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[12] This is similar to the error in American call valuation, when the stock pays a dividend before expiration, *i.e.* the optimal exercise policy can change up to the dividend date and will therefore add value.

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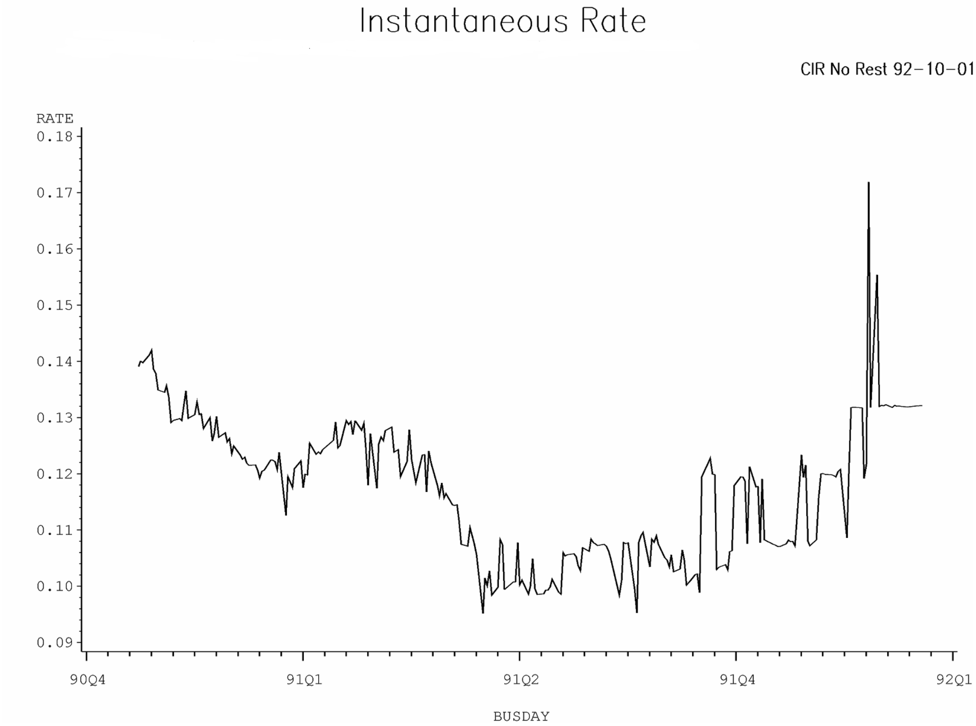
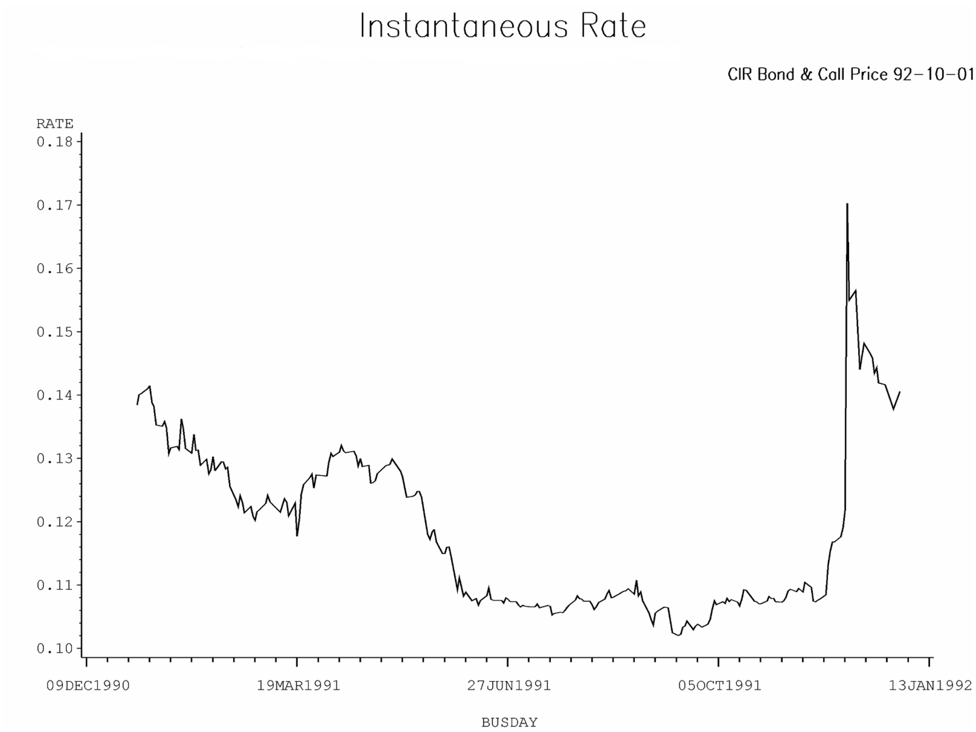
FIG. 7.1 Times-series of cross-section estimates of the instantaneous rate

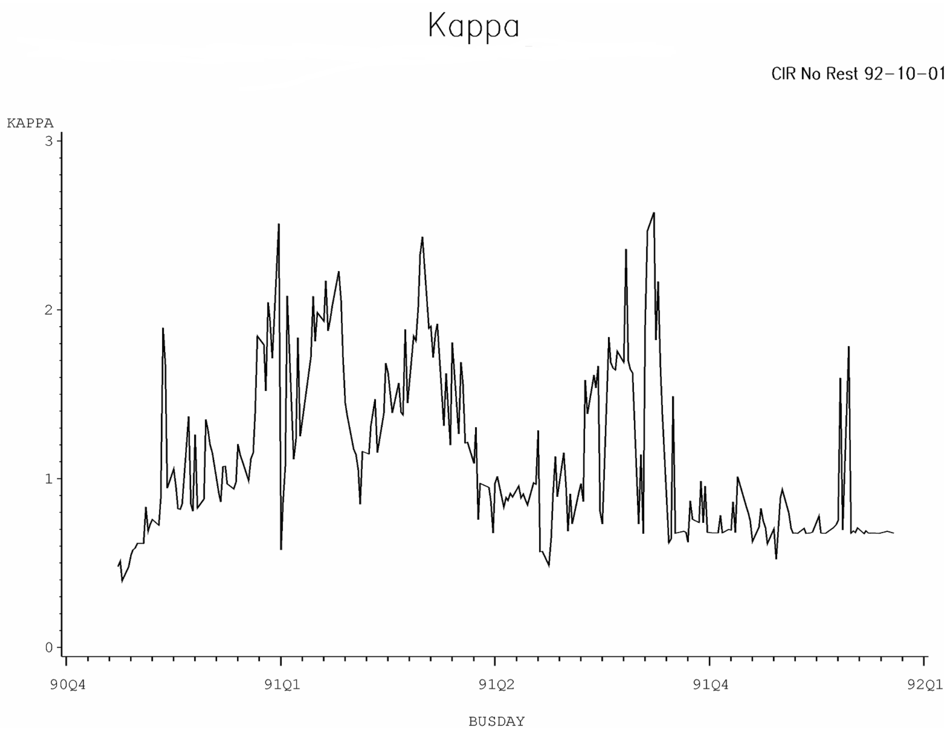


Instantaneous rate ρ with Bonds only.

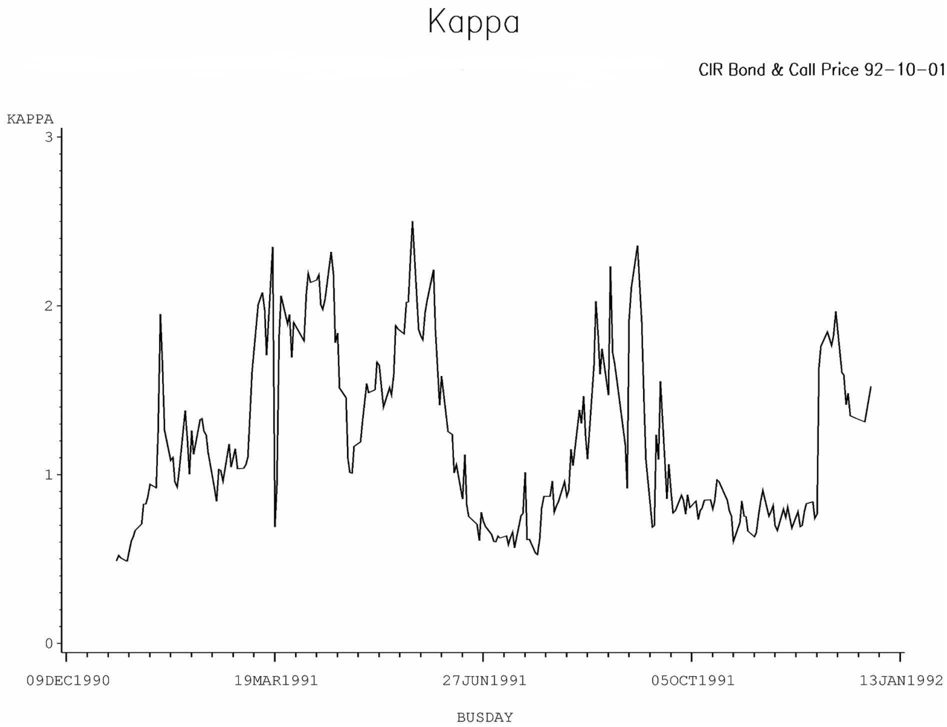


Instantaneous rate ρ with bonds and options.

FIG. 7.2 Times-series of cross-section estimates of the mean reversion rate

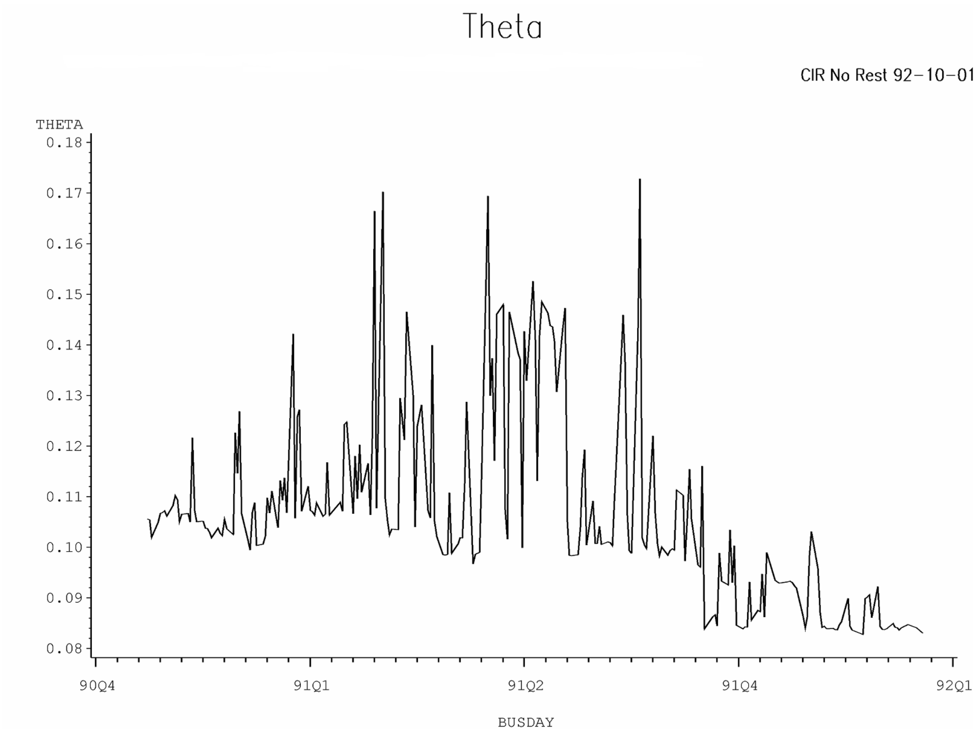


Mean reversion κ with bonds only.

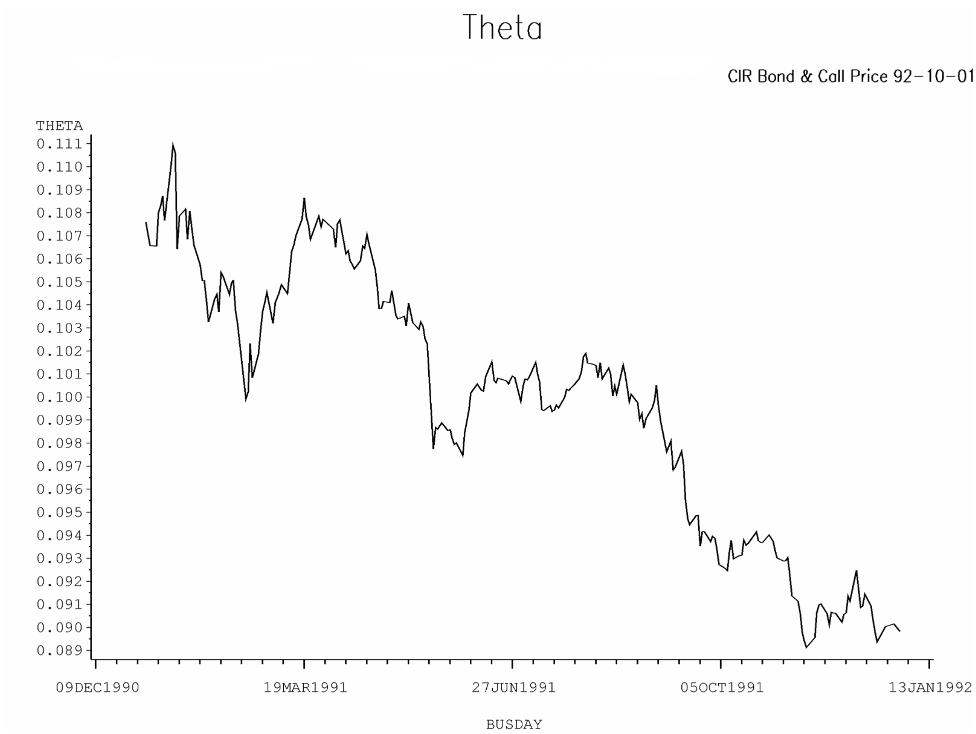


Mean reversion κ with bonds and options.

FIG. 7.3 Times-series of cross-section estimates of the long-term mean rate

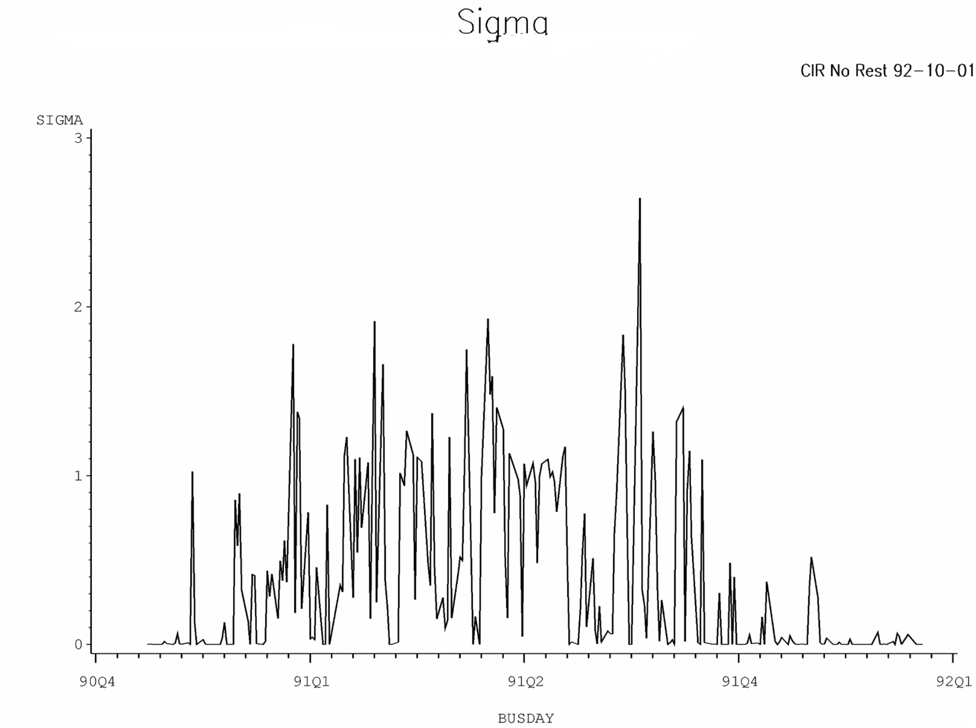


Long-term mean θ with bonds only.

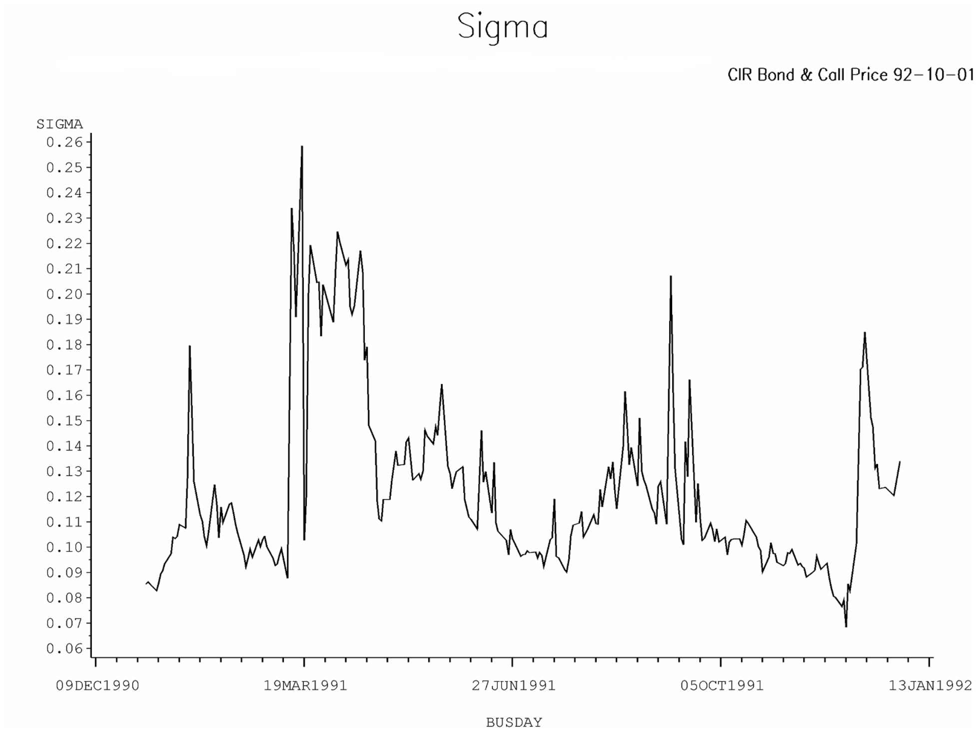


Long-term mean θ with bonds and options (note the difference in scales for θ).

FIG. 7.4 Times-series of cross-section estimates of the volatility parameter



Volatility σ with bonds only.



Volatility σ with bonds and options (note the difference in scales for σ).

FIG. 7.5 Parameter estimates from Brown and Barone

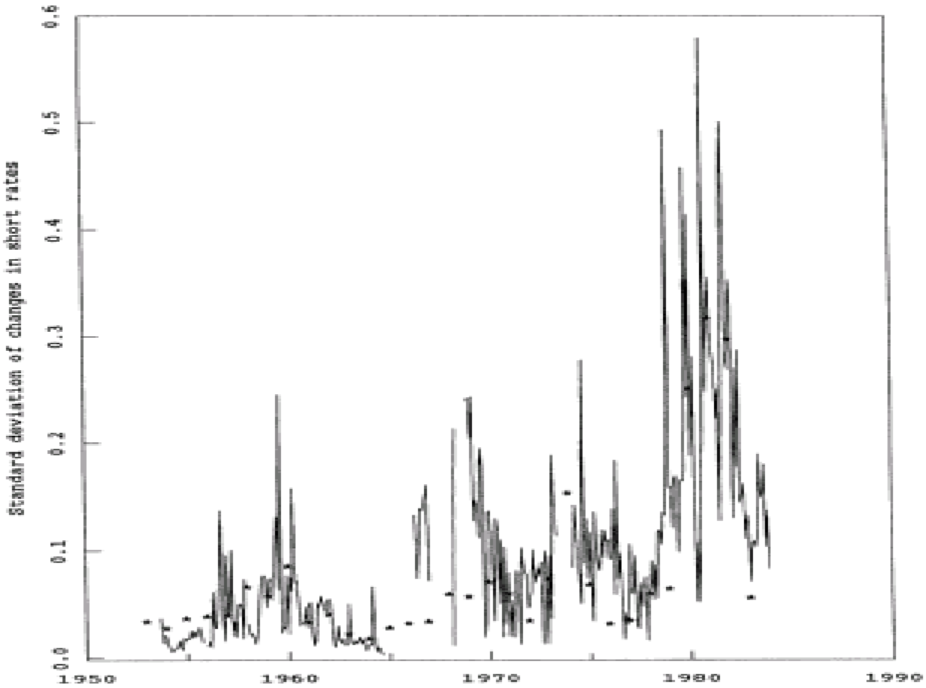
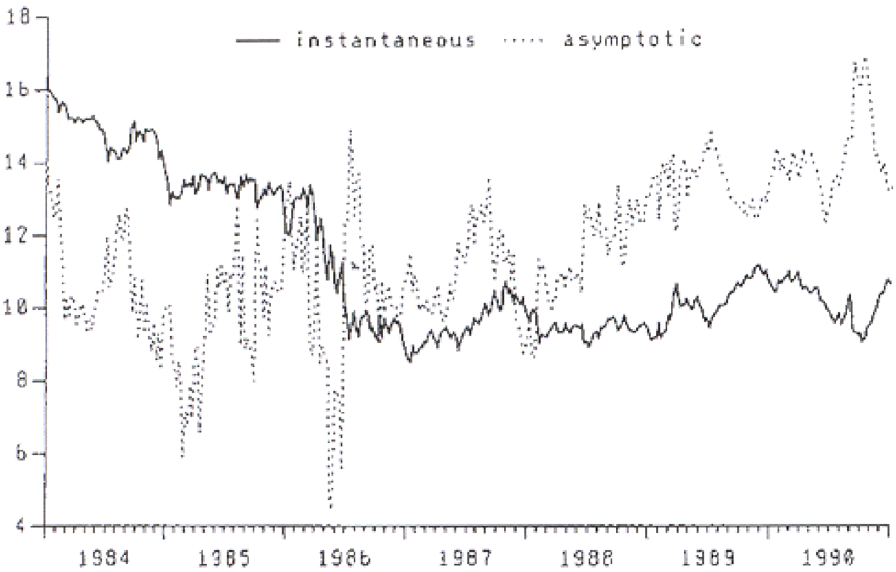


Figure 1. Implied and time series estimates of standard deviation.

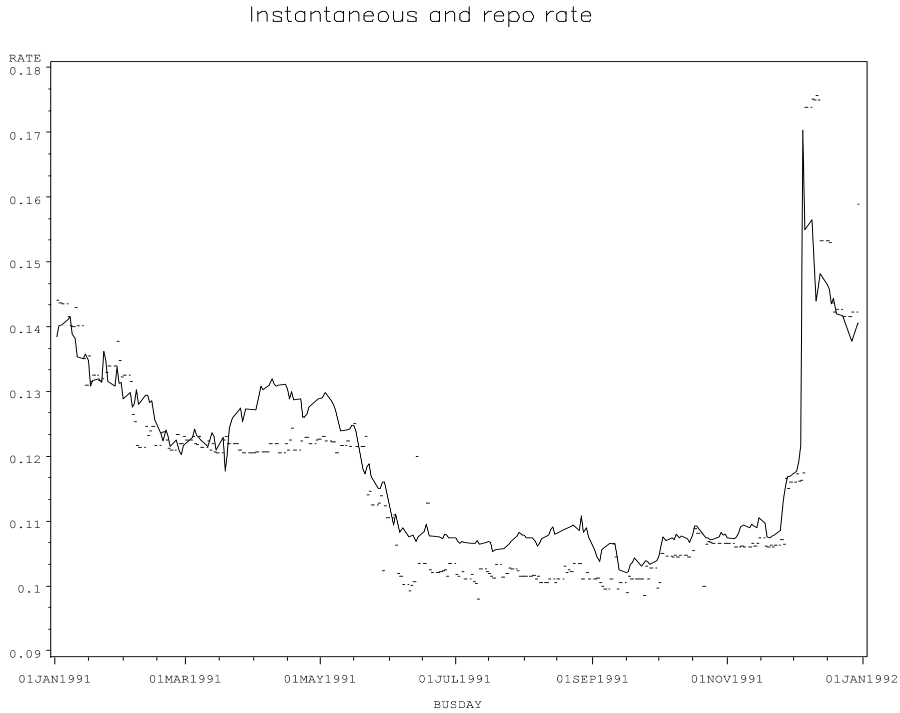
Brown and Dybwig (1986) estimate (p. 624) of the standard deviation of the instantaneous rate.

FIGURE 2 ▪ Instantaneous and Long-Term Asymptotic Interest Rates (Percentage Values)

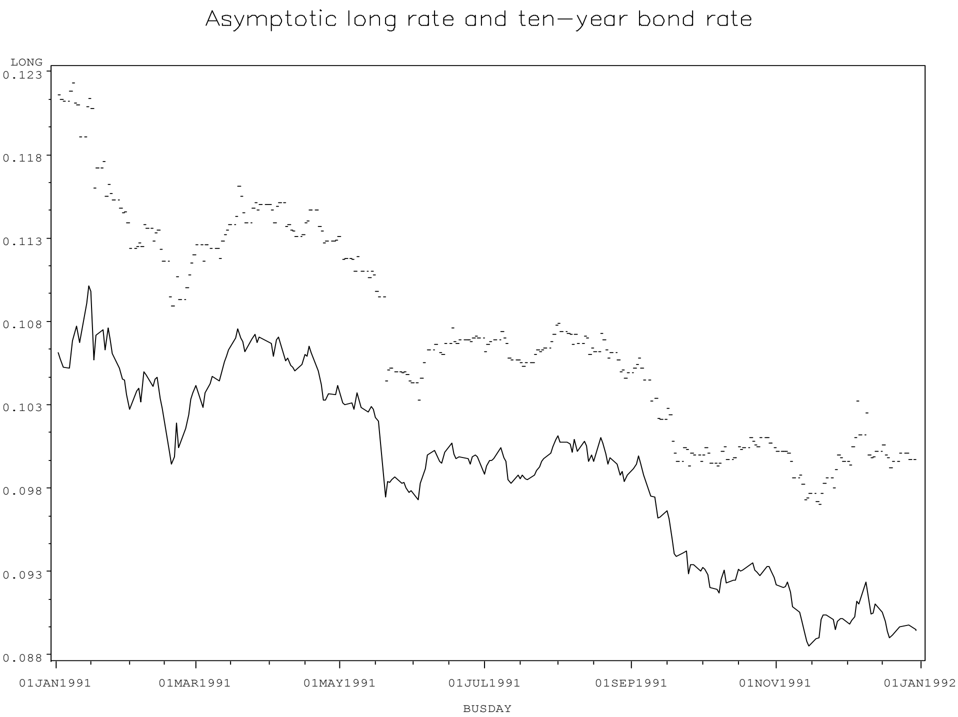


Barone *et al* (1991) (p. 93) instantaneous and long-term rate.

FIG. 7.6 Estimated and actual short and long rates



Estimated instantaneous rate ρ (solid) and actual over-night repo rate (dashed).



Estimated long-term rate $R(\infty) = \gamma(\alpha - \beta)$ (solid) and the actual ten-year bond (dashed).