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Non Utility Maximizing Behaviour: Probabilistic Choice in a Budget Set "Box". Properties of Expected Demand Functions

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Non utility maximizing behaviour:

Probabilistic choice in a budget set "box".

Properties of expected demand functions.

Abstract. *In this paper we use some(even a convex) probabilistic frequency functions in two choice variables defined over the budget set" box" and calculate the expected demand to study its properties* ***The expected demands have own price negativity , are normal goods and are homogeneous of degree zero*.*** *The detailed properties of deterministic demand functions can be replaced with similar properties for some expected demand functions the latter found* ***with fewer and behaviourally less restrictive assumptions****. To assume a deterministic utility function to be maximized is more restrictive in a behavioural sense than assuming random choice between some boundaries.*

Keywords: Non-maximising behaviour, Bounded rationality, Random choice, Expected demand.

JEL classification: C60, D01, D11

Introduction

In the theory of choice based on maximization of a strictly increasing and strictly quasiconcave utility function it is proved that demand functions are homogeneous of degree zero and that their substitution matrix is symmetric and negative semidefinite. In this paper we use some probabilistic frequency functions in two choice variables defined over the budget set" box" and calculate the expected demand to study its properties. We assume two sets of choice frequency functions

We first try **a uniform distribution defined on a budget set "box**" with an upper bound budget line and lower bounds on the consumption of both of the goods.

We then choose some simple increasing frequency functions guided by **the ordinary microeconomic textbook preferences** such as perfect complements, Cobb-Douglas, quasilinear and linear, all in quasiconcave utility form and concave preferences in **convex** utility form.
Some motivations for the choice of distribution can be found in section 2A.

* In this text the words referring to traditional theory like normal goods or own price negativity etc should be seen as average (expected) properties.

We find that
Homogeneity of degree zero holds for <u>all</u> expected demand functions.
The expected demand functions corresponding to the chosen frequency functions also have own price negativity and are normal goods.
Note that the convex frequency(utility) function(concave preferences)also have these properties and as such extends them outside the class of quasiconcave (utility) functions used in traditional theory.
For the Cobb-Douglas and linear frequency functions the expected substitution matrix is also symmetric and negative definite.
The calculated average expenditure expressions and are relatively simple (price independent)for the perfect complements, Cobb-Douglas and linear frequency functions and more complex(price dependent) for the others.
We compare the expected solution (all interior solutions $x>0$, $y>0$)with the utility maximising solution for one income price combination. For the same income price combination we also calculate some own price elasticities and find the uniform and perfect complements to be inelastic (>-1), Cobb-Douglas to be unitary elastic(=-1), quasilinear is elastic (<-1) the linear is more elastic and the convex even more elastic. The average expenditure range from 0,75m up to 0,93m. The latter number comes from the uniform distribution (the average expenditure is relative close to m (budget) due to the tight lower bounds). **Identify lower bounds and the choice and/or preferences inside the “box” matter less.**
There is no need to assume a utility maximising consumer to get these central results of economic theory if we accept that they are average (expected) results.
If one for various reasons don’t want to use preference (utility) maximization but still want to use the budget idea and find economic results similar to traditional theory one way we suggest is to use a probabilistic choice approach to find expected demand functions.
One reason that comes to mind is to separate the study of consumer choice into a behavioural part (consumer behaviour) and normative (prescriptive) part. The normative part may contain improvement of consumer choice by better information of prices and quality, optimal search theories of prices and quality, utility maximization with more complex preferences and constraints, static or dynamic, deterministic or stochastic etc. **The use of optimization methods in economics, operations analysis, applied mathematics etc reflects the important aspect of helping forming better decisions in the world but does not necessarily mean that they are good models for explaining (describing) actual economic behaviour. See below for some reflections of leading economists.**
The idea above is of course possible to use with other bounding upper constraints such as **time constraints**. To introduce quantity taxes and subsidies in the budget line to allow for **economic policy** is also possible. It might further be useful in other areas of economics where bounding constraints are essential and the maximization assumption is questionable.

Contents

We start in section 1 by introducing the properties of demand functions in utility maximizing theory and refer to some critical remarks and alternative ideas by some economists.

In section 2A we search for properties of expected demand through a look at traditional theory. We also discuss the choice of frequency functions.

In section 2B results for a uniform distribution defined over a lower and upper bounded set are given. In Section 2C we find properties of expected demand functions using a probabilistic choice idea for some specific frequency functions.

In the appendix we present some calculations (mostly integrations) in more detail.

An expression for average expenditure can be found in the appendix.

We also in the appendix give expected values, comparison with the utility maximising solution, own price elasticity values and average expenditure values for one income price combination.

1 Properties of demand functions in the traditional theory of choice

In the theory of consumer behaviour we find the following properties of demand functions

The demand functions

$x(p_x, p_y, m)$

$y(p_x, p_y, m)$

are

i)homogeneous of degree 0 in prices and money(budget) $(\mathrm{p}_{\mathrm{x}}, p_y, m) = (p, m)$

(Varian 1992 p 99) and ii)

the substitution matrix

$$\left(\frac{\partial \mathrm{x}_{\mathrm{j}}(p,m)}{\partial p_i} + \frac{\partial x_j(p,m)}{\partial m} x_i\right)$$

is a symmetric, negative semidefinite matrix.

Varian 1992 p 123 (note the change of notation here due to the quotation–in the rest of the text we use (x ,y) for the two goods)

The details of the two goods case are taken up in section 2.

These traditional results are proved using the assumption of preference (utility) maximization subject to a budget constraint.

Some critical remarks against the mainstream ideas, extensions and alternatives have been given.

We quote Varian 1992 p 123

“this is a rather nonintuitive result: a particular combination of price and income derivatives has to result in a negative semidefinite matrix. However it follows inexorably from the logic of maximizing behaviour.”

Stigler (1961) (Nobel price winner) on price dispersion as one measure of ignorance about the market and the introduction of a probabilistic search theory.

Herbert Simon (1965) (Nobel price winner) on “bounded rational behaviour”.

Barten (1969) on empirical tests of the traditional theory of demand.
According to Klevmarken (1979) the classical theory of demand is rejected.

Kahneman - Tversky (1974) (Kahneman Nobel price winner) on behavioural elements in economic choice.
We now have a new field of research in economics – behavioural economics and subfields such as behavioural finance.

Myerson (1999) p 1069 “This assumption of perfect rationality is certainly imperfect as a description of real human behaviour”
(Myerson Nobel price winner)

2 Properties of expected demand functions using a probabilistic choice idea.

2A Searching for properties of expected demand through a look at traditional theory and reflections on the choice of frequency functions.

The results of traditional utility maximizing approach in the case of two goods

The demand functions are homogeneous of degree zero in prices and budget

$$x(\lambda p_x, \lambda p_y, \lambda m) = x(p_x, p_y, m) \text{ for } \lambda > 0$$
$$y(\lambda p_x, \lambda p_y, \lambda m) = y(p_x, p_y, m) \text{ for } \lambda > 0$$

and the substitution matrix

$$\begin{bmatrix} \frac{\partial x}{\partial p_x} & \frac{\partial x}{\partial p_y} \\ \frac{\partial y}{\partial p_x} & \frac{\partial y}{\partial p_y} \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} \frac{\partial x}{\partial m} & \frac{\partial y}{\partial m} \end{bmatrix}$$

is symmetric and negative semidefinite.
In the traditional theory the interior solutions (x>0, y>0) satifies the budget balancedness property
$$p_x x(p_x, p_y, m) + p_y y(p_x, p_y, m) = m$$
This property differentiated gives **the conditions of Engel and Cournot aggregation.** Engel and Cournot aggregation plus symmetry of the substitution matrix can be used to prove homogeneity. In that sense budget balancedness plus negative semi definiteness and symmetry of the substitution matrix are the independent properties of demand functions.

These properties of demand functions are usually proved using an increasing quasi concave utility function. For more details see Shone 1975 pp 88-91 and Jehle – Reny 2001 pp 82-83.

Search for properties of expected demand functions

One way is to replace x and y with their expected values E(x) and E(y) the **reference properties** could be

homogeneity

$$E(x(\lambda p_x, \lambda p_y, \lambda m)) = E(x(p_x, p_y, m)) \text{ for } \lambda > 0$$
$$E(y(\lambda p_x, \lambda p_y, \lambda m)) = E(y(p_x, p_y, m)) \text{ for } \lambda > 0$$

and symmetry and negative semidefiniteness of the following matrix of expected values

$$\begin{bmatrix} \frac{\partial Ex}{\partial p_x} & \frac{\partial Ex}{\partial p_y} \\ \frac{\partial Ey}{\partial p_x} & \frac{\partial Ey}{\partial p_y} \end{bmatrix} + \begin{bmatrix} Ex \\ Ey \end{bmatrix} \begin{bmatrix} \frac{\partial Ex}{\partial m} & \frac{\partial Ey}{\partial m} \end{bmatrix}$$

In the probabilistic approach we will have the expected expenditure
$E(p_x x + p_y y) < m$
so **the budget balancedness property in expected values**
$E(p_x x + p_y y) = m$
is not valid (and therefore not the average Engel and Cournot restrictions).
On the other hand we might find the other properties above in some cases.

What type of choice frequency functions might be reasonable?

i) Specify outer limits as in section 2B and assume a uniform distribution. The latter assumption might reflect our **ignorance** of what determines the choice inside the “box”. It could also be that choice is equally likely to take place everywhere inside the “box”. The assumption also makes for simple calculations. If the “choice” set is very small, as when we have tight lower bounding constraints, a uniform distribution could be used as an **approximation (**see appendix for some numerical values**)**. We are concentrating on the upper and lower bounds of the budget set “box” . If we know that choice is taken place in the “box” of the budget set we know that expected choice changes when the walls (budget line) and lower bounds of the “box” changes. The focus is on the boundaries of the “box” and not on the preferences inside it (assuming they exist).

ii) Specify monotonous increasing frequency functions as in section 2C to relate to traditional utility maximizing theories**. If preferences with such regular properties exists this might be reflected in choice frequencies.** The consumer may strive for a maximization of his preferences seen in the probability of choice.

iii)Use specific frequency functions found elsewhere such as theories from behavioural sciences or behavioural economics. These functions can be integrated over the “box” to relate them to economic variables. . In waiting for a “final theory”, which of course is an illusion, probability formulated choice gives a link between traditional theory of negativity and homogeneity and theories of behaviour within the “box”.
If a theory is based on monetary variables like price dependent preferences in utility theory we can have price dependent frequency functions and integrate them to find expected demand. The result might be non-homogenous or even non-negative expected demand functions **but if we know more then we know more**. One case of non-homogeneity is the money illusion (Fehr and Tyran 2001 p1240).

iv) Estimate choice frequencies.
We do not assume that choice is decided by utility maximization which usually gives a choice at the border of the budget set. If choice reflects attempts to maximize utility then the estimated frequency function will be heavily weighted and non-symmetric near the chosen point at the border of the budget set.

2B Lower and upper bounds on consumption and a uniform choice distribution.

We try **a uniform distribution on a budget set “box**” with an upper bound budget line and lower bounds on the consumption of both of the goods. Lower bounds will reduce the set of possible choices(choice set). In addition to the upper bound budget line we also assume that the consumption of x and y is bounded below by
$x_0, x \geq x_0$ $y \geq y_0$ where x_0 and y_0 can be seen as a function of shiftvariables s
if one wishes (one can then use the chain rule to obtain sensitivity wrt s).
We name these lower levels “subsistence” levels, **S levels** for short .One early example on the use of lower levels is Stone-Geary preferences (Hey 2003 p 80).
We integrate (to find expected values) over the set.
$D = \left((x, y) in R^2 : p_x x + p_y y \leq m \text{ and } x \geq x_0, y \geq y_0\right)$
The results in relation to the lower bounds are

Positive in terms of x_0 $(\frac{\partial E(x)}{\partial x_0} > 0)$.
In words: A higher S – level of x will increase the consumption of x on average.
In other words: shift variables that increases the S-level will increase the consumption of x on average.
Negative in terms of y_0 $(\frac{\partial E(x)}{\partial y_0} < 0)$
In words: A higher S – level of y will decrease the consumption of x on average.
In other words: shift variables that increases the S-level of y will decrease the consumption of x on average. Drug addicts will decrease their consumption of other goods when their S-level increases.

Similar result are found for E(y)
The calculations are found in section A 5.
The results in relation to the budget line upper bounds are given in 2C.

2C Specific frequency functions

We next calculate the expected demand functions for some (probability) frequency functions and look at their properties. We have chosen both for reasons of simplicity and for connections to economic textbook examples the following frequency functions. All the chosen functions are positive monotonous in the choice variables, that is we expect choice to take place in the direction of the upper constraint (in this case the budget constraint). **If preferences of such regularity exists then this might be reflected in choice frequencies.**

i)The fixed proportions (perfect complements) frequency $f(x, y) = K \min(x, y)$
The function is symmetric and homogeneous in the variables.
ii)Cobb-Douglas frequency $f(x, y) = Kx^2 y$
The choice probability increases more with x than it increases with y.
The function is nonsymmetric , homogeneous and positive monotonous in the choice variables.
iii) The quasi linear frequency function f(x,y)=K($\sqrt{\mathrm{x}} + y$)
The function is nonsymmetric and nonhomogeneous in the variables.
iv)Linear density function f(x,y)=K(x+y)
The choice probability increases linearly with x and y.
The function is symmetric and homogeneous in the variables.
v) Convex frequency f(x,y)=K$(x^2 + y$) ("concave "preferences)
Note that this function is not quasiconcave.
The choice probability increases more with x than with y.
The function is nonsymmetric and nonhomogeneous.
The constant K (normalization constant) is explained below

Our idea is to focus on the budget set D of the consumer and to assume some (probability)frequency functions of choice f(x, y) defined in this set.
The budget set D is

D=$\left\{(x, y) \text{ in } \mathrm{R}^2 : p_x x + p_y x_2 \leq m \text{ and } \mathrm{x} \geq 0, y \geq 0\right\}$

For each one of the frequency functions given earlier we find the constant K by integrating the functions over D to find the volume bounded by the function. We then find K by putting this volume equal to one (more details in the appendix).
We then integrate once more to find the expected consumption of good x , E(x) and the same for good y, E(y).
After finding the expressions of E(x) and E(y) which are functions of the prices of x and y and the income (budget) m

$E(x) = f(p_x, p_y, m)$

$E(y) = g(p_x, p_y, m)$

we differentiate to find sensitivity results.

Results

The results for all chosen frequency functions plus the uniform distribution are

i) The goods x and y are on average normal goods.

$\frac{\partial E(x)}{\partial m}$>0 and $\frac{\partial E(y)}{\partial m}$ >0

ii) they are on average strictly negatively related to its own prices

$\frac{\partial E(x)}{\partial p_x}$ <0 and $\frac{\partial E(y)}{\partial p_y} < 0$

iii) the cross- derivatives

$\frac{\partial E(x)}{\partial p_y} \geq 0,\ \frac{\partial E(y)}{\partial p_x} \geq 0$
.
are on average non-negative (gross substitutes).
An exception to iii) is the perfect complements distribution and
the uniform distribution with positive lower bounds (x and y are complementary through the lower bounds). where

$\frac{\partial E(x)}{\partial p_y} < 0,\ \frac{\partial E(y)}{\partial p_x} < 0$.

Note that all solutions are **inner solutions** by construction (x>0, y>0) whereas the utility maximising solution in two cases generally are border solutions (either x=0,or y=0) and that **the results are valid for the non-quasiconcave frequency function**.

They are homogeneous of degree zero in prices and budget

$E(x(\lambda p_x, \lambda p_y, \lambda m)) = E(x(p_x, p_y, m))$ for $\lambda > 0$

$E(y(\lambda p_x, \lambda p_y, \lambda m)) = E(y(p_x, p_y, m))$ for $\lambda > 0$

In **elasticity form** the homogeneity condition give us

$El_{Exm} + El_{Exp_x} + El_{Exp_y} = 0$

where $El_{Exm} = \frac{\partial E(x)}{\partial m}\frac{m}{E(x)}$

$El_{Eym} + El_{Eyp_x} + El_{Eyp_y} = 0$

where $El_{Eym} = \frac{\partial E(y)}{\partial m}\frac{m}{E(y)}$

This elasticity condition is the same as in the utility maximizing case if we replace x with E(x.). For elasticity forms in traditional theory see (Shone 1975 p 91).
We also found a symmetric negative definite substitution matrix for the Cobb-Douglas frequency and the linear frequency.
Homogeneity is valid for all frequency functions that are independent of monetary variables when we start. This implies f(x,y)= K g(x,y) where g(x,y) is independent of monetary variables.
Other examples we have calculated give the same results as above.
We tried one simple example negative monotone in the choice variables that is we expected choice to take place in the direction of the lower constraint (in this case low consumption).Another simple example negative monotone in one choice variable and positive monotone in the other gave no different result.
Examples with more convexity and increasing give the same result.
So far we have not been able to prove that the results of negativity are valid for all frequency functions (like homogeneity is). It seems reasonable (my calculations hint to) that a large class of frequency functions positive monotonous in the choice variables will give negativity. A mathematician might help here in the future.
There is no reason to believe that the result is valid for more complex frequency functions.

Comments on the examples given in the appendix
We note that for simple expressions of the frequency functions such as the perfect complements, the Cobb-Douglas or the linear we get average expenditure independent of prices whereas the more complex expressions of convex and quasilinear do not have this property.
In the numerical calculations we find a regular pattern.
The own price elasticities goes from inelastic to more and more elastic (from uniform and perfect complements to convex)..The average expenditure range from 0,75m up to 0,93m. The latter number comes from the uniform distribution (the average expenditure is relative close to m due to the tight lower bounds). **Identify lower bounds and the preferences and choices inside the "box" matter less.**
The uniform frequency function expected demand also has a symmetric substitution matrix which is negative definite for the case where the lower bounds are zero.
The substitution matrix is not symmetric if the lower bounds are not zero but the diagonal is negative**. This example shows that some results of traditional theory with x replaced by E(x) is not valid for all frequency functions.**

Aggregation

If two households meet the same prices the expected aggregate demand for good x

$$EX\ \ (p_x, p_{y,} m_1, m_2) = Ex_1(p_x, p_{y,} m_1) + Ex_2(p_x, p_{y,} m_2)$$

is homogeneous of degree zero in prices and money and strictly negative in the price of x for the chosen frequency functions. Any two households may have different frequency functions among the set we have chosen.

Appendix

Contents

A1 Calculation of expected demand functions and their properties.

For each frequency function f(x, y) we first integrate over the budget set
D= $\left\{(x, y) \text{ in } \mathrm{R}^2 : p_x x + p_y x_2 \le m \text{ and } \mathrm{x} \ge 0, y \ge 0\right\}$
to find the constant K. $(\iint_D Kf(x, y)dxdy = 1)$

To make the calculations somewhat simpler we do some change of variables.
We put x=u $\frac{\mathrm{m}}{\mathrm{p_x}}$ and y=v $\frac{\mathrm{m}}{\mathrm{p_y}}$ to integrate over the set
$\mathrm{D}' = \left((\mathrm{u,v}) \in \mathrm{R} : \mathrm{u} \ge 0,\ \mathrm{v} \ge 0, \mathrm{u+v} \le 1\right)$
We then have $\iint f(x, y)dxdy = \frac{m^2}{p_x p_y} \iint_{D'} f(x(u), y(v))dudv$
where the Jacobian of the transformation is taken outside the integral on the right side. For change of variable formulas see Buck (1956) p 244.

A2 Homogeneity of degree zero a general property

The change of variables where
we put x=u $\frac{\mathrm{m}}{\mathrm{p_x}}$ and y=v $\frac{\mathrm{m}}{\mathrm{p_y}}$
shows how the transformation connecting (x, y) space and (u, v) space is homogeneous in degree zero in prices and budget. . We assume f(x, y) = K g(x, y) where g(x,y) is independent of monetary variables and the monetary dependence enters through the normalization constant K.

We then have $\iint f(x, y)dxdy = \frac{m^2}{p_x p_y} \iint_{D'} f(x(u), y(v))dudv$

homogeneous of degree zero and x f(x, y) as well.

After integrating and finding K we do two more integrations to find E(x) and E(y) using the same change of variables.
After finding them we differentiate them to find sensitivity results.
Since the derivations are laborious but follow the same pattern we illustrate by taking the example of convex frequency function f(x, y) = K $(x^2 + y)$ in section A 4.

A 3 More on the chosen frequency functions and numerical values
The general results are given earlier in section 2C.
Here we mainly give more details such as the explicit expressions of frequency functions, of the expected demands and of average expenditure.
We also calculate numerical values for one income price combination.
For the same income price combination we find the utility maximising solution.

The fixed proportions (perfect complements) frequency function.

f(x, y)=K min(x, y)= $\dfrac{\min(x,y)}{\dfrac{\dfrac{m}{p_x^2}^2 \dfrac{m}{p_y^2}^2}{6(\dfrac{m}{p_x}+\dfrac{m}{p_y})}}$

$$E(x) = \frac{\frac{m}{p_x}}{4}\left(1+\frac{\frac{m}{p_y}}{(\frac{m}{p_x}+\frac{m}{p_y})}\right)$$

$$E(y) = \frac{\frac{m}{p_y}}{4}\left(1+\frac{\frac{m}{p_x}}{(\frac{m}{p_x}+\frac{m}{p_y})}\right)$$

The average expenditure
$E(p_x x + p_y y) = \dfrac{3m}{4}$
which is price independent.

Numerical values.

For the numerical values m=100, $p_x = 10, p_y = 5$ we find

E(x) = 4,166. E(y) = 6,666.The utility maximising solution x=y=6,666.

The own price elasticity $\mathrm{El}_{\mathrm{Exp}_x} = \frac{\partial E(x) p_x}{\partial p_x E(x)} = -0,867$

The Cobb-Douglas frequency function.

The Cobb-Douglas freqency function

$$\mathrm{K}x^2 y = \frac{x^2 y}{\frac{m^2 m^3}{p_x p_y 60 p_x^2 p_y}}$$

$$E(x) = \frac{m}{2p_x}$$

$$E(y) = \frac{m}{3p_y}$$

The average expenditure is

$$\mathrm{E}(p_x x + p_y y) = \frac{5m}{6} < m$$

For the Cobb-Douglas frequency function we find that the expected demand has a symmetric substitution matrix which is negative definite.

$$\begin{bmatrix} \frac{\partial Ex}{\partial p_x} & \frac{\partial Ex}{\partial p_y} \\ \frac{\partial Ey}{\partial p_x} & \frac{\partial Ey}{\partial p_y} \end{bmatrix} + \begin{bmatrix} Ex \\ Ey \end{bmatrix} \begin{bmatrix} \frac{\partial Ex}{\partial m} & \frac{\partial Ey}{\partial m} \end{bmatrix}$$

Using a Cobb-Douglas frequency leads to simple calculations and can be used with higher exponents to get closer to the deterministic budgetline.
Example

$$f(x, y) = Kx^3 y^2$$

implies average expenditure

$$E(p_x x + p_y y) = \frac{7m}{8} > \frac{5m}{6}$$

Numerical values.

The numerical values for m=100, $p_x = 10$ and $p_y = 5$ are

E(x)=5, E(y)=$6\frac{2}{3}$

The utility maximising solution is x=$6\frac{2}{3}$, y=$6\frac{2}{3}$.

The own price elasticity $\mathrm{El}_{\mathrm{Exp}_x} = -1$

The average expenditure =0,83m

The quasilinear frequency function.

$$K(\sqrt{x}+y)=\frac{\sqrt{x}+y}{\frac{m^2}{p_x p_y}\frac{1}{3}(\sqrt{\frac{m}{p_x}}\frac{4}{5}+\frac{m}{p_y}\frac{1}{2})}$$

$$E(x)=\frac{\frac{m}{p_x}(\sqrt{\frac{m}{p_x}}\frac{4}{35}+\frac{m}{p_y}\frac{1}{24})}{\frac{1}{3}(\sqrt{\frac{m}{p_x}}\frac{4}{5}+\frac{m}{p_y}\frac{1}{2})}$$

$$E(y)=\frac{\frac{m}{p_y}(\sqrt{\frac{m}{p_x}}\frac{8}{35}+\frac{m}{p_y}\frac{1}{4})}{(\sqrt{\frac{m}{p_x}}\frac{4}{5}+\frac{m}{p_y}\frac{1}{2})}$$

The average expenditure

$$E(p_x x+p_y y)=\frac{m(\sqrt{\frac{m}{p_x}}\frac{5}{7}+\frac{m}{p_y}\frac{3}{8})}{(\sqrt{\frac{m}{p_x}}\frac{4}{5}+\frac{m}{p_y}\frac{1}{2})}$$

Numerical values.

The numerical values for m=100. $p_x = 10, \; p_y = 5$ are

E(x)=2,86 E(y)=9,13

The utility maximizing solution is

x=0,06, y=19.88

The own price elasticity $El_{Exp_x} = -1.05$

The average expenditure =0,77m

The linear frequency function.

The linear frequency function

$$\mathrm{K}(x+y)=\frac{x+y}{\frac{m^2}{6p_x p_y}\left(\frac{m}{p_x}+\frac{m}{p_y}\right)}$$

$$E(x)=\frac{\frac{m}{p_x}\left(\frac{m}{p_x}+\frac{m}{2p_y}\right)}{2\left(\frac{m}{p_x}+\frac{m}{p_y}\right)}=\frac{\frac{m}{p_x}\left(2\frac{m}{p_x}+\frac{m}{p_y}\right)}{4\left(\frac{m}{p_x}+\frac{m}{p_y}\right)}$$

$$E(y)=\frac{\frac{m}{p_y}\left(\frac{m}{2p_x}+\frac{m}{p_y}\right)}{2\left(\frac{m}{p_x}+\frac{m}{p_y}\right)}=\frac{\frac{m}{p_y}\left(\frac{m}{p_x}+2\frac{m}{p_y}\right)}{4\left(\frac{m}{p_x}+\frac{m}{p_y}\right)}$$

The average expenditure $E(p_x x+p_y y)=\frac{3m}{4}=0,75m$

The substitution matrix is symmetric and negative definite.

Numerical values.

For m=100, $\mathrm{p_x}=10, p_y=5$ we have E(x)=$3\frac{1}{3}, E(y)=8\frac{1}{3}$.

The utility maximising solution is x=0, y=20 for comparison.

The own price elasticity $\mathrm{El}_{\mathrm{Exp_x}}=-1,16$

The convex frequency function.

The expected demand for good x

$$E(x)=\frac{m(\frac{m^2 3}{p_x^2 5}+\frac{m}{p_y 2})}{p_x\ (\frac{m^2}{p_x^2}+\frac{m}{p_y}2)}$$

We also give the expected demand for good y

$$E(y)=\frac{m(\frac{m^2 1}{p_x^2 5}+\frac{m}{p_y})}{p_y\ (\frac{m^2}{p_x^2}+\frac{m}{p_y}2)}$$

Calculating the average expenditure

$$E(p_x x+p_y y)=\frac{m(\frac{4}{5}\frac{m^2}{p_x^2}+\frac{m}{p_y}\frac{3}{2})}{(\frac{m^2}{p_x^2}+\frac{m}{p_y}2)}$$

Numerical values.

The numerical values for the income price combination

$m=100,\ p_x=10,\ p_y=5$ are

$E(x)=5,\ E(y)=5\frac{5}{7}$

The utility maximising solution is x=10,y=0.

The own price elasticity $El_{Exp_x}=\frac{\partial E(x)p_x}{\partial p_x E(x)}=-1,285$

Average expenditure = 0, 78 m

A4 The convex frequency function f(x, y) = K $(x^2 + y\ )$ calculations.

Step 1 Finding the constant K
We integrate

$$\iint_D K(x^2 + y) = \frac{m^2}{p_x p_y}\iint_{D'} K(\frac{m^2}{p_x^2}u^2 + \frac{m}{p_y}v)dudv$$

using the variable transformation above.

$$\iint_D K(x^2 + y)dxdy = \frac{m^2}{p_x p_y}\iint_{D'} K(\frac{m^2}{p_x^2}u^2 + \frac{m}{p_y}v)dudv$$

$$\frac{Km^2}{p_x p_y}\int_0^1 du\int_0^{1-u}(\frac{m^2}{p_x^2}u^2 + \frac{m}{p_y}v)dv = \frac{Km^2}{p_x p_y}\int_0^1 du\ \mathrm{I}_0^{1-u}(\frac{m^2}{p_x^2}u^2 v + \frac{m}{p_y}\frac{v^2}{2}) =$$

$$\frac{Km^2}{p_x p_y}\int_0^1 du\ (\frac{m^2}{p_x^2}u^2(1-u) + \frac{m}{p_y}\frac{(1-u)^2}{2}) =$$

$$\frac{Km^2}{p_x p_y}\int_0^1 du\ (\frac{m^2}{p_x^2}(u^2 - u^3) + \frac{m}{2p_y}(1 + u^2 - 2u) =$$

$$\frac{Km^2}{p_x p_y} I_0^1\ (\frac{m^2}{p_x^2}(\frac{u^3}{3} - \frac{u^4}{4}) + \frac{m}{2p_y}(u + \frac{u^3}{3} - \frac{2}{2}u^2) = \frac{Km^2}{p_x p_y}\ (\frac{m^2}{p_x^2}(\frac{1}{3} - \frac{1}{4}) + \frac{m}{2p_y}(1 + \frac{1}{3} - 1) =$$

$$\frac{Km^2}{p_x p_y}\ (\frac{m^2}{p_x^2}\frac{1}{12} + \frac{m}{6p_y}) = \frac{Km^2}{6p_x p_y}\ (\frac{m^2}{p_x^2}\frac{1}{2} + \frac{m}{p_y}) = 1$$

which will give us K= $\dfrac{1}{\dfrac{m^2}{6p_x p_y}\ (\dfrac{m^2}{p_x^2}\dfrac{1}{2} + \dfrac{m}{p_y})}$

and the density function K($\mathrm{x}^2 + y$)= $\dfrac{\mathrm{x}^2 + y}{\dfrac{m^2}{6p_x p_y}\ (\dfrac{m^2}{p_x^2}\dfrac{1}{2} + \dfrac{m}{p_y})}$

Step 2 Finding E(x)

$$E(x)=K\iint_D x(x^2+y)dxdy=\frac{Km^2}{p_x p_y}\iint_{D'}\frac{m}{p_x}u(\frac{m^2}{p_x^2}u^2+\frac{m}{p_y}v)dudv=\frac{Km^3}{p_x^2 p_y}\int_0^1 du\int_0^{1-u}(\frac{m^2}{p_x^2}u^3+\frac{m}{p_y}uv)dv=$$

$$\frac{Km^3}{p_x^2 p_y}\int_0^1 du\ \mathrm{I}_0^{1-u}(\frac{m^2}{p_x^2}u^3 v+\frac{m}{p_y}\frac{v^2}{2})=$$

$$\frac{Km^3}{p_x^2 p_y}\int_0^1 du\ (\frac{m^2}{p_x^2}(u^3-u^4)+\frac{m}{2p_y}(u+u^3-2u^2)=$$

$$\frac{Km^3}{p_x^2 p_y}I_0^1\ (\frac{m^2}{p_x^2}(\frac{u^4}{4}-\frac{u^5}{5})+\frac{m}{2p_y}(\frac{u^2}{2}+\frac{u^4}{4}-\frac{2}{3}u^3)=\frac{Km^3}{p_x^2 p_y}\ (\frac{m^2}{p_x^2}(\frac{1}{4}-\frac{1}{5})+\frac{m}{2p_y}(\frac{1}{2}+\frac{1}{4}-\frac{2}{3}))=$$

$$\frac{Km^3}{4p_x^2 p_y}(\frac{m^2}{p_x^2}\frac{1}{5}+\frac{m}{6p_y})$$

After inserting K and simplifying we obtain the expected demand for good x

$$\mathrm{E(x)}=\frac{\mathrm{m}(\frac{m^2 3}{p_x^2 5}+\frac{m}{p_y 2})}{p_x\ (\frac{m^2}{p_x^2}+\frac{m}{p_y}2)}$$

We also give the expected demand for good y

$$\mathrm{E(y)}=\frac{\mathrm{m}(\frac{m^2 1}{p_x^2 5}+\frac{m}{p_y})}{p_y\ (\frac{m^2}{p_x^2}+\frac{m}{p_y}2)}$$

Calculating the average expenditure

$$\mathrm{E(p_x}x+p_y y)=\frac{m(\frac{4}{5}\frac{m^2}{p_x^2}+\frac{m}{p_y}\frac{3}{2})}{(\frac{m^2}{p_x^2}+\frac{m}{p_y}2)}$$

A5 Upper and lower bounds on the consumption of both goods and a uniform distribution. Calculation of expected demand, sensitivity results and numerical values.

$$E(x) = \iint xf(x,y)dxdy$$

where f(x,y)=K is the uniform density function and the integral

is taken over the budget set D

$$D = \left((x,y) in R^2 : p_x x + p_y y \le m \text{ and } x \ge x_0 = k\frac{m}{p_x}, y \ge y_0 = l\frac{m}{p_y} \right)$$

Using the variable transformation

$$x = \frac{m}{p_x}(s+k), \text{ y=}\frac{m}{p_y}(t+l)$$

We change the set D into the set D´

$$D' = \{(s,t) \text{ in } R^2 : \text{ s+t} \le \text{a=1-k-l and s} \ge 0, t \ge 0\}$$

The Jacobian of the transformation is $\dfrac{m^2}{p_x p_y}$

Step 1 Calculation of the constant K

$$\frac{m^2}{p_x p_y}K\iint_{D'} dsdt = \frac{m^2}{p_x p_y}K\int_0^a ds\int_0^{a-s} dt\,\frac{m^2}{p_x p_y}K\int_0^a dsI_0^{a-s}t =$$

$$\frac{m^2}{p_x p_y}K\int_0^a ds(a-s) = \frac{m^2}{p_x p_y}KI_0^a as - \frac{s^2}{2} = \frac{m^2}{p_x p_y}K\frac{a^2}{2} = 1$$

which give us K= $\dfrac{1}{\dfrac{m^2}{p_x p_y}\dfrac{a^2}{2}}$

Step 2 Calculation of the expected value of x (E(x))

$$E(x) = \frac{Km^2}{p_x p_y}\iint_{D'}\frac{m}{p_x}(s+k)dsdt = \frac{Km^3}{p_x^2 p_y}\int_0^a ds\int_0^{a-s}(s+k)dt =$$

$$\frac{Km^3}{p_x^2 p_y}\int_0^a (s+k)(a-s)ds = \frac{Km^3}{p_x^2 p_y}I_0^a a\frac{s^2}{2} + aks - \frac{s^3}{3} - \frac{ks^2}{2} = \frac{Km^3}{p_x^2 p_y}(\frac{a^3}{6} + \frac{a^2k}{2})$$

Inserting K and simplifying we get

$$E(x) = \frac{m(\frac{a}{3}+k)}{p_x} = \frac{m(\frac{1-l-k}{3}+k)}{p_x} = \frac{1}{3p_x}(m - p_y y_0) + \frac{2}{3}x_0$$

$$E(y) = \frac{m(\frac{a}{3}+l)}{py} = \frac{m(\frac{1-l-k}{3}+l)}{p_y} = \frac{1}{3p_y}(m - p_x x_0) + \frac{2}{3}y_0$$

The average expenditure is

$E(p_x x + p_y y) = \frac{1}{3}(2m + p_x x_0 + p_y y_0) < m$,

which is price independent if $x_0 = y_0 = 0$.

Sensitivity results.

Relation between the expected value of x and change in the lower border of x .

We look for $\frac{\partial E(x)}{\partial x_0}$ which we find to be positive .The expected demand increases when the lower constraint x_0 increase.

Relation between the expected value of x and change in the lower border of y . We look for $\frac{\partial E(x)}{\partial y_0}$ which we find to be negative. The expected demand of x decreases when the lower constraint of good y (y_0) increases.

We note here that

$$\frac{\partial E(x)}{\partial p_y} < 0, \quad \frac{\partial E(y)}{\partial p_x} < 0$$

for positive lower bounds (x and y are complementary through the lower bounds) in contrast to the other case without lower bounds where.

$$\frac{\partial E(x)}{\partial p_y} = 0, \quad \frac{\partial E(y)}{\partial p_x} = 0$$

Note that the other chosen frequency functions gave us non negative crossderivatives.

The uniform frequency function expected demand also has a symmetric substitution matrix which is negative definite for the case where the lower bounds are zero.

The substitution matrix is not symmetric if the lower bounds are not zero but the diagonal is negative**. This example shows that results of traditional theory with x replaced by E(x) is not valid for all frequency functions.** We have already noted that the average Engel and Cournot restrictions are not valid.

Numerical values.

Numerical values for m=100, $p_x = 10$, $p_y = 5$, k=0,5 and l=0,3 are
E(x)=$5\frac{2}{3}$, E(y)=$7\frac{1}{3}$

The own price elasticity $El_{Exp_x} = \frac{\partial E(x) p_x}{\partial p_x E(x)} = -0,41$
Average expenditure 0,933m < m.
Note the relative closeness to m due to the tight lower border. **Identify lower bounds and the choice and/or preferences inside the "box" matter less.**

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